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1 Comparative Study of Homogenization Techniques for  
2 Evaluating the Bearing Capacity of Bimsoils under  
3 Shallow Foundations

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9 **Abstract**

10 Bimsoils, characterized by discrete blocks within a finer-grained matrix, pose  
11 challenges in evaluating the bearing capacity of shallow foundations due to their  
12 spatial variability. To address this, a binary random field coupled with a Finite-  
13 Element model is used to simulate the variability of bimsoils. This study focuses  
14 on investigating the effect of the blocks spatial fraction ( $\gamma$ ), the isotropic and  
15 anisotropic spatial correlations, and the undrained shear strength ( $c_u$ ) ratio  
16 between the matrix and the blocks. The results reveal that the coefficients  
17 of variation (CV) for the bearing capacity ( $q_u$ ), at a given  $\gamma$ , reach nearly  
18 20% due to the diverse spatial configurations. This dispersion is attributed  
19 to the development of distinct failure mechanisms. However, optimizing the  
20 local average area used to evaluate  $\gamma$  can help reduce these CV values. The  
21 average  $q_u$  for a given  $\gamma$  can be accurately determined using the Bruggeman  
22 symmetric effective medium (BEM) equation, ensuring safe design compared to  
23 traditional homogenization techniques. The BEM equation considers  $\gamma$  and the  
24  $c_u$  ratio, providing an accurate estimation of bearing capacity for an equivalent  
25 homogeneous model suitable for probabilistic analyses.

26 *Keywords:* Bimsoils, Spatial variability, Binary random fields, Bearing  
27 capacity, Homogenization theories, Finite element model

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28 **1. INTRODUCTION**

29 Bimsoils (Block-in-Matrix soils), consisting of geotechnically complex forma-  
30 tions, present unique challenges in civil engineering due to their heterogeneous  
31 nature and spatial variability of mechanical parameters (Napoli et al., 2022).  
32 The interaction between the blocks and the matrix, along with their contrast-  
33 ing geotechnical parameters, necessitates the development of appropriate ho-  
34 mogenization techniques to accurately evaluate the bearing capacity of these  
35 soils.

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36 Conventional geotechnical analyses often oversimplify bimsoils by assuming  
37 homogeneity, disregarding the inherent complexities arising from the presence  
38 of discrete blocks within the matrix. Some authors have suggested using only  
39 the shear strength parameters derived from the weaker matrix when the spatial  
40 fraction of blocks is small (Medley, 2001; Medley and Sanz Rehermann, 2004).  
41 This simplified approach can lead to inaccurate predictions, excessively conser-  
42 vative designs and compromised safety margins. Lindquist (1994) and Sonmez  
43 et al. (2009), among others, proposed statistical relationships to adjust the val-  
44 ues of uniaxial compressive strength, friction angle and cohesion for bimsoils by  
45 utilizing their matrix values as a function of the spatial fraction. The mechanical  
46 influence of increasing the spatial fraction primarily arises from the formation  
47 of tortuous failure surfaces that circumnavigate the blocks. The following has  
48 been studied on the effect of slope stability by Medley and Sanz Rehermann  
49 (2004) and Montoya-Araque et al. (2020), among others. Bearing capacity anal-  
50 ysis on bimsoils, however, remains limited, with only a few examples available  
51 (Campos-Muñoz et al., 2018; Schmüdderich et al., 2021).

52 In probabilistic studies, the effects of spatial variability are often compared  
53 to homogeneous models. However, most comparisons rely on traditional homog-  
54 enization techniques such as arithmetic or harmonic averages, which generally  
55 fail to consider the effects of heterogeneous counterparts. Therefore, probabilis-  
56 tic analyses are necessary. While significant progress has been made in numerical  
57 optimization and random field generation, accurate homogenization techniques  
58 can provide valuable insights into the behavior of heterogeneous deposits from  
59 an engineering perspective. Studies on the random heterogeneity of soil proper-  
60 ties assert that phenomena governed by highly non-linear constitutive laws are  
61 affected the most. Nobahar and Popescu (2000), Griffiths and Fenton (2001),  
62 Li et al. (2015) Popescu et al. (2005a), among others, have studied the effects  
63 of inherent random soil heterogeneity on the bearing capacity ( $q_u$ ) of shallow  
64 foundations. All of them have used Monte Carlo Simulation (MCS) considering  
65 a continuous heterogeneous field. However, bimsoils consist of two very different  
66 soil types hence spatial variability effects could be better understood if binary  
67 random fields are considered. These fields have already been utilized to analyze  
68 the effects of settlement on structures founded on improved liquefiable soils by  
69 Montoya-Noguera and Lopez-Caballero (2016).

70 This paper presents a comprehensive comparative study of homogenization  
71 techniques for evaluating the bearing capacity of bimsoils under shallow founda-  
72 tions. The study specifically examines three aspects: (1) the effect of the  
73 spatial fraction of the blocks, (2) the influence of isotropic and anisotropic spa-  
74 tial correlations, and (3) the impact of the contrast on the block and the matrix  
75 mechanical properties. These aspects are crucial in understanding the behavior  
76 of bimsoils and their implications for geotechnical design. By considering dif-  
77 ferent block arrangements, sizes, and mechanical properties, the study provides  
78 insights into the effects of spatial variation on the bearing capacity of shal-  
79 low foundations. Furthermore, the investigation explores the use of different  
80 homogenization theories to account for the average effects of spatial variation.

81 **2. Models and Theories**

82 This section presents first the model for generating binary random fields to  
 83 simulate bimsoils, followed by the different homogenisation theories used in this  
 84 paper. Finally the numerical model for the assessment of bearing capacity is  
 85 shown.

86 *2.1. Binary random field (BRF)*

87 The binary random field (BRF) is generated with the homogeneous auto-  
 88 logistic model derived by [Bartlett and Besag \(1969\)](#). It is a nearest-neighbor  
 89 model defined as a conditional probability, instead of a joint probability distri-  
 90 bution as done with the Gaussian Markov random field models ([Whittle, 1963](#)).  
 91 That means it treats dependence directly through the so-called *autocovariate*,  
 92 which is a function of the observations themselves. The one-sided approxima-  
 93 tion of the conditional autoregressive binary model in two dimensions under  
 94 the assumption of homogeneity was derived from the Ising model to a 1st or-  
 95 der Markov serie by [Honjo \(1985\)](#). The assumption of homogeneity implies  
 96 that the underlying spatial process is stationary, meaning that the probability  
 97 of a material being at a particular location is constant across the entire study  
 98 area. According to [Hughes et al. \(2011\)](#), this model is straightforward to imple-  
 99 ment and fast to compute as the probability of a material at a specific location  
 100 depends on the presence or absence of the material in only two nearest neighbor-  
 101 ing locations. The model will be briefly discussed; however, more information  
 102 can be found in [Bartlett and Besag \(1969\)](#); [Besag \(1972\)](#); [Honjo \(1985\)](#) and  
 103 [Montoya-Noguera and Lopez-Caballero \(2016\)](#).

104 The binary mixture used to model the heterogeneous zone is defined by  
 105 three parameters: the spatial fraction ( $\gamma$ ) and the auto-regressive coefficients  
 106 in the horizontal and vertical directions, respectively ( $\beta_H$  and  $\beta_V$ ). Under the  
 107 condition of homogeneity,  $\gamma$  is equal to the expectation of  $x_{ij}$ , a value of the  
 108 binary random variable  $X_{ij}$  where  $i$  and  $j$  denote the position in the horizontal  
 109 and vertical direction, respectively, in a 2D coordinate system. Following the  
 110 one-sided approximation, this expectation is given by:

$$E[x_{ij}|x_{i-1,j}, x_{i,j-1}] = \left[ 1 - \frac{1}{2}(\beta_H + \beta_V) \right] \cdot \gamma + \frac{1}{2} (\beta_H \cdot x_{i-1,j} + \beta_V \cdot x_{i,j-1}) \quad (1)$$

111 Thus, the probability of which material is in the position  $(i, j)$  depends on  
 112 the nearest-neighboring binary variables  $x_{i-1,j}$  (left neighbor) and  $x_{i,j-1}$  (upper  
 113 neighbor). This one-sided approximation approach assumes that the conditional  
 114 probabilities depend only on the states of two neighboring variables, not on  
 115 the states of the variables in the opposite direction. This allows for a more  
 116 tractable computation of the expectation. Which makes the model easy and  
 117 quick to compute. The spatial fraction is defined as the areal ratio of blocks  
 118 with respect to the total area, i.e.  $\gamma = N_{blocks}/(N_{blocks} + N_{matrix})$  where  $N_m$   
 119 is the number of elements of material  $m$ . Thus, a  $\gamma$  value of 0 indicates a pure

120 matrix soil while a value of 1 indicates only blocks. For engineering applications,  
 121  $\gamma$  is related to the areal block proportion in two dimensions; hence, it can be  
 122 calculated from the sum of the areas occupied by blocks with respect to the  
 123 total area. Napoli et al. (2020) showed that the areal block proportions may  
 124 vary significantly for a given volume of bimsoil under analysis. However, since a  
 125 2D analysis is carried out for the practical application, the spatial fraction will  
 126 be used herein.

