# Barlaam's Refutation of the Chapters Added to Ptolemy's Harmonics. A Critical Edition

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# Barlaam's Refutation of the Chapters Added to Ptolemy's *Harmonics*. A Critical Edition*

ABSTRACT: The article presents a critical edition, with a translation and an introduction, of the "refutation" of chapters 14–16 of Book III of Ptolemy's *Harmonica*, which were not handed down in the manuscript tradition. Chapters 14 and 15 were added in Byzantine times, the origin of chapter 16 is uncertain. The "refutation" was authored by the 14th-century scholar and polemicist Barlaam of Seminara, one of the two leading characters in the Palamite and hesychast controversies. In this way, the last item of Barlaam's scientific writings can now be read in a critical edition. The present edition explains in detail the background of Barlaam's work, describes all of its manuscript witnesses, and reconstructs a stemma codicum.

KEYWORDS: Barlaam of Seminara, Ptolemy's *Harmonica*, refutation, Byzantine reception of ancient Greek harmonic theory

## INTRODUCTION

Barlaam of Seminara (†1348), born in Calabria, was a noted personality in Palaiologan Byzantium, also serving as an imperial ambassador. He was one of the two leading characters in the Palamite and hesychast controversies, where he opposed the monk Gregorius Palamas[1]. After being defeated, in 1341 Barlaam left Constantinople for the Latin West, settled by the papal Curia in Avignon, and converted to Catholicism. It is tempting to speculate that he fell victim to the Black Death pandemic.

Many of Barlaam's works were collected in volume 151 of Migne's *Patrologia Graeca*; most of them have also been published in a critical edition[2]. Barlaam also composed five scientific works[3]:

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* Reproductions of most manuscripts mentioned in this article can be found through the website https://pinakes. irht.cnrs.fr/, which also provides additional bibliography. We are grateful to Daniele Bianconi, Ciro Giacomelli, Anna Gioffreda, and Francesco Valerio for their expertises, and to Matthieu Cassin for his logistic support. FA had access to some of the manuscripts thanks to the digital repository of the project Sin-aps (Alexander von Humboldt-Professorship, FAU Erlangen-Nürnberg). FA translated and annotated the text; SDM prepared the critical text and wrote the four sections that precede it.

[1] For a first orientation on the Palamite and hesychast controversies, see A. RIGO, Gregorio Palamas e oltre. Studi e documenti sulle controversie teologiche del XIV secolo bizantino (*Orientalia Venetiana* 16). Firenze 2004; A. FYRIGOS, Dalla controversia palamitica alla polemica esicastica (con un'edizione critica delle Epistole greche di Barlaam). Roma 2005, 67–97; N. RUSSELL, The Hesychast Controversy, in: The Cambridge Intellectual History of Byzantium, ed. A. Kaldellis – N. Siniossoglou. Cambridge 2017, 494–508. On Barlaam, see S. IMPELLIZZERI, Barlaam, in: Dizionario Biografico degli Italiani. VI. Roma 1964, 392–397; *PLP* 2284; A. FYRIGOS, Barlaam Calabro: l'uomo, l'opera, il pensiero. Atti del Convegno internazionale, Reggio Calabria, Seminara, Gerace, 10-11-12 dicembre 1999. Roma 2001; FYRIGOS, Dalla controversia palamitica 161–169. On Barlaam's philosophical stance, see most recently M. TRIZIO, «Una è la verità che pervade ogni cosa». La sapienza profana nelle opere perdute di Barlaam Calabro, in: Byzantine Theology and its Philosophical Background, ed. A. Rigo (*Byzantioς* 4). Turnhout 2011, 108–140, with bibliography.

[2] See the next footnote for the scientific works. Other editions include C. GIANNELLI, Un progetto di Barlaam Calabro per l'unione delle Chiese, in: Miscellanea Giovanni Mercati. III (*StT* 123). Città del Vaticano 1946, 157–208; R. E. SINKEWICZ, The Solutions Addressed by Barlaam the Calabrian to George Lapithes and their Philosophical Context. *MS* 43 (1981) 151–217 (this contains a complete list of Barlaam's writings, with references to the editions to that date; see also, more recently, FYRIGOS, Dalla controversia palamitica 169–182); T. M. KOLBABA, Barlaam the Calabrian. Three Treatises on Papal Primacy. Introduction, Edition, and Translation. *REB* 53 (1995) 41–115; and the complete edition A. FYRIGOS, Barlaam Calabro. Opere contro i Latini. Introduzione, storia dei testi, edizione critica, traduzione e indici (*StT* 347). I–II. Città del Vaticano 1998; FYRIGOS, Dalla controversia palamitica.

a treatise of number theory, in six books (*Logistikē*); two short texts in which he computed the circumstances of the solar eclipses of 1333 and 1337 (*De eclipsi* I and II); a pamphlet on the determination of the date of Easter (*De paschate*); a number-theoretical rewriting of the first ten propositions of Book II of Euclid's *Elements* (*Demonstratio*); a tract in which he showed that the last three chapters of Ptolemy's *Harmonica* as they are handed down by a part of the manuscript tradition cannot be authentic (*Refutatio*). Barlaam was a scientific polemicist: *De eclipsi* I and II, *De paschate*, and *Refutatio* have Nikephoros Gregoras as their polemical target[4]; the *Demonstratio* covertly criticises George Pachymeres' approach to the same subject-matter.

The present article contains a critical edition of the *Refutatio* accompanied by an annotated translation. Until now, this work could be read only in the text of Franz's 1840 edition, which was reprinted with corrections in Düring's 1930 edition of Ptolemy's *Harmonica*[5]. The present edition is preceded by four sections, whose contents are as follows: an outline of the background of the *Refutatio*; an analysis of the structure and of the style of the treatise; a detailed description of its manuscript witnesses; and a discussion of the stemmatic relations between these manuscripts. An iconographic complement reproduces the beginning of the *Refutatio* in the prototype of the tradition, and the diagram associated with the treatise in several witnesses.

## A BACKGROUND TO THE *REFUTATIO*

A division of the monochord, or canonic division, is a rational method for locating the notes of specific harmonic intervals. Several procedures were put forward in Greek antiquity to locate the notes; individual authors and harmonic schools differed as to the rational tools to be used to carry out the division and as to the exact location of the notes (*phthongoi*)[6]. If we use numerical ratios to represent musical intervals and focus on the interval of one octave, this is compounded of a fifth and a fourth: $^{2}/_{1} = (^{3}/_{2})(^{4}/_{3})$; the compounding intervals differ by an epogdoic tone: $(^{3}/_{2}):(^{4}/_{3}) = {}^{9}/_{8}$. Thus, an octave is compounded of two intervals of a fourth "disjoined" by a tone: $^{2}/_{1} = (^{4}/_{3})(^{9}/_{8})(^{4}/_{3})$. These three "conjoined"—that is, sharing a bounding note—intervals are bounded by four notes: these are the "standing" (*hestōtes*) notes in an octave. Each of the intervals of a fourth within an octave was called a "tetrachord". One of the essential tasks of Greek harmonic theorists was to

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[3] See the following editions: P. CARELOS, Barlaam tou Kalabrou. Logistikē. Barlaam von Seminara, Logistiké (*Corpus philosophorum Medii Ævi. Philosophi byzantini* 8). Athens – Paris – Bruxelles 1996 (*Logistikē*); J. MOGENET – A. TIHON – D. DONNET, Barlaam de Seminara, Traités sur les éclipses de soleil de 1333 et 1337. Louvain 1977 (*De eclipsi* I and II); A. TIHON, Barlaam de Seminara. Traité sur la date de Pâques. *Byz* 81 (2011) 362–411 (*De paschate*); F. ACERBI, Barlaam's Paraphrase of Euclid, *Elements* II.1–10. A Critical Edition. *JOeB* 72 (2022) 1–62 (*Demonstratio*), which also contains summaries of all these works. On the grounds of allusions in some of Palamas' writings, Barlaam's scientific activity can be limited to *ca*. 1330–37, see FYRIGOS, Dalla controversia palamitica 170–172.

[4] Assessments of the Gregoras-Barlaam controversy are found in MOGENET – TIHON – DONNET, Barlaam 150–157 and in TIHON, Barlaam, which also contains a detailed description of Gregoras' contribution to Easter Computi. Gregoras' computation of the solar eclipse of July 1330 is edited in J. MOGENET – A. TIHON – R. ROYEZ – A. BERG, Nicéphore Grégoras, Calcul de l'éclipse de soleil du 16 juillet 1330 (*Corpus des Astronomes Byzantins* 1). Amsterdam 1983. Gregoras wrote three pamphlets against Barlaam: possibly the *Antilogia*, certainly the *Philomathēs* and the *Phlorentios*: see the editions in P. L. M. LEONE, Nicephori Gregorae «Antilogia» et «Solutiones Quaestionum». *Byz* 40 (1970) 471–516; P. L. M. LEONE, Il Philomathēs ē Peri hybristōn di Niceforo Gregora. *RSBN* 8–9 (1971–72) 171–201; P. L. M. LEONE, Niceforo Gregora, Fiorenzo o Intorno alla sapienza (*Byzantina et Neo-Hellenica Neapolitana* 4). Napoli 1975.

[5] See J. FRANZ, De musicis graecis commentatio. Berolini 1840, 14–23 (based on the manuscript Napoli, Biblioteca Nazionale Centrale Vittorio Emanuele III, III.C.3 [1330–50; *Diktyon* 46279]), reprinted in I. DÜRING, Die Harmonielehre des Klaudios Ptolemaios. Göteborg 1930, 112–121 with corrections coming from manuscripts that also contain Ptolemy's *Harmonica*, among which our witnesses $P^1$ and V.

[6] On canonic division, the genera, the "systems", and, more generally, on ancient Greek harmonic theory see A. BARKER, Three Approaches to Canonic Division. *Apeiron* 24.4 (1991) 49–83; A. BARKER, The Science of Harmonics in Classical Greece. Cambridge 2007; D. E. CREESE, The Monochord in Ancient Greek Harmonic Science. Cambridge 2010; S. HAGEL, Ancient Greek Music. A New Technical History. Cambridge 2010.

locate two more notes within each tetrachord; these are the "movable" (*kinoumenoi*) notes, whose different positions produce the standard genera (enharmonic, chromatic, and diatonic) and their shades. Traditionally—and very much as the fixed notes in the construction just outlined, which are pairwise set one fifth apart—the movable notes within the two disjoined tetrachords that make an octave were placed one fifth apart; thus, only the movable notes within either of the tetrachords had to be located. The outcome of this procedure is a canonic division over the span of one octave; the structure can be enlarged by suitably joining other tetrachords and single notes to this core octave.

The so-called "(greater) perfect system"[7] spans two octaves, that is—as $^4/_1 = (^2/_1)(^2/_1)$—an interval with which a quadruple ratio is associated. The numerical ratios associated with the tetrachords and to the epogdoic tones that compound the double octave of the perfect system are arranged as follows: $(^4/_3)(^4/_3)(^9/_8)(^4/_3)(^4/_3)(^9/_8)$[8]. These six conjoined intervals are bounded by seven standing notes. These notes are named by means of a genus-plus-species denomination: the generic name of the tetrachord, in the genitive plural, qualifies the specific name of the note. The names of the tetrachords are, from highest to lowest, *hyperbolaiōn*, *diezeugmenōn*, *mesōn*, *hypatōn*. Accordingly, and again from highest to lowest, the names of the seven standing notes of the perfect system are *nētē hyperbolaiōn*, *nētē diezeugmenōn*, *paramesē*, *mesē*, *hypatē mesōn*, *hypatē hypatōn*, *proslambanomenos*. The lowest note does not belong to any tetrachord; it was "added" (hence its name) one epogdoic tone below the *hypatē hypatōn* in order to complete the span of a double octave.

The most accomplished Greek technical treatise of harmonic theory is Ptolemy's *Harmonica*[9], in three Books[10]. Book I first expounds introductory matters on methods and aims of harmonics, then goes deeper into notes, concords and associated intervals and ratios, and into the divisions of the genera and of the tetrachords. In this Book, Ptolemy scrutinises the approaches of the Pythagoreans and of Aristoxenus and his school. Book II begins by treating again genera; it passes then to systems, and to clarifying the notions of "form" (*eidos*) of a concord, and of "position" (*thesis*) and "function" (*dynamis*) of a note; chapters II 7–11 investigate into *tonoi* and their modulations; chapters II 12–13 expound specific issues of canonic division; chapters II 14 and 15 set out the divisions of the octave in the three genera according to several harmonic theorists, and the divisions of the genera in each *tonos*, respectively. *Harm*. III 1–2 show how to realise the double octave of the perfect system on a *kanōn*.

*Harm*. III 3–13 are of a radically different nature[11]. They hardly include any technical argument, but outline—on the grounds of their sharing a mathematical structure (the *harmonia*) which gives an appropriate form to the underlying matter—a comparison between harmonic items, features of the human soul, and relative positions and motions of the heavenly bodies (the Sun, the Moon, and the planets). *Harm*. III 3–4 set out the general coordinates of this comparison. As for the human soul, *Harm*. III 5–7 set a parallel between harmonic concords, genera, modulations of the *tonoi*, and primary divisions of the soul, the forms of knowledge, and the changing attitudes of the soul in

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[7] In general, a "system" is a harmonic structure compounded of at least two intervals. All Greek harmonic theorists define a system in this way, see for instance Aristoxenus, *El. harm*. I, 21, 6–7 Da Rios.

[8] The core octave of the perfect system is the one associated with the compounded ratio $(^4/_3)(^9/_8)(^4/_3)$.

[9] The best introduction to Ptolemy (fl. AD 127–160) and his works is G. TOOMER, Ptolemy, in: Dictionary of Scientific Biography, ed. Ch. C. Gillispie. I–XVII. New York 1970 XI 186–206. The reference edition of Ptolemy's *Harmonica* is DÜRING, Die Harmonielehre; see also the emendations proposed in B. ALEXANDERSON, Textual Remarks on Ptolemy's Harmonica and Porphyry's Commentary (*Studia Graeca et Latina Gothoburgensia* XXVII). Göteborg 1969. The reference translation is A. BARKER, Greek Musical Writings. I–II. Cambridge 1984–89 II 275–391; see also J. SOLOMON, Ptolemy, *Harmonics*. Translation and Commentary. Leiden – Boston – Köln 2000. A comprehensive study of the *Harmonica* is A. BARKER, Scientific Method in Ptolemy's *Harmonics*. Cambridge 2000.

[10] Very detailed descriptions of the contents of Ptolemy's *Harmonica* are in T. J. MATHIESEN, Apollos' Lyre. Greek Music and Music Theory in Antiquity and the Middle Ages. Lincoln (NE) and London 1999, 434–494, and in CREESE, The Monochord 283–355. A useful overview of the *Harmonica* is in BARKER, Greek Musical Writings II 270–275.

[11] Read the bewilderment at Ptolemy's arguments in *Harm*. III 5–13 in BARKER, Scientific Method 268–269.

specific moments of life, respectively. As for heavenly matters, *Harm*. III 8 shows that the (periodic) structure of the perfect system and the circle of the zodiac can be most naturally mapped into one another; III 9 maps concords and discords into the astrological aspects (this section is quite technical); III 10–12 set a parallel between the sequence of the notes, the genera, the modulations of the *tonoi*, and the motions of the heavenly bodies in length (the diurnal motion), in depth (the anomalistic motion), and in breadth (the motion along the zodiac)[12], respectively; III 13 expounds a correspondence between the tetrachords and the tones in the perfect system and the "phases" of the heavenly bodies with respect to the Sun.

The last three chapters of Ptolemy's *Harmonica* are missing in the manuscript tradition; their presence in Ptolemy's original plan is proved by the fact that their titles are independently preserved in the *pinakes* of the treatise. These titles are as follows: III 14, κατὰ τίνας ἂν πρώτους ἀριθμοὺς παραβληθεῖεν οἱ τοῦ τελείου συστήματος ἑστῶτες φθόγγοι ταῖς πρώταις τῶν ἐν τῷ κόσμῳ σφαί-ραις "According to which first numbers the standing notes of the perfect system ought to be mapped into the first heavenly spheres"; III 15, πῶς ἂν λαμβάνοιντο διὰ τῶν ἀριθμῶν οἱ τῶν οἰκείων κινήσεων λόγοι "How the ratios of the motions proper <to each sphere> ought to be found in terms of numbers" ; III 16, πῶς ἂν αἱ τῶν πλανωμένων συνοικειώσεις παραβάλλοιντο ταῖς τῶν φθόγγων "How the affinities of the planets ought to be mapped into those of the notes".

