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► **To cite this version:**

| Rene-Louis Clerc. THE PERFECT R-NARCISSISTIC NUMBERS. 2023. hal-04229895v2

HAL Id: hal-04229895

<https://hal.science/hal-04229895v2>

Preprint submitted on 13 Oct 2023

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THE PERFECT R-NARCISSISTIC NUMBERS

René-Louis Clerc (september 2023) ((*))

ABSTRACT-A brief definition of perfect r-narcissistic numbers (or rppdi), a natural extension of the classic Armstrong numbers (or perfect narcissistic numbers or ppdi), a demonstration of their finiteness and the list of 15 rppdi greater than 1 currently calculated.

- DEFINITION.

Given a k-digit number m ($k > 0$), we note:

$$m = \sum_{i=1}^{i=k} a_i 10^{k-i}, S_p(m) = \sum_{i=1}^{i=k} a_i^p, p \geq 1.$$

A k-digit number m is perfect r-narcissistic (or rppdi) if there exists an integer $r > 1$ such that the sum of the powers k of the digits of m^r is equal to m .

The case $r = 1$ corresponds to the classic perfect narcissistic numbers or Armstrong numbers, or ppdi ([1]); we know that there are 88 such numbers.

An rppdi is still a k-digit fixed point of the S_k^r transformation ([2])

$$m \rightarrow S_k^r(m) = S_k(m^r)$$

We showed in [2] that these S_k^r transformations are pleasant (cf. Remark 1 below), that is to say they are everywhere convergent and have a finite number of attractors and in particular a finite number of fixed points with k digits (let us observe that S_k^r has a finite number of fixed points, some of which have exactly k digits, cf. Remark 2 below).

There is therefore a finite number of rppdi, each of these numbers being defined by a unique r greater than 1; however, there can theoretically be r-narcissistic numbers for several different values of r (but we can reasonably conjecture that this is not the case).

For the 15 calculated rppdi (numbers from 1 to 10 digits) greater than 1, the corresponding $r (> 1)$ is unique; only the trivial solution 1 is r-narcissistic for all r .

For example, note 7(4) to indicate that 7 is 4-narcissistic.

- PROPERTY.

There are a finite number of perfect r-narcissistic numbers.

The 15 smallest perfect r-narcissistic numbers ($r \geq 2$) greater than 1 are:

7(4), 8(3), 9(2),

180(6), 205(2),

38998(2), 45994(2), 89080(2),

726191(2),

5540343(3), 7491889(2), 8690141(3),

167535050(3), 749387107(4).

9945245922(3).

-REMARK 1: demonstration of S_k^r pleasant transformation for any (k, r) .

Any p -digit number being strictly less than 10^p , it comes

$$S_k^r(m) \leq r p 9^k$$

For any pair (k, r) of strictly positive and finite integers, there exists an upper bound $M = r 9^k$ such that for any integer m with p digits we have the increase

$$S_k^r(m) \leq p M.$$

Consequently, as soon as the number pM has strictly less than p digits, the transformation S_k^r cannot have a fixed point at p or more than p digits (since we cannot ensure $S_k^r(f) = f$); nor can it then have a cycle with elements with p or more than p digits (iteration by S_k^r would not make it possible to find an element of strictly more than p digits).

This is ensured as soon as p satisfies the inequality $10^{p-1}/p > M$ and there obviously always exists a smallest p having this property for any pair (k, r) .

We will note $p^* = p^*(k, r)$ the smallest p such that $10^{p-1}/p > M$.

The function $10^{p-1}/p$ being increasing, for all $p \geq p^*$, we always have $p M < 10^{p-1}$ and the iteration by $S < sub>k^r$ of a number with such p digits always provides a number with at most $p-1$ digits (each iteration causes the transform to lose at least one digit).

Let us observe that for all k and all r strictly positive, we will always have $p^* \geq 3$.

It follows that all the attractors of S_k^r are necessarily made up of numbers contained in the finite interval $[0, 10^{p^*-1}[$, since this p^* is still the smallest p such that the upper bound $p M$ is strictly less than 10^{p-1} (i.e. again, has at most $(p-1)$ digits).

The transformations S_k^r are everywhere convergent in \mathbb{N} towards a finite number of attractors and are therefore all pleasant transformations, the convergence interval being $[0, 10^{p^*-1}[$; any number it contains has all its iterations in it and any number outside it ends up there after a finite number of iterations.

It follows that it is enough to seek the convergence of all integers strictly less than 10^{p^*-1} to determine all the attractors of S_k^r .

The pleasant nature of S_k^r transformations implies in particular that there are only a finite number of rppdi.

- REMARK 2: examples of fixed points of S_k^r .

We will observe that if the transformations S_k^r , $k \geq 1$, $r \geq 2$, have many fixed points with strictly more than k digits, they have quite few with exactly k digits ...

S_3^6 has the 6-narcissistic number 180 but also the fixed point 3172;

S_5^2 has the three 2-narcissistic numbers 38998, 45994, 89080 but also the fixed points 128230 and 244353;

S_5^3 no 5-digit 3-narcissistic number, but the fixed points 137023, 163451, 192412, 233722;

S_3^4 no 3-digit 4-narcissistic number, but the fixed point 2110;

S_2^4 no 2-digit 4-narcissistic number, but fixed points 130 and 235...

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This article was published in HAL Open Archive on 05/10/2023 ([hal-04229895](https://hal.archives-ouvertes.fr/hal-04229895)).

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