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A quantitative study of systematic uncertainties due to QED corrections in accurate Compton polarimetry experiments

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ABSTRACT: Several new high-energy physics accelerators will exploit beam polarization as a core part of their program. In several cases the beam polarization needs to be accurately measured with a precision better than one per-mille. At this level of precision, α^3 QED corrections must be accounted for. In this paper, we estimate the related correction for the detectors considered for several projects as ILC and FCC-ee. Two different techniques to extract the beam polarization are investigated and found to provide complementary information. The related measurements are dominated by different sources of systematic uncertainties, either related to QED corrections or likely to uncontrolled variations of experimental conditions at the per-mille level. It is found in particular that the measurement of the spatial distribution of photons, besides experimental challenges, is more sensitive to QED corrections than the technique consisting in measuring electrons spatial and energy distribution.

KEYWORDS: Accelerator Subsystems and Technologies; Instrumentation for particle accelerators and storage rings - high energy (linear accelerators, synchrotrons); Beam-line instrumentation (beam position and profile monitors, beam-intensity monitors, bunch length monitors)

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1 Introduction

The use of polarized beams of leptons is considered in many future high energy collider [1–6]. An accurate measurement of the beams polarization will be a key tool in the optimization of these accelerators, essential to monitor on short time scales the evolution of the polarization and in several cases critical to extract physics results with the high energy physics detectors [7, 8]. Inverse Compton scattering will be used in these projects to provide this information.

At electron-proton (as LHeC) and electron-ion colliders (as EIC), the ability to polarize beam is essential to understand in detail the nucleon structure. The polarization measurement of the electron beam at EIC poses some specific challenges. It will be performed during acceleration in the Rapid Cycling Synchrotron and also while the beam is stored in the final colliding ring. A precision polarization measurement of less than one percent per bunch every minute of beams up to 18 GeV is needed [1]. A similar strategy is expected to be used at the e^+e^- collider CEPC but with a larger beam energy of at least 45.65 GeV for studies of electroweak physics [5], where polarization is expected to be measured with statistical precision of 1% within a minute. The ability at CEPC to polarize beams allows to compensate the impact on the physics measurements due to a reduced luminosity compared to the FCC-ee project. For this project, the polarization of pilot bunches will allow to continuously measure with extreme precision the energy of the beams by means of the resonant depolarization technique, formerly used at LEP [9]. The currently estimated statistical precision is about 0.1% every 30 seconds for pilot bunches. At linear colliders, the ability to collide polarized beams is a key asset for direct and indirect signs of new physics [10]. More recently, an upgrade of SuperKEKB with polarized electron beams has been suggested to extend the scope of

physics measurements at the Belle II detector [11]. The expected statistical precision is about 1% per bunch every five minutes.

So far most of the advanced studies on Compton polarimetry were performed for ILC [12–16] with a design similar to that used in the past at SLC [17]. The target systematic uncertainty budget is of 0.25% for this project [3]. The statistical precision will be of 1% in four seconds for each bunch, or 0.1% in one second averaged over all bunches. More recently, a preliminary concept for a transverse Compton polarimeter at the CEPC has been proposed [18], improving on a proposal that was proposed several years ago for ILC [19]. In the past years, a totally new proposal of three-dimensional (3D) polarimeter [20] has been introduced in the context of precise electron and positron beam polarimetry for the accurate calibration of beam energies by resonant depolarization [9] at FCC-ee. It is expected to be performed at the running energies of the Z-pole but also at the WW threshold. In the context of this project, high accuracy, about 10^{-5} or better, on the determination of forward-backward asymmetries is expected [6]. In order not to spoil it, a possible residual longitudinal lepton beam polarization at the collider’s IP must be measured. It could stem for instance from a natural build-up of nearly transverse polarization, which longitudinal projection may not vanish exactly [21, 22]. First estimates suggest that the longitudinal lepton beam polarization should be constrained to $|P_z| < 10^{-4}$ or better at every IP [23]. In this case including QED corrections to Compton scattering in the extraction of the beam polarization will be essential, as will be shown in this paper.

In all these studies of future polarimeters, at different stage of development, statistical precision and systematic uncertainties related to experimental sources, in particular spin transport from the Compton interaction point to the collider interaction point, have been studied or are currently subject to scrutiny. However, it is noticeable that the effect of QED corrections in the extraction of the electron beam polarization lacks quantitative estimates, though it could be a limiting systematic uncertainties in some of these projects where high-energy beams are employed. The Klein-Nishina cross-section was obtained in 1929 [24]. Virtual corrections were considered by Brown and Feynman [25], and the real two photon emission, double Compton scattering in Refs [26, 27]. Complete α^3 corrections were further computed in Refs. [28–31]. Very recently the total cross-section at next to leading order has been provided with an analytical expression [32]. As it is explicitly shown in Refs. [28], the complete α^3 QED correction to Compton scattering lies at the per-mille level at the Z pole. To the best of our knowledge, α^4 to leading order correction to Compton scattering are not computed. Thus, if it is in principle possible to correct for the bias induced by QED corrections on the extraction of beam polarization, the procedure to assign a related systematic uncertainty would be arguable given the unknown scale of the next order correction. We aim in this paper at revisiting the estimates performed in Ref. [28]. The goal is to quantitatively estimate the bias on the extraction of longitudinal and transverse polarization due to QED corrections for various detector concepts in order to guide further studies.

In addition, a relatively large laser intensity, as considered in some projects, may induce further corrections to the Compton cross-section. Indeed several photons in the initial state may participate in the interaction and thus modify the physics of Compton scattering. Cross-section, including polarization terms, for asymptotic cases where the laser is either fully circularly or fully linearly polarized has been considered [33]. One thus investigates its effect on the estimation of electron beam polarization.

This article is organized as follows. In section 3 one considers several detector implementations based on existing designs for ILC and FCC-ee and quantify the contribution of QED corrections to the systematics uncertainties for various beam energies. Before concluding, one investigates in Section 4 the impact of relatively high laser pulse energies that are foreseen to be used in some of these conceptual Compton polarimeters.