The missing chapters *Harm*. III 14–15 were restored by Nikephoros Gregoras, and added to his own recension of the *Harmonica*. As for chapter III 16, its origin is unclear; its text is witnessed in some manuscripts only. In these manuscripts, what Byzantine scholars came to identify with chapter III 16 is included, obviously out of place, in III 9[13]. It is a safe guess—Barlaam is explicit about this—that Gregoras himself extracted "*Harm*. III 16" from *Harm*. III 9 and made it the last missing chapter. That chapters 14 and 15 were added by Gregoras is borne out by partly autograph scholia in the manuscript Paris, Bibliothèque nationale de France, Coislin 173 (*ca*. 1334; *Diktyon* 49312), see in particular ff. 32r and 108r[14]. Par. Coislin 173, which was annotated and amply glossed by Gregoras himself, is also the oldest witness—and the only independent one—of Gregoras' recension of the *Harmonica*, and *a fortiori* of *Harm.* III 14–15 too. The oldest witness—and the only independent one—of the text of "*Harm*. III 16" as located in the middle of III 9 is the manuscript München, Bayerische Staatsbibliothek, Cod. graec. 361a (end of 13th century; *Diktyon* 72898)[15], f. 40v. The *Harmonica* in this manuscript was also annotated by Gregoras; "*Harm*. III 16" was here collated by Gregoras' pupil Philotheos of Selymbria against a witness carrying Gregoras' recension[16]. An omission in the text of "*Harm*. III 16" witnessed by Monac. gr. 361a and a variant reading in its title suggest that Gregoras could not have used this manuscript as his model for the text he

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[12] The Greek terms are κατὰ μῆκος (*kata mēkos*, transl. "in length"), κατὰ βάθος (*kata bathos*, transl. "in depth"), and κατὰ πλάτος (*kata platos*, transl. "in breadth"). We adopt the translations in N. M. SWERDLOW, Ptolemy's *Harmonics* and the 'Tones of the Universe' in the *Canobic Inscription*, in: Studies in the History of the Exact Sciences in Honour of David Pingree, ed. Ch. Burnett – J. P. Hogendijk – K. Plofker – M. Yano. Leiden 2004, 137–180: 158–160.

[13] Of course, one must disregard the manuscripts in which *Harm*. III 14–16 are witnessed by contamination with Gregoras' recension. One such manuscript is mentioned just below.

[14] See DÜRING, Die Harmonielehre LXXVIII–LXXXVIII for this and other documents. Gregoras speaks of his recension of Ptolemy's *Harmonica* in a letter to Michael Kaloeidas: P. L. M. LEONE, *Nicephori Gregorae epistulae*. I–II. Matino 1982 II 301.95–302.107. The date given for Par. Coislin 173 coincides with the date of one of the watermarks; the letter to Kaloeidas is posterior to AD 1331/2.

[15] On Monac. gr. 361a, see ACERBI – GIOFFREDA, Manoscritti scientifici, *passim*; F. ACERBI – A. GIOFFREDA, Harmonica Membra Disjecta. *GRBS* 59 (2019) 646–662, and in particular 659–661. Monac. gr. 361a was once one and the same manuscript with the oldest portion of the manuscript Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 2338 (end of 13th century; *Diktyon* 68969). For the late history of this manuscript, see L. CALVIÉ, Un manuscrit médiéval d'anciens musicographes grecs: le Vaticano, BAV, gr. 2338. *Script* 74 (2020) 219–250.

[16] On f. 42r–v, Philotheos of Selymbria added *Harm*. III 11–15 (numbered as III 12–16) taking them from Gregoras' recension, but as for III 16 he referred to the text on f. 40r.

relocated as *Harm*. III 16. Accordingly, Düring makes Par. Coislin 173 depend on a hyparchetype to which Monac. gr. 361a is also subordinated.

The contents of the added chapters can be summarised as follows[17].

*Harm*. III 14. The zodiac is divided into 360 degrees; the astrological aspects with respect to the Sun are associated with its subdivisions (opposition to 180 degrees, trine to 120 degrees, quartile to 90, sextile to 60). The perfect system is mapped into these items, as follows: *proslambanomenos* is mapped into the aspect (*stasis*) at 180 degrees and hence into opposition, *hypatē mesōn* into the aspect at 120 degrees and hence into trine, *nētē diezeugmenōn* into the aspect at 90 degrees and hence into quartile, *nētē hyperbolaiōn* into the aspect at 60 degrees and hence into sextile. The two standing notes that bound the disjunctive tone are mapped into the point from which the degrees are measured.

*Harm*. III 15. The sesquialter, sesquitertian, double, and quadruple ratios show up both in the numerical values associated with aspects (with 90 acting as a mean, these values are 90 to 60, 120 to 90, which make the double ratio once compounded, and 360 to 90) and in concords (a fifth, a fourth, which make the homophone of an octave once compounded, and a double octave). Alternatively, 120, 90, and 60 degrees reduce to 4, 3, and 2 if measured in zodiacal signs; these terms, 3 acting as a mean, make the sesquitertian, sesquialter, and double ratios. In its turn, the number of zodiacal signs, namely, 12, makes with 3 a quadruple ratio, which is also associated with a double octave. Alternatively, one might set out the same mapping by using the angles of the polygons associated with the aspects, but a fuller account of this is left to another occasion[18].

*Harm*. III 16. We shall not describe the astrological folklore expounded in this chapter; we only give the mapping between the planets and some specific notes: Saturn to *nētē hyperbolaiōn*, Jupiter to *nētē diezeugmenōn*, Mars to *nētē synēmmenōn*[19], Sun to *paramesē*, Venus to *mesē*, Moon to *hypatē mesōn*.

Barlaam's *Refutatio* is an *anaskeuē* "refutation" of *Harm.* III 14–16 insofar as Barlaam did not set out to prove that these chapters are not Ptolemy's—for he is explicit about this in the very first lines of his tract, while not mentioning Gregoras (henceforth "the opponent", in this context)—but that they *could not* be Ptolemy's because of their shortcomings.

In the prototype of the manuscript tradition—which does not contain Ptolemy's *Harmonica*—the *Refutatio* is preceded by the added chapters *Harm*. III 14–16. This is a constitutive feature of the *Refutatio*, for Barlaam refers to these chapters as preceding his own text. He also states: "in the oldest copies we have found [the last chapter] as part of the text somewhere in the middle of the third <book> of the harmonics", which is exactly what one finds in the family whose prototype is Monac. gr. 361a. A collation of the text of *Harm*. III 14–16 in the prototype of the *Refutatio* (see the Appendix) exhibits the same variant readings as Monac. gr. 361a. As Gregoras' and Philotheos of Selymbria's interventions in this manuscript show that the codex was available in highbrow milieux in Constantinople exactly when Barlaam composed his *Refutatio*, we may apply a principle of economy and suppose that Barlaam has had access to this very manuscript too.

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[17] A thorough discussion of the astronomical underpinnings of Book III of Ptolemy's *Harmonica* is found in SWERDLOW, Ptolemy's *Harmonics* 151–165, and see 165–176 for *Harm*. III 14–16. That part of the contents of the lost *Harm*. III 14 can be recovered from some of the Byzantine musical excerpts known as *Excerpta Neapolitana*, which in their turn nearly coincide with the relevant part of Ptolemy's *Inscriptio Canobi*, is just Swerdlow's conjecture. As we shall explain in a moment, "*Harm*. III 16" does expound a mapping between notes and planets that (after a plausible integration) coincides with part of the one set out in the *Inscriptio Canobi* itself (see below), but we do not see how the scenario can be ruled out in which the connection between "*Harm*. III 16", the *Excerpta Neapolitana*, and the *Inscriptio Canobi* goes the other way around (see also below). This makes the flimsy ground on which Swerdlow's conjecture rests evaporate.

[18] This is trivial, since these angles subtend the arcs resulting from the division of the zodiac into 360 parts.

[19] This note belongs to the so-called "lesser perfect system"; see for instance BARKER, Three Approaches 58 and 60–61, and CREESE, The Monochord 21.

Barlaam's *Refutatio* settled the issue of authenticity of *Harm.* III 14–16 once and for all. This was already recognised by Johannes Kepler in his annotated translation of *Harm.* III 3–16[20]. Kepler also elaborated a reconstruction of *Harm.* III 14–16.

## STRUCTURE AND STYLE OF THE *REFUTATIO*

Barlaam states six requirements that should be fulfilled if the added chapters are to be regarded as authentic, and shows that none of these requirements is met. The six requirements are: first, the added chapters should not repeat what Ptolemy has expounded in previous chapters; second, the added chapters should contain particular, and not general, arguments, for, at the end of III 13, Ptolemy states that this is exactly what he will do in the subsequent chapters; third, the mappings set out in the added chapters should be in agreement with the observative data available in Ptolemy's times; fourth, these mappings should be "fitting" (*oikeioi*), that is, they should not map dissimilar items into one another; fifth, the language should be correct and it should fit the subject-matter; sixth, the contents of the added chapters should fit their titles.

The six requirements are discussed unevenly. After their presentation at lines 15–41 (they are summarised again in the wind-up at lines 314–321), the first is allotted lines 42–95; the second, lines 90–95; the third, 96–113; the fourth, 113–159; the fifth, 160–194; the sixth, which includes a clarification of the meaning of "prime number" (lines 197–211) and a geometric argument (259–283) about the straight line being the shortest path between two points, 195–297. Contrary to Barlaam's initial claim, *Harm*. III 16 is treated separately, in a dozen lines (298–308) and by means of an argument which does not correspond to any of the six requirements (the other argument is about the contents not fitting the title). This is followed by a new attack on the opponent, about his making III 16 preceded by unfitting material (lines 309–313). The *Refutatio* is opened by a statement of its aims (3–9) and by a quick overview of the status of the added chapters in the manuscript tradition (9–14); it is closed by the beginning of Barlaam's reconstruction of the missing chapters (322–329). The *Refutatio* breaks up in the middle of a sentence, but we cannot exclude that Barlaam deliberately left his tract incomplete, maybe as an allusion to the incompleteness of the *Harmonica*.

Barlaam's tract is a compendium of refutation strategies. Some of his refutations are very subtle, some are straightforward arguments, some are weak, some prove too much, some are tendentious or squarely against the rules of the refutation game. We give a summary in what follows, and provide detail in the footnotes to the translation. There is a gross mistake in the mapping the opponent sets out in *Harm*. III 14–15, for this mapping associates relations (the aspects) to terms (the notes)—which are logically and factually inhomogeneous items (that this is the case is proved at lines 113–130 of the *Refutatio*)[21]—and it does that in a way that is patently at variance with Ptolemy's similar mapping in *Harm*. III 9. Pointing out such an error would allow the autenticity's case for chapters III 14–15 to settle rapidly. Barlaam wants to do more: he wants to show that his opponent argues inconsistently and that he is unable to master basic technical terms of the three disciplines involved, namely, harmonics, arithmetic, and astronomy. For this reason, Barlaam fires on his target from several positions, and using different dialectical weapons.

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[20] See Ch. FRISCH, Joannis Kepleri astronomi opera omnia. V. Francofurti a. M. et Erlangae 1864, 392 Nota I. Kepler asserts that he owned a transcription of the *Refutatio*, *ex dono J. G. Herwarti*. This manuscript is lost. On Kepler's interest in harmonics see B. STEPHENSON, The Music of the Heavens. Kepler's Harmonic Astronomy. Princeton 1994, in particular ch. VII. Düring's attempts at showing that "*Harm*. III 16" was authored by Ptolemy (DÜRING, Die Harmonielehre LXXXIV–LXXXVIII) are unconvincing: "*Harm*. III 16" expounds standard astrological lore and must be a scholium—possibly linked with the *Excerpta Neapolitana*—that found its way into the text.

[21] The opponent considers only planetary aspects with respect to the Sun, and keeps the position of the Sun fixed. However, and as Barlaam points out, an aspect is a relation in its essence, and cannot be reduced to the position of only one of the two heavenly bodies involved in this relation.

The fifth requirement, about lexicon and phrasing, is not met by the opponent's use of "music" instead of "harmonics", "arithmetical" as referred to heavenly matters, "proportional" as referred to ratios, and by his conflating a place and the fact that something lies in that place. The counterargument about prime numbers (197–211), which Barlaam categorises under the sixth requirement, also amounts to showing that the opponent did not know the meaning of "prime" as predicated of numbers. The same can be said of the counterargument about the "first spheres", that the opponent incorrectly identifies with the heavenly bodies they carry (lines 247–256). The lexical section also contains a kind of refutation that will be met elsewhere: the opponent overspecifies his arguments, which are thereby flawed. The point here is that the opponent refers to the "perfect system" but he maps the heavenly items only into the standing notes of this system, or just into its bounding notes (which would require using simply "double octave"). Other examples of the opponent's overshooting, that is, regarding specific features as necessary while they are not, are pointed out by Barlaam. One such feature is dividing the zodiac into 360 parts, a move that Barlaam repeatedly attacks as unnecessary and hence fallacious (see lines 56–73, 73–78, 203–209, and 235–246).

The opponent is also charged with establishing his mapping without supporting it by means of observations (96–102) or of pieces of evidence (103–113), with using "the two standing notes that contain the disjunctive tone […] as one, even if they are two and differentiated as to function" (151–153), and with incorrectly measuring distances along arcs and not by means of straight lines (256–293).

As for the logic of his counterarguments, Barlaam shows that his opponent's choice of the sets to be mapped into one another (namely, notes and aspects) is incorrect, first by arguing that the two sets do not share any essential property (113–130), and then by proving that the assumption that a mapping between them obtains entails either an impossible conclusion (lines 130–134), or two conclusions that are logically incompatible (134–139, 139–151 and 239–243), or a conclusion that, while possibly being true, contradicts the intended assumption (151–159). The latter argument—a little logical masterpiece indeed—is what the ancient sceptical tradition called *peritropē*.

Barlaam's terminology and argumentative patterns show that he was perfectly at ease with number theory, harmonics, geometry, and, most notably, with Aristotelian doctrines and with dialectical techniques[22]. The *Refutatio* sets out the arguments at a very fast pace, enlivening them by steep rhetorical *climax* (Barlaam spends four rhetorical questions, three of which in the core section at lines 113–130), and bristles with lexical resonances and wordplays. Sometimes, Barlaam slightly cheats, which is part of the rules of the game after all.

## MANUSCRIPT WITNESSES OF THE *REFUTATIO*

The manuscript witnesses of Barlaam's *Refutatio* are listed below; they are arranged in alphabetical order of the names of the towns where the several libraries are located, which coincides with an increasing *Diktyon* number; most of the *sigla* are standard in the editions of Barlaam's scientific treatises. We describe in detail such family relations among manuscript witnesses as can be established on the basis of the contents of the manuscripts and on the stemmas established in critical editions of treatises other than the *Refutatio*[23]. It is not difficult to see that most family relations of

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[22] One must not forget that the Palamite controversy began with Palamas and Barlaam disagreeing about what kind of Aristotelian syllogism allows us to know God's attributes. On this issue, see K. Ierodiakonou, The Anti-Logical Movement in the Fourteenth Century, in: Byzantine Philosophy and its Ancient Sources, ed. K. Ierodiakonou. Oxford 2002, 219–236, in particular 225–236.

[23] Editions are cited on the first occurrence of a treatise in the whole list. For descriptions of some of these manuscripts in the editions of Barlaam's scientific treatises other that the *Refutatio*, and for their stemmatic relationships, a reference to the relevant pages of Carelos, Barlaam (*Logistikē*), Mogenet – Tihon – Donnet, Barlaam (*De eclipsi* I and II), Tihon, Barlaam (*De paschate*), and Acerbi, Barlaam's Paraphrase (*Demonstratio*) is understood. We use the abbreviations and num-

the tradition of Barlaam's *Refutatio* can be anticipated in this way. This is one of the reasons why the descriptions of the manuscript witnesses are very detailed.

L[1]. Leiden, Universiteitsbibliotheek, B.P.G. 16D (*Diktyon* 37637), 1651, copyist <Gerard Langbaine the Elder>[24]. The *Refutatio*, without a title, is on ff. 103v–111r (*des*. 246 ἐκ τῶν αὐτῶν τἀναντία, see the edition below); it is preceded on f. 103r by *Harm*. III 16 and, on f. 103v, by a scholium and a blank space of 6 lines[25]. Other works in the manuscript: ff. 3r–101v Porphyry, *In Ptolemaei Harmonica I–II 7*[26]; 102r *diagramma harmonicum*.