127 Because of the simple derivation of the autocorrelation function ( $\rho$ ). Honjo  
 128 (1985) stated that the one-sided approximation is preferred to the general case.  
 129 Thus,  $\rho$  is equal to  $\beta_H^i \cdot \beta_V^j$ , where  $\beta_H$  and  $\beta_V$  have a direct physical interpre-  
 130 tation as they give the one-step correlation of the process in the horizontal and  
 131 vertical direction, respectively. Note that the auto-regressive coefficients are  
 132 dimensionless, as they depend on the spatial discretization, i.e. the total size of  
 133 the finite element model divided by the number of elements. Figure 1 shows  $\rho$   
 134 for different  $\beta$  values with  $\beta_H$  equal to  $\beta_V$ . Note that the correlation structure  
 135 shows an exponential decay, which is expected as it was assumed the Markov  
 136 definition of order 1.

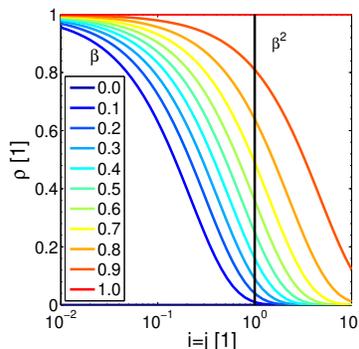


Figure 1: Autocorrelation function ( $\rho$ ) for different isotropic correlations ( $\beta_H=\beta_V$ ).

137 For each element, the generated probability is not a binary number, so it is  
 138 compared to a random number ( $u_{ij}$ ) that follows a uniform distribution function  
 139 between 0 and 1, where each element is independent. This process, known as  
 140 binarization, makes use of MCS to converge to a given  $\gamma$  value, thus preserving  
 141 the underlying probability without being too sensitive to outliers. This model  
 142 offers valuable insights into spatial dependence and binary outcomes, however  
 143 it has some limitations. Due to the one-sided approximation, the model is  
 144 sensitive to boundary effects thus the obtained spatial fraction may differ from  
 145 the target one. Honjo (1985) stated that this limitation is negligible if the size  
 146 of the model is large enough. To avoid this, a tolerance error was set to 1%  
 147 between the target and the obtained spatial fraction. In addition, as  $\beta_H$  and  $\beta_V$   
 148 define an exponential correlation structure, the field is highly sensitive even for  
 149 lower values (as shown in Figure 1 (Honjo, 1985)). In this regard, the number of  
 150 simulations required to achieve convergence will be higher for higher correlations

151 and for anisotropy. This aspect will be evaluated in the next section.

152 Contrary to other algorithms used for modeling bimsoils (e.g. [Suarez-Burgoa](#)  
 153 [et al. \(2019\)](#) and [Schmüdderich et al. \(2021\)](#)), the blocks do not have circular  
 154 shapes. Due to the spatial correlation, the bimsoils generated correspond to  
 155 polydisperse 2D block distributions where the block sizes are nonuniform. In  
 156 bimsoils not only the block area proportion (spatial fraction) but also the size  
 157 distribution plays an important role. When the isotropic correlation increases,  
 158 the binary soil tends to show a more concentrated spatial distribution in clusters.  
 159 The mean equivalent block size ratio ( $BSR_{eq}$ ) was evaluated for the different  
 160 isotropic correlations and the different spatial fractions and the results are shown  
 161 in Figure 2. As defined by [Schmüdderich et al. \(2021\)](#), the  $BSR$  value is the  
 162 ratio between the block diameter and the footing width for a monodisperse  
 163 distribution of spherical blocks. However, as the model generates nonuniform  
 164 blocks, the equivalent diameter was obtained by clustering analysis using the  
 165 code for connected components in binary image from the Image Processing  
 166 Toolbox in Matlab named `bwconncomp`. Then, the mean value of all clusters  
 167 for 100 distributions was determined. The results range from 0.08 to 0.2 for a  $\gamma$   
 168 value of 0.1 and increases to 0.3 for  $\gamma=0.5$ , where the  $BSR_{eq}$  value increases with  
 169 the spatial correlation. But, for  $\gamma$  values above 0.5, the size of the block clusters  
 170 increases exponentially and is higher for lower spatial correlations. For instance,  
 171 for  $\gamma=0.9$ , the  $BSR_{eq}$  value is 1.76 for  $\beta_H=\beta_V=0.9$  and 3.35 for  $\beta_H=\beta_V=0.1$ .

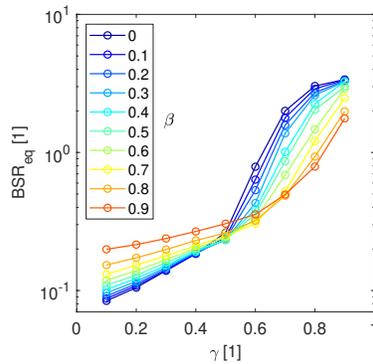


Figure 2: Equivalent block size ratio ( $BSR_{eq}$ ) for different isotropic correlations ( $\beta_H=\beta_V$ ).

172 Lastly, the BRF model simulates only welded bimsoils, thus no difference  
 173 in strength is assumed for the interface between the matrix and the blocks.  
 174 [Napoli et al. \(2022\)](#) avoided the classification between welded and unwelded  
 175 bimrocks, used for example by [Sonmez et al. \(2009\)](#), as it can be extremely  
 176 difficult to estimate the interface strength. However, [Schmüdderich et al. \(2021\)](#)  
 177 evaluated numerically this effect on the bearing capacity and found lower results  
 178 for unwelded bimsoils, compared to welded ones, because the lower strength at  
 179 the interface conditions the failure surface to pass through these weaker areas.  
 180 For welded bimsoils, the failure surface is only conditioned by the location of  
 181 the blocks for a given  $\gamma$  value. Hence more variability will be expected for  $\gamma$

182 values near 0.5. This aspect will be explored in the next section.

### 183 2.2. Homogenisation theories

184 Traditional homogenisation theories are often used to describe geotechnical  
 185 properties. For example, the work on spatial variability effect on bearing  
 186 capacity of [Popescu et al. \(2005a\)](#) often compares the average results of the het-  
 187 erogeneous soil models with the “corresponding homogeneous soil”. According  
 188 to the authors, the homogenisation is the mean value of the Monte Carlo sim-  
 189 ulations; although this is only true for vertically layered materials (i.e. parallel  
 190 to the bearing capacity) described by classical homogenisation theories. If, on  
 191 the contrary, the layers are horizontal (i.e perpendicular or serial) the effective  
 192 properties of the homogeneous model would be a harmonic average. It is clear  
 193 that for random fields, these are only extreme cases which are known as [Wiener](#)  
 194 (1912) bounds. The bearing capacity ( $q_u$ ) of the equivalent homogeneous model  
 195 will be described as:

- Parallel (arithmetic average) :

$$q_{u \parallel} = (1 - \gamma) \cdot q_{u \text{ matrix}} + \gamma \cdot q_{u \text{ block}} \quad (2)$$

- Serial (harmonic average):

$$q_{u \perp} = \frac{1}{\frac{(1-\gamma)}{q_{u \text{ matrix}}} + \frac{\gamma}{q_{u \text{ block}}}} \quad (3)$$

196 where  $q_{u \text{ matrix}}$  and  $q_{u \text{ block}}$  are the bearing capacity values for a pure matrix soil  
 197 (i.e. when the spatial fraction  $\gamma=0$ ) or a pure block soil (i.e.  $\gamma=1$ ), respectively.  
 198 Among the common types of averages, there is also the geometric average, which  
 199 lies between the arithmetic and harmonic ones. It favours low values, although  
 200 not as drastically as does a harmonic average. [Griffiths and Fenton \(2001\)](#) eval-  
 201 uated these averages for a lognormally distributed random field of soil strength  
 202 over a domain of about the size of the plastically deformed bearing failure region.  
 203 According to these authors, the geometric average showed the best agreement.  
 204 However, an empirical adjustment for the mean value was used in the Prandtl’s  
 205 formula.