2 Compton scattering

At the Born-level the Compton cross-section including beams polarization terms but summing over scattered particles spins, reads [34]

$$\frac{d\sigma^{(0)}}{dy_e d\phi} = \frac{r_e^2}{x_e} \left(F_0^{(0)}(r, y_e) + P_L F_L^{(0)}(r, y_e, \phi) + P_C (P_z F_{C,z}^{(0)}(r, y_e) + P_\perp F_{C,\perp}^{(0)}(r, y_e, \phi)) \right). \quad (2.1)$$

The functions $F_i^{(0)}$ are given by

$$F_0^{(0)}(r, y_e) = 1 - y_e + \frac{1}{1 - y_e} - 4r(1 - r), \quad (2.2)$$

$$F_L^{(0)}(r, y_e, \phi) = -4r(1 - r) \cos 2(\phi - \phi_L), \quad (2.3)$$

$$F_{C,z}^{(0)}(r, y_e) = y_e \frac{(2 - y_e)(1 - 2r)}{1 - y_e}, \quad (2.4)$$

$$F_{C,\perp,\phi}^{(0)}(r, y_e) = -2y_e \sqrt{r(1 - r)} \cos(\phi - \phi_\perp); \quad (2.5)$$

and

$$y_e = \frac{E_\gamma}{E_e}, r = \frac{y_e}{x_e(1 - y_e)}, \text{ and } x_e = \frac{2E_e h\nu_0(1 + \beta \cos \theta_{\text{in}})}{m_e^2 c^4}, \quad (2.6)$$

where E_γ is the scattered photon energy, E_e the initial electron energy, $h\nu_0$ the initial laser photon energy, $\pi + \theta_{\text{in}}$ the beams crossing angle, m_e the electron rest mass and c the speed of light in vacuum. The degree of circular polarization of the laser is denoted by P_C and the degree of linear polarization by P_L . The orientation of the linear component of the laser polarization in the laboratory reference frame, defined by the electron beam direction, and the vertical axis pointing to the sun, is given by ϕ_L . The longitudinal component to the electron beam polarization is denoted by P_z , and the transverse polarization by P_\perp . The orientation of the transverse component of the electron beam polarization is denoted by ϕ_\perp , with the convention that $\phi_\perp = 0$ when the transverse contribution to the polarization is horizontal, orthogonal to the beam direction and the direction of the sun. The angle ϕ denotes the azimuthal angle of the emitted photon. The range of scattered of reduced photon energy y_e is restricted to $[0, \frac{x_e}{1+x_e}]$ to energy conservation.

Several comments are already in order. Sensitivity to the electron beam polarization is only obtained by use of a circularly polarized laser beam. The linear polarization of the laser contributes as a nuisance parameter that is only of a serious matter when the transverse distribution of the photons is measured, due to the ϕ dependence of this term. Since only product of P_C and P_z or P_\perp contribute, any extraction of electron beam polarization involves unavoidable systematic uncertainty for the knowledge of the laser polarization itself. In the following we will assume that it is perfectly known to the required precision. If one is interested in knowing the longitudinal

polarization of the electron beam, it is sufficient to measure the energy of the photon or equivalently the scattered electron by means of energy conservation in the $e^- + \gamma \rightarrow e^- + \gamma$ interaction. Indeed the contribution of transverse polarization vanishes after integration in the transverse plane of the scattered particles. If however one is interested in measuring the transverse polarization of the electron beam, the transverse distribution of the photons and electrons must be measured or at least the energy of the scattered particles in different but well-chosen space regions, as it was done in the past [35].

The energy of the emitted photon is related to its polar angle θ with respect to the axis of the incoming electron.

$$E_\gamma = \frac{E_e h\nu_0 (1 + \beta \cos(\theta_{\text{in}}))}{h\nu_0 (1 + \cos(\theta - \theta_{\text{in}}) + E_e (1 - \beta \cos(\theta)))} \approx \frac{2\gamma^2 h\nu_0 (1 + \cos(\theta_{\text{in}}))}{1 + \frac{2\gamma h\nu_0 (1 + \cos(\theta_{\text{in}}))}{m_e c^2} + \gamma^2 \theta^2}, \quad (2.7)$$

where $\gamma = E_e/m_e c^2$ is the Lorentz boost of the initial electron in the laboratory frame. The approximation accounts for the recoil of the electron that slightly red-shifts the energy of the scattered photon when $4\gamma h\nu_0 = O(m_e c^2)$ to ensure $E_\gamma < E_e$ when $\gamma \rightarrow \infty$, as it is the case in high energy electron beam polarimetry. The relation is unequivocal for given initial electron and photon energies and interaction angle. This provides the possibility to indirectly retrieve sensitivity to the photon energy by measuring its position in the transverse plane. This technique is being proposed at FCC-ee [20] to extract the longitudinal electron polarization also from the measurement of the photon distribution.

In order to give some insights on the sensitivity of the Compton scattering technique for measuring the longitudinal electron beam polarization, the Born-level cross section as function of the photon energy for two different beam energies is given in Figure 1. Similar curves as function of the scattered electron energy can be obtained very easily by energy conservation, and not shown here for brevity. We remark that at very high electron beam energies and close to the threshold of Compton scattering i.e. the high-end of the scattered photon spectrum, the polarized term has a magnitude similar to that of the unpolarized term. It can be understood by observing that when $P_C P_z = +1$ (see dashed light red curve), the cross-section nearly vanishes. Clearly most of the sensitivity to the electron polarization lies close to the kinematic threshold. It is also found on this figure that the polarized component vanishes around 200 GeV (20 GeV) for a 250 GeV (45.65 GeV) electron beam, where the three curves cross.

3 Simulation results

In the present study we employ the FORTRAN package COMRAD written by Morris Swartz [28]. Results obtained with this package were found compatible with those obtained by Dinner and Dittmaier later on [29]. Ten million events are generated with COMRAD for each assumption in terms of energy and beam polarization, either vertical or longitudinal. Event weights are then accumulated in the same fashion as that described in Ref. [28] to fill histograms corresponding to each detector type. It has been checked that the distributions shown in Ref. [28] were numerically reproduced by running COMRAD on a modern laptop.

Considered detector types are the following:

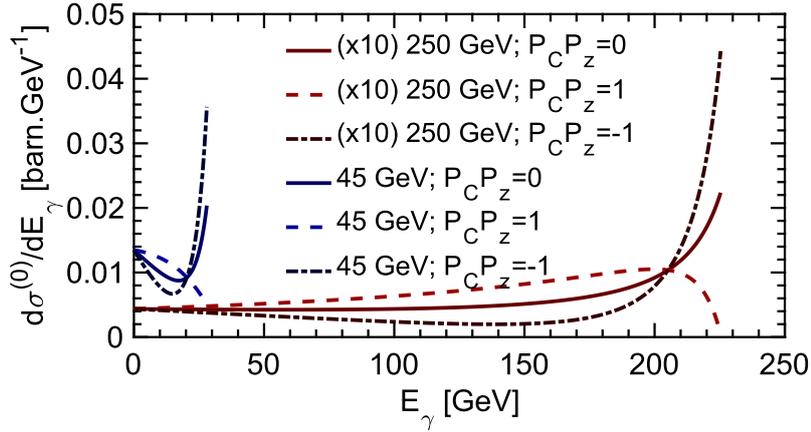


Figure 1. (Colors online) The Born-level differential cross section versus the scattered photon energy for 45.65 GeV (blue curves) and 250 GeV (scaled up by a factor 10 to ease reading, red curves) electron beams. For these two beam energies, it is shown for unpolarized ($P_C P_z = 0$, plain lines), and fully longitudinally polarized ($P_C P_z = -1$, opposite helicity for the photon and electron, in dash-dotted lines and $P_C P_z = +1$ in dashed lines) electron beams.