This manuscript is one of the outcomes of a copying campaign of musical treatises coordinated in Oxford by John Selden on behalf of Marcus Meibom (see the manuscript Utrecht, Universiteitsbibliotheek 1.A.1 [Tiele 11; *Diktyon* 64497] for Ptolemy's *Harmonica*; the apograph containing Manuel Bryennios' *Harmonica* is apparently lost[27]). The copies were collated with other Oxford witnesses, and partly carried out personally, by Gerard Langbaine (notes on f. 1r; the second note is in the hand of John Selden). Meibom immediately drew apographs of these exemplars: these are the manuscripts Utrecht, Universiteitsbibliotheek 1.A.2 (Tiele 12; *Diktyon* 64498) for Ptolemy's *Harmonica*; Leid. Periz. Q° 22 (our witness $L^2$) for Porphyry's commentary thereon; Leiden, Universiteitsbibliotheek, B.P.G. 16E (*Diktyon* 37638) for Manuel Bryennios' *Harmonica*. As stated on f. 1r of $L^1$, its model for Porphyry was the manuscript Oxford, Magdalen College, gr. 13 (composite; 2nd half of 16th century; copyists Nikolaos Tourrianos, ff. 1r–138r, and Sophianos Melissenos[28], ff. 143r–273r; *Diktyon* 48706), collated with two Bodleian witnesses (these are listed in De Meyier's catalogue). From f. 103r on, the antigraph of $L^1$ was the manuscript Oxon. New College 299 (our witness $O^2$), as stated in a note located on f. 103r, before the Greek text: *Libru(m) 3tium et ultimum Harmonicorum Claudij Ptolomaei (in Codice MS° in Collegio quod Oxonij vulgò Novum audit) excipiunt haec parabysmata*. The model was already incomplete, as confirmed by a note appended to the Greek text on f. 111r: *Desiderantur reliqua ob folium extremum excisum* […], with a final reference to a peculiarity of the model: […] *tanta est in MS° Codice et chartae*

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berings in SINKEWICZ, The Solutions 185–194 for Barlaam's non-scientific works: Or. = Greek discourses; AL = Antilatin treatises; EG = Greek Letters.

[24] Langbaine copied only ff. 1r, 95r, 103r–111r. Two other hands penned ff. 3r–18v, 47r–94v, 96r–102r, and ff. 19r–46v, respectively; John Selden added a note on f. 1r. See K. A. DE MEYIER, Codices Bibliothecae Publicae graeci (*Bibliotheca Universitatis Leidensis. Codices Manuscripti* VIII). Lugduni Batavorum 1965, 18–19; T. J. MATHIESEN, Ancient Greek Music Theory. A Catalogue Raisonné of Manuscripts. München 1988 no. 276. This manuscript was originally paginated from f. 3r on; page numbers are absent from f. 102v (blank).

[25] The scholium, a citation from Diodorus Siculus I.16.1 autograph of Nikephoros Gregoras in Par. Coislin 173, is edited in DÜRING, Die Harmonielehre XCIX–C and in F. ACERBI, I problemi aritmetici attribuiti a Demetrio Cidone e Isacco Argiro. *Estudios Bizantinos* 5 (2017) 131–206 Testo 6, second transcript. The text, which in this position can be traced back at least to the model of the manuscript Oxon. New College 299 (our witness $O^2$)—see below—reads here as follows: φησὶν ὁ Διόδωρος ὁ Σικελιώτης ὅτι Αἰγύπτιος ὁ Ἑρμῆς ἐποίησε πρῶτος λύραν τετράχορδον (τρίχορδον 173) μιμησάμενος τὰς κατ' ἐνιαυτὸν ὥρας· τρεῖς γὰρ ὑπεστήσατο φθόγγους, ὀξύν, βαρὺν καὶ μέσον, ὀξὺν μὲν ἀπὸ τοῦ θέρους βαρὺν δὲ ἀπὸ τοῦ χειμῶνος μέσον δὲ ἀπὸ τοῦ ἔαρος καὶ τοῦ φθινοπώρου "Diodoros the Sicilian asserts that Hermes the Egyptian first made a four-chord lyra, by imitation of the yearly seasons: for he conceived three kinds of sound, high, low, and intermediate, high from Summer, low from Winter, intermediate from Spring and Autumn".

[26] The reference edition of Porphyry is I. DÜRING, Porphyrios Kommentar zur Harmonielehre des Ptolemaios (*Göteborgs Högskolas Årsskrift* 38). Göteborg 1932. The recent edition M. RAFFA, Porphyrius, Commentarius in Claudii Ptolemaei Harmonica. Berlin–Boston 2016 depends on Düring's as for the textual tradition.

[27] On Meibom and Bryennios see most recently A. WEDDIGEN, Marcus Meiboms Briefe an Marquard Gude und andere. Studie zu einem gescheiterten Editionsversuch der *Harmonica* des Manuel Bryennios. *Wolfenbütteler Renaissance-Mitteilungen* 39 (2018) 29–66.

[28] For these two copyists, see *RGK* I 319, II 438, III 520, and G. DE ANDRÉS, El cretense Nicolás de la Torre, copista griego de Felipe II. Biografía, Documentos, Copias, Facsímiles. Madrid 1969; *RGK* I 362 and P. CAROLLA, A proposito di stemmi multipli: Andrea Darmario e i suoi collaboratori. Con 22 nuovi manoscritti di Sofiano Melisseno tra Parigi e Lisbona, in: Griechisch-byzantinische Handschriftenforschung, hrsg. von C. Brockmann – D. Deckers – D. Harlfinger – S. Valente. Berlin – Boston 2020, 381–394: 382–388, respectively.

*amplitudo, et characterum exilitas*[29]. A letter by R. W. Hunt to De Meyier, dated 15 March 1955, is inserted between ff. I and II of $L^1$; the letter confirms that the relevant quire of Oxon. New College 299 had been manipulated (see also *infra*): "The 10th. quire, which begins on fol. 68, originally had 8 leaves. The last 5 are now missing, of which the first has been torn out and the remaining 4 cut out and the stubs oversewn together". As for Porphyry's commentary, Düring, almost certainly without collating the manuscripts, confirms the family relations just outlined: DÜRING, Porphyrios Kommentar X–XI, XXIV–XXVI, XXIX (stemma); he also shows that the manuscript Oxford, Magdalen College, gr. 13 is a copy of the very prolific manuscript Venezia, Biblioteca Nazionale Marciana, gr. Z. 322 (coll. 711; 1468–72; copyist <Iohannes Rhosos>; *Diktyon* 69628).

$L^2$. Leiden, Universiteitsbibliotheek, Periz. Q° 22 (*Diktyon* 37873), *ca*. 1651, copyist <Marcus Meibom>[30]. The *Refutatio*, without a title, is on ff. 126v–132v (*des*. 246 ἐκ τῶν αὐτῶν τἀναντία); it is preceded on f. 126r by *Harm*. III 16 and by the same scholium as in our witness $L^1$. Other works in the manuscript: ff. 2r–124v Porphyry, *In Ptolemaei Harmonica I–II 7*; 125r–v *diagrammata harmonica*[31].

This is Meibom's conformal apograph of Leid. B.P.G. 16D (our witness $L^1$); he also copied Langbaine's working notes and *marginalia* to the *Refutatio*. Meibom wrote the following note by the side of the first line of the *Refutatio*, which begins on a new page: *inter haec et praecedentia spatium 6 vers. erat relictu(m)*.

A. Milano, Biblioteca Ambrosiana, E 76 sup. (gr. 292; *Diktyon* 42700)[32], composite, *ca*. 1340, Barlaam's copyist III[33]. The *Refutatio*, with the standard title, is on ff. 180r–190r; it is preceded on ff. 178v–180r, headed τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics", by *Harm*. III 14–16. Other works in the manuscript: ff. 1r–107v Ptolemy, *Harmonica* I–III 16[34]; 108r–110v *varia mathematica*; 111r–172r Barlaam, *Logistikē*

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[29] To give an idea of the *exilitas* of the script of Oxon. New College 299, Aristotle's *Organon* takes 43 folios only.

[30] See K. A. DE MEYIER, Codices Perizoniani (*Bibliotheca Universitatis Leidensis. Codices Manuscripti* IV). Lugduni Batavorum 1946, 76; MATHIESEN, Ancient Greek Music Theory no. 283. This manuscript was originally paginated from f. 2r on; page numbers begin again with 1 on f. 126r. Marcus Meibom (*ca*. 1630–1710) is an interesting yet little-studied figure of *clericus vagans*. The manuscripts he copied are found in Leiden (these were bought in bulk in 1752) and in Utrecht (the town where he died); see DE MEYIER, Codices Bibliothecae Publicae graeci, X–XI and *ad locos*. These items are the manuscripts Leiden, Universiteitsbibliotheek, B.P.G. 2B, 16C, 16E, 16G–H*, 37A, f. 4–5, 67A (*Diktyon* 37626, 37636, 37638, 37640–42, 37668, 37701) and Periz. Q° 22 et Q° 39 (*Diktyon* 37873 and 37878; in the latter, the annotations only), and the manuscripts Utrecht, Universiteitsbibliotheek I.E.1, I.G.1, I.A.2, I.A.8–10, I.C.1 (Tiele 3, 5, 12–16; *Diktyon* 64520, 64528, 64498, 64502–4, 64507). A bio-bibliographic synthesis on Meibom can be read in R. HÖISTAD, Marcus Meibom and the Lost Codex Meibomianus. *Eranos* 83 (1985) 103–112: 103–105; detail on incidents that marked Meibom's life is given in J.-Ch.-F. HÖFER (éd.), Nouvelle Biographie Générale. Tome Trente-Quatrième. Paris 1861 cc. 764–766; see also R. J. LEONHARDT, Meibom, Marcus, in: *Geschichte der Altertumswissenschaften*. *Biographisches Lexicon*, hrsg. von P. Kuhlmann – H. Schneider (*Der neue Pauly, Supplemente, Band* 6). Stuttgart-Weimar 2012 cc. 803–804.

[31] The first diagram is identical to that on f. 102r of $L^1$; the second diagram reproduces the diagram, in Langbaine's hand, located on f. 95r of $L^1$.

[32] The two Milan manuscripts are described in E. MARTINI – D. BASSI, Catalogus codicum Graecorum Bibliothecae Ambrosianae. I–II. Milano 1906, 326–328 and 836–839, respectively; see also MATHIESEN, Ancient Greek Music Theory nos. 178 and 182. On Pinelli's library, see M. GRENDLER, A Greek Collection in Padua: The Library of Gian Vincenzo Pinelli (1535–1601). *Renaissance Quarterly* 33 (1980) 386–416 (407–408 on Manuel Moros, see below) and, more recently, A. M. RAUGEI, Gian Vincenzo Pinelli e la sua biblioteca (*Cahiers d'Humanisme et Renaissance* 151). Genève 2018.

[33] On this copyist, who penned ff. 111r–190r and 291r–305v, see A. GIOFFREDA, Su scrittura, libri e collaboratori di Barlaam calabro. *Segno e Testo* 14 (2016) 361–378. Folios 191r–290r of Ambr. E 76 sup. were copied by Barlaam's copyist II, who is also found in Marc. gr. Z. 332 (see below); other hands are engaged in the copy, for instance on ff. 108r–110v. Folios 2r–v, 8r–10v, 38v–40v are in the hand of Theodoros Rentios: S. MARTINELLI TEMPESTA, Per un repertorio dei copisti greci in Ambrosiana, in: Miscellanea Graecolatina I, a cura di F. Gallo. Roma 2013, 101–153: 140; see also *RGK* III 215 (a copy is subscribed in 1557).

[34] The edition of Ptolemy's *Harmonica* is DÜRING, Die Harmonielehre.

I–VI; 172r–178r Barlaam, *Demonstratio*; 191r–243r Barlaam, *De Spiritu Sancto adv. Latinos* (AL 1–6); 243r–247v Barlaam, *De principatu Papae* (AL 7); 247v–252r Barlaam, *Legatus seu de Spiritu Sancto* (AL 11); 252r–254r Barlaam, *Confutatio dogmatis Latinorum* (AL 10); 254v–267v Barlaam, *Solutiones ad Georgium Lapitham*; 268r–283v Barlaam, *Oratio de concordia* (Or. 1); 283v–290r Barlaam, *Oratio ad Synodum de unione* (Or. 2); 291r–294v Barlaam, *De eclipsi* I; 294v–300v Barlaam, *De eclipsi* II; 301r–v *scholium de cyclo lunari et computationes astronomicae*; 302r–305v, 290v (*alia manu*) Barlaam, *De paschate*.

Barlaam's works in this manuscript are penned by two of his collaborators. Contrary to what happened in Marc. gr. Z. 332 (our witness M), he did not revise his own texts, which exhibit typical copying mistakes. A is the sole independent witness for the *varia mathematica* it carries on ff. 108r–110v: ACERBI, Barlaam's Paraphrase 5, and Appendix 2 for the edition. A is a copy of the manuscript Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 185 (composite; 1330–50; *Diktyon* 66816), and a stemmatic sibling of Oxon. New College 299 (our witness $O^2$), as for Ptolemy's *Harmonica*: DÜRING, Die Harmonielehre LXV and LXIX (stemma). A is a direct copy of M as for the *Demonstratio*, the *Logistikē*, and *De eclipsi* I and II.

L. Milano, Biblioteca Ambrosiana, R 117 sup. (gr. 724; *Diktyon* 43201), composite, *ca*. 1565, copyist <Manuel Moros>[35]. The *Refutatio*, with the standard title, is on ff. 133r–140v; it is preceded on ff. 132r–133r by *Harm*. III 14–16, headed τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works in the manuscript: ff. 3r–47r Herennios, *in Aristotelis Metaphysica*; 52r–81v [Georgius Codinos], *De officiis*; 86r–127r Barlaam, *Logistikē* I–VI; 127r–131v Barlaam, *Demonstratio*; 140v–143v Barlaam, *De eclipsi* I; 143v–148r Barlaam, *De eclipsi* II; 148r–149r *scholium de cyclo lunari et computationes astronomicae*; 149v–152v Barlaam, *De paschate*; 152v–155r *varia mathematica*; 158r–188r Dionysius of Halicarnassus, *De Thucydide*; 191r–195r, 196r–200r, 203r–207r (triplex) *De comoedia*; 209r–225r, 247r–262v (duplex), 265r–266r Georgius Hamartolos, *Chronicon* (fragm.); 227r–235r Iohannes Scylitzes, *Synopsis Historica* (excerpts) *cum additamenta Theodori Scutariotae*; 237r–239v Theodoros Gaza, *Epistula de origine Turcarum*; 241r–v, 242r–v (duplex) Polyaenus, *Stratagemata* (excerpt); 243r–244v *apographum inscriptionis*; 269r–270v, 273r–274v (duplex) *Vita Aristotelis*; 277r–323v Origenes, *Contra Celsum cum Gennadii Scholarii scholia*.

This codex is a recent assemblage of parts of several manuscripts. The Barlaam block was entirely copied by Manuel Moros, one of Pinelli's favourite calligraphists, using Ambr. E 76 sup. as a model.

m. Moskva, Gosudarstvennyj Istoričeskij Musej, Sinod. gr. 315 (Vlad. 441; *Diktyon* 43940), 1590–1600, copyist <Maximus Margounios>[36]. The *Refutatio*, with the standard title, is on ff. 297v–303v;

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[35] On Moros, see *RGK* I 252 (which confirms the identification by Martini and Bassi), II 348, III 417; P. GÉHIN, Évagre le Pontique dans un recueil de mélanges grammaticaux du fonds Pinelli, l'Ambr. C 69 sup., in: Nuove ricerche sui manoscritti greci dell'Ambrosiana. Atti del Convegno Milano, 5–6 giugno 2003, a cura di C. M. Mazzucchi – C. Pasini. Milano, 2004, 265–313. The four hands engaged in the copy are distributed as follows: Camillo Zanetti on ff. 3–47, 191–241, 243–323; an anonymous hand supplies f. 242; Maximus Margounios on ff. 52–81 (†1602; *RGK* I 259 [which identifies the copyist], II 356, III 427; G. FEDALTO, Massimo Margunio e il suo commento al "De trinitate" di S. Agostino [1588]. Brescia 1967; F. CICCOLELLA, Maximos Margounios and Anacreontic Poetry: An Introductory Study, in: Greeks, Books and Libraries in Renaissance Venice, ed. R. M. Piccione [*Transmission* 1]. Berlin – Boston 2021, 147–160, with recent bibliography), who also copied the entire Mosq. Mus. Hist. gr. 315 (our witness m) and was engaged in the copy of Vat. gr. 1756 (our witness v); Manuel Moros on ff. 86–155 and 171–188. Folios 158–170 were excised from the Aldine no. 1559 of Dionysius of Halicarnassus.

[36] Margounios penned the entire manuscript with the exception of ff. 16r–26v, which contain Arethas' first pamphlet. The best description of this manuscript is found in L. G. WESTERINK, Arethae archiepiscopi Caesariensis scripta minora. I–II.

it is preceded on f. 297r–v by *Harm*. III 14–16, headed τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works in the manuscript: ff. 1–13v Leo medicus, *Conspectus medicinae*[37]; 16r–129r Arethas, *Opuscula*; 135r–159r Barlaam, *epistulae tres* (EG 1–3); 161r–251v Demetrios Cydones, *De processione Spiritus Sancti ad amicum*; 252r–255r Manuel Chrysoloras, *De processione Spiritus Sancti*[38]; 257r–290v Barlaam, *Logistikē* I–VI; 294r–296v Barlaam, *De paschate*; 290v–294r Barlaam, *Demonstratio*; 303v–307v Barlaam, *epistulae quinque* (EG 4–8); 307v–309r Barlaam, *De eclipsi* I; 309v–312v Barlaam, *De eclipsi* II; 313r *scholium de cyclo lunari et computationes astronomicae*; 317r–321v Nikephoros Gregoras, *excerptum e Byzantina Historia X 8*; 321v–327r Nikephoros Gregoras, *In annuntiationem Deiparae*; 327r–328r Nikephoros Gregoras, *excerpta*; 331r–334v Nikephoros Gregoras, *Epistula XVI*[39]; 334v–339v Nikephoros Gregoras, *Oratio in Deiparam*[40]; 341r–351v Nikephoros Gregoras, *Vita Iohannis episcopi Heracleensis*; 360r–366r Maximus Planudes, *De compassione*; 366v–401r Maximus Planudes, *Laudatio Sanct. Petri et Pauli*; 401v–419v Maximus Planudes, *In Sanctum Diomedem*; 420r–440r Maximus Planudes, *Basilikos logos*[41]; 440r–442v *excerpta e Planudis operis et epistulis*; 442v–443r Maximus Planudes, *Versus politici*.