206 Another case that can be exactly modelled as homogeneous consists of  
 207 concentric-shell structures, i.e. one material coating the other like spheres of  
 208 different size. In this case, the properties can be described by the [Hashin and](#)  
 209 [Shtrikman \(1962\)](#) equation:

$$q_{u \text{ HS}^+} = q_{u \text{ block}} + \frac{1 - \gamma}{\frac{1}{q_{u \text{ matrix}} - q_{u \text{ block}}} + \frac{\gamma}{d \cdot q_{u \text{ block}}}} \quad (4)$$

$$q_{u \text{ HS}^-} = q_{u \text{ matrix}} + \frac{\gamma}{\frac{1}{q_{u \text{ block}} - q_{u \text{ matrix}}} + \frac{1 - \gamma}{d \cdot q_{u \text{ matrix}}}} \quad (5)$$

210 where  $d$  is the dimensionality. This parameter binds the model to fluctuate  
 211 between the Wiener bounds; hence, when  $d$  is equal to unity, they become the  
 212 parallel bound and as it tends to infinity they approach the perpendicular one.  
 213 Actually, HS bounds are narrower than the Wiener bounds and are often used as  
 214 they are simple and intuitive. However, they still give wide predictions, specially  
 215 if the ratio between the material properties is big.

216 Besides these traditional homogenisation theories, another approach consists  
 217 in identifying an effective property -in this case  $q_{u\,eff}$ - for which the average be-  
 218 haviour of the heterogeneous model remains unchanged. Among them, probably  
 219 the most common is the [Bruggeman \(1935\)](#) effective medium (BEM) equation,  
 220 which is a classical theory of conduction in mixtures also known as the symmet-  
 221 ric effective medium approximation. When the properties of both materials are  
 222 of similar magnitude, the BEM equation is:

$$(1-\gamma)\frac{q_{u\,matrix} - q_{u\,eff}}{q_{u\,matrix} + (d_{BEM} - 1) \cdot q_{u\,eff}} + \gamma\frac{q_{u\,block} - q_{u\,eff}}{q_{u\,block} + (d_{BEM} - 1) \cdot q_{u\,eff}} = 0 \quad (6)$$

223 where  $d_{BEM}$  is equal to half the mean number of bonds presented at any site  
 224 of the network ([Kirkpatrick, 1973](#)). The BEM equation gives a more precise  
 225 solution as it accounts for each material independently. An important aspect of  
 226 this equation is the adaptability of the  $d_{BEM}$  parameter for each bimsoil. Thus,  
 227 contrary to the geometric average recommended by [Fenton and Griffiths \(2002\)](#),  
 228 the BEM equation can be fitted to different bimsoils as will be seen later on.

### 229 2.3. Numerical model

230 A finite element model (FEM) is used to evaluate the bearing capacity of  
 231 a shallow foundation. Two-dimensional plane-strain analyzes are performed  
 232 with the general purpose finite element code GEFDyn ([Aubry and Modaressi,](#)  
 233 [1996](#)). The numerical model uses quadrilateral isoparametric elements with  
 234 eight nodes. A rigid perfectly plastic model is implemented to simulate the be-  
 235 havior of purely cohesive soils. While the Young modulus (E) and the Poisson’s  
 236 ratio ( $\nu$ ) influence the computed settlement, the bearing capacity of a footing  
 237 depends primarily on the undrained shear strength ( $c_u$ ) ([Griffiths et al., 2002](#)).  
 238 Thus in the present study, to simplify the analyses, E and  $\nu$  are kept constant  
 239 and equal to 100MPa and 0.3, respectively. Each simulation of the binary ran-  
 240 dom field is used as input for  $c_u$  at each element on the model. Hence, for  
 241 binary value ( $x_{ij}$ ) equal to 0, the matrix is used with, initially,  $c_{u\,matrix}$  equal  
 242 to 20kPa; and for  $x_{ij}=1$ , block is used with  $c_{u\,block} = 100$  kPa.

243 Bimsoils are characterized by discrete blocks within a finer-grained matrix.  
 244 As stated by [Medley and Zekkos \(2011\)](#), the term “geotechnically significant  
 245 blocks” means that there is a mechanical contrast between blocks and matrix  
 246 that forces a tortuous failure surface around the blocks. Limiting values to define  
 247 that mechanical contrast are given for types of failure and different parameters  
 248 as shown in Table 1. The authors also state that relatively modest block-matrix  
 249 mechanical contrast is necessary for a block-in-matrix mass to be considered.

Failure	Criterion	Value
Triaxially induced shears	$E_{block}/E_{matrix}$	$\geq 2.0$
Deflect failure surfaces	$\tan \phi_{block}/\tan \phi_{matrix}$	$\geq 1.5 - 2.0$
Unconfined Compressive Strength	$UCS_{block}/UCS_{matrix}$	$\geq 1.5$

Table 1: Block-matrix strength contrast using estimates of weakest block and matrix mechanical parameters (Medley and Zekkos, 2011; Kalender et al., 2014)

250 Below these values, the failure surface would have an increased tendency to  
 251 pass through the blocks rather than around them. If the bearing capacity is  
 252 assumed to be controlled by the failure on undrained conditions, i.e. the loading  
 253 rate of the foundation's construction is higher than the pore water pressure  
 254 dissipation, the  $c_u$  value is equal to half the Unconfined Compressive Strength  
 255 (UCS). Results will be presented first for a strength contrast of 5 and in section  
 256 4.3 lower and higher contrast ratios will be tested.

257 A schema of the model is shown in Figure 3. The dimensions are given by the  
 258 width of the base (B), in this case 4m, and are taken from the recommendations  
 259 of Griffiths and Fenton (2001): a depth of 2B (8m) and a width of 5B (20m).  
 260 The size of the elements is 0.25 m in both directions which corresponds to B/16,  
 261 which was also used by Chen et al. (2012). A rigid bedrock underlies the soil  
 262 hence no vertical displacements are allowed on the bottom of the model and  
 263 as only vertical loads are applied, horizontal displacements are inhibited on the  
 264 lateral boundaries, as shown by equations 7 and 8.

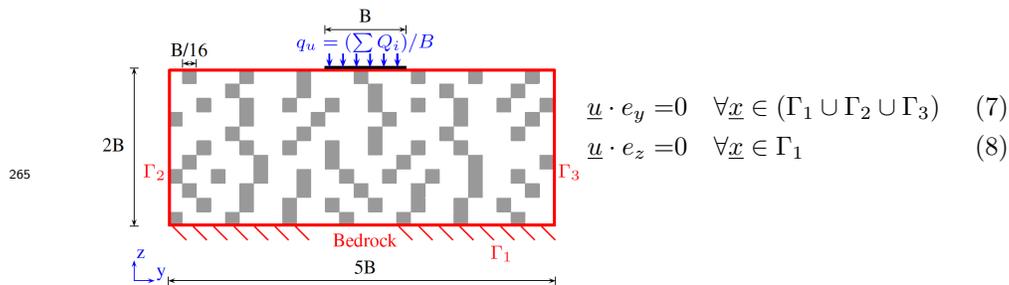


Figure 3: Schema of the numerical model

266 The test consists of applying an increasing vertical displacement ( $u_z$ ) at  
 267 the center of the foundation (i.e. at one node) with free rotations allowed for  
 268 the foundation. As the latter is rigid, a uniform displacement is considered in  
 269 this interface. The ultimate bearing capacity is taken when  $u_z/B = 2.5\%$  (10  
 270 cm) and is equal to the sum of the nodal forces ( $Q_i$ ) in the interface between  
 271 the soil and the structure's foundation divided by the width of the base (i.e.  
 272  $q_u = (\sum Q_i)/B$ ).

273 For a homogeneous weightless soil, the bearing capacity is given by the  
 274 Prandtl's solution as  $q_u = N_c \cdot c_u$ , where  $N_c$  is the dimensionless bearing capacity

275 factor and equals  $2 + \pi$  or 5.14 (Prandtl, 1921). The finite element analysis of  
276 the homogeneous cases, specifically when considering only the matrix or only  
277 the blocks, revealed a relative difference of 6.85% and 14.7%, respectively, in  
278 comparison to Prandtl’s solution. The former is one percentual point lower than  
279 the results of Chen et al. (2012) with the same properties and mesh size and  
280 similar to those of Popescu et al. (2005b). These differences are due to the  
281 assumptions in the model, for example, concentration of shear stresses at the  
282 soil-structure interface, gradual development of plastic zones, as well as inherent  
283 approximations induced by the numerical methods which are further explained  
284 by Nobahar (2003).

285 Furthermore, the difference with respect to Prandtl’s solution depends on the  
286 mesh size. Refining the mesh can assist in reducing these differences, albeit they  
287 cannot be entirely eliminated. However, this refinement comes at the expense  
288 of increased computational resources needed for the subsequent random-field  
289 simulations. Other researchers have evaluated the impact of increasing mesh  
290 discretization, i.e., the number of elements in each direction. For instance,  
291 Chen et al. (2012) demonstrated an approximately 50% decrease in numerical  
292 error when the mesh discretization was increased by a factor of 4.

