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# THE PERFECT R-NARCISSISTIC NUMBERS

René-Louis Clerc (september 2023) ([\(\\*\)](#))

ABSTRACT-A brief definition of perfect r-narcissistic numbers (or rppdi), a natural extension of the classic Armstrong numbers (or perfect narcissistic numbers or ppdi), a demonstration of their finiteness and the list of 14 rppdi greater than 1 currently calculated.

- DEFINITION.

Given a k-digit number  $m$  ( $k > 0$ ), we note:

$$m = \sum_{i=1}^{i=k} a_i 10^{k-i}, S_p(m) = \sum_{i=1}^{i=k} a_i^p, p \geq 1.$$

A k-digit number  $m$  is perfect r-narcissistic (or rppdi) if there exists an integer  $r > 1$  such that the sum of the powers  $k$  of the digits of  $m^r$  is equal to  $m$ .

The case  $r = 1$  corresponds to the classic perfect narcissistic numbers or Armstrong numbers, or ppdi ([1]); we know that there are 88 such numbers.

An rppdi is still a k-digit fixed point of the  $S_k^r$  transformation ([2])

$$m \rightarrow S_k^r(m) = S_k(m^r)$$

We showed in [2] that these  $S_k^r$  transformations are pleasant (cf. Remark 1 below), that is to say they are everywhere convergent and have a finite number of attractors and in particular a finite number of fixed points with  $k$  digits (let us observe that  $S_k^r$  has a finite number of fixed points, some of which have exactly  $k$  digits, cf. Remark 2 below).

There is therefore a finite number of rppdi, each of these numbers being defined by a unique  $r$  greater than 1; however, there can theoretically be r-narcissistic numbers for several different values of  $r$  (but we can reasonably conjecture that this is not the case).

For the 14 calculated rppdi (numbers from 1 to 9 digits) greater than 1, the corresponding  $r (> 1)$  is unique; only the trivial solution 1 is r-narcissistic for all  $r$ .

For example, note 7(4) to indicate that 7 is 4-narcissistic.

- PROPERTY.

There are a finite number of perfect r-narcissistic numbers.

The 14 smallest perfect r-narcissistic numbers ( $r \geq 2$ ) greater than 1 are:

7(4), 8(3), 9(2),

180(6), 205(2),

38998(2), 45994(2), 89080(2),

726191(2),

5540343(3), 7491889(2), 8690141(3),

167535050(3), 749387107(4).

-REMARK 1: demonstration of  $S_k^r$  pleasant transformation for any  $(k, r)$ .

Any  $p$ -digit number being strictly less than  $10^p$ , it comes

$$S_k^r(m) \leq rp^k$$

For any pair  $(k, r)$  of strictly positive and finite integers, there exists an upper bound  $M = r9^k$  such that for any integer  $m$  with  $p$  digits we have the increase

$$S_k^r(m) \leq pM.$$

Consequently, as soon as the number  $pM$  has strictly less than  $p$  digits, the transformation  $S_k^r$  cannot have a fixed point at  $p$  or more than  $p$  digits (since we cannot ensure  $S_k^r(f) = f$ ); nor can it then have a cycle with elements with  $p$  or more than  $p$  digits (iteration by  $S_k^r$  would not make it possible to find an element of strictly more than  $p$  digits).

This is ensured as soon as  $p$  satisfies the inequality  $10^{p-1}/p > M$  and there obviously always exists a smallest  $p$  having this property for any pair  $(k, r)$ .

We will note  $p^* = p^*(k, r)$  the smallest  $p$  such that  $10^{p-1}/p > M$ .

The function  $10^{p-1}/p$  being increasing, for all  $p \geq p^*$ , we always have  $pM < 10^{p-1}$  and the iteration by  $S_k^r$  of a number with such  $p$  digits always provides a number with at most  $p-1$  digits (each iteration causes the transform to lose at least one digit).

Let us observe that for all  $k$  and all  $r$  strictly positive, we will always have  $p^* \geq 3$ .

It follows that all the attractors of  $S_k^r$  are necessarily made up of numbers contained in the finite interval  $[0, 10^{p^*-1}[$ , since this  $p^*$  is still the smallest  $p$  such that the upper bound  $p M$  is strictly less than  $10^{p-1}$  (i.e. again, has at most  $(p-1)$  digits).

The transformations  $S_k^r$  are everywhere convergent in  $\mathbb{N}$  towards a finite number of attractors and are therefore all pleasant transformations, the convergence interval being  $[0, 10^{p^*-1}[$ ; any number it contains has all its iterations in it and any number outside it ends up there after a finite number of iterations.

It follows that it is enough to seek the convergence of all integers strictly less than  $10^{p^*-1}$  to determine all the attractors of  $S_k^r$ .

The pleasant nature of  $S_k^r$  transformations implies in particular that there are only a finite number of rppdi.

- REMARK 2: examples of fixed points of  $S_k^r$ .

We will observe that if the transformations  $S_k^r$ ,  $k \geq 1$ ,  $r \geq 2$ , have many fixed points with strictly more than  $k$  digits, they have quite few with exactly  $k$  digits ...

$S_3^6$  has the 6-narcissistic number 180 but also the fixed point 3172;

$S_5^2$  has the three 2-narcissistic numbers 38998, 45994, 89080 but also the fixed points 128230 and 244353;

$S_5^3$  no 5-digit 3-narcissistic number, but the fixed points 137023, 163451, 192412, 233722;

$S_3^4$  no 3-digit 4-narcissistic number, but the fixed point 2110;

$S_2^4$  no 2-digit 4-narcissistic number, but fixed points 130 and 235...

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## REFERENCES

[1] Armstrong numbers, [OEIS A005188](https://oeis.org/A005188).

[2] R.L.Clerc, Les transformations agréables et une nouvelle classe de nombres narcissiques parfaits, p.1-17, (<https://hal.archives-ouvertes.fr/hal-03619147>), 2022.