Maximus Margounios collaborated with Alvise Lollino in the copy of Vat. gr. 1756 (our witness v). WESTERINK, Arethae archiepiscopi IX–XVIII shows that several works contained in Mosq. Mus. Hist. gr. 315 were copied from manuscripts held in Venice: Barlaam's works from Marc. gr. Z. 332 (our witness M); the two treatises on the procession of the Holy Spirit from the manuscript Venezia, Biblioteca Nazionale Marciana, gr. Z. 157 (coll. 399; 1442; *Diktyon* 69628); Gregoras' and Planudes' works from two manuscripts held in the library of the monastery of St Anton in Venice, destroyed by a fire in 1687.

N. Napoli, Biblioteca Nazionale Centrale Vittorio Emanuele III, III.C.3 (*Diktyon* 46279), once a unitary whole with the manuscript Napoli, Biblioteca Nazionale Centrale Vittorio Emanuele III, III.C.4 (*Diktyon* 46280), *ca*. 1321–26, a single copyist for most of both manuscripts, but not for the *Refutatio*, which was added later[42]. The *Refutatio*, with the standard title, is on ff. 103v–107v; it is preced-

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Lipsiae 1968–72 I IX–XVIII. This volume of the edition of Arethas' writings exactly comprises all works contained in the Moscow manuscript (57 items), which is the sole witness of most of them.

37 Leo medicus is edited in F. Z. ERMERINS, Anecdota Medica Graeca. Lugduni Batavorum 1840, 79–275.

38 See A. SPOURLACOU, Einai o Manouēl Chrysolōras o suggrapheus tou ergou Kephalaia oti kai ek tou uiou to agion pneuma ekporeuetai. *Θησαυρίσματα* 2 (1963) 88–117.

39 See the edition in S. BEZDEKI, Nicephori Gregorae epistulae XC. *Ephemeris Dacoromana* 2 (1924) 239–377: 303–311.

40 This item is edited in L. G. WESTERINK, Nikephoros Gregoras, Dankrede an die Mutter Gottes. *Helikon* 7 (1967) 259–271.

41 These four Planudean works are edited in *PG* 147, 985–1016 and 1017–1112; L. G. WESTERINK, Trois textes inédits sur Saint Diomède de Nicée. *AnBoll* 84 (1966) 161–227; L. G. WESTERINK, Le *Basilikos* de Maxime Planude. *BSl* 27 (1966) 98–103; 28 (1967) 54–67; 29 (1968) 34–50, respectively.

42 See M. FORMENTIN – F. RICHETTI – L. SIBEN, Catalogus codicum Graecorum Bibliothecae nationalis Neapolitanae. Volumen III. Roma 2015, 89–91; MATHIESEN, Ancient Greek Music Theory no. 202. As the quire numbers show, Neap. III.C.4 comprises quires 1–16 of the original whole, Neap. III.C.3 comprises quires 17–30. Other hands can be found at ff. 1r–v, 2r–v, 3r–v, 65v–66r, 79r, 103r–107v of Neap. III.C.3. The editions of the works contained in this manuscript are as follows: Nikomachos and *excerpta*, S. DI MAMBRO, Nicomaco, Manuale di Armonica (*Mathematica Graeca Antiqua* 5). Pisa – Roma 2023; Planudes, *Versus heroici*, A. STÜCKELBERGER, Planudes und die *Geographia* des Ptolemaios. *Museum Helveticum* 53 (1996) 197–205, and see also F. PONTANI, The World on a Fingernail: An Unknown Byzantine Map, Planudes, and Ptolemy. *Traditio* 65 (2010) 177–200, and C. M. MAZZUCCHI, Il Tolomeo Ambr. D 527 inf. e i versi di Massimo Planude sulle carte della *Geografia* (Ambr. A 199 sup.), in: Miscellanea Graecolatina I, a cura di F. Gallo. Roma 2013, 259–266; Planudes, *epigramma*, il secondo in C. F. A. NOBBE, Claudii Ptolemaei Geographia. Tomus I. Lipsiae 1843, XXXIII no. VII; the first of the *notae harmonicae*, K. VON JAN, Musici Scriptores Graeci. Lipsiae 1895, 411, 2–412, 8 (= sects. 1–2 of the so-called *Excerpta Neapolitana*, for which see now F. ACERBI – S. PANTERI, Eratosthenes in the *Excerpta Neapolitana*. *GRBS* 59 [2019] 663–679); [Plutarch], A. MERIANI, Ps.-Plutarchi De musica. Berlin – Boston 2024; Theon of Smyrna, E. HILLER, Theoni Smyrnaei philosophi platonici Expositio rerum mathematicarum ad legendum Platonem uti-

ed on f. 103r–v by *Harm*. III 14–16, headed Βαρλαὰμ μοναχοῦ τοῦ Καλαβροῦ τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "by the monk Barlaam of Calabria, chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works in the manuscript: ff. 1r–v Porphyry, *In Ptolemaei Harmonica*, proemium; 2r–v *excerpta e Nicomachi Harmonico encheiridio*; 3r–v Maximus Planudes, *Versus heroici in Ptolemaei Geographiam* and *epigramma in Ptolemaeum*; 4r–v *notae harmonicae*; 5r–51r Ptolemy, *Harmonica* I–III 14 (*des*. 109, 11 Düring); 51v–52v *diagrammata harmonica*; 53r–65r [Plutarch], *De musica*; 65r–94r Theon of Smyrna, *Expositio* (*des*. 119, 21 Hiller); 94v–101v Nikomachos, *Harmonicum encheiridion*; 101v–102v Maximus Planudes, *excerpta ex commentario Macrobii in Somnium Scipionis graece verso*.

N and its complement Neap. III.C.4 are independent witnesses of some of the treatises of harmonic theory they contain: for Ptolemy, see DÜRING, Die Harmonielehre LIV and LXIX (stemma); for Nikomachos, see DI MAMBRO, Nicomaco, sect. II.3.2.

$O^1$. Oxford, Bodleian Library, Savile 1 (*Diktyon* 48424), composite, 1589, copyist <James Darlymple of Ayr>[43]. A very short summary of the *Refutatio*, with the title Βαρλαὰμ μοναχοῦ ἀνασκευὴ εἰς τὰ κτλ. "by the monk Barlaam, refutation of the, etc." and including *incipit* and *desinit*, is on ff. 318v–319r; it is preceded on f. 318v by long *incipit* and *desinit* of *Harm*. III 14–16, headed τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works in the manuscript: ff. 1r–195v Sextus Empiricus, *Adversus mathematicos*; 195v–199r Dissoi logoi; 200r–222r Euclid, *Data*; 224r–227v Autolycus, *De sphaera mota*; 228r–240r Euclid, *Phaenomena*; 240r–258r Theodosius, *De diebus et noctibus*; 258v–264r Theodosius, *De habitationibus*; 264r–275r Autolycus, *De ortibus et occasibus*; 276r–318r Barlaam, *Logistikē* I–VI; 318r–v Barlaam, *De paschate* (summary); 318v–319r Barlaam, *Refutatio* (summary).

The manuscript ends with very short summaries of Barlaam's *De paschate* and *Refutatio*. The only other witness that adopts the ordering *Logistikē – De paschate – Refutatio* is Marc. gr. Z. 332 (our witness M). The only editor who determines the stemmatic position of $O^1$ is Carelos (Barlaam LXV–LXVIII), who makes it a copy of the manuscript Oxford, Bodleian Library, Savile 6 (composite; *ca*. 1582; copyist of the scientific part Henry Savile; *Diktyon* 48429), in its turn a copy of Marc. gr. Z. 332. As Oxon. Savile 6 begins with Book III and does not bear any trace of mutilation, and as two of the three separative innovations of $O^1$ are Carelos' false readings (likewise, two of the four conjunctive innovations are Carelos' false readings in $O^1$), and the third one just shows that Oxon. Savile 6 cannot be a copy of $O^1$, we conclude that the two Oxford manuscripts are independent witnesses of the *Logistikē*.

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lium. Leipzig 1878; Planudes, *excerpta*, A. MEGAS, Macrobii commentariorum in Somnium Scipionis libri duo in linguam graecam translati. Thessalonike 1995.

[43] See F. MADAN – H. H. E. CRASTER – N. DENHOLM-YOUNG, A Summary Catalogue of Western Manuscripts in the Bodleian Library at Oxford. Vol. II part II. Oxford 1937 no. 6548. The *Data* (+ 1 blank folio) is copied in four ternions, numbered from α΄ το δ΄; the Barlaam section is paginated from 1 to 87. For the copyist, see *RGK* I 142 (identification in this manuscript). See also J.-L. QUANTIN, Historical Criticism, Confessional Controversy, and Self-Censorship: Henry Savile and the *Lives* of John Chrysostom. *Erudition and the Republic of Letters* 6 (2021) 138–223: n. 69 on 157–158. The editions of the works contained in this manuscript are as follows: Sextus, H. MUTSCHMANN – J. MAU, Sexti Empirici Opera. Vol. II–III. Lipsiae 1914–61; Dissoi Logoi, E. WEBER, Dissoi Logoi: Eine Ausgabe der sogenannten Dialexies, in: Philologisch-Historische Beiträge Curt Wachsmuth zum sechzigsten Geburtstag überreicht. Leipzig 1897, 33–51; Euclid's *Data* and *Phaenomena*, HEIBERG – MENGE, Euclidis opera omnia VI and VIII, respectively; Autolycus, J. MOGENET, Autolycus de Pitane. Histoire du texte suivie de l'édition critique des traités de la sphère en mouvement et des levers et couchers (*Université de Louvain, Recueil de travaux d'histoire et de philologie, 3e série* 37). Louvain 1950; Theodosius, R. FECHT, Theodosii De Habitationibus Liber. De Diebus et Noctibus Libri duo (*Abhandlungen der Gesellschaft der Wissenschaften zu Göttingen, Philologisch-Historische Klasse, Neue Folge bd.* XIX,4). Berlin 1927.

$O^2$. Oxford, New College 299 (*Diktyon* 48768), watermark range 1364–72, one single copyist for the entire manuscript[44]. The *Refutatio*, without a title, is on f. 70r–v (*des*. *mut*. 246 ἐκ τῶν αὐτῶν τἀναντία); it is preceded on ff. 69v–70r, in continuity with Ptolemy's treatise, by *Harm*. III 14–16, by the same scholium as in our witnesses $L^1$ and $L^2$, and by a blank space of 2 lines. Other works in the manuscript: ff. 1r–51r Aristotle, *Organon*, of which 1r–2v Porphyry, *Isagōgē*, 2v–6r *Categoriae*; 6r–8r *De interpretatione*; 10r–20v *Analytica priora* I–II; 20v–27r *Analytica posteriora* I–II; 32r–45v *Topica*, 47r–51r *Sophistici elenchi*; 52r–59r Nikomachos, *Introductio arithmetica* I–II; 59v *diagrammata harmonica*; 60r–70r Ptolemy, *Harmonica* I–III 16; 71r–83v Ptolemy, *Apotelesmatica* I–IV; 84r–v Porphyry, *Introductio in Ptolemaei Apotelesmatica* 47–55; 85r–v [Ptolemy], *Centiloquium*; 86r–v text tit. περὶ τῶν τῆς σελήνης ἡμερινῶν παρόδων τίνα σημαίνει καθ' ἑκάστην αὐτῆς ἐν τοῖς τῶν ζῳδίων δεκανοῖς διάφορον πάροδον καὶ σημασίαν "about the daily passages of the Moon, what is a sign of in the decans of the zodiac for each of its different passages and indications"; 87r–89r John Philoponus, *De usu astrolabii*; 89v–90v Anonymus, *De constructione astrolabii*; 90v–91v text tit. περὶ τῶν ἑπτὰ τῆς γῆς κλιμάτων "about the three terrestrial *klimata*".

Because of a loss of folios, the *Refutatio* ends abruptly with the last line of f. 70v. The same curtailed text is carried by our witnesses $L^1$ and $L^2$. Düring shows that $O^2$ is a copy of Vat. gr. 185 and a stemmatic sibling of Ambr. E 76 sup. (our witness A) as for Ptolemy's *Harmonica*. Vat. gr. 185 is the earliest witness in which the scholium that precedes the *Refutatio* in our witnesses $L^1$, $L^2$, and $O^2$ is located at the end of Ptolemy's *Harmonica*, after the added chapters III 14–16; the variant reading τετράχορδον in the scholium also appears to originate in Vat. gr. 185. Hübner states that $O^2$ is an obvious copy of the manuscript Oxford, Bodleian Library, Auct. T.5.4 (Misc. 266; 4th quarter of the 13th century; *Diktyon* 47214) as for Ptolemy's *Apotelesmatica*; Boer does not identify a model in the case of Ptolemy's *Centiloquium*[45].

$P^1$. Paris, Bibliothèque nationale de France, gr. 2381 (*Diktyon* 52013), mm 295×220; composite, *ca*. 1371–73 (certainly before 1392: tables on ff. 100r and 101r; later note of the main hand on f. 104v;

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[44] This in-folio, mm 420×295, comprises 92 folios written on 57 lines per page; watermarks "hunting horn", like Briquet *huchet* 7645 (1364), "griffon", like Briquet *griffon* 7453 (1370–76) and 7454 (1372); these dates are confirmed by a paleographic evaluation of the hand; the quire numbers run from α΄ to ιβ΄, but the folios carrying numbers ς΄ and ιβ΄ have been excised; the composition (the production units are separated by a vertical bar) is $1^8$ (8) | $2^8$ (24) | $1^6$ (30) | $2^8$ (46) | $1^{6-1}$ (51) $1^8$ (59) | $1^8$ (67) $1^{8-5}$ (70) | $2^8$ (86) | $1^{8-2}$ (92). Folios 8v–9r, 27v–31r, 46r–v, 51v, 92r–v are blank. Folio 9v contains a few lines of text, followed by a representation of Aristotle; the same depiction is found on f. 31v, which is otherwise blank. See H. O. COXE, Catalogus codicum mss. qui in collegiis aulisque Oxoniensibus hodie adservantur. Oxonii 1852 Codies mss. Collegii Novi 107–108; Catalogus Codicum Astrologorum Graecorum. I–XII. Bruxelles 1898–1953 IX 1 (S. WEINSTOCK) 96–97; MATHIESEN, Ancient Greek Music Theory no. 152. The hand has been dated on our request by C. Giacomelli, D. Bianconi, and A. Gioffreda, *per litteras*. The editions of the works contained in this manuscript are as follows: no truly critical edition exists of any of the treatises traditionally included in Aristotle's *Organon*; for Porphyry, see A. BUSSE, Porphyrii Isagoge et in Aristotelis Categorias commentarium (*CAG* IV 1). Berlin 1887; for an overall assessment of the Aristotelian works, with a complete bibliography, see C. GIACOMELLI, Circolazione e lettori dell'*Organon* di Aristotele. Indagini sui *codices vetustissimi*. *Segno e Testo* 21 (2023); Nikomachos, R. HOCHE, Nicomachi Geraseni pythagorei Introductionis Arithmeticae libri II. Lipsiae 1866 (not a critical edition); Ptolemy's *Apotelesmatica*, W. HÜBNER, Claudii Ptolemaei opera quae exstant omnia. III 1. Apotelesmatika. Lipsiae 1998; Porphyry, Catalogus Codicum Astrologorum Graecorum V 4 (Ae. BOER – S. WEINSTOCK), 187–228 (not a critical edition); [Ptolemy], Centiloquium, Fr. LAMMERT – Ae. BOER, Claudii Ptolemaei opera quae exstant omnia. III 2. De iudicandi facultate et animi principatu. Pseudo-Ptolemaei Fructus sive Centiloquium. Lipsiae 1952; text on f. 86r–v, J. CAMERARIUS, Astrologica, Norimbergae 1532, 31–36; Philoponus, C. JARRY, Jean Philopon, Traité de l'Astrolabe. Paris 2015 (the editor lists $O^2$ among the witnesses but he does not discuss its stemmatic position) and A. STÜCKELBERGER, Ioannes Philoponus, De usu astrolabii eiusque constructione. Berlin 2015 (not a critical edition). We were unable to find an exact parallel to the text copied on ff. 89v–90v in published or described Byzantine treatises on the astrolabe; see A. TIHON, Traités byzantins sur l'astrolabe. *Physis* 32 (1995) 323–357 for a synthesis. This text, however, intermittently coincides with Argyros' treatise.