293 Another factor of mesh discretization in the accuracy of predicting  $q_u$ , is  
294 capturing the concentration of plastic strain within narrower zones. As the  
295 footing is assumed rigid, the vertical stress distribution is non-uniform and  
296 stress concentrates near the edges of the footing, resulting in higher plastic  
297 strains. However, considering this effect would introduce additional complexity  
298 to the generation of the random field, which is beyond the scope of this study.  
299 Considering the extensive number of random field simulations performed, the  
300 chosen mesh size was deemed both efficient and suitable for the purposes of this  
301 study.

### 302 3. Coupling the binary random field with FEM

303 The spatial discretization is used to analyse the heterogeneous deposit. The  
304 spatial fraction ( $\gamma$ ) is varied from 0.1 to 0.9 (9 values) and 50 spatial distributions  
305 per value were done. Different auto-regressive coefficients on both directions  
306 were tested to analyse the effect of the correlation length.

307 As an example, the deformed mesh at failure of two spatial distributions  
308 for the same spatial fraction ( $\gamma = 0.5$ ) and the same auto-regressive coefficients  
309 ( $\beta_H = \beta_V = 0.9$ ) are shown in Figure 4. The deformation is scaled by a factor  
310 of 100 to improve visualization. They correspond to the extreme values of  
311 bearing capacity found for this set of values: the minimum shown in Figure 4a  
312 ( $q_u=148$  kPa) and the maximum, in Figure 4b ( $q_u=287$  kPa). The soil matrix,  
313 which has a smaller  $c_u$ , is in red color and the blocks are in blue. Similar  
314 to the results of Fenton and Griffiths (2002); Schmüdderich et al. (2021), the  
315 failure surfaces are certainly not symmetric and only approximately follow a  
316 log-spiral on different sides. It is clear that for the mesh that presented the  
317 minimum  $q_u$  the soil matrix is mostly concentrated near the foundation which  
318 has triggered a non-symmetric failure mechanism; while for the other one, the

319 blocks under the foundation interconnect from the surface to the deeper zone.  
 320 The spatial variability is not simply affecting the value of the bearing capacity  
 321 but it modifies the basic form of the failure mechanism. These two distributions  
 322 presented very different  $q_u$  value, thus a convergence analysis was performed to  
 323 choose the sufficient number of simulations per  $\gamma$  value.

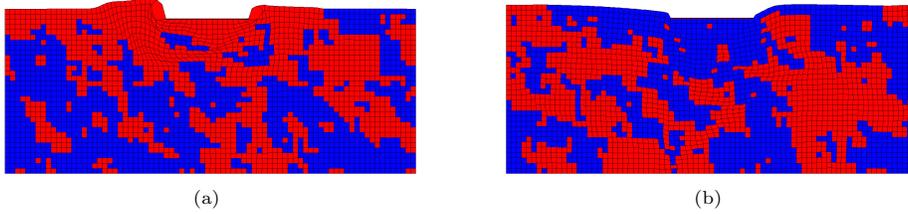


Figure 4: Deformed mesh at failure for two distribution of bimsoils with  $\gamma = 0.5$  and  $\beta_H = \beta_V = 0.9$ : a) minimum  $q_u$  (148 kPa) and b) maximum  $q_u$  (287 kPa). The matrix is depicted in red ( $c_{u\ matrix} = 20\text{kPa}$ ) and the blocks in blue ( $c_{u\ block} = 100\text{kPa}$ ). The deformation is scaled by a factor of 100.

324 The convergence of both the mean and the standard deviation of the bearing  
 325 capacity is shown for one case ( $\gamma = 0.7$  and  $\beta_H = \beta_V = 0.9$ ) in Figure 5. The  
 326 average values are shown in red and the confidence intervals, in dotted blue  
 327 lines. The latter are obtained with the t-student and  $\chi^2$  statistical models  
 328 with 5% confidence level for the mean and standard deviation, respectively. 200  
 329 spatial distributions were tested; although, after approximately 50 the statistical  
 330 convergence appears to be stable hence is sufficient for the application considered  
 331 in this work. As expected the mean value converges more easily and presents  
 332 less variation than the standard deviation. In contrast, the standard deviation  
 333 has more variation and wider confidence intervals. Here on, a maximum of 50  
 334 simulations are performed for each case.

335

### 336 3.1. Effect of the spatial fraction of blocks

337 The evolution of  $q_u$  as a function of  $\gamma$  is shown in Figure 6 for different  
 338 correlation values. For the sake of brevity only four cases are presented: the un-  
 339 correlated model (i.e.  $\beta_H = \beta_V = 0$ ) and 3 other values of isotropic correlation.  
 340 The box-and-whiskers plot is useful to show scalar-value statistics because of  
 341 the large amount of uncertainty information compared to mean and standard  
 342 deviations. Additionally, due to its flattened shape, box plots are better when it  
 343 is desired to compare the uncertainties in a number of related variables (Helton  
 344 et al., 2006). The box is composed of 3 quartiles, corresponding to 25, 50 and  
 345 75% of data and the whiskers are the lowest and highest values within 1.5IQR  
 346 (Inter-quartile range). Values outside the whiskers are outliers and are drawn  
 347 as blue dots. The mean values are in red and joined by the curve.

348 It is seen in Figure 6a that when no correlation is used less dispersion is  
 349 presented. Whereas, when a high correlation is used, like shown in Figure

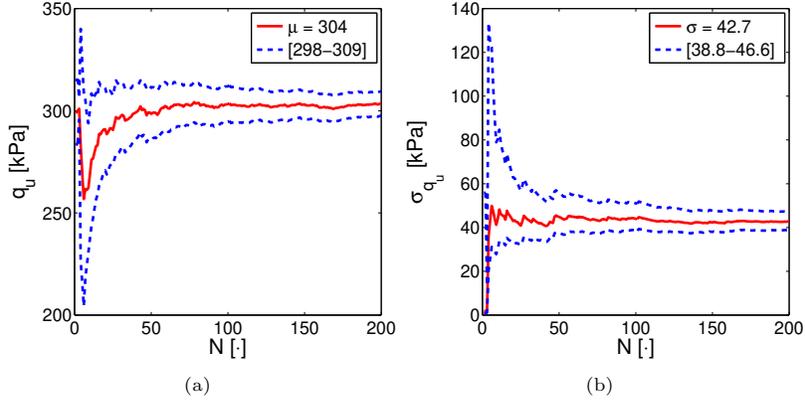


Figure 5: Convergence of the a) mean and b) standard deviation of the bearing capacity ( $q_u$ ) for  $\gamma = 0.7$  and  $\beta_H = \beta_V = 0.9$ . The average values are shown in red and the confidence intervals, in dotted blue lines.

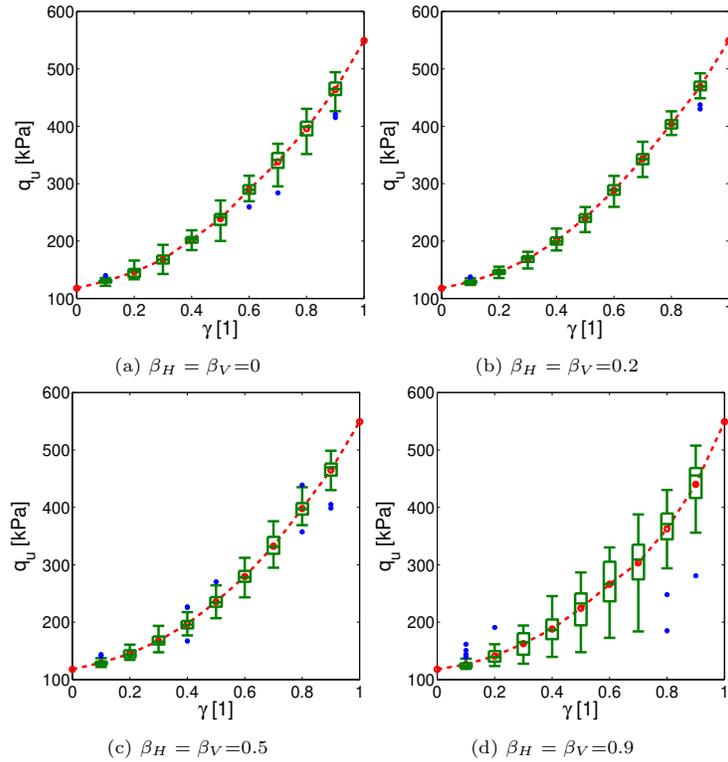


Figure 6: Models with different spatial fraction  $\gamma$  and different isotropic correlations

350 6d, the dispersion increases. Additionally, the dispersion is higher for spatial  
351 fractions around 0.5; though, it seems that the spatial configuration is of key  
352 importance when a similar fraction of both materials is used. This could be  
353 explained by the higher interaction between the mixture and the blocks and  
354 the resulting failure surfaces. When the isotropic correlation increases, the soils  
355 are “packed” in clusters and therefore each distribution will have a different  
356 behaviour.