1De ILC electron counting Cherenkov detector made of 18 channels located from 2 to 20 cm away from the main beam after a magnetic chicane as described in Ref. [12]. It is designed to measure longitudinal polarization. It is a simple detector that is similar to that used at SLC, and thus a well proven technique to measure with precision the longitudinal polarization of electron and positron beams. It is quite insensitive to low energy backgrounds. It is insensitive to the transverse polarization of the beam.

1Dγ A detector similar to that of HERA LPOL2 made of a single scintillating crystal designed to measure the scattered photons energies to extract longitudinal polarization as described in Ref. [36]. It has been demonstrated that a fit of the photon spectrum allows to extract the longitudinal polarization with high precision provided that the energy distribution of backgrounds are well understood. Sensitivity to the transverse polarization may be obtained by realizing a split of this detector in two vertically superimposed elements. However this strategy seems very challenging and is not considered further in this article.

2De FCC-ee 2D segmented counting detector for electrons as proposed in Ref. [20]. It is similar in concept as those proposed for CEPC [18] and previously at ILC [19]. Placed after a dipolar magnetic field, this detector mainly acts as a spectrometer in the horizontal dimension. This detector is similar to **1De** with the advantage of being sensitive to the transverse polarization thanks to its vertical sampling. The drawback resides in the need for a relatively small size of pixels in the vertical direction and overall a larger number of pixels.

2Dγ FCC-ee segmented counting detector for photons as proposed in Ref. [20]. The idea is not to measure directly the energy of the photons but rather their scattering angle in the transverse plane. Compton scattering indeed exhibits an unequivocal relation between photons polar angle and their energy. This distribution allows in principle to extract all components of

the polarization vector of the beam. This strategy is likely more difficult due to the need of measuring distribution of photons in a narrow spot of few millimeters at most.

Beside the choice of the detector technology, it is worth noting that there are generally two strategies employed to measure electron beam polarization. One consists in measuring left-right asymmetries in the detectors by changing the sign of the circular polarization of the laser beam, see for instance [17]. However this needs corrections for any fluctuation of the interaction luminosity in between subsequent flips of laser polarization, that may occur due to fluctuations in beam sizes, crossing angle, alignments, laser intensity and electron beam charge at interaction point, but also changes in background levels and shapes or detector response. The inversion of the circular laser polarization may not be perfect either. A more involved approach consists in extracting more information by a fit of measured energy or spatial distributions of scattered particles [35, 36]. It allows to in principle correct for the aforementioned effects in real time from the data itself but requires a very detailed modeling of the detector and backgrounds. Both approaches have in principle a different sensitivity to QED corrections since they do not probe the same part of the phase-space of the scattered particles.

3.1 Longitudinal polarization

The effect of QED correction on Compton scattering for the extraction of the electron/positron beam polarization has been evaluated by Swartz [28] assuming that an experimental asymmetry is obtained by changing the sign of the circular laser polarization. Following mostly the same notations, one denotes by $\sigma_{i,u/p}$ the (u) unpolarized and (p) polarized terms of the cross section for Compton scattering in the channel i of the detector. It is the sum of Born-level $\sigma_{i,u/p}^{(0)}$ and α^3 correction $\sigma_{i,u/p}^{(1)}$ contributions. The experimental asymmetry reads

$$\mathcal{A}_i^{\text{exp}} = P \mathcal{A}_i = \frac{\sigma_{i,p}}{\sigma_{i,u}}, \quad (3.1)$$

or at Born-level

$$\mathcal{A}_i^{\text{exp}, (0)} = P \mathcal{A}_i^{(0)} = \frac{\sigma_{i,p}^{(0)}}{\sigma_{i,u}^{(0)}}, \quad (3.2)$$

where $P \rightarrow P_C P_z$ denotes the expected longitudinal beam polarization. The estimated beam polarization using exclusively data measured in bin i is written $\widehat{P}_i = P + \delta \widehat{P}_i$. It follows that [28]

$$\delta \widehat{P}_i = P \frac{\mathcal{A}_i - \mathcal{A}_i^{(0)}}{\mathcal{A}_i^{(0)}}. \quad (3.3)$$

This is essentially the technique that was used at SLC. For a 45 GeV beams it corresponds to a relative bias of approximately 0.1 % on the estimation of the beam polarization. It is found to be very similar over the bins close to large displacement values Δx . This could be judged suitable for an accurate extraction of the beam polarization but this technique is very sensitive to experimental conditions. Indeed precise control on any relative change of luminosity during measurements is necessary to reach the per-mille level or better. Likely more difficult to handle are possible changes in background conditions to the required precision. It was one of the lessons from the HERA

polarimeter where background parameterization had to be left free to allow for a robust extraction of polarization [36].

In this later experiment, Baudrand *et al.* performed a fit of the energy distribution without relying on asymmetries. This is possible thanks to the fact that the energy dependence of the polarized and unpolarized contributions to the differential cross-section is different. Before turning to analyze results obtained by a fit of the energy distribution of the scattered particles, it is interesting to understand the contributions bin by bin. One thus constructs a χ^2 variable for each channel i of the detector

$$\chi_i^2 = \frac{[\sigma_{i,u} + P\sigma_{i,p} - (\sigma_{i,u}^{(0)} + \widehat{P}_i\sigma_{i,p}^{(0)})]^2}{\sigma_{i,u} + P\sigma_{i,p}} = \frac{[\sigma_{i,u}^{(1)} + P\sigma_{i,p}^{(1)} - \widehat{\delta P}_i\sigma_{i,p}^{(0)}]^2}{\sigma_{i,u} + P\sigma_{i,p}}. \quad (3.4)$$

Since the estimation of \widehat{P}_i , or equivalently $\widehat{\delta P}_i$, is a linear problem, one obtains for the bin i

$$\widehat{\delta P}_i = \frac{\sigma_{i,u}^{(1)} + P\sigma_{i,p}^{(1)}}{\sigma_{i,p}^{(0)}}. \quad (3.5)$$

These biases are shown in Figure 2 for a beam energy of 45.65 GeV. The large biases observed around 20 GeV (detector 1D γ) or around 0.1 m (detector 1De) are related to the nearly vanishing contribution of the polarized term $\sigma_{i,p}^{(0)}$ to the cross-section in these bins. It is shown that results only differ slightly for unity and vanishing beam polarizations.