[45] See DÜRING, Die Harmonielehre LXV and LXIX (stemma); HÜBNER, Apotelesmatika XVIII and XXV (stemma); BOER, Pseudo-Ptolemaei Fructus XXIX–XXX and XXXII (stemma).

watermark range 1358–93)[46]. The *Refutatio*, with the title Βαρλαὰμ ἀνασκευὴ εἰς τὰ κτλ. "by Barlaam, refutation of the, etc.", is on ff. 32r–35r. Other works in the manuscript: ff. 1r–2v *notae chronologicae et metrologicae*; 3r–12v Maximus Planudes, *Psephophoria secundum Indos*; 13r–30v Barlaam, *Logistikē* I–VI; 30v–32r Barlaam, *Demonstratio*; 35v–41v Gregorius Palamas, *Physica, theologica moralia et practica capita CL*[47]; 41v–46v Gregorius Palamas, *Pro Hesychastis orationes duo*[48]; 46v *excerpta theologica*; 47r–62r Cleomedes, *Caelestia cum scholiis Pediasimi*; 55r marg. [Apollonius], *On finding two mean proportionals*[49]; 56r marg. Anatolius, *De generatione*; 56r marg. *nota astrologica*; 56r marg. [Melampos], *De divinatione ex naevis*[50]; 56v marg. *geographica et astronomica varia*; 62r Hermes Trismegistos, *De partibus hominis*; 62r *Oneirocriticon e Danielis psalmis*; 62r *excerptum e Galeni De dignotione ex insomniis*[51]; 62v Nicholas Rhabdas, *Methodus de arithmeticis et geometricis medietatibus, et problemata arithmetica octo*; 63r–v *De Persici astrolabii usu capita XX–XXXIX*; 64r–77v *Aratea, astrologica et brontologica varia*[52]; 78r–79v Anonymous and Demetrius Triclinius, *De lunae schematismis*; 80r *De climatibus*[53]; 80v *excerpta ex Adamantii De ventis*[54]; 81r–85v Iohannes Pediasimos, *Geometria*; 85v–86r *notae et tabulae metrologicae chronologicae astrologicae* (dated to 1371–73); 86r–88v [Aristotle], *De mundo*; 93r–96v Alexander of Aphrodisias medicus, *Quaestiones et solutiones physicae*[55]; 96v–99r [Philo], *De mundo*[56]; 99r–v

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[46] A very detailed description of this manuscript (here completed) is found in Catalogus Codicum Astrologorum Graecorum VIII 3 (P. Boudreaux) 43–59. See also P. Schreiner, Die byzantinischen Kleinchroniken. I–III (*CFHB* 12). Wien 1975–79 I 191–192; P. Caballero Sánchez, El Comentario de Juan Pediásimo a los «Cuerpos celestes» de Cleomedes (*Nueva Roma* 48). Madrid 2018, 107–110 (watermarks and identification of the main copyist); Mathiesen, Ancient Greek Music Theory no. 75. The composition of Par. gr. 2381 is as follows: $1^{2}$ (2, marked α′ by a later hand, a1–a2), $1^{10}$ (12, β′, a3–a12) | $2^{8}$ (28, γ′–δ′, b1–b8 and c1–c8), $1^{10}$ (38, ε′, d1–d10), $1^{8}$ (46, ς′, e1–e8) | $2^{8}$ (62, ζ′–η′, f1–f8 and g1–g8) | $0^{5}$ (67, ιβ′, h1–h5) | $0^{6}$ (73, ιε′, i1–i6) | $0^{4}$ (77, h2–3, h7–8) | $0^{3}$ (80, h9–11) | $1^{8}$ (88, ιζ′, l1–l8) | $0^{4}$ (92) | $1^{8+2}$ (102, ff. 100–101 are unwarrantedly added to a quire, ιη′, m1–m10) | $0^{7}$ (109, n1–n7), where $0^{x}$ denotes *x* loose folios now bound together, | a junction, namely, a change of quire coinciding with a change of work. The Greek quire numbers are placed in the middle of the lower margin of the verso of the last folio of a quire. The Latin quire and page markers are placed in the lower outer corner of the recto of each folio. Folios 100 and 103 are bound with recto and verso interchanged. Folios 89–92 are blank. The last words of the last page are μετὰ γ φύλλα (solar symbol). The editions of the works contained in Par. gr. 2381 are sometimes listed in clusters; to simplify the references, some of these editions are given in the final assessment of the manuscript.

[47] This text is edited in R. E. Sinkewicz, Saint Gregory Palamas, The One Hundred and Fifty Chapters (*Studies and Texts* 83). Toronto 1988.

[48] These are parts 2 and 3 of the first Triad of Palamas' treatises edited in J. Meyendorff, Grégoire Palamas. Défense des saints hésychastes (*Spicilegium Sacrum Lovaniense. Études et documents* 30). Louvain 1973.

[49] This is a method for finding two mean proportional lines between two given straight lines, witnessed in several sources and variously assigned to Hero of Alexandria or to Apollonius; see W. R. Knorr, Textual Studies in Ancient and Medieval Geometry. Boston – Basel – Berlin 1989, 11–28 and 41–61, and earlier, and paying attention to Byzantine authors, V. De Falco, Sul problema delico. *Rivista Indo-Greco-Italica* 9 (1925) 41–56. The proof in Par. gr. 2381 is an abridged version of Knorr's text PK, transmitted in late witnesses of Philoponus' *in APo*.

[50] The first of these three texts is edited in Catalogus Codicum Astrologorum Graecorum VIII 3, 188, the third is edited in J. Franz, Scriptores physiognomoniae veteres. Altenburgi 1780, 501–508, and now in S. Costanza, Una versione bizantina e una metafrasi neogreca dello ps. Melampo *De Naevis*. *Byz* 83 (2013) 83–102.

[51] The first of these three texts is edited in H. Diels, Beiträge zur Zuckungsliteratur des Okzidents und Orients. I. Die griechischen Zuckungsbücher (Melampus Peri palmōn), Abhandlungen der königlich preussischen Akademie der Wissenschaften. Philosophisch-historische Klasse. Berlin 1907 IV, 41–42; for the third see C. G. Kühn, Claudii Galeni opera omnia. I–XX. Lipsiae 1821–33 VI 832–835.

[52] This sequence of extracts is very accurately described, and partly edited, in Catalogus Codicum Astrologorum Graecorum VIII 3, 47–53 and 189–191, to which we refer.

[53] This text is edited in J. A. Cramer, Anecdota Graeca e codicibus manuscriptis bibliothecae Regiae Parisiensis. I. Oxonii 1839, 362, 1–364, 7 μέρεσι.

[54] These excerpts are edited in V. Rose, Anecdota Graeca et Gaecolatina. I–II. Berlin 1864–70 I 49–52.

[55] This is a version of the compilation of problems edited in J. L. Ideler, Physici et medici Graeci minores. I–II. Berolini 1841–42 I 3–80.

[Aristotle], *De virtute*; 99v, 102r–v Theophylact Simocatta, *Dialogus de quaestionibus physicis*[57]; 100r–101v *tabulae (partim vacuae) et notae variae*[58]; 103v–r Barlaam, *De paschate*; 104r–105v line 2 Matthew Blastares, *Computus Paschalis*[59]; 105v line 3–107r line 5 Michael Psellos, *Opus chronologicum* (excerpts); 107r line 6–v *notae physiognomonicae*; 108r to line 14 *notae computisticae*; 108r line 15–v line 11 a.i. *notae theologicae*; 108v line 10 a.i. – 109r line 11 *De astris*; 109r–v *notae astronomicae*.

This is a highbrow manuscript written for personal use by a distinguished scholar, who possibly added the quire comprising ff. 3–12 (Planudes) to his notebook (our doubts come from the fact that our scholar did not leave traces in the added quire); he also briefly collaborated with another copyist on f. 88v. As first remarked by P. Caballero Sánchez, the main copyist also penned the so-called *textus tripartitus* of Manuel Bryennios' *Harmonica*, whose three *membra disiecta* we read in the manuscripts Paris, Bibliothèque nationale de France, gr. 2549 (*Diktyon* 52181), ff. 43r–46v and 75v–78v, Madrid, Biblioteca nacional de España 4625 (*Diktyon* 40105), ff. 2r, 68r–71v, and 122v–123v, and München, Bayerische Staatsbibliothek, Cod. graec. 487 (*Diktyon* 44935), ff. 272r–289r[60]. As is to be expected, the scientific texts contained in Par. gr. 2381, possibly by the intermediation of hyparchetypes, have very important witnesses as ancestors. This is the case for the following treatises. Planudes: the ancestor of $P^1$ is the manuscript Firenze, Biblioteca Medicea Laurenziana, Ashb. 1599 (14th century; *Diktyon* 15767); Barlaam: the ancestor is the manuscript Marc. gr. Z. 332 (our witness M); Cleomedes: the ancestor is the manuscript Edinburgh, National Library of Scotland, Adv. 18.7.15 (*ca*. 1290; copied for the most part by Maximus Planudes; *Diktyon* 13730)[61]. Par. gr. 2381 is an independent witness of Triclinius' treatise and of Pediasimos' scholia to Cleomedes; it is the only witness of Rhabdas' short logistic text[62]. The presence of Barlaam's *De paschate* in $P^1$ is not recorded in the standard edition (the folio is bound with recto and verso interchanged), nor is it the fragment from Psellos' chronological treatise[63].

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[56] This work is a compendium of Philo's *De aeternitate mundi*: see F. CUMONT, Philonis De aeternitate mundi. Berolini 1891, XXVII; L. COHN – P. WENDLAND, Philonis Alexandrini opera quae supersunt. I–VI. Berolini 1896–1915 VI XXXIV–XXXVI. The edition can be read in K. E. RICHTER, Philonis Judaei opera omnia. I–VIII. Lipsiae 1828–30 VI 148–174.

[57] This text is edited in IDELER, Physici et medici I 168, 1–177, 16, and L. MASSA POSITANO, Teofilatto Simocata. Questioni naturali. Napoli 1965, 7, 1–26, 15.

[58] The contents of these folios are as follows: f. 100v, day (Nov. 8 and 23, Dec. 8, Jan. 6, Feb. 5, Mar. 6, Mar. 11, Jun. 2, Dec. 19 AM 6881 [= AD 1372]), hour, longitude, distance from nodes of new and full Moons; definition of the base of the Moon; notes and expense reports for trips to Rhodes and back to Constantinople, and to Cyprus; f. 100r, incomplete (only the first row of tabulated values is inserted) table of the yearly mean longitude, mean anomaly, and double elongation of the Moon, years AM 6879–6900 [= AD 1371–90]; table (referred to long. 72º) of the mean longitude, mean anomaly, and double elongation of the Moon, for 1 to 10, 20 to 90, 100 to 300 days, 1 year of 365 days and 1 year of 366 days; astrological *thema*; f. 101r, table of the yearly anomaly and apogee of the Sun, years AM 6879–6900 [= AD 1371–90]; table of the anomaly and apogee of the Sun, for 1 to 10, 20 to 90, 100 to 300 days, 1 year of 365 days and 1 year of 366 days; reason for taking (0)0;59,8 as the value of the mean daily motion in longitude of the Sun, with associated tabular computations; f. 101v, tables 2a (hours) and 2b (both incomplete) according to the list in R. LEURQUIN, La *Tribiblos astronomique* de Théodore Méliténiote (Vat.gr. 792). *Janus* 72 (1985) 257–282: 270–276, as in the manuscript Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 210 (middle of 14th century; *Diktyon* 66841), f. 50r (only the last two columns).

[59] This extract coincides with G. RHALLES – M. POTLES, Syntagma tōn theiōn kai hierōn kanonōn kata stoicheion. VI. Athēnai, 404–425 (= *PG* CXLV 65–104) 404–419, 8.

[60] See G. H. JONKER, De textu Bryennii tripartito. *Mnemosyne* 19 (1966) 399–400; G. H. JONKER, Manouēl Bryenniou Harmonika. The Harmonics of Manuel Bryennius. Groningen 1970, 36, 37, 40, 46–47; B. MONDRAIN, Les écritures dans les manuscrits byzantins du XIVe siècle. Quelques problématiques. *RSBN* 44 (2007) 157–196: 194 and n. 70, who identified a fourth limb in the composite manuscript München, Bayerische Staatsbibliothek, Cod. graec. 489 (*Diktyon* 44937), f. 246r–v and provides references to other manuscripts where the hand of the copyist of Par. gr. 2381 can be found.

[61] See ALLARD, Maxime Planude, Le grand calcul selon les Indiens. Louvain-la-Neuve 1981, 12–14 (Planudes); ACERBI, Barlaam's Paraphrase, and below for Barlaam; R. B. TODD, Cleomedis Caelestia (Meteora). Leipzig 1990 X (Cleomedes).

[62] See the edition in F. ACERBI, A New Logistic Text of Nicholas Rhabdas. *Byz* 92 (2022) 17–45.

[63] For Pediasimos' scholia, see CABALLERO SÁNCHEZ, El Comentario 139 (stemma) and 165–166. For the Anonymous and Triclinius, see A. WASSERSTEIN, An Unpublished Treatise by Demetrius Triclinius on Lunar Theory. *JOeB* 16 (1967) 153–174 and ACERBI, I problemi aritmetici 136 n. 16 and Testo 2. The excerpts from Psellos' treatise are sects. 1–3 and 21–22 (but other material is added); see G. REDL, La chronologie appliquée de Michel Psellos. *Byz* 4 (1927–28) 197–236 and G.

P[4]. Paris, Bibliothèque nationale de France, gr. 2397 (*Diktyon* 52029), 2nd quarter of the 16th century, copyist <Konstantinos Mesobotes>[64]. The *Refutatio*, without a title, is on ff. 1r–7r. Other works in the manuscript: ff. 7v–9v Anonymus, *De astrolabio*; 9v–15r, Nikephoros Gregoras, *De constructione astrolabii*, the so-called version A; 15r–v *scholium Macarii in Gregorae astrolabium*; 19r–27r Ptolemy, *Psephophoria*; 27r–54r Theon of Alexandria, *In Ptolemaei Tabulas Manuales commentarium parvum*; 54r–v *nota astronomica*[65]; 55r–62r *excerpta e Gemini Introductione* (the entire ch. IV, V, XV, III, XVIII); 62r *methodus de radice quadrata*; 62r–100r Proclus, *Hypotyposis*; 100v–103v *notae astronomicae*; 105v–117r John Philoponus, *De usu astrolabii*; 117r–v *notae tres de usu astrolabii*.

Obviously copied from disparate models, P[4] stems from the manuscript Oxford, Bodleian Library, Auct. T.4.20 (Misc. 258; middle of 14th century; *Diktyon* 47206) as for Theon: TIHON, Le "Petit Commentaire" 81–82 and 75 (stemma). Ptolemy's primer on the *Handy Tables* derives from the manuscript Firenze, Biblioteca Medicea Laurenziana, Plut. 28.7 (2nd half of 14th century, *Diktyon* 16188): HEIBERG, Claudii Ptolemaei II CLXXVII, with the additional evidence of the square root method placed after excerpts from Geminus in both manuscripts. No other edition mentions P[4] or determines its stemmatic position.

V[1]. Vatican City, Biblioteca Apostolica Vaticana, Urb. gr. 77 (*Diktyon* 66544), a unitary whole of independent quire sequences[66], 1549–55, copyist <Johannes Franciscus from Candia>[67]. The *Refutatio*, with the standard title, is on ff. 177v–185r; it is preceded on ff. 176v–177v by *Harm*. III 14–16, headed Βαρλαὰμ μοναχοῦ τοῦ Καλαβροῦ τὰ ὑπό τινων προστεθέντα κεφάλεα ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "by the monk Barlaam of Calabria, chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works

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REDL, La chronologie appliquée de Michel Psellos (suite). *Byz* 5 (1929–30) 229–286. Pediasimos' *Geometria* is published in G. FRIEDLEIN, Die Geometrie des Pediasimus. Programm Ansbach 1866.