357 The dispersion in mechanical properties is commonly quantified by the coef-  
358 ficient of variation (CV) and defined as the ratio between the standard deviation  
359 and the mean value. Thus, CV is a normalized quantity and it is of great use  
360 in probabilistic analysis. Representative values of CV can be found in the liter-  
361 ature based on laboratory data, *in-situ* tests or engineering judgement (Phoon  
362 and Kulhawy, 1999). Typical CV values are around 10 to almost 60 % for  
363 site-specific undrained shear strength in clays and below 40 % for normalized  
364 strength (i.e. divided by the vertical effective stress) (ISSMGE-TC304, 2021).  
365 In this analysis, the CV of the bearing capacity ( $q_u$ ), due to discrete spatial  
366 variability was calculated for the different spatial fractions and isotropic cor-  
367 relations.

368 Figure 7 shows the CV value as a function of the spatial fraction for each  
369 correlation and for all the distributions tested. It is interesting to note how  
370 CV increases with the degree of correlation and it is more important for spatial  
371 fractions near 0.5. In general, compared to the ranges given by ISSMGE-TC304  
372 (2021), the values are very low (i.e. below 20%). This is due to the fact that the  
373 variability measured is only induced by the spatial variability, referred by other  
374 authors as inherent random heterogeneity, and it does not take into account the  
375 measurement errors and uncertainty in physical parameters, present in experi-  
376 mental data (Wang et al., 2016). According to these values, thus, it seems that  
377 uncertainties due to spatial variability are lower to those related to the strength  
378 values (induced by measurement, statistical and transformation errors). Note  
379 that the compared ranges are for material properties and the one calculated is  
380 for the structure response. However, as for this case the relation between the  
381 initial property (i.e.  $c_u$ ) and the response ( $q_u$ ) is only a constant value, this  
382 comparison is still valid.

### 383 3.2. Effect of anisotropic correlation

384 An important effort is evidenced in geotechnical engineering in order to  
385 quantify the correlation length in the spatial variability. Jones et al. (2002) and  
386 more recently ISSMGE-TC304 (2021) present a literature review of the scale of  
387 fluctuation in horizontal and vertical direction mostly from *in-situ* tests, however  
388 the amount of information concerning these values is limited in comparison to  
389 the CV of inherent variability. In general, the correlation length in the horizontal  
390 direction is between 10 and 20 times larger than in the vertical one. The degree  
391 of anisotropy in bimsoils is influenced by various factors, including the processes  
392 involved in their formation, as well as the superposition and interaction of these  
393 processes (Napoli et al., 2022). At the moment, very little information can be  
394 found hence a parametric analysis was performed for different correlations. For

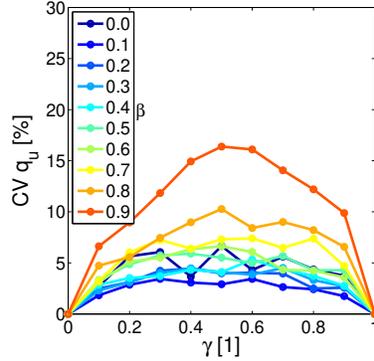


Figure 7: Coefficient of variation (CV) of the bearing capacity ( $q_u$ ) in bimsoils as a function of the spatial fraction ( $\gamma$ ) for different isotropic correlations

395 a spatial fraction of 0.4, only one correlation was changed from 0.1 to 0.9 while  
 396 the other one was fixed and equal to 0.1. The mean and CV of the bearing  
 397 capacity are shown in Figure 8. It appears that the anisotropic correlation is  
 398 inversely related to the mean and directly related to the CV results. However,  
 399 the effect of the correlation is more important for the CV value (e.g. for  $\beta = 1$ ,  
 400 CV is twice the one without correlation; while the decrease in the mean value is  
 401 only 2%). For this case, the vertical correlation presents in general higher values  
 402 for both mean and CV. For the sake of brevity, the results are only shown for  
 403 one spatial fraction, nevertheless other values were tested and they presented  
 404 the same trends.

#### 405 4. Homogeneous equivalent models

406 Homogeneous equivalent models and their effective properties are interesting  
 407 from an engineering point of view. With the aid of widely used computer soft-  
 408 ware such as Microsoft Excel, performing Monte Carlo Simulation (MCS)-based  
 409 probabilistic analysis for geotechnical applications is becoming more convenient  
 410 and straight-forward (Wang and Huang, 2012; Wang et al., 2016). Is it possible  
 411 to obtain the same probability density functions (PDF) of the bearing capacity  
 412 using binary random fields (BRF) and homogeneous MCS?

##### 413 4.1. Monte-Carlo simulations

414 To develop the homogeneous equivalent model, three steps are performed.  
 415 A flowchart of this procedure is shown in Figure 9. First, the results from the  
 416 BRF analysis are used to generate the empirical cumulative density function  
 417 (CDF) of the bearing capacity ( $q_u$ ). Second, the CDF of the effective strength  
 418 ( $c_u^{eff}(\gamma)$ ) is linearly calculated from the previous results considering the Prandtl  
 419 solution. The third step consists of using the obtained CDF of  $c_u^{eff}$  to generate  
 420 multiple homogeneous fields with Monte Carlo simulations and evaluate the

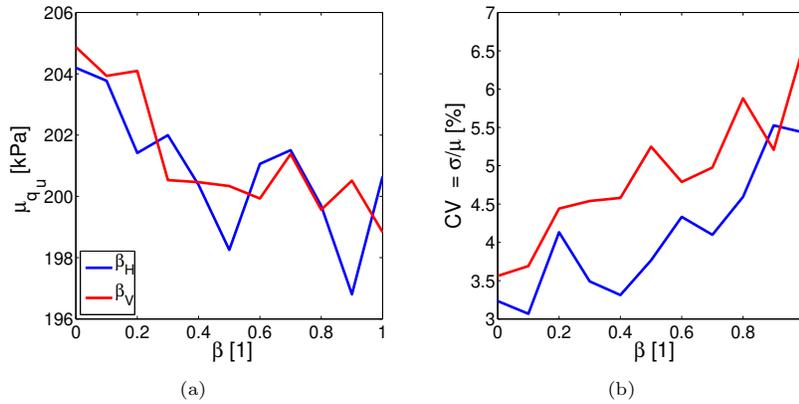


Figure 8: Anisotropic correlation effect on the a) mean and b) CV of the bearing capacity for bimsoils with a spatial fraction of 0.4. The coefficient in the horizontal ( $\beta_H$ ) or vertical ( $\beta_V$ ) correlation is varied while the other correlation is fixed to 0.1.

421 resulting statistical properties of the bearing capacity. Hence, addressing the  
 422 question regarding the feasibility of achieving identical mean and dispersion  
 423 using homogeneous models that can faithfully replicate the effect of the spatial  
 424 variability on the bearing capacity observed in bimsoils. The procedure is shown  
 425 in detail for different spatial fractions and isotropic correlations.

#### 4.1.1. CDF of the bearing capacity from BRF results

427 First, the results from the spatial variability simulated with BRF are used  
 428 to calculate the empirical CDF. Figure 10 shows the normalized CDF of the  
 429 bearing capacity for  $\gamma$  of 0.1, 0.5 and 0.9 and all different correlations. As it  
 430 is seen once more, the isotropic correlation plays an important role in the CV  
 431 (refer to the steepness in the CDF in figures 10b and 10c). Additionally, for  $\gamma$   
 432 of 0.9, the type of the distribution changes; though, for instance, the results for  
 433  $\beta$  of 0.9 have a lognormal distribution -as it presents a positive skewness- while  
 434 for  $\beta$  of 0.1, a normal distribution could be more adequate.

435 Jones et al. (2002) summarized the inherent variability on strength charac-  
 436 teristics given by *in-situ* and laboratory measurements and suggest a lognormal  
 437 PDF for undrained shear strength ( $c_u$ ) in clays and a normal one for  $c_u$  in  
 438 silty-clays. Hence, as the  $c_u$  decreases, the distribution shifts from lognormal to  
 439 normal, as it does with the results for  $\gamma$  of 0.9. An accurate probability func-  
 440 tion could be very useful to construct fragility curves that take into account the  
 441 spatial variability uncertainty in bimsoils. However the success of this kind of  
 442 analysis requires an accurate safety limit or threshold related to the determinis-  
 443 tic properties. In the next section, an effort to find an homogeneous equivalent  
 444 model will be presented.