These curves show that the drawback of extracting the polarization directly from energy distributions without constructing asymmetries is that the systematic uncertainty related to the theory only slightly depends on the magnitude of the polarization and is not vanishing for a zero polarization. This could be a limitation when dealing with the goal of measuring accurately small polarization values, as needed when diagnosis of colliding beams is needed at FCC-ee [23]. Reaching this specific goal will likely involve both techniques (the measurement of asymmetries and the fit of the distributions) to cross-check results. Indeed the characteristic energy dependence of the polarization bias may be used as a handle towards validating high accuracy measurements.

As we can see in Figure 2 the bias on the extraction of beam polarization depends in general on the binning scheme. The investigated detector types allow to measure directly or indirectly the energy distribution of the scattered particles. Photons energies are measured for detector 1D γ , positions of electrons after a deflecting magnetic field in case of 1De and 2De, and position of photons in a pixel detector 2D γ . The latter corresponds to an indirect measurement of the photons energies due to the unequivocal energy-angle relation for the scattered photons. As expected the scale of the bias is found to depend significantly on the binning, compare detectors 1De and 2De, especially close to $\sigma_{i,p}^{(0)} \simeq 0$.

The few last bins in electron detectors are usually considered to extract the beam polarization in case of detector 1De [17, 28]. It is striking that the bias changes significantly from bin to bin in this region, due to the sharp behavior of the correction to the unpolarized term of the cross-section in this region. The use of asymmetries provides a more theoretically robust approach since, in an ideal situation, it cancels this unpolarized component. In this latter case the extracted polarization is biased in a similar fashion for bins close to Compton edge at a 0.1% level approximately. It is interesting to note that the bias obtained by looking at distributions and not asymmetries shows a

sign flip in this region, and that the value of the bias exhibits an opposite sign compared to that extracted with asymmetries. This is again understood as an effect related to the contribution of the unpolarized cross-section using this method.

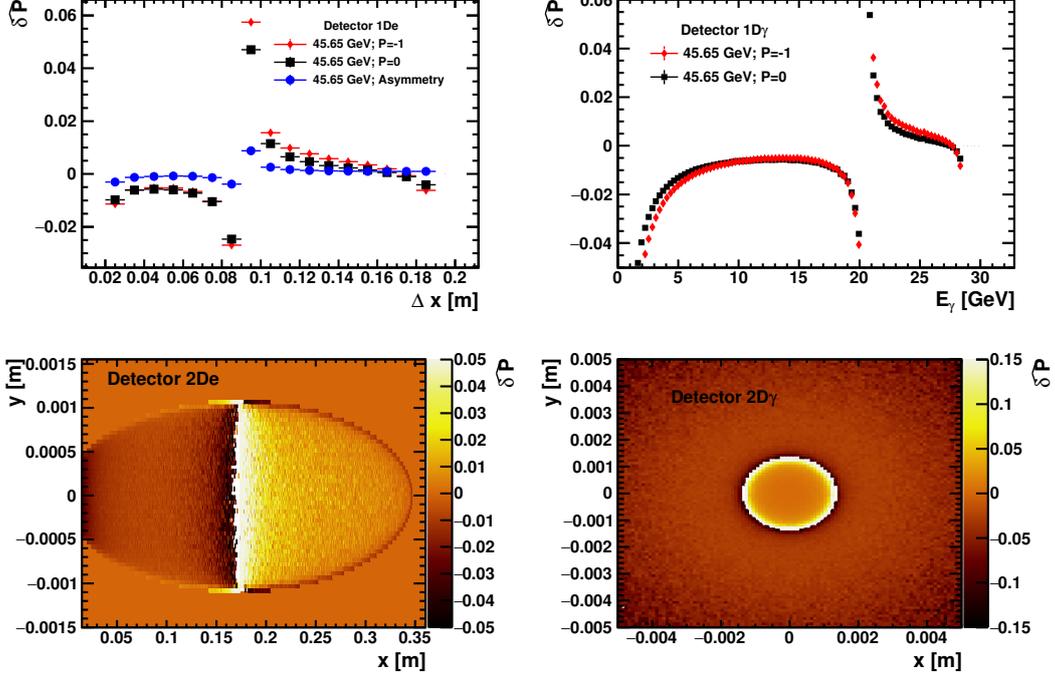


Figure 2. (Colors online) Contribution to the bias due to α^3 QED correction when not included in the fit model for the extraction of longitudinal polarization for $P = 0$ (black squares) and $P = -1$ (red diamonds) in the case of detector **1De** (top, left) and detector **1D γ** (top, right) as function of measurement bins (horizontal position of electrons in top left plot, and photon energy for the top right plot). Similar results obtained when fitting two-dimensional spatial distributions is shown for detectors, **2De** (bottom left) and **2D γ** (bottom right) as function of the measurement bins. Error bars related to statistical uncertainty of the weighted Monte Carlo procedure are included in the top plots and found small. The bias obtained for the measurement of asymmetries while ignoring α^3 QED correction in the model is given for the detector **1De** in each measured bin (blue dots).

The theoretical calculation necessarily involves a cancellation of virtual photon correction and soft photon correction, that involves a cut-off energy E_γ^{min} below which the events with two photon emission $e^- \gamma \gamma$ are indistinguishable from the $e^- \gamma$ events [28]. This threshold, defined in the center of mass frame, is to some extent related to experimental cutoffs in the detection. We varied this threshold in the range [0.1, 2] keV, with its nominal value set at 1 keV, that approximately corresponds to a 100 MeV cut-off in the laboratory reference frame at 45 GeV. Its effect is shown in Fig. 3 for the case of detector **1De** on four bins near the Compton edge for both initially fully longitudinally polarized and unpolarized electron beams. It is also shown that in the case of the measurement of asymmetries, the bias is robust against such a change in the cut-off value despite the already mentioned experimental caveats, it changes by less than 10^{-5} .