64 See A. TIHON, Le "Petit Commentaire" de Théon d'Alexandrie aux Tables Faciles de Ptolémée (*StT* 282). Città del Vaticano 1978, 68–69. On the copyist, see *RGK* I 224, II 315 (identification in this manuscript), III 363; S. MARTINELLI TEMPESTA, Alcune riflessioni sulla produzione scritta di Costantino Mesobote da codici Ambrosiani, in: Griechisch-byzantinische Handschriftenforschung, hrsg. von C. Brockmann – D. Deckers – D. Harlfinger – S. Valente. Berlin – Boston 2020, 215–232. The editions of the works contained in this manuscript are as follows: Anonymus, A. DELATTE, Anecdota Atheniensia et alia. Tome II. Textes grecs relatifs à l'histoire des sciences. Liège – Paris 1939, 254–262; Gregoras and scholium Macarii, C. JARRY, Nicéphore Grégoras, Isaac Argyros, Deux traités byzantins de construction de l'astrolabe. Paris 2021; Ptolemy, HEIBERG, Claudii Ptolemaei II 159–185; Theon, TIHON, Le "Petit Commentaire"; Geminus, J. AUJAC, Géminos, Introduction aux Phénomènes. Paris 1975; Proclus, C. MANITIUS, Procli Diadochi Hypotyposis Astronomicarum Positionum. Leipzig 1909; *methodus*, J. L. HEIBERG, Kleine Anecdota zur byzantinischen Mathematik. *Zeitschrift für Mathematik und Physik. Historisch-literarische Abtheilung* 33 (1888) 161–170: 164; *notae tres*, JARRY, Jean Philopon 49–50.

65 The *notae astronomicae* on ff. 54r–v, 100v–103v, and 117r–v are nos. 22, 16, 30, 79, 21, 13, 1, 39 in the list of TIHON, Le "Petit Commentaire" 359–369.

66 This is borne out by the composition, kindly determined by Francesco Valerio on our request (the production units are separated by a vertical bar): $1^{12}$ (12) | $4^{8}$ (44) | $7^{8}$ (100) $1^{4}$ (104) | $7^{8}$ (160) $1^{4}$ (164) | $1^{8}$ (172) $1^{12}$ (184) $1^{4}$ (188) | $2^{8}$ (204) | $7^{8}$ (260) $1^{2}$ (262) | $1^{8}$ (270) | $1^{10}$ (280) | $2^{8}$ (296) | $1^{8}$ (304) $1^{6}$ (310) | $1^{1}$ $1^{2}$ $1^{4}$ $1^{1}$ (318) $4^{8}$ (350) $1^{2}$ (352). Despite its composition, the quire made of ff. 311–318 does not exhibit any textual perturbations.

67 See C. STORNAJOLO, Codices Urbinates Graeci Bibliothecae Vaticanae. Romae 1895, 103–106; MATHIESEN, Ancient Greek Music Theory no. 255. For the copyist (but f. 185r appears to be in a different hand), see *RGK* I 188 (identification in this manuscript), II 247, III 312; R. S. STEFEC, Die griechische Bibliothek des Angelo Vadio da Rimini. *Römische Historische Mitteilungen* 54 (2012) 95–184: 101–102 n. 32. The editions of the works contained in this manuscript are as follows: for most of them, see already VON JAN, Musici; Cleonides, J. SOLOMON, Cleonides: Eisagōgē harmonikē; Critical Edition, Translation, and Commentary. PhD Thesis, University of North Carolina 1980; Euclid, A. BARBERA, The Euclidean Division of the Canon. Greek and Latin Sources. Lincoln (NE) and London 1991; Aristoxenus, R. DA RIOS, Aristoxeni Elementa Harmonica. Roma 1954, and R. WESTPHAL, Aristoxenos von Tarent. Melik und Rhythmik des classischen Hellenenthums. II. Band. Leipzig 1893, 68–95; Aristides Quintilianus, R. P. WINNINGTON-INGRAM, Aristidis Quintiliani De Musica libri tres. Lipsiae 1963; Dionysios, Ch. TERZĒS, Dionisiou technē mousikē: kritikē ekdosē. Athēna 2008; Mesomedes; *Anonymus Bellermann*, D. NAJOCK, Anonyma De Musica scripta Bellermanniana. Lipsiae 1975.

in the manuscript: ff. 1r–7v [Euclid] *immo* <Cleonides>, *Introductio harmonica*; 8r–12r Euclid, *Sectio canonis*; 13r–39r Aristoxenus, *Elementa harmonica* I–III; 39r–43r Aristoxenus, *Elementa rhythmica* II 1–36; 45r–102v Ptolemy, *Harmonica* I–III 14 (*des*. 109, 11 Düring); 105r–163r Porphyry, *In Ptolemaei Harmonica I 1–4* (*des*. 90, 5 düring); 165v–174v Nikomachos, *Harmonicum encheiridion*; 174v–176r Maximus Planudes, *excerpta ex commentario Macrobii in Somnium Scipionis graece verso*; 189r–192v Theon of Smyrna, *Expositio* (46, 20–57, 6 Hiller); 193r–201r [Pappus] *immo* <Cleonides>, *Introductio harmonica*; 205r–261v Aristides Quintilianus, *De musica*; 263r–268r Bacchius the Elder, *Ars harmonica*; 268r–270v [Bacchius the Elder] *immo* Dionysios, *Ars harmonica*; 272r–v Mesomedes *Hymna tria*; 273v–280r *Anonymus Bellermann* 1–82, 105; 280v [Ptolemy], *Harmonica* III 16; 281r–294r Alypius, *Introductio harmonica*; 297r–307v Gaudentius, *Introductio harmonica*; 311r–352r Theon of Smyrna, *Expositio* (*des*. 119, 21 Hiller).

Almost all models of this manuscript have been identified; the exception is the first version of Cleonides; the second version seems to be copied from the manuscript Vatican City, Biblioteca Apostolica Vaticana, Barb. gr. 265 (1535–63; copyist <Johannes Honorios>; *Diktyon* 64811): J. SOLOMON, Vaticanus gr. 2338 and the Eisagōgē harmonikē. *Philologus* 127 (1983) 247–253: 249 and 253 (stemma). The models are Marc. gr. Z. 322 for Euclid: BARBERA, The Euclidean Division 77 and 78 (stemma); the manuscript Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 191 (*ca*. 1270; *Diktyon* 66822; on this manuscript see F. ACERBI – A. GIOFFREDA, Manoscritti scientifici della prima età paleologa in scrittura arcaizzante. *Scripta* 12 [2019] 9–52 *passim*) for Aristoxenus' *Elementa harmonica*: DA RIOS, Aristoxeni Elementa Harmonica LXXXVII–LXXXIX and CVI (stemma); Neap. III.C.3 (our witness N) for Ptolemy and Porphyry: DÜRING, Die Harmonielehre LIV and LXIX (stemma) and DÜRING, Porphyrios Kommentar XIX–XX and XXIX (stemma), and for Nikomachos: DI MAMBRO, Nicomaco, sect. II.3.2; the manuscript Neap. III.C.4 (middle of 14th century; once one and the same manuscript with Neap. III.C.3) for Aristides Quintilianus: WINNINGTON-INGRAM, Aristidis Quintiliani XII, for Dionysios: TERZĒS, Dionisiou technē mousikē LX and LXVII (stemma), for Mesomedes: E. HEITSCH, Die Mesomedes-Überlieferung (*Nachrichten der Akademie der Wissenschaften zu Göttingen, Philologisch-Historiche Klasse* 1959,3). Göttingen 1959, 43, and for the *Anonymus Bellermann*: NAJOCK, Anonyma De Musica X and XIX (stemma). For $V^1$ stemming from the two Naples manuscripts, see already VON JAN, Musici LXXVI.

V. Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 187 (*Diktyon* 66818), middle of 14th century, one single copyist for the entire manuscript[68]. The *Refutatio*, with the title Βαρλαὰμ μοναχοῦ καὶ φιλοσόφου λόγος ἀνασκευαστικὸς εἰς τὰ κτλ. "by the monk and philosopher Barlaam, refutation discourse of the, etc.", is on ff. 71r–81v; it is preceded on ff. 69r–71r by *Harm*. III 14–16, in continuity with Ptolemy's treatise but headed τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works in the manuscript: ff. 2r–71r Ptolemy, *Harmonica* I–III 16; 82r–161r Porphyry, *In Ptolemaei Harmonica I* (*des*. 150, 22 Düring); 162r–214r Barlaam, *Logistikē* I–VI; 214v–220r Barlaam, *Demonstratio*; 220r–223r Barlaam, *De eclipsi* I; 223r–227v Barlaam, *De eclipsi* II (incomplete); 228r–v Barlaam, *De paschate* (fragm.)[69].

V is a copy of Monac. gr. 361a as for Ptolemy, and an independent witness of Porphyry: DÜRING, Die Harmonielehre LIV–LVI and LXIX (stemma) and Düring, Porphyrios Kommentar XX–XXI and XXIX (stemma).

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[68] Descriptions of this manuscript and of the subsequent two items are found in G. MERCATI – P. FRANCHI DE' CAVALIERI, Codices Vaticani graeci. Codices 1–329. Romae 1923, 217–218, 218–219, and 232–233; see also MATHIESEN, Ancient Greek Music Theory nos. 211, 212, and 217. Vat. gr. 187 and 188 were owned by Angelo Colocci: G. CARDINALI, Il profeta e il monsignore: quarantasette nuovi manoscritti (e tredici nuovi stampati) di Angelo Colocci nella Vaticana e alla Nazionale di Parigi, in: Libri, scritture e testi greci. Giornata di studio in ricordo di Mons. Paul Canart (*StT* 554), a cura di C. Pasini – F. D'Aiuto. Città del Vaticano 2022, 259–334: 285–287.

[69] *De eclipsi* II *des*. *mut*. MOGENET – TIHON – DONNET, Barlaam 76 line 269 ἀπὸ τῆς; *De paschate inc*. *mut*. TIHON, Barlaam 378 sect. 26 πανσελήνους.

The editions of *Logistikē*, *Demonstratio*, *De eclipsi* I and II confirm that V stems from M; however, the editors of *De eclipsi* I and II give reasons to posit a further witness between M and V: MOGENET – TIHON – DONNET, Barlaam 40–44.

$V^2$. Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 188 (*Diktyon* 66819), 2nd quarter of 16th century, copyist <Michael Rhosaitos>[70]. The *Refutatio*, with the standard title, is on ff. 49v–53v (*des*. 175 ἤτοι ἐν ᾧ τόπῳ); it is preceded on ff. 48r–49r by *Harm*. III 14–16, in continuity with Ptolemy's treatise but headed τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works in the manuscript: ff. 1r–49r Ptolemy, *Harmonica* I–III 16.

The *Refutatio* ends at the same point as in our witness $V^3$. $V^2$ is a copy of $V^3$ as for Ptolemy: DÜRING, Die Harmonielehre XXXIV–XXXV, XXXVI, LVI, and LXIX (stemma).

$V^3$. Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 196 (*Diktyon* 66827), composite but of unitary conception, 1st half of 14th century, several copyists[71]. The *Refutatio*, with the standard title, is on ff. 74r–77r (*des*. 175 ἤτοι ἐν ᾧ τόπῳ); it is preceded on ff. 73r–74v by *Harm*. III 14–16, in continuity with Ptolemy's treatise but added by a different hand (the same as the *Refutatio*) and headed τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works in the manuscript: ff. 1r–30v Nikomachos, *Introductio arithmetica* I–II; 31r–73r Ptolemy, *Harmonica* I–III 14 (*des*. 109, 11 Düring); 78r–229v Euclid, *Elements* I–XII, without diagrams from X 7 (f. 163r) on.

The *Refutatio* ends at the same points as in our witness $V^2$. $V^3$ is a copy of Monac. gr. 361a as for Ptolemy: DÜRING, Die Harmonielehre XXXVI, LVI, and LXIX (stemma). B. Vitrac shows that, as for the *Elements*, $V^3$ is a copy of the manuscript Paris, Bibliothèque nationale de France, gr. 2466 (3rd quarter of 12th century; *Diktyon* 68385) from f. 149 (beginning of *Elem*. VIII 10) on[72].

v. Vatican City, Biblioteca Apostolica Vaticana, Vat. gr. 1756 (*Diktyon* 68385), composite, end of 16th – beginning of 17th century, copyist <Alvise Lollino>[73]. The *Refutatio*, with title ἀνασκευὴ εἰς τὰ προστεθέντα τρία κεφάλαια "refutation of the three added chapters", is on ff. 158r–167v; it is preceded on ff. 156r–158r by *Harm*. III 14–16, headed τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "by the monk Barlaam of Calabria, the chapters appended by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works in the manuscript: ff. 1r–88v Bessarion, *Opera theologica quinque*; 89r–127v

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[70] For this copyist, see *RGK* II 391, III 467 (identification in this manuscript).

[71] The composition of $V^3$ is as follows (the production units are separated by a vertical bar): $3^8$ (24) $1^{8-1}$ (30a) (no original quire numbers) | $5^8$ (70) $1^{8-1}$ (77) (original quire numbers α′–ς′) | $5^8$ (117) $1^{10-1}$ (126) $1^8$ (134) $1^{8-1}$ (141) $1^{8-1}$ (148) $10^8$ (228) (original quire numbers α′–ιθ′) $1^2$ (231, including 229a). The hand that copied the *Refutatio* is found on ff. 1r–12v, 73r11–75v20, 76r–98r21 αὐτὰ δὴ, 137r–142v (D. BIANCONI, Tessalonica nell'età dei Paleologi. Le pratiche intellettuali nel riflesso della cultura scritta [*Dossiers byzantins* 5]. Paris 2005, 134 n. 51); a different hand is at work in the last thirteen lines of f. 75v.

[72] See B. VITRAC, A propos de l'histoire du texte des *Éléments* d'Euclide : Préalables à une nouvelle édition critique. 2022. hal-03328161 Annexe 10G.

[73] A detailed description of this manuscript is found in P. CANART, Codices Vaticani graeci. Codices 1745-1962. I–II. In Bibliotheca Vaticana 1970–73 I 54–62. Nine hands are engaged in the copy; one of these copyists is Maximus Margounios, who also penned Mosq. Mus. Hist gr. 315 (our witness m). On Lollino, see P. CANART, Alvise Lollino et ses amis grecs. *Studi Veneziani* 12 (1970) 553–587; P. CANART, Les Vaticani Graeci 1487-1962. Notes et documents pour l'histoire d'un fonds de manuscrits de la Bibliothèque Vaticane (*StT* 284). Città del Vaticano 1979 *passim* and in particular 41–78.

*epistulae variorum, praesertim Bessarionis et Plethonis*; 129r–142v *excerpta ex Aristotelis HA a Plethone collecta*; 145r–146r Theodoros Gaza, *epistula ad fratres Andronicum et Demetrium*; 148r–153r Barlaam, *De eclipsi* II; 169r–170v Vicellius, *Seismologium*; 171r–177v Libanius, *Declamatio XXVI*; 179v–189v Barlaam, *Logistikē* I[74]; 190r–192v Barlaam, *ex epistulis excerpta* (EG 3, 4, 8, 1, 3)[75]; 195r–204r Barlaam, *Demonstratio*; 207r–223r Barlaam *epistulae duo* (EG 2–3); 231r–278v *Operae quinque ad hesychasticam controversiam pertinentia*; 279r–294r Barlaam, *epistula ad Gregorium Palamam* (EG 1); 295r–297r Barlaam, *De paschate* (compendium); 298r–306v Barlaam, *epistulae quinque* (EG 5–8, 4); 307r–346r Demetrios Cydones, *De processione Spiritus Sancti*[76]; 347r–348v; Demetrios Cydones, *De processione Spiritus Sancti ad amicum* (incomplete); 350r–352r Hippolytus, *De universo* (fragm.).

Lollino, who copied Barlaam's works that precede the *Demonstratio*, notes on ff. 191v–192r that the model of the excerpted letters also contains, in this order, Barlaam, *Logistikē* I–VI, *Demonstratio*, *De paschate*, *Refutatio*, *De eclipsi* I and II. The only manuscripts that fit this description are Mosq. Mus. Hist. gr. 315 (our witness m, copied by Maximus Margounios, who was also engaged in the copy of Vat. gr. 1756) and Marc. gr. Z. 332 (M); M, m, and v are also the only witnesses that contain Barlaam's letters. The editions of *Logistikē*, *Demonstratio*, *De paschate*, *De eclipsi* I and II confirm that v is a direct copy of M.

M. Venezia, Biblioteca Nazionale Marciana, gr. Z. 332 (coll. 643; *Diktyon* 69803), watermark range 1335–38, Barlaam's copyist II[77]. The *Refutatio*, with the standard title, is on ff. 73r–85r; it is preceded on ff. 73r–74v by Ptolemy, *Harm*. III 14–16, headed τὰ ὑπό τινων προστεθέντα κεφάλαια ταῖς τρισὶ τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν ἁρμονικῶν Πτολεμαίου "chapters added by someone to the last three titles of the third <book> of Ptolemy's harmonics". Other works in the manuscript: ff. 1r–61r Barlaam, *Logistikē* I–VI; 61v–67r Barlaam, *Demonstratio*; 67r–71v Barlaam, *De paschate*; 85r–140v Barlaam, *epistulae octo* (EG 4–8, 1–3); 142r–145v Barlaam, *De eclipsi* I; 146r–152v Barlaam, *De eclipsi* II; 153r–v *scholium de cyclo lunari et computationes astronomicae*.

As is well known[78], M contains Barlaam's edition of some of his own writings; he did not copy the manuscript himself, but he revised it. M is the prototype of the entire tradition as for the *Demonstratio*, *De paschate*, *De eclipsi* I and II, and the *Letters*; it is the prototype of most of the tradition as for the *Logistikē*.