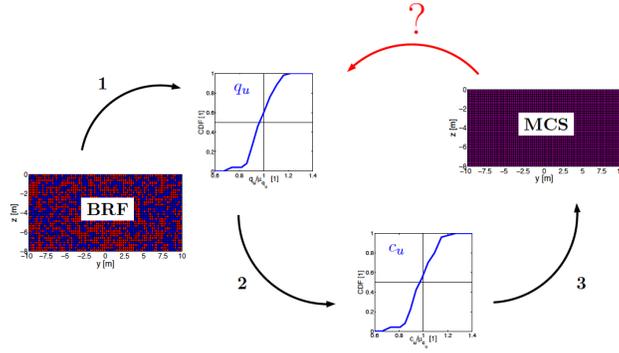


Figure 9: Flowchart of the procedure to generate the homogeneous equivalent model with the same probability distribution

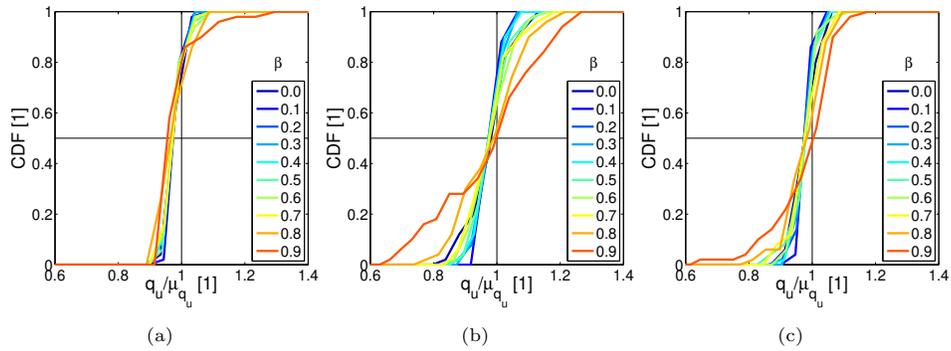


Figure 10: Experimental cumulative density function (CDF) of the bearing capacity for : a)  $\gamma=0.1$ , b)  $\gamma=0.5$  and c)  $\gamma=0.9$

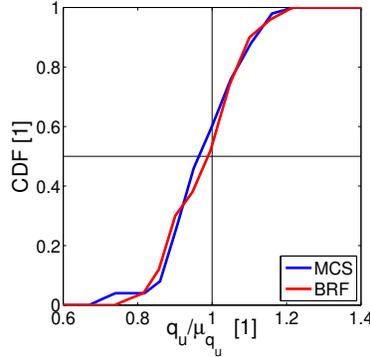


Figure 11: Comparison of the normalized CDF of  $q_u$  obtained from the homogeneous Monte Carlo simulations (MCS) and the one obtained from the initial binary random field (BRF) for  $\gamma$  of 0.5 and  $\beta_H$  and  $\beta_V$  equal to 0.8

445 *4.1.2. CDF of the equivalent undrained shear strength for homogeneous models*

446 Second, as  $q_u$  depends only on the undrained shear strength, the effective  
 447 property ( $c_u^{eff}$ ) is equal to  $\mu_{q_u}(\gamma)/5.14$ , where  $\mu$ , indicates the mean value of  
 448 all the  $q_u$  values resulting of the spatially variable models for a specified  $\gamma$ . It  
 449 is reasonable to believe that the same numerical error - i.e. 10% in average  
 450 - with respect to the Prandtl solution is present in all cases; therefore, for the  
 451 homogeneous model the error is deducted from the effective property (otherwise  
 452 it would be counted twice). Thus, the resulting CDF of  $q_u$  from the first step  
 453 is divided by 5.14 and corrected due to numerical errors to obtain the CDF for  
 454  $c_u^{eff}$  that will be used as input in the next step.

455 *4.1.3. CDF of the bearing capacity from homogeneous MCS*

456 Monte Carlo simulations involve generating a large number of random sam-  
 457 ples from a known probability distribution to approximate various statistical  
 458 properties of a system. In this context, the CDF of the  $c_u^{eff}$  is used to simulate  
 459 200 homogeneous fields. The results of the MCS are used to obtain the CDF  
 460 of  $q_u$ . Figure 11 compares the empirical CDF of  $q_u$  from the BRF models (step  
 461 1) and from the homogeneous MCS (step 3). For the sake of brevity, results  
 462 are only shown for one set of spatial fraction and correlations. It corresponds  
 463 to  $\gamma$  of 0.5 and  $\beta_H$  and  $\beta_V$  equal to 0.8. The resulting CDF from the MCS  
 464 matches very well the one from the BRF analysis. Hence the effect of spatial  
 465 variability can be included in homogeneous analysis with the use of probability  
 466 functions. Now, the question relies on how to describe such function in a general  
 467 case according to the soil parameters.

468 *4.2. Homogenisation theories*

469 In engineering practice, it is common to use deterministic homogenisation  
 470 theories to simplify the average effects of spatial variation. Figure 12a shows

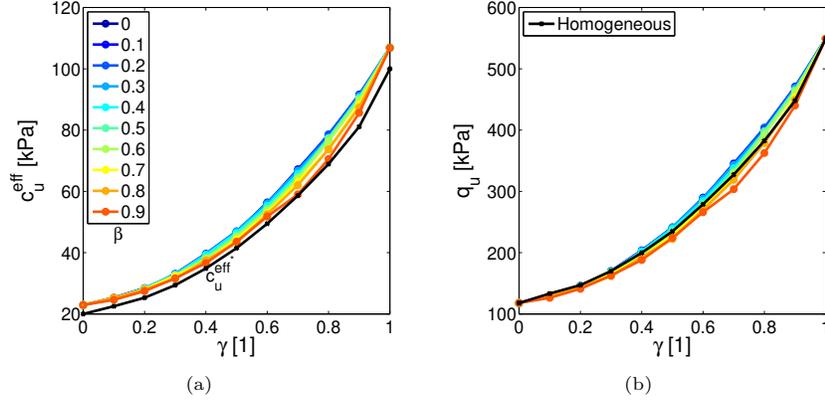


Figure 12: Input and Output of the homogeneous equivalent model : a) Effective property ( $c_u^{eff} = \mu_{q_u}(\gamma)/5.14$ ) for all isotropic correlations and average corrected value ( $c_u^{eff*}$ ), in black, used as input and resulting b) Bearing capacity  $q_u$  as a function of the spatial fraction ( $\gamma$ ) in black compared to the average of the heterogeneous models

471 the mean of  $c_u^{eff}$  as a function of  $\gamma$  for all the different correlations. As in  
 472 the previous analysis, the undrained shear strength is corrected from the FE  
 473 error. The average corrected effective property ( $c_u^{eff*}$ ), shown in the figure in  
 474 black, was used for the deterministic homogeneous equivalent model. In Figure  
 475 12b, the results are compared with the  $q_u$  mean values of all the correlations.  
 476 As expected, the homogeneous models have the same behaviour and could be  
 477 described by an equation. It is important to note that the homogeneous model  
 478 takes into account all the correlations tested and their respective dispersion;  
 479 however the general behaviour with respect to the spatial fraction (i.e. the  
 480 shape of the function) is the same for all cases hence the equation should be  
 481 similar.

482 The traditional homogenisation theories are compared to the numerical re-  
 483 sults in Figure 13a. It shows the mean  $q_u$  values normalized by the  $q_{u\ block}$  as  
 484 a function of  $\gamma$  and the Wiener and HS bounds. As it is shown, if the homoge-  
 485 neous models take the upper Wiener bound described by the arithmetic average  
 486 (i.e. diagonal straight line in Figure 13a) as the effective property, the heteroge-  
 487 neous model will always present lower resistance (i.e. lower  $q_u$  values). Overall,  
 488 results are inside the HS bounds with a  $d$  value of 2. However, for lower spatial  
 489 fractions ( $\gamma < 0.4$ ) the mean values are even lower than the  $HS^-$  bound and  
 490 almost similar to the lower Wiener bound described by the harmonic average.  
 491 Nonetheless, this harmonic average is significantly lower than the numerical  
 492 results for higher  $\gamma$  values.

493 In contrast, as it can be seen in Figure 13b, the mean values are well fitted  
 494 by the BEM equation in which, for the case tested,  $d_{BEM}$  was found by a root-  
 495 mean-square fit to a value of 1.5. Though if an homogeneous equivalent model  
 496 should be compared with discrete spatial heterogeneous models, BEM should

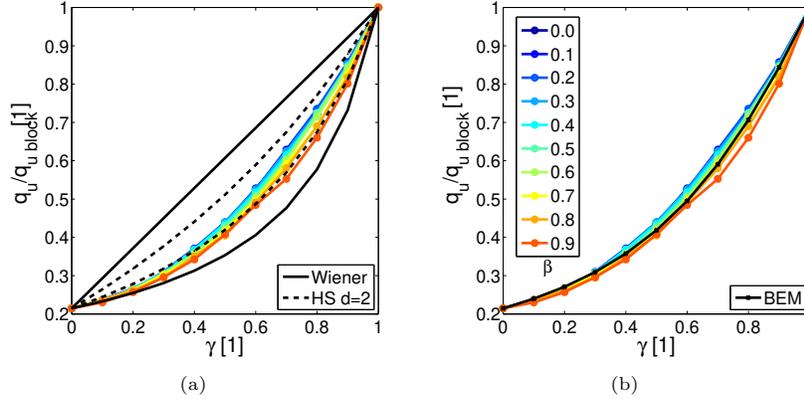


Figure 13: Heterogeneous mean normalized  $q_u$  values compared to a) traditional homogenisation and b) Bruggeman symmetric effective medium theories ( $d_{BEM} = 1.5$ )

497 be used.