Since the Born-level polarized contribution to the cross-section does vanish in some regions of

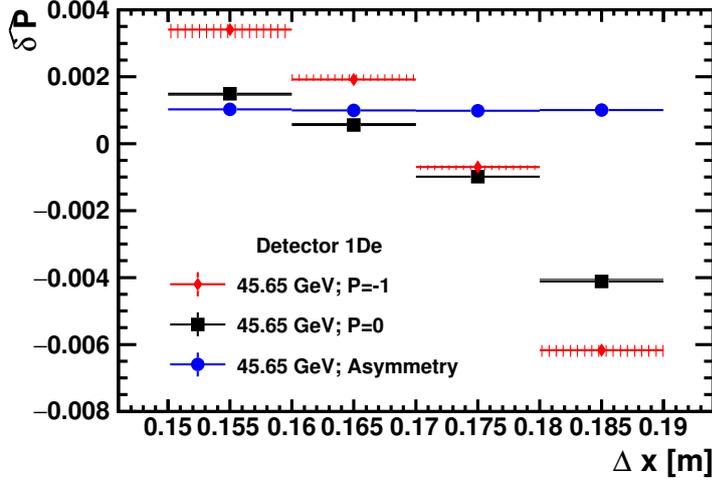


Figure 3. (Colors online) Zoom on the four bins close to the Compton kinematical edged for the detector **1De** (top left plot of Figure 4). Error bars related to statistical uncertainty of the weighted Monte Carlo procedure are found small. The hatched areas correspond to the variation of bias when changing the cutoff in the range [0.1, 2] keV in the center of mass frame. It is visible for some bins for the red curve ($P = -1$) and hardly distinguishable for the black curve ($P = 0$). The measurement of the asymmetry is robust against this change of cut-off and shown in blue dots.

the phase space, and accordingly the bias on the extraction of polarization gets large and depends on detector configuration, see Figure 2, different results when performing fits of the spectrum as a whole are expected. This is what has been made or proposed for some projects [18, 20, 36]. One thus introduces a global χ^2 obtained by considering the measured distribution as a whole,

$$\chi^2 = \sum_{i=1}^{N_{\text{bins}}} \frac{[\sigma_{i,u}^{(1)} + P\sigma_{i,p}^{(1)} - \widehat{\delta P}\sigma_{i,p}^{(0)}]^2}{\sigma_{i,u} + P\sigma_{i,p}}, \quad (3.6)$$

where now $\widehat{\delta P}$ is the overall bias on the polarization extraction. It reads

$$\widehat{\delta P} = \frac{\sum_{i=1}^{N_{\text{bins}}} \sigma_{i,p}^{(0)} \frac{\sigma_{i,u}^{(1)} + P\sigma_{i,p}^{(1)}}{\sigma_{i,u} + P\sigma_{i,p}}}{\sum_{i=1}^{N_{\text{bins}}} \frac{(\sigma_{i,p}^{(0)})^2}{\sigma_{i,u} + P\sigma_{i,p}}}. \quad (3.7)$$

The variation of the bias as a function of the initial longitudinal polarization is given in Fig. 4 for the four detector types. It is noteworthy that measurements based on detectors **1De**, **1D γ** and **2De** provide similar results and are less sensitive to the QED corrections compared to the one based on the measurement of the spatial distribution of the photons. Indeed that measurement is less direct compared to the others. Biases at the level of 0.1% at 45.65 GeV are obtained for detectors **1De**, **1D γ** and **2De** and 0.5% for detector **2D γ** . These values increase to 0.1-0.3% at 80 GeV and further at 250 GeV. For granular photon detectors as those expected to be used for FCC-ee and CEPC, the bias scales up from 0.5% for a fully longitudinally polarized beam at 45.65 GeV (black curve of Figure 4 left) to 2% at 80 GeV (not shown). The influence of the generator cutoff is also investigated

and shown in Fig. 4 (right). It is noticeable that the bias on the estimated polarization is significantly smaller when fitting the distribution as a whole compared to using the data near the Compton edge. For reference, the relative bias resulting from a measurement of the asymmetry, described at the beginning of this section, in the most sensitive bin to the electron beam polarization is shown for 45.65, 80 and 250 GeV in dash-dotted lines in the Figure. Interestingly, the obtained bias is of the same scale for both techniques at 45.65 GeV. This statement gets less and less true when beam energy is increased. This is consistent with the result obtained by Swartz at 500 GeV [28]. It is due to the fact that the Born-level asymmetry increases and that the QED correction is reduced when increasing energy. It would be of interest to investigate higher order QED corrections to investigate if this statement remains valid at high energies to per-mille or better accuracy.

The fit of the cross-section is also found to provide smaller biases against variations of generator cut-off compared to what is observed in the last bin, compare Figures 3 and 4 (right). This can be explained by the fact that this effect is found much smaller in most bins compared to the bin at the threshold, see Figure 3. A last remark is in order. The extraction of the electron beam polarization may be performed varying the degree of circular polarization of the laser from -1 to 1. This would allow to measure the curves shown on Figure 4, providing a handle on assigning systematic uncertainties. There is some interest in operating most of the time in a regime where the product of the circular laser polarization and that of the electron beam is close to one, if large longitudinal electron beam polarization can be obtained.

The fit of the distribution may thus be considered as a viable alternative to constructing asymmetries showing similar robustness against QED corrections and with the advantage of a less stringent need to experimentally control and correct for fake asymmetries that could come from natural fluctuations in luminosity and background but with the need to understand in details the measured experimental distributions. Beside providing similar accuracy related to QED corrections as the asymmetry, it also provides an experimental handle on the systematic uncertainties that are otherwise difficult to confirm experimentally.

3.2 Transverse polarization

In the case of CEPC and FCC-ee transverse polarimeters are expected to be implemented. The sensitivity of the extraction of the transverse beam polarization must then be evaluated. In this case we apply a similar approach as the one developed for longitudinal polarization except that now the contribution to the polarized cross section is calculated assuming that the electron beam is fully vertically polarized, $\phi_{\perp} = \frac{\pi}{2}$ in Equation 2.1. The Equations 3.5 and 3.7 are used, where now $P \rightarrow P_C P_{\perp}$ and the $\sigma_{i,P}^{(n)}$ are numerically different since different terms contribute to the cross-section. Indeed at the Born level $F_{C,\perp}^{(0)}$ contributes instead of $F_{C,z}^{(0)}$, see Equation 2.1. Results for $\widehat{\delta P}_i$ from Equation 3.5 are given in Figure 5.

Similarly the fit of the distribution can be made to extract the polarization. The resulting bias obtained with Equations 3.7 scales as P_{\perp} , contrary to the case of longitudinal polarization. Thus the curves for $\widehat{\delta P}$ are now crossing the zero point for an expected vanishing transverse polarization, see Fig. 6. This is expected since, for a purely circularly polarized laser, the only contribution to the transverse asymmetry comes from the term $F_{C,\perp}^{(0)}$ of Equation 2.1. It suggests that the procedure may allow to measure a vanishing transverse polarization with excellent accuracy, provided that all