## THE RELATIONSHIPS BETWEEN THE MANUSCRIPT WITNESSES OF THE *REFUTATIO*

Establishing the relationships between the manuscript witnesses of the *Refutatio* is eased by a number of facts: Barlaam's scientific writings were often copied as a corpus; all of these writings have been published in a critical edition; these editions reconstruct one and the same stemma for all witnesses the edited works share; in all these stemmas, Marc. gr. Z. 332, whose text was revised by Barlaam himself, is the prototype of the entire tradition. With one exception, our edition confirms

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[74] The proemium is missing, the correct folio order is 180–181, 186, 184–185, 182, 187–189 (no folio is numbered 183), see CARELOS, Barlaam XLII.

[75] See FYRIGOS, Dalla controversia palamitica 29–32.

[76] See M. RACKL, Die ungedruckte Verteidigungsschrift des Demetrios Kydones für Thomas von Aquin gegen Neilos Kabasilas. *Divus Thomas* 2nd series 7 (1920) 303–317.

[77] This copyist is responsible for ff. 1r–140v of Marc. gr. Z. 332 and ff. 191r–290r of Ambr. E 76 sup.; ff. 142r–152v of Marc. gr. Z. 332 were copied by Barlaam's copyist I; f. 153r–v is entirely in Barlaam's hand: GIOFFREDA, Su scrittura. On this manuscript, see E. MIONI, Bibliothecae Divi Marci Venetiarum codices graeci manuscripti. Volumen II. Thesaurus Antiquus. Codices 300–625. Roma 1985, 60–61; MATHIESEN, Ancient Greek Music Theory no. 265.

[78] See MOGENET – TIHON – DONNET, Barlaam 46–49; FYRIGOS, Dalla controversia palamitica 23–24, 43; TIHON, Barlaam 363, 408–410; ACERBI, Barlaam's Paraphrase 24–31; GIOFFREDA, Su scrittura.

all the family relations between manuscript witnesses that were established in the critical editions of Barlaam's scientific works. However, organising the tradition of the *Refutatio* will prove tricky.

The tradition of the *Refutatio* is rooted in Marc. gr. Z. 332 (M) as its prototype; it comprises four families and two copies of M that were not further copied. These two isolated copies are Mosq. Mus. Hist. gr. 315 (m) and Vat. gr. 1756 (v). The four families are led by Ambr. E 76 sup. (A), whose copy is Ambr. R 117 sup. (L); by Neap. III.C.3 (N), whose copy is Vat. Urb. gr. 77 ($V^1$); by the common ancestor of Vat. gr. 187 (V) and Vat. gr. 196 ($V^3$), a copy of the latter being Vat. gr. 188 ($V^2$); and by Par. gr. 2381 ($P^1$), whose copy is Oxon. New College 299 ($O^2$), whose independent copies are Par. gr. 2397 ($P^4$) and Leid. B.P.G. 16D ($L^1$), a copy of the latter being Leid. Periz. Q° 22 ($L^2$). Oxon. Bodl. Savile 1 ($O^1$) contains only a very short Latin summary of the *Refutatio*, but its stemmatic position can be reasonably guessed on the grounds of the contents of the manuscript. Franz's 1840 edition was based on Neap. III.C.3 (N), as the editor himself states[79].

The variant readings we call "characteristic" or "peculiar" are the *Leitfehler* and are not shared by other (families of) manuscript witnesses. The variant readings of any witness are listed by taking the text of its model as a reference; accordingly, we shall sometimes omit pointing out that the witness under scrutiny reproduces all innovations of its model. If a manuscript carries characteristic readings, the minor variant readings it exhibits are listed in reduced font size; these minor variant readings are categorised by the kind of innovation. The following features are not recorded among the variant readings: presence or absence of elision and of movable *ny* and *sigma*; use of standard symbols; differences in spelling (like οὐδὲ μίαν vs. οὐδεμίαν), accent, and punctuation; writing numerals by means of numeral letters or by spelling them in full. All variant readings are identified by the number of the line in which they occur.

### *The ancestor of the tradition: Marc. gr. Z. 332 (M)*

The copy of the *Refutatio* in Marc. gr. Z. 332 (M) is not flawless. There are six mistakes: 43 τὰς$^2$ 248 αὐτοῖς 274 ΑΗΒ 277 ΑΓ 278 ΑΒΓ; a second, wrong letter Θ (*lege* Κ) in the diagram (the problems with lettering are noteworthy)[80]. There is also a misspelling: 52 διορίσθω. Note also the possible lapse 119 διαστάσει (which we do not correct) and the correction at 147 διπλασιεπιδιτρίτῳ. Seventeen other corrections, nearly half of which are words written above the line, were made by the main hand.

### *The family of Ambr. E 76 sup. (A) and Ambr. R 117 sup. (L)*

Ambr. E 76 sup. (A) is a fairly correct copy of Marc. gr. Z. 332 (M). Taking the text of Marc. gr. Z. 332 (M) as a reference, its innovations are 22 ὑπάρχειν (very likely a misreading of the compendium in M) 55 ἢ] ἡ 86 κατομαῖς 92 σχημάτων 100 λογιστικῆς (but λογικῆς after correction in M) 111 ἀπέδωκεν 133–134 τὸ πεπερασμένον (note also the subsequent τῷ corrected from τὸ) 151 τοιοῦτον 219 ἀριθμοὺς 237 ἀριθὸς 272 ΔΓ ἐλάβωμεν (but ΔΓΕ λάβωμεν after correction in M) 277 om. τοῦ (written above the line in M) 292 om. τῶν$^2$ 300 ἑξοῖς, and the omission of the diagram. Note also the capital letter in red at the beginning of line 79. If we exclude M, Ambr. E 76 sup. is the earliest manuscript of the *Refutatio*: it cannot be a copy of any extant witness apart from M. The readings listed above and the discussions in the editions of the *Demonstratio*, of *De eclipsi* I and II, and of the *Logistikē* show that Ambr. E 76 sup. is a copy of Marc. gr. Z. 332[81].

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[79] FRANZ, De musicis graecis 11. Barlaam's text is preceded on 12–14 by *Harm*. III 14–16, headed by the standard title.

[80] The diagrams found in a number of witnesses are reproduced in the Iconographic Complement at the end of this paper.

[81] See MOGENET – TIHON – DONNET, Barlaam 22–24; CARELOS, Barlaam LVI–LVIII; ACERBI, Barlaam's Paraphrase 24–25.

Ambr. R 117 sup. (L) reproduces almost all readings of Ambr. E 76 sup.; its restorations are 86 κατατομαῖς 219 ἀριθμὸς 237 ἀριθμὸς. L is a very good copy of A; excluding the restorations just listed, the innovations of L with respect to A are 24 διήλεκται 26 γινομένων 30 ἐπισκέψασθαι 97 bis τοῦτο καὶ 99 τίνων] τόνων 104 τὴν] τὸν 140 om. καὶ 145 δὶς] διὸ 166 μουσικὸν] μουσικὴν 178 ἐπαγαγὴν 221 ἀποιεῖμαι 233 τῇ[1]] τῶν 240 τὴν[2]] τῇ 245 ὁμοιότητος] ὁμοιότητα 264 ἐπιζεύχθω 320 ἀπαγγέλων.

### *The family of Neap. III.C.3 (N) and Vat. Urb. gr. 77 ($V^1$)*

Neap. III.C.3 (N) and Vat. gr. 187 (V) raise an interesting methodological problem, which we shall tackle in the following subsection. Taking the text of Marc. gr. Z. 332 (M) as a reference, the innovations of N are the restorations 52 διωρίσθω 248 αὐταῖς 274 AKB 277 AΓB 278 AΓB and letter K in the diagram (that is, N corrects almost all mistakes of M), the mistakes 28 τὸ[2]] τὸν 62 om. γὰρ 66 τεμνομένῳ 85 ἁρμονιῶν 90 τὰ] τὸ 104 δεῖ] δὴ 107 παρέσχε 136 ποιήσοιε 137 δόξῃ 146 om. τῶν 147 and 149 μέσον 182 τῷ[1]] τὸ 197 ἐπαγγελομένου 204 παραλειφθέντες 227 συσχηματισμοῖς 227 τῶν ἀριθμῶν 228 πάντα] τἄλλα (shared with V) 232 παραβαλομένου 239 διάμετρος 247 ἐπαγγελομένου 287 om. τοῦ 320 ἀπαγγέλων 328 λέγομεν, the variants 213 τασσομένου (shared with V) 222 τοιοῦτον, and the inversion 97 καὶ τοῦτο.

Vat. Urb. gr. 77 ($V^1$) is an obvious copy of Neap. III.C.3: reading 2 πρῶτον in $V^1$ for the compendium $\alpha^{\chi}$ (= μοναχοῦ) in N already suggests that we are on the right track. $V^1$ reproduces all innovations of its model, sometimes elaborating on the mistake (147 and 149 μέσην). $V^1$ is a careless copy; the copyist was unable to follow Barlaam's argument (103 ἐξαρκεῖ] ἐκ σαρκὶ may suffice), was frequently unable to decipher N's script, had lost contact with the phonological system of ancient Greek (recall 1 κεφάλεα), and seems to take naps from time to time (trivial mistakes tend to cluster). Misspellings and syntagms supplied in the margins abound. The main variant readings of $V^1$ with respect to N are the omissions 6 τε καὶ ἁρμονικαῖς 14 καὶ τῶν 67 ἢ εἰς τόσα 278 ἢν 314 ἂν 319 τε. We refrain from listing the other innovations.

### *The family of Vat. gr. 187 (V), Vat. gr. 196 ($V^3$), and Vat. gr. 188 ($V^2$)*

Vat. gr. 187 (V) was copied shortly after the composition of Barlaam's work. Its text is characterised by the variant 1–2 Βαρλαὰμ μοναχοῦ καὶ φιλοσόφου λόγος ἀνασκευαστικὸς — ἁρμονικῶν. Note also the interventions of the first hand 111 ἀποδέδωκεν] ἀπο– e corr., 228 μεταβάλοι] –λάβοι corr. s.l. m.1, and 235 μεταβληθέντος] –ληφθ– corr. s.l. m.1, and the reading 200 οὐδενὸς γὰρ] οὐδεν {sp. 3 litt}, pointing to a quirk in the model. Other variant readings are as follows.

*Restorations*. 52 διωρίσθω 248 αὐταῖς 274 AKB 277 AB 278 AB *Omissions*. 3 δὲ 67 εἰς[3] 150 λόγον 110 πρὸς ἀλλήλους 287 τοῦ *Mistakes*. 20 ἁρμονικοῖς 37 ἰδίαν] οἰκείαν 67 ρκ] ρ 68 ὁποιοῦν 105 τὰς] τὰ 142 ρπ πρὸς τὸν ξ 151 τὸν] τὸ 165 ὁποίας 180 ἐπόντα 198 ἀριθμητικοὺς 228 πάντα] τἄλλα (shared with N) 242 τὴν μὲν, τῇ ὑπάτῃ 257–258 δ' ἔτι 271 ἔσθω 272 σημείῳ 287 αὐτῷ 311 ἐπηγάγετο *Additions*. 12 τοῦ Πτολεμαίου 94 ἐν οἷς καὶ 275 ἔσονται αἱ *Dittographies*. 274 τούτου *Variants*. 30 ἐπισκέψασθαι 35 ἴσχει] ἔχει 56 ἐπειδὴ 103 οὐδὲ 132 ἐστὶν] εἰσὶν 213 τασσομένου (shared with N)

The main feature of V's text is that it corrects five mistakes in the text of Marc. gr. Z. 332, but not the wrong lettering of the diagram.

Vat. gr. 187 shares several—and crucial—variant readings with Vat. gr. 196 ($V^3$). The latter is characterised by the fact of ending abruptly, at the beginning of the ninth line of a page, with 175 ἤτοι ἐν ᾧ τόπῳ. The characteristic readings shared by Vat. gr. 196 and Vat. gr. 187 are as follows (note that none of the restorations is shared).

*Omissions*. 3 δὲ 110 πρὸς ἀλλήλους *Mistakes*. 20 ἁρμονικοῖς 37 ἰδίαν] οἰκείαν 67 ρκ] ρ 68 ὁποιοῦν *Additions*. 12 τοῦ Πτολεμαίου 94 ἐν οἷς καὶ *Variants*. 30 ἐπισκέψασθαι

These conjunctive readings are formalised in a hyparchetype. Additional innovations of Vat. gr. 196 are as follows.

*Omissions*. 145 δὲ *Mistakes*. 31 συμπτώματι 101 κεφάλεα 108 ἐνεργηκωτάτους 111 παραδέδωκεν (recall the correction in V) 147 διπλασιεπιτρίτῳ 165–166 τὸ δὲ, ὁ Πτολεμαῖος *Additions*. 139 δὲ καὶ

No codicologically conspicuous feature of Marc. gr. Z. 332 or of Vat. gr. 187 (V) helps explain why the text in Vat. gr. 196 ($V^3$) breaks off where it does. We conclude that this relates to a feature of the hyparchetype—most likely, a loss of folios that occurred between the copying of V and the copying of $V^3$. This also entails that we may identify the hyparchetype with a lost manuscript[82]. The presence of a lost manuscript from which Vat. gr. 187 derives confirms the conclusion drawn by the editors of *De eclipsi* I and II on the sole basis of the kinds of mistakes occurring in the Vatican manuscript.

Let us now tackle the methodological problem we have alluded to above. As the text in Marc. gr. Z. 332 (M) is not flawless, and as Neap. III.C.3 (N) and Vat. gr. 187 (V) do not carry M's mistakes but they contribute several innovations of their own, these three witnesses exhibit nearly disjoint sets of innovations. The genealogical method would seemingly force us to organise M, N, and V as independent witnesses of a stemma without an archetype[83]; this contradictis the stemma whose structure we are explaining. In fact, a sensible application of the genealogical method is here required. First, let's free ourselves from the myth of the "best text": an "original" may contain mistakes[84]. Second, correcting M's mistakes is very easy (although they were not so obvious to all copyists): for a misspelling is just a misspelling; 248 αὐτοῖς does not find any masculine or neuter referent in the preceding clauses; the three lettering mistakes in the text are exposed by the mathematical context; a diagram with two letters Θ cries out for revenge. Third, the two mistakes whose correction is not univocal are amended in different ways by N and V: 277 ΑΓΒ 278 ΑΓΒ (which we adopt) versus 277 AB 278 AB, respectively. Fourth, the lettering of the diagram is restored in N only. Fifth, 52 διορίσθω is also corrected in Par. gr. 2381 ($P^1$). Sixth, 52 διορίσθω in M is not corrected in Vat. gr. 196 ($V^3$), which strongly suggests that V (partly) corrects on its own initiative. Seventh, Barlaam's other works in V derive, directly or indirectly, from M. These seven facts make us believe that the most economical explanation of the *varia lectio* is the one we adopt: M is a prototype that contains mistakes; N, V, and partly $P^1$, correct them independently. The issue is complicated by the two innovations shared by N and V: if 213 τασσομένου is a polygenetic trifle, 228 πάντα] τἄλλα is no trifle, even if it might be polygenetic and it makes the resulting designator τἄλλα τὰ περιττά asyntactic. We shall resist using this mistake as the sole basis for positing a hyparchetype above N and the hyparchetype $VV^3$. Likewise, we do not make the lettering mistake in the diagram separate N from the rest of the tradition, thereby grouping the latter under the umbrella of a hyparchetype. There is really no point in multiplying hyparchetypes on the basis of a single shared innovation, when a more economical stemma explains the same *varia lectio*.

Vat. gr. 188 ($V^2$) is a copy of Vat. gr. 196 ($V^3$); the text of the former ends exactly where the text of the latter ends, and this suffices to prove the filiation. $V^2$ also inherits all innovations of $V^3$.

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[82] This remark is not idle: see the discussion in F. ACERBI, Editing Scientific Texts. Lessons from Greek and Byzantine Textual Traditions. *The Vatican Library Review* 2 (2023). This paper also explains the underpinnings of the discussion in the next paragraph.

[83] There would be no archetype because no innovation is shared by the entire tradition. The reading 43 τὰς$^2$ is shared by the entire tradition, but we correct it hesitatingly.

[84] This is very frequently the case if the author is Byzantine: see F. ACERBI, Riscrivere Tolomeo a Bisanzio. Concezione ed appropriazione delle *Tabulae Novae* di Isacco Argiro. *Bollettino dei Classici* 44 (2023).

A characteristic innovation of $V^2$ is the omission of 36–37 τοῖς ὑποκειμένοις — οἰκείως. The other variant readings of $V^2$ with respect to $V^3$ are as follows.