#### 498 4.3. Effect of the contrast in the matrix and block properties

499 The spatial variability effect depends on the matrix and block soil properties;  
500 hence, seven more strength ratios ( $c_{u,block}/c_{u,matrix}$ ) were used and the normal-  
501 ized mean and CV values are shown in Figure 14. No correlation was introduced,  
502 which was found to increase the variability, and its effects have been formerly  
503 addressed. The undrained shear strength ( $c_u$ ) of each set and the respective  
504 ratios are shown in Table 2. In Figure 14a, the lower and upper quartiles (i.e.  
505 25 and 75%) are also shown however the dispersion is very low compared to the  
506 mean value. Note that as the ratio increases, the shape of the model changes  
507 thus it could only be represented by an equation with an additional parameter,  
508 such as the  $d_{BEM}$  of equation 6. Moreover, for the last two cases, where the  
509 ratios are repeated but with different  $c_u$  values, the normalized behavior does  
510 not change. Hence, it depends more on the contrast between the materials and  
511 not on the values used.

512 Figure 14b shows the CV values of all the  $c_u$  ratios tested. In general, as  
513 the contrast between the block and matrix strength increases, there is a corre-  
514 sponding increase in dispersion, akin to what is observed with higher levels of  
515 isotropic correlation. This phenomenon is expected because a greater difference  
516 in strength implies a more pronounced change in the failure surface. Notably,  
517 the CV values tend to be higher for spatial fractions near 0.5, with the exception  
518 of the two cases with a contrast ratio of 20. The increase in dispersion nearing  
519 equal fractions of blocks and matrix might be related to the clustering effect that  
520 produces an increase in equivalent block size as shown in Figure 2. As for the  
521 cases of a  $c_u$  ratio of 20, the largest CV values are observed for spatial fractions  
522 of about 0.2 possibly because when the blocks occupy only a small portion of  
523 the material, the strength of the matrix dominates the bearing capacity in the

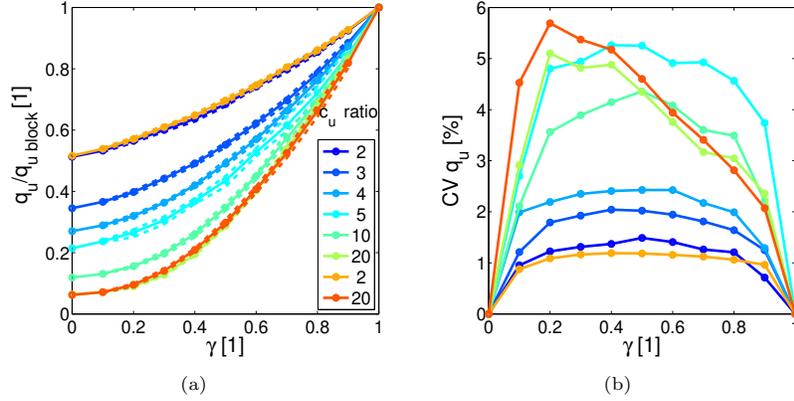


Figure 14: Different bimsoils  $c_u$  ratios tested: a) mean and percentiles 25 and 75% and b) CV values

ratio	$c_{u\ matrix}$ [kPa]	$c_{u\ block}$ [kPa]
2	20	40
3	20	60
4	20	80
5	20	100
10	10	100
20	10	200
2	30	60
20	5	100

Table 2: Sets of parameters tested

524 model thus inducing more intricate failure surfaces. Nevertheless, it is worth  
525 highlighting that for all cases, the CV remains below 6%, which is less than half  
526 of the maximum value attributable to isotropic correlation, of almost 16% as  
527 depicted in Figure 7.

528 Finally, the BEM equation was used to fit the bearing capacity of the differ-  
529 ent sets of soils tested. The parameter  $d_{BEM}$  was found by the minimization of  
530 the root-mean-square error. The results are shown in Figure 15. It is interesting  
531 to note that the changes in the shape of the curves from the bearing capacity  
532 ratio as function of the spatial fraction, shown in Figure 15a, for the different  
533  $c_u$  ratios, agree very well with the  $d_{BEM}$  proposed. Figure 15b shows the re-  
534 lationship between the proposed values for  $d_{BEM}$  and the  $c_u$  ratio. A power  
535 equation appears to fit the data with a  $R^2=0.99$ , as following:

$$d_{BEM} = (c_{u\ matrix}/c_{u\ block})^{0.28} \quad (9)$$

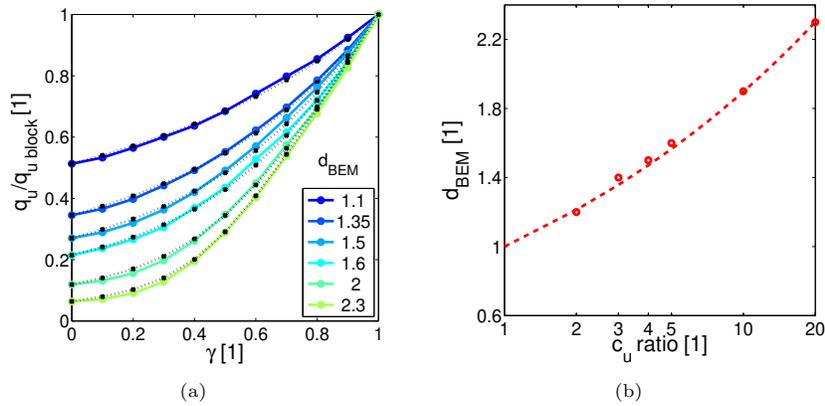


Figure 15: BEM equation for the different  $c_u$  ratios tested: a) bearing capacity ratio as function of the spatial fraction and b)  $d_{BEM}$  values as a function of the  $c_u$  ratio.

#### 536 4.4. Effect of the effective loaded zone

537 As depicted in Figure 4, it becomes evident that not the entire strength of  
538 the soil deposit is crucial when evaluating the bearing capacity ( $q_u$ ) of the foun-  
539 dation. Instead, there appears to be a defined region for which the average shear  
540 strength is related the most with the  $q_u$  value. The existence of an equivalent  
541 homogeneous soil deposit that could reproduce (statistically) the same response  
542 as a spatially variable deposit is encouraging for practical purposes. Pioneering  
543 work by Vanmarcke (1977) stated that the performance of geotechnical struc-  
544 tures is controlled by the average over a certain size of line, area or volume,  
545 referred as the Local Average (LA), rather than by the soil parameter value at  
546 a single point. In fact, LA is recommended in practical design codes, such as  
547 Eurocode 7 (CEN, 2004), to determine characteristic values of geotechnical pa-  
548 rameters. LA applications in the assessment of the bearing capacity have been  
549 studied by Asaoka and Matsuo (1983), Griffiths and Fenton (2001), Kasama  
550 et al. (2012) and Honjo and Otake (2013), among others. The former proposed  
551 appropriate sizes of local averages to evaluate the effects of spatial variability  
552 for various geotechnical structures. Concerning the bearing capacity of shallow  
553 foundation in cohesive soils, a size of  $L_y \times L_z = 2B \times 0.7B$  was suggested. It  
554 corresponds to the rectangle where is located the majority of the plastic zone  
555 obtained by the Prandtl-type ultimate bearing capacity equation. This size was  
556 said to be independent of the random field characteristics and of the absolute  
557 undrained strength of the soils. Thus, the averaging area size purely depends  
558 on the mechanism controlling the limit state, but not on the spatial variability  
559 properties of the soil.