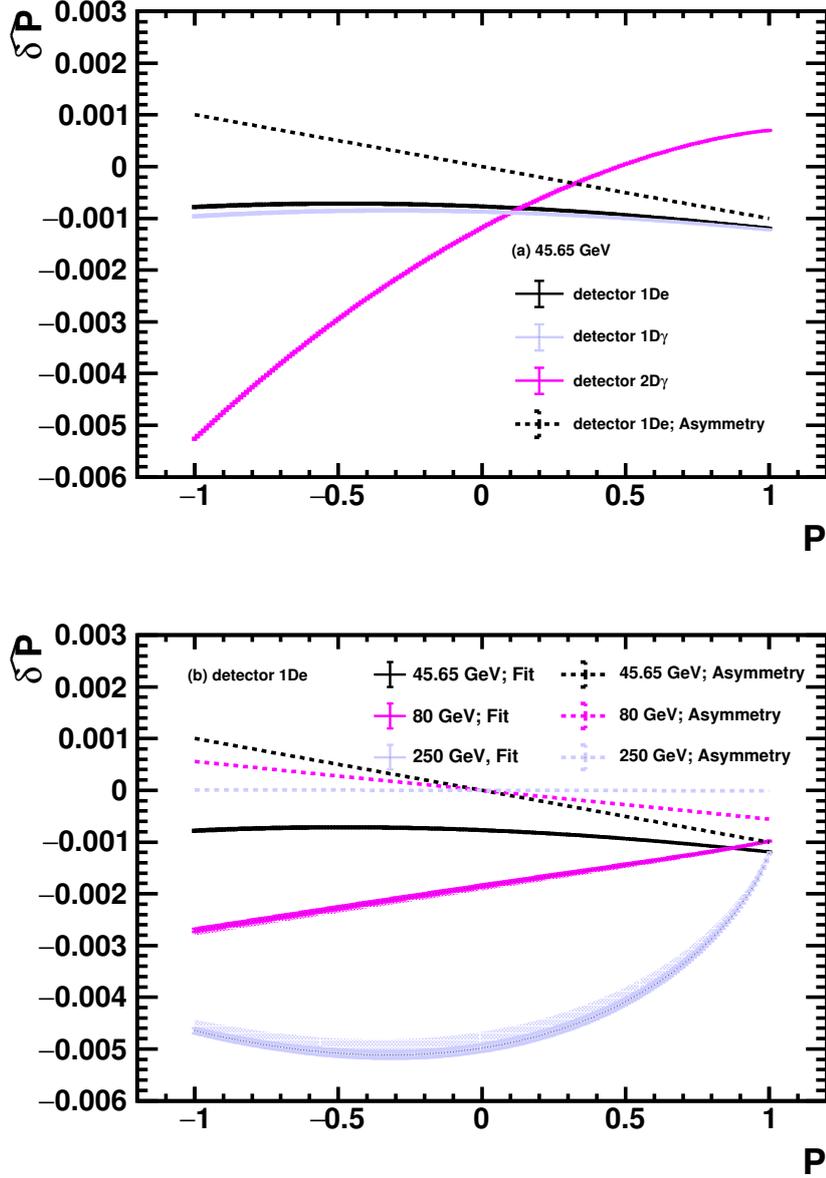


Figure 4. (Colors online) (Top, a) Bias due to α^3 QED correction when not included in the model for the extraction of longitudinal polarization as a function of the expected longitudinal polarization in the case of detectors **1De** (black), **1D γ** (magenta, grey) and **2D γ** (light blue, light grey) for a 45.65 GeV beam (left) for the fit-based technique and for the asymmetry technique (dashed black line). The curve corresponding to detector **2De** is nearly superimposed on that for detector **1De** and not shown for the clarity of the picture. Curves corresponding to detectors **1De**, **2De** and **1D γ** are similar. Detector **2D γ** allows to indirectly measure the energy of the scattered photon but is more sensitive to QED corrections. (Bottom, b) the same quantity is shown for detector **1De** at 45.65 GeV (black), 80 GeV (magenta, grey) and 250 GeV (light blue, light grey) on the figure on the right. Results obtained based on the asymmetry technique are shown as dash-dotted lines with the same color code on the right. Error bars related to statistical uncertainty of the weighted Monte Carlo procedure are included in the curves and found small. Grey, light-red and light-blue bands are related to the variation of the generator cut-off, see text for details. They are hardly distinguishable from the line thickness, except at 250 GeV using the fitting technique.

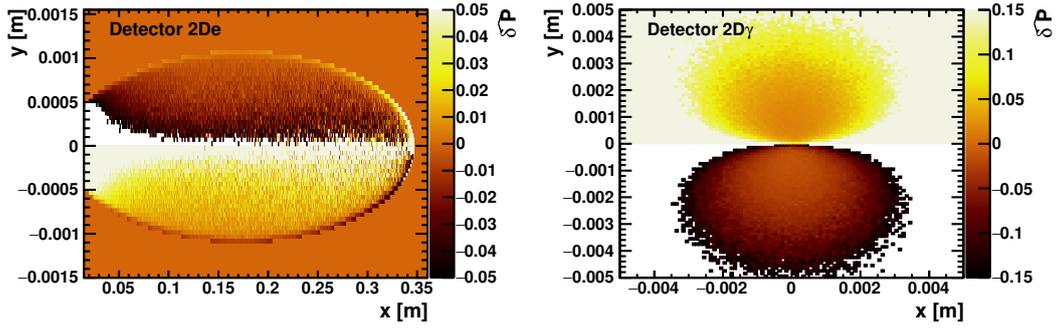


Figure 5. (Colors online) Contribution to the bias due to α^3 QED correction when not included in the model for the extraction of transverse polarization for $P_{\perp} = 1$ in the case of detector **2De** (left) and **2D γ** (right) as function of the measurement bins. The bias clearly shows up exclusively as a transverse asymmetry as it is the strictly case at the Born level, see Equation 2.1.

experimental effects are under control. It appears that the measurement of electrons is more robust against QED corrections compared to measurement of photons. For a typical expected polarization of $P = 0.1$, the bias amounts to 0.01% (0.005%) and 0.04% (0.1%) for electrons and photons, respectively, at 45.65 GeV (80 GeV). The fact that the bias gets smaller when increasing energy for electrons is related to the observation made for the case of longitudinal polarization, where the effect of α^3 QED correction gets smaller on the asymmetry. For the measurement of the transverse asymmetry of the photons, the increase in the bias is likely related to the fact that the spot size on the detector is inversely proportional to the beam energy, thus the detector is less sensitive in general to the polarization. A look at the precision of the detector with a fixed pixel size when increasing the beam energy is in order to conclude on that aspect. This is out of the scope of this paper.