*Restorations*. 52 διωρίσθω *Omissions*. 121 γὰρ *Mistakes*. 3 ὑμᾶς 15 ἔκτινα 24 προσκαιδεκάτου 32 τοινῶν 35 ἔσχει 37 ἔχει 76 ἐνελλιπῶς 83 διπλασίωνι 84 διπλάσιον 99 λόγος] μόγος 101 ὑποθέσεις 101 ἐπίγαγέ 104 ὁμηότητος 106 διαμέτρων (διαμέτρῳ[[ν]] $V^3$) 112 φάσεων 126 ἀνάφορος 132 ὅμοιο 167 πραγματίαν *Dittographies*. 93–94 καὶ τετραγωνίζουσι· καὶ ἐξαγωνίζουσι *Variants*. 103 οὐδὲν μίαν 138 ἔγνωκεν

### *The family of Par. gr. 2381 ($P^1$), Oxon. New College 299 ($O^2$), Par. gr. 2397 ($P^4$), Leid. B.P.G. 16D ($L^1$), and Leid. Periz. Q° 22 ($L^2$)*

This family presents an interesting state of affairs. The learned scholar who penned Par. gr. 2381 ($P^1$) made systematic use of abbreviations, compendia, and signs of all kinds. His system is consistent and most of the time unambiguous, yet it requires time and care to be mastered. His copy of the *Refutatio* is quite correct; he sensibly amended the text; less frequently, he committed a mistake. Shortly after its making, the *Refutatio* in $P^1$ was copied in Oxon. New College 299 ($O^2$). Confronted with a text bristling with abbreviations, the copyist of $O^2$ decided to reproduce it conformally (one shared omission shows that the two witnesses are genealogically linked; three omissions in $O^2$ only make the filiation direction obvious); when he did not do that, Barlaam's original text got modified more often than not. Somewhere between the middle of the 16th century and 1651, the folio containing the last third of the *Refutatio* was torn out from $O^2$. Before that time span, $O^2$ was copied (the three characteristic omissions included) by Konstantinos Mesobotes in Par. gr. 2397 ($P^4$); in 1651, an irremediably defective $O^2$ was copied in Leid. B.P.G. 16D ($L^1$) by Gerard Langbaine the Elder. Both copyists decided to resolve all abbreviations and signs. The outcome is acceptable in the latter case; it is a real disaster in the former case. Mesobotes' copy is so bad as not to allow us to get a likely picture of the last third of the *Refutatio* as it was reproduced in $O^2$.

Before setting out the variant readings of the above-mentioned witnesses, let us point out that a lesson of method can be drawn from the overabundant *varia lectio* associated with this family. The lesson to be drawn is as follows: whenever a text passes through a stage of copying in which abbreviations are used systematically, the *exact* reading of well-defined parts of the original text may be beyond recovery. This happens whenever equally acceptable readings correspond to one and the same abbreviated word or syntagm. As most, if not all, Greek mathematical and astronomical texts have passed through at least one stage of copying in which abbreviations were systematically used[85], we must come to terms with the fact that the exact *dictum* of all of, say, Euclid's *Elements* cannot in principle be recovered. A typical example are the competing forms of Χπλασίων vs. Χπλάσιος: the former is used in Marc. gr. Z. 332, the latter in Oxon. New College 299 and in its copies, which resolved in this way the ambiguous abbreviations $Χπ^{λ}$ occurring in Par. gr. 2381. Replying that this sounds trifling (as in fact it is) just confirms that the philology of Greek and Byzantine mathematical texts may not be interested in the *exact* reading of the "original". Conversely, this phenomenon corroborates the contention that conservative textual criticism is the only sensible stance on editing technical texts[86].

Let us start with Par. gr. 2381 ($P^1$). Its characteristic reading is the omission 136–137 εἴ τις — παραβολὴν. Note also the reading 84 /συν\τεθεῖσαν, where the preverbal συν is most unusually

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[85] See the evidence adduced in F. ACERBI, Topographie du Vat. gr. 1594, in: La «collection philosophique» face à l'histoire. Péripéties et tradition, a cura di D. Bianconi – F. Ronconi. Spoleto 2022, 239–321: 271 n. 80.

[86] On the last point, see already P. TANNERY, review of Heronis Alexandrini opera quae supersunt omnia. *Journal des Savants* (March 1903) 147–157 (April 1904) 203–211, reprinted in ID., Mémoires Scientifiques. I–XVII. Toulouse – Paris 1912–50 III 131–157: 133.

written below the line. Other innovations are as follows (we take it that most of the variant readings are deliberate corrections; we categorise them accordingly).

*Restorations*. 52 διωρίσθω *Corrections*. 4 τινων] τινος 24 διαλέξασθαι 30 ἐπισκέψασθαι 58 ὡς] ὅτι 59 συμβαίνειν 138 παραβάλλειν 172 ἀριθμητὰ (but marg. μήπο [!] ὀφείλει λέγειν ἀριθμητικὰ) 290 ὅταν *Omissions*. 2 μοναχοῦ 50 οὖν 51 τὰ 69 τῆς$^{2}$ 73 χρείαν 81 μὲν 132 ἐστὶν 186 αὐτῇ 263 οἷον 271 καὶ$^{1}$ 273 δὲ 273 καὶ$^{2}$ 287 τοῦ 310 πρὸ *Mistakes*. 237 τούτῳ sed marg. τοῦτο m.1 *Additions*. 17 πρῶτον μὲν 125 ἢ καὶ 149 τῶν διεζευγμένων *Variants*. 1–2 Βαρλαὰμ ἀνασκευὴ — ἁρμονικῶν 218 ἐλάττους *Inversions*. 64 τμήματα ἴσα 145 τὸν ρπ ποιεῖ 218 οὐ περιέχουσι τοὺς τοιούτους λόγους 271 γραφόμενος (κύκλος) 310 ἔχειν βουλομένου 312 δεῖν ᾠήθη *Lettering*. 272 ΔΓ

Oxon. New College 299 ($O^{2}$) inherits all innovations of Par. gr. 2381 and exhibits three characteristic omissions: 1–2 Βαρλαὰμ ἀνασκευὴ — ἁρμονικῶν, 132–133 τὰ δὲ — συμφωνίᾳ, and 178–180 ἐπαγαγεῖν — διαστάσεων, and of course, in its present state, the entire portion of text from 246 συμπεραίνεσθαι on. The copyist of $O^{2}$ did not understand that the συν between the lines in Par. gr. 2381 must be attached to the verb placed above it; accordingly, he wrote 84 τεθεῖσαν. The copyist of $O^{2}$ struggled with $P^{1}$'s abbreviation system; most of the additional variant readings in the Oxford manuscript come from resolving abbreviated forms in $P^{1}$.

*Omissions*. 19 πάλιν 171 ὡς] sp. 2 litt. 245 γὰρ *Mistakes*. 22 ὑπάρχουσι 23 μερικῶν λαμβανάνομεν 38 ἰούσης 38 ἀναιρεῖν 48 τετράγωνος 59 τῶν τξ ἀριθμῶν 63 τηρεῖ 64 ὁμοιώτης 69 δυνάμεως 87 γε] τε 102 ἀνάρμαστον 104 προσθῆναι 112 ἑκάστου 116 ἱστάμενα 133 συντεθέντι 134 τῷ] τὸ 135 τῇ$^{2}$] τῷ 149 περιέχουσι 153 ἔνι 155 τῆς εἰρημένης διαστάσεως 167 οὐδαμῶς 171 ἀριθμητὰ] ἀριθμητικὴ 173 ὅταν 185 λέγεσθαι 197 ὁ $\alpha^{ov}$ ἐχρήσατο ἀριθμῷ 211 ἀριθμοῖς] ἀριθμῷ 213 τάττομεν 214 κατὰ] μετὰ 216 τάσσομεν 218 ἀριθμοὶ] ἀριθμοὺς 219 τούτων 222 τοιοῦτον 226 τοιαῦται 235 ἔχουσιν 241 τετραπλάσιος 243 διπλάσιος *Additions*. 42 τῷ $\theta^{\omega}$ 83 ἐν τῷ 202 ὁ (ἀριθμὸς) 233 τῶν διεζευγμένων *Dittographies*. 177 πρὸς *Variants*. 13 ποιήσομεν 77 ἀριθμῶν μόνων ἀντιμεταληφθέντων 77 τοῦ$^{1}$] τῶν *Inversions*.

Leid. B.P.G. 16D ($L^{1}$) is a copy of $O^{2}$ where all abbreviations are resolved. $L^{1}$ inherits all readings of $O^{2}$. Langbaine also added some marginal notes. In these notes, corrections are proposed (these are preceded by ἴσως); original readings are recorded (preceded by γρ.) when the correction is placed in the main text or, most frequently, when the signs used in $O^{2}$ are reproduced on their first occurrence. On one occasion, Langbaine proposes a wrong correction because he has misunderstood an abbreviation: 142 τριπ$^{\lambda}$ γὰρ ὁ ρπ τ(οῦ) ξ is read as τριπλάσιος γὰρ ὁ ρπ τξ and is consequently amended to τριπλάσιος γὰρ ὁ ρκ {marg. γρ. ρπ} τξ. Among the *marginalia* related to abbreviations, one reads *hoc est ½ et ⅓ et ¼ et ⅙* by the side of a sequence of abbreviations of aliquot parts. Additional innovations of $L^{1}$ are as follows.

*Restorations*. 23 λαμβανομένης 69 δύναμιν 73 ἀναγκαίως … ὁμοιότητα 149 τῇ νήτῃ 177 semel πρὸς 226 τοιαύτας *Corrections*. 38 ἰούσης] οὔσης 38 ἀναιρεῖν] ἀνευρεῖν 116 ἱστάμενα] διιστάμενα 125 (κέντρου)] τεταρτημορίου perperam 134 τὸ] τῷ 135 τῷ] τῇ$^{2}$ 153 ἔνι] ἐνὶ 185 λέγεσθαι] λέγεται 197 ὁ $\alpha^{ov}$ ἐχρήσατο ἀριθμῷ] οὐ πρώτοις ἔχρ. ἀριθμοῖς 218 ἀριθμοὺς] ἀριθμοὶ *Omissions*. 108 τε 145 δὲ 170 ἐστὶ 174 τὰ$^{1}$ 176 τῶν *Mistakes*. 5 ἐναρμοθέντα 19 ἕτερον 26 ἐν οὐρανῷ κινήσεσιν 26 ἐξομοιώσαντες 32 μεταλήψεως 34 παραβεβληθῆναι 40 τῶν … σκοπῶν 43 τοῦ] τῶν 48 τρίγωνος 60–61 τῶν τξ ἀριθμῶν … προσληφθέντων … τούτων 62 ὁμοιότης 65 ἡμικύκλου 71 ὁμοιότητα πανταχοῦ 80 ἁρμονικῇ 90 πρότερα 92 κοινῇ 100 αἰσθητῆς 100 ἐχόμενον 107 ἑκατέρῳ 116 Πτολεμαῖος ἐμμελέστερον 117 παραλογότατον 119 ὁμοίους 171–172 ἀριθμουμένων καὶ μετρουμένων 175 ᾗτε 192 διάστημα 192 συστήματα 193 αὐτῶν 199 μόνος μόνῳ 213 αὐτὴν 228 μεταβαλεῖ *Additions*. 88 δύο γὰρ 236 αὐτὸν 237 ἔτι] ὡς *Variants*. 22 προτέροις 75 ἐθελήσῃ 77–78 τοῦ] τῶν (*nonies*) 107 διπλάσιον 140 ἀναλόγοις 142 τριπλάσιος 155 ἀντιστρέφεται 181 διπλάσιον 182 τετραπλάσιον 185 διπλάσιος 189 τετραπλάσιον 190 τετραπλάσιον 197 δεῖξαι 202 τέταρτον] τεταρτημόρια 208 μόνου 231 τρίτον] τριτημόριον 231 $\varsigma^{ov}$] ἑκτημόριον 234 τεταρτημόριον 234 ἑκτημόριον 240 τριπλάσιος

As seen in the previous section, Marcus Meibom copied conformally Leid. B.P.G. 16D in Leid. Periz. Qº 22 ($L^2$), thereby reproducing the marginalia of the former manuscript. He also put forward a couple of corrections of obviously mistaken readings in its model: 5 ἐναρμοθέντα] ἐναρμοσθέντα and 119 ὁμοίους] ὁμοίως. Additional variant readings are the mistake 133 τῷ[1]] τὸ and the omission 220 τοὺς[2].

Par. gr. 2397 ($P^4$) inherits all innovations of $O^2$ and exhibits three characteristic omissions: 27–31 πιθανῶς — συμπτωμάτων, 93–94 καὶ τετραγωνίζουσι· καὶ ἐξαγωνίζουσι, 113–115 οὐδεμιᾷ — τὰ μὲν, 150–151 ἡ γὰρ — λόγον. The diagram is not present. There are more than one hundred additional variant readings, among which the systematic misreading of forms of προσλαμβανόμενος as forms of προλαμβανόμενος, and likewise τέλειος instead of τέλος. The reader will forgive us for not listing all these innovations.

### *Mosq. Mus. Hist. gr. 315 (m)*

With Vat. gr. 1756 (v), Mosq. Mus. Hist. gr. 315 (m) is the most recent witness of the *Demonstratio*; no extant witness other than v can be a copy of m. The text of Mosq. Mus. Hist. gr. 315 (m) is almost flawless: Margounios strove for accuracy[87]. The only variant readings are 9 δύο 225 τὴν παραβολὴν 247 ἐοικέναι 265 ΔΓΕ corr. e δὲ ΓΕ 272 ΔΓ ἐλάβομεν 322 ὀρθῶς] καλῶς. The diagram is present but it is torn off for the most part; what remains exhibits the incorrect lettering that affects the diagram in most witnesses.

Mosq. Mus. Hist. gr. 315 does not share any of the characteristic readings of the other independent copies of Marc. gr. Z. 332. Therefore, it is an independent copy of it[88].

### *Vat. gr. 1756 (v)*

With Mosq. Mus. Hist. gr. 315 (m), Vat. gr. 1756 (v) is the most recent witness of the *Demonstratio*; no extant witness other than m can be a copy of v. Apparently, Lollino got increasingly tired of copying Barlaam's text: Vat. gr. 1756 is characterised by a series of long omissions (1–2 ταῖς — μοναχοῦ, 30–32 ἐπισκέψεσθαι — εἰληφότα, 34–36 οὐ — παραβαλεῖν, 69 ἢ τῆς τριγώνου — τοιούτων, 185–186 ἀνάλογον δὲ αὐτῇ οὐδαμῶς, 188–189 σύστημα — ἄκρους, 192–193 ὥστε — λόγον, 205–206 πρῶτοι εἰσὶ — λόγους, 252–253 ὡσαύτως — ὁμοίως, 257 φανερὸν — κεφάλαιον, 277–278 ἐλάττων — ἀχθήσεται, 279 καὶ — τινά, 288–289 ὥστε — εἴληφεν). Towards the end of the tract, this degenerates into omissions of entire sections and into plain rewriting by elimination of syntagms perceived as superfluous. The last 36 lines of the text become (we keep the punctuation of Marc. gr. Z. 332):

ἀλλὰ μὲν οὐδὲ ἐν τῷ βῳ κεφαλαίῳ ἀψάμενος φαίνεται τῆς ἐπιγραφῆς· ἡ μὲν γὰρ, τὰς κινήσεις βούλεται τῶν πρώτων σφαιρῶν ἀναγαγεῖν, κατὰ τὰ τάχη, ὑπὸ λόγους ἀριθμητικούς· τούτῳ δὲ, οὔτε περὶ τῶν κινήσεων, οὔτε περὶ τῶν λόγων ἔστι τις λόγος· ἀλλὰ πάλιν τὰ ἐπὶ τοῦ ζῳδιακοῦ, ὑπόκειται αὐτῷ, τῷ λόγῳ·

περὶ τοῦ γου κεφαλαίου, ὅτι μὲν οὐκ ἔστι Πτολεμαίου, δῆλον ἐκ τοῦ τὸν τοῦ Ἄρεος ἀναλόγως ἔχειν τῇ νήτῃ συνημμένων λαμβάνειν· ὅτι δὲ οὐ καλῶς τῇ ἐπιγραφῇ ἥρμοσται, φανερὸν ἔκ τοῦ. τὸν Ἑρμῆν παραλελεῖφθαι πλανώμενον ὄντα, καὶ τοῦ. τοὺς ἑστῶτας μόνον τῶν φθόγγων παρειλῆφθαι, τῆς ἐπιγραφῆς μήτε τινῶν πλανώμενον λεγούσης, ἁπλῶς καὶ καθόλου καὶ πλανωμένων καὶ φθόγγων·

ὃ δὲ μάλιστα θαυμάζω, ὅτι μηδεμίαν τὸ κεφάλαιον ἀναφορὰν ἔχει πρὸς τὰ προσεχῶς εἰρημένα, οὐδὲν γάρ ἐστι τῶν ἐν τῷ βῳ κεφαλαίῳ τοιοῦτον, πρὸς ὃ ἂν. ἀνάγοιτο τὰ ἐν τῷ τρίτῳ·

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[87] This confirms the remarks in WESTERINK, Arethae XVI–XVII and XXII.