560 For the binary random field used to simulate bimsoils presented in this analy-  
561 sis, an optimization procedure was established to find the LA area. For the sake  
562 of brevity, only the case of maximum variation is presented, i.e.  $\beta_H = \beta_V = 0.9$ ;  
563 however, the other cases were also analysed and the results were similar. The

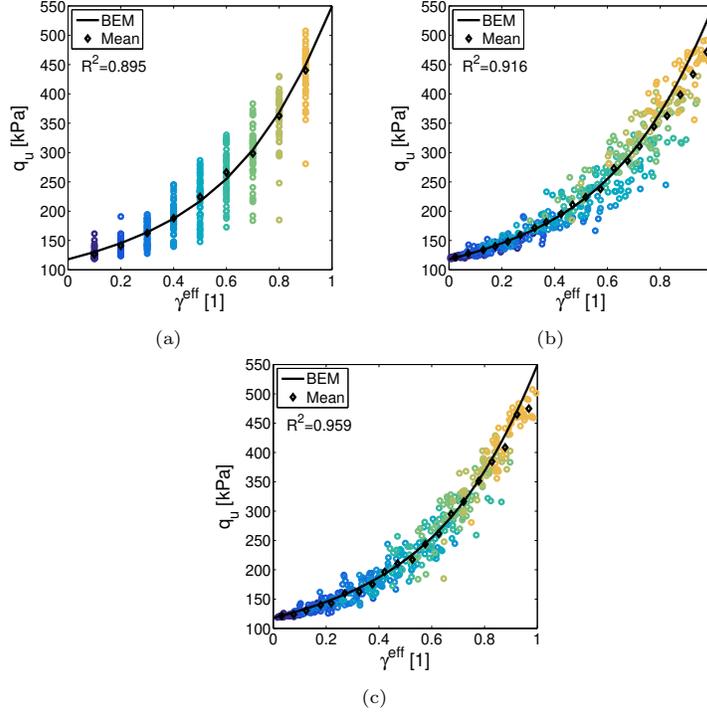


Figure 16: Bearing capacity as a function of the  $\gamma^{eff}$  : a) evaluated in all the deposit ( $5B \times 2B$ ), b) at the suggested area by Honjo and Otake (2013) ( $2B \times 0.7B$ ) and c) the optimized area ( $2.4B \times B$ ).

564 optimization consisted in maximizing the coefficient of determination ( $R^2$ ) of  
 565 the data fitted to the previously obtained BEM equation. In the fit, the bearing  
 566 capacity values are related to the effective spatial fraction ( $\gamma^{eff}$ ), calculated  
 567 at the area below the foundation of size  $L_y \times L_z$ . The distances  $L_y/2$  and  $L_z$   
 568 were varied every element size ( $B/16$ ). As an example, three cases are shown  
 569 in Figure 16 : a) evaluated in all the deposit ( $5B \times 2B$ ), b) at the suggested  
 570 area from Honjo and Otake (2013) ( $2B \times 0.7B$ ) and c) the optimized area found  
 571 ( $2.4B \times 1.1B$ ). Globally, the dispersion is reduced when using an appropriate  
 572  $\gamma^{eff}$ , although some dispersion is still present for the optimized area. Concerning  
 573 the mean values evaluated at different bins, the BEM equation is in general  
 574 appropriate for the three cases, except for higher  $\gamma^{eff}$  as shown in Figure 16b.

575 A summary of the optimization is shown in Figure 17. The  $R^2$  values for  
 576 each area tested are mapped in the mesh. As the results are vertically sym-  
 577 metric, only half of the mesh is shown. Even if the  $R^2$  value for the area of  
 578  $2B \times 0.7B$ , shown in dashed box, is high ( $R^2 = 0.916$ ), it is improved for the  
 579 optimized area ( $R^2 = 0.959$ ), as shown in the solid box. The remaining box,  
 580 in dash-dot, corresponds to the entire plastic zone of the Prandtl-type ulti-

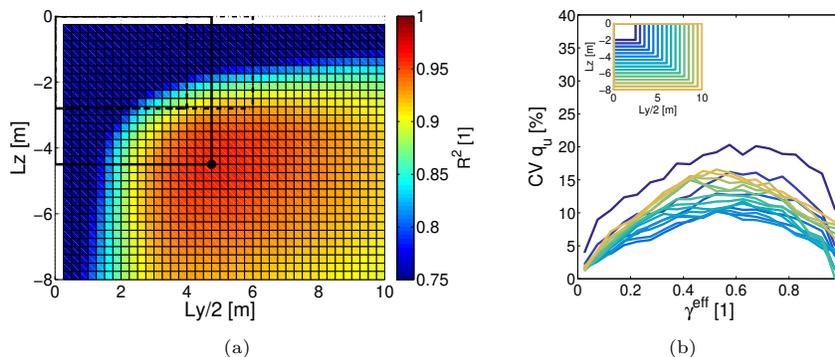


Figure 17: Optimization summary results: a) Half mesh for the coefficient of determination  $R^2$  values of varying LA area and b) CV reduction due to LA area in bimsoils

581 mate bearing capacity equation and presented a smaller  $R^2$  value. Figure 17b  
 582 demonstrates that optimizing the LA area leads to a significant reduction in  
 583 the coefficient of variation (CV), nearly halving its value. The CV reduction  
 584 was also evaluated for the other isotropic correlations tested; however it was  
 585 smaller and is not shown here to avoid repetition. While the CV is reduced  
 586 when the optimized area is used, there is still a remaining dispersion due to the  
 587 different failure mechanisms induced by the spatial variability. Compared to  
 588 the initial CV values, from Figure 7, the maximum value is still higher for this  
 589 isotropic correlation. In other words, the variation due to the spatial correlation  
 590 in bimsoils is more important than that of the local average.

## 591 5. Conclusions

592 Homogeneous equivalent models can present the same  $q_u$  values when the  
 593 accurate  $c_u$  value of bimsoils is taken as input. Because of its capability to  
 594 account for the contrast in matrix and block mechanical properties, the Brugge-  
 595 man effective medium (BEM) equation appears to accurately predict the mean  
 596 bearing capacity for varying spatial fractions ( $\gamma$ ). On the contrary, traditional  
 597 homogenisation techniques, such as the arithmetic or harmonic averages, will  
 598 either overestimate or underestimate the resistance of bimsoils. However, for  
 599 a given  $\gamma$  and a given correlation, the spatial distribution affects the  $q_u$  with  
 600 coefficients of variation (CV) up to almost 20%. This CV can be reduced if the  
 601 optimized LA area is taken into account, although it can not be avoided as it is  
 602 due to the radically different failure mechanisms (surfaces) that are developed.  
 603 Lastly, these variations can be also obtained with Monte Carlo simulations on  
 604 homogeneous models which could be of great use to account for the effects of  
 605 bimsoils spatial variability on the bearing capacity in a simple reliability-based  
 606 design.

607 The results shown in this analysis have taken into account the results dif-  
 608 ference with the Prandtl's solution. First, it has been measured as the ability

609 of the finite element method to reflect the actual behaviour of an homogeneous  
610 (ideal) soil and it has been subtracted before applying the Prandtl's formula.  
611 It has been assumed that both the finite element method and the theoretical  
612 formula are sufficiently reasonable approximations to the behaviour of soils to  
613 allow the investigation of the major features of stochastic bimsoils behaviour  
614 under loading from a rigid foundation. Note that the effects of the spatial vari-  
615 ability in the CV of  $q_u$  have been evaluated independently of these assumptions  
616 and are associated with traditional usage of this engineering problem.

617 The numerical model of binary spatial variability applied in a probabilistic  
618 framework appears to properly include heterogeneity on the bimsoils. Other  
619 cases of discrete spatial variability, such as soil-mixing for liquefaction mitiga-  
620 tion, have been also analyzed with this model in order to identify homogeneous  
621 equivalent models (Montoya-Noguera and Lopez-Caballero, 2016). Homoge-  
622 neous equivalent models with the BEM equation were successfully calibrated  
623 for eight sets of contrast between matrix and block undrained shear strength  
624 values to evaluate the bearing capacity under undrained conditions. For fur-  
625 ther research, other values for contrast in strength and stiffness properties be-  
626 tween the blocks and matrix may be tested evaluating also the settlement and  
627 other engineering demand parameters. Also, other constitutive models may  
628 be implemented, representing for example a purely frictional matrix as done  
629 by Schmüdderich et al. (2021). Additionally, a continuous random field model  
630 could be included in order to account for inherent variability in the matrix and  
631 the block behavior. Bimsoils were simulated with a binary random field (BRF)  
632 model and a uniform mesh was used. Employing adaptively refined meshes such  
633 as that presented by Schmüdderich et al. (2021) may bypass the computational  
634 resources while at the same time enhancing accuracy by utilizing a smaller total  
635 number of elements. This technique is of even greater importance in scenarios  
636 where irregularly shaped blocks can impede the development of regular fail-  
637 ure surfaces. This could be an improvement for further studies. The BRF  
638 model produced polydispersed clusters with irregular shapes of different sizes  
639 controlled partly by the correlation coefficients, although the model could be  
640 enhanced to produce regular shapes of fixed sizes. Finally, the results presented  
641 are limited to welded bimsoils but unwelded bimsoils could be also studied with  
642 this numerical model and with adaptively refined meshes.

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