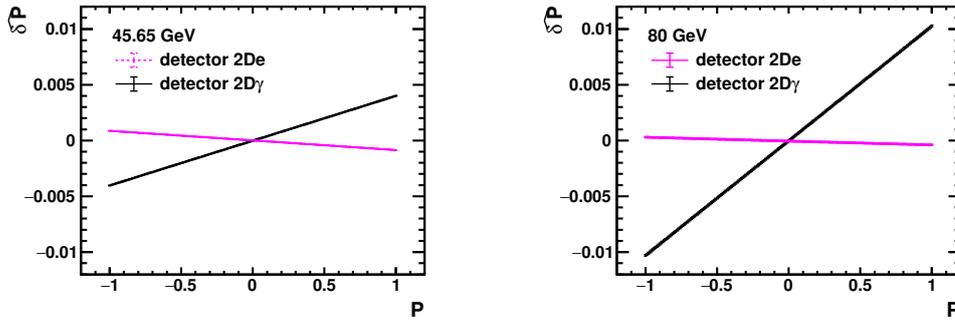


Figure 6. (Colors online) Bias due to α^3 QED correction when not included in the model for the extraction of transverse polarization as a function of the expected transverse polarization in the case of detector **2De** (magenta or grey in black and white rendering) and **2D γ** (black) for a 45.65 GeV beam (left) and 80 GeV beam (right). Error bars related to statistical uncertainty of the weighted Monte Carlo procedure are included in the curves and found small.

As a summary, transverse polarization may be extracted with very high accuracy with a systematic relative uncertainty that amounts to approximately 0.1% at 45.65 GeV. This statement

holds when the electron detector is used. Unfortunately this detector is not well suited to measure the horizontal polarization term [20] due to the presence of the dipole magnetic field to realize this spectrometric measurement. Horizontal polarization can however be obtained by a fit of the photon distribution [20] and will thus be affected by a 0.5% relative systematic uncertainty at 45.65 GeV. This is the first assessment of QED related systematic uncertainties for this type of measurements. One essential measurement of FCC-ee polarimeters will be to measure the vertical beam polarization for pilot bunches [9]. The resonant depolarization technique consisting in scanning the spin precession frequency and to look for the point when beam polarization vanishes. As shown here, QED corrections intervene as a relative correction on the measurement of the polarization, and will thus not significantly affect this procedure.

4 Strong-field QED

The needs of the FCC-ee project involve the use of relatively high energy laser pulse of few millijoules focused down to few hundreds of micrometers with pulse duration possibly as low as few picoseconds, allowing the use of modern Yb mode-lock laser technology [37]. It has been shown in the past that strong-field QED [38, 39] may contribute to per-mille level in the energy spread of low-energy Compton scattering sources of high energy photons [40]. These results are derived from the calculation of Compton scattering in the strong field for monochromatic plane waves [33] and approximated in the limit where the intensity of the strong field is small. A similar procedure can be applied here. It is thus expected to induce a small correction to observables of interest. Since high-accuracy is needed for FCC-ee where electroweak asymmetries are expected to be measured with an accuracy well below one per-mille, it may also become an essential effect to account for. Here we assume, still, that other sources of systematic uncertainties can be controlled to a similar level, including spin transport in the ring and residual turn by turn beam-beam effects on the spin [41]. These have been studied to some extent in the case of the ILC [14]. A dedicated study of the influence of these effects on the accuracy with which polarization will be measured at FCC-ee remains to be done but is out of the scope of this paper. It motivates the investigation of these effects in the context of FCC-ee.

4.1 Multiple interactions

First there is the possibility of having an electron interacting twice (or more) with the laser field. Accounting for this effect, dubbed here multiple interactions, implies that (i) the electron energy is strongly modified for the second interaction and (ii) the electron spin orientation is modified. Both effects likely bias the extraction of the polarization by (i) modifying the spatial distribution of the measured electrons (ii) effectively modifying the polarization of the electron beam to be measured. A simplified model can be drawn to estimate its magnitude [40]. The probability for an electron to interact twice approximately reads for small crossing angles

$$P_{int} \approx \frac{2\sqrt{\sigma_{t,l}^2 + \sigma_{t,e}^2} \sigma_C U}{E_\lambda (2\pi)^{3/2} \sigma_{x,l} \sigma_{y,l} \sigma_{t,l}}, \quad (4.1)$$

where σ_C is the Compton cross-section. It is numerically found that this probability lies within 10^{-5} when green ($\lambda = 515$ nm, $E_\lambda = 2.4$ eV) laser pulses reach $U = 10$ mJ, $\sigma_{x,l} = \sigma_{y,l} = 100$ μ m

RMS transverse sizes, $\sigma_{t,l} = 3$ ps pulse duration and while colliding with a $\sigma_{t,e} = 3$ ps electron beam with $200 \mu\text{m}$ RMS transverse size. On that respect, Compton polarimetry can be considered accurate and this effect fairly ignored. It may however be relevant in very specific cases where Joule-class, picosecond and focused laser beams are considered [42].

4.2 Shift and spread of the Compton edge

The Compton cross-section is modified in the presence of strong electromagnetic fields as those provided by lasers. This effect has been computed, see for instance Ref. [33], and has been observed experimentally [43]. The electron mass gets effectively shifted in the presence of the strong electromagnetic field. This shift is related to the density of laser photons seen by the electrons, and since this density is not homogeneous, there is a spread in this shift when considering the electron beam as a whole. It induces thus a red-shifting and broadening of the Compton edge. This Compton edge is used to extract the beam energy from the Compton spectrum, eventually with high precision using electrons and a spectrometer [20]. This shift must thus be accounted for in order not to spoil the accuracy. It is numerically found of the order of $\eta^2 \approx 3.65 \times 10^{-19} I [\text{W}/\text{cm}^2] \lambda [\mu\text{m}^2] \approx 2 \times 10^{-7}$, with $I = U / ((2\pi)^{3/2} \sigma_{x,l} \sigma_{y,l} \sigma_{t,l})$ and when using the same parameters as in the previous subsection. This effect can be fairly neglected in the extraction of the energy of the beam, even when high accuracy is looked for.

4.3 Modification of the scattered particles energy distribution

For the sake of completeness, one also investigates a possible bias on the extraction of electron beam polarization since the strong field also affects the overall energy distribution of scattered photons and electrons. If linear Compton scattering cross-sections are used to extract the polarization it can result in a bias. The bias is estimated by using the energy distribution of photons or electrons as follows. Since one is interested in the limit where $\eta^2 \ll 1$ and that n -photon absorption cross-sections scale as $\eta^{2(n-1)}$, only one- and two-photon absorption must be considered when computing a first order correction to linear Compton scattering. In the case of Compton polarimetry one is mostly interested in pure circularly polarized lasers despite a small residual ellipticity may be present. Complete QED calculation of two-photon absorption, with possibly different helicities and momenta, is not available in the literature to the best of our knowledge. However absorption of two identical photons (momentum and helicity) is computed both for purely circularly polarized laser and purely linearly polarized laser, clearly stating that crossed polarization terms are not accounted for. We will thus assume that the laser is purely and totally circularly polarized $P_C = 1$ in the following. For simplicity, one restrict ourselves to energy distributions integrated in the transverse plane (i.e. detector 1De or 1D γ), that are thus insensitive to transverse electron beam polarization. One defines,

$$\frac{d\sigma}{dy_e} \approx \frac{d\sigma^{(0)}}{dy_e} + \frac{d\sigma^{(1)}}{dy_e} \quad (4.2)$$

where $\frac{d\sigma^{(0)}}{dy_e}$ is given in Equation 2.1 and [33]

$$\frac{d\sigma^{(1)}}{dy_e} \approx \frac{2\pi r_e^2}{x_e} \eta^2 \left(F_0^{(1)}(r, y_e) + P_C P_z F_{C,z}^{(1)}(r, y_e) \right) \quad (4.3)$$

is the first order correction in η^2 to the cross-section. The functions $F_i^{(1)}$ are given by

$$F_0^{(1)}(r, y_e) = r(2-r) \left(1 - y_e + \frac{1}{1-y_e} + \frac{r^2(4-r)}{2-r} \right) \quad (4.4)$$

$$F_{C,z}^{(1)}(r, y_e) = -\frac{(3-r)r^2(2-y_e)y_e}{1-y_e}. \quad (4.5)$$

It must be noted that the range of scattered photon energy is modified in the presence of a strong laser field by $[0, \frac{nx_e}{1+\eta^2+nx_e}]$, where n denotes the number of absorbed photons in the initial state.

In order to provide a first estimate of possible biases occurring from strong field effects, one is assuming that the whole energy range of scattered photons (or equivalently electrons) is measured. It allows to construct a χ^2 quantity from which biases are obtained:

$$\begin{aligned} \chi^2 &= \int_0^{\frac{x_e}{1+\eta^2+x_e}} \frac{[\frac{d\sigma}{dy_e}(P_C, P_z) - \frac{d\sigma_0}{dy_e}(\widehat{P}_z)]^2}{\frac{d\sigma}{dy_e}(P_C, P_z)} dy_e \\ &= \int_0^{\frac{x_e}{1+\eta^2+x_e}} \frac{[\eta^2 (F_0^{(1)} + P_C P_z F_{C,z}^{(1)}) - P_C \widehat{\delta P}_z F_{C,z}^{(0)}]^2}{F_0^{(0+1)} + P_C P_z F_{C,z}^{(0+1)}} dy_e, \end{aligned} \quad (4.6)$$

where $F_i^{(0+1)}$ stands for $F_i^{(0)} + \eta^2 F_i^{(1)}$. It must be noted that the first order correction to the cross-section includes both terms with one and two laser-photon absorption. The term originating from two laser photon absorption would induce the production of photons with $y > \frac{x_e}{1+\eta^2+x_e}$. This would imply that, in absence of backgrounds, this contribution could be measured and constrained from the data itself. However, it is likely that background would prevent the experiment to be sensitive to these events, given their probability. For the current study, it is enough to perform the integration in the range $y < \frac{x_e}{1+\eta^2+x_e}$.

One defines the following integral quantities that are useful to write more compact expression in the following of this paper:

$$\mathcal{I}_{n,z} = \int_0^{\frac{x_e}{1+\eta^2+x_e}} \frac{(F_0^{(1)} + P_C P_z F_{C,z}^{(1)}) F_{C,z}^{(0)}}{F_0^{(0+1)} + P_C P_z F_{C,z}^{(0+1)}} dy_e, \quad (4.7)$$

$$\mathcal{I}_{z,z} = \int_0^{\frac{x_e}{1+\eta^2+x_e}} \frac{(F_{C,z}^{(0)})^2}{F_0^{(0+1)} + P_C P_z F_{C,z}^{(0+1)}} dy_e. \quad (4.8)$$

One has implicitly dropped the dependence on P_C and P_z in the definition of these integrals. The bias then reads

$$\widehat{\delta P}_z = \eta^2 \frac{\mathcal{I}_{n,z}}{\mathcal{I}_{z,z}}. \quad (4.9)$$

It is numerically found below 6×10^{-7} in the whole electron beam polarization range for $\eta^2 \approx 2 \times 10^{-7}$.

5 Summary

Various types of detectors are being considered for Compton polarimeters to be installed in upgrades or new high energy colliders. These are expected to provide precise and accurate information on the beam polarization at the per-mille level at linear colliders or an order of magnitude below at FCC-ee. In order to reach this goal, QED corrections must be accounted for. We have revisited numerical estimations of QED corrections that were performed three decades ago and adapted the simulation to actual detector geometries that are foreseen for these future Compton polarimeters. In particular the ability to measure polarization with pixelized sensor was not considered in previous works on the subject. Several measurements strategies have been studied. We have observed that biases on the extraction of longitudinal and transverse polarization are in the range of one to several per-mille depending on the measurement strategy for 45 GeV beams. In particular we show that fits of energy distributions of scattered particles exhibit biases of similar magnitude as the technique consisting of measuring forward-backward asymmetries. On one hand, the latter presents the advantage of having a systematic uncertainty proportional to the true beam polarization but suffers from fluctuating experimental conditions that may not be precisely enough under control. On the other hand, the former allows to extract from the data itself scale and shape parameters without relying on the assumption that beam and background conditions are stable. In this case QED corrections have specific shapes versus the true beam polarization multiplied by the degree of circular polarization of the laser. Indeed higher order QED corrections are neglected, and thus measuring it may provide an interesting handle to validate the simulations and the systematic uncertainty related to QED corrections. Both techniques are thus complementary and judged useful to consider for future experimental implementations. For FCC-ee, 10^{-4} accuracy on nearly vanishing beam longitudinal polarization needs to be reached in order to preserve the accuracy on electroweak forward-backward asymmetries. In that specific case the asymmetry technique allows to nicely get rid of the QED related systematic uncertainty but requires to control perfectly experimental conditions. It will likely require that the sign of the circular laser polarization can be flipped at high frequency with similar accuracy. If not realized, fit of the spatial distributions including the α^3 QED-correction and a dedicated assessment of the next order QED-correction would be required. It further implies that that spin transport systematic uncertainties are under control at a similar level. We also observed that the measurement of the spatial distribution of photons is more sensitive to QED corrections than the corresponding energy distributions. Reaching per-mille accuracy on the determination of the horizontal polarization is thus not reachable without implementing a QED correction. This is particularly the case for the three-dimensional polarimeter proposed at FCC-ee. However assessing the related systematic uncertainty to that correction remains a challenge to date. It may require to account for α^3 QED effects in the fitting procedure itself and to estimate possible unaccounted for effects stemming from higher-order QED corrections. Effects related to strong-field QED have also been assessed in a quantitative manner for the first time and can be safely ignored in all cases.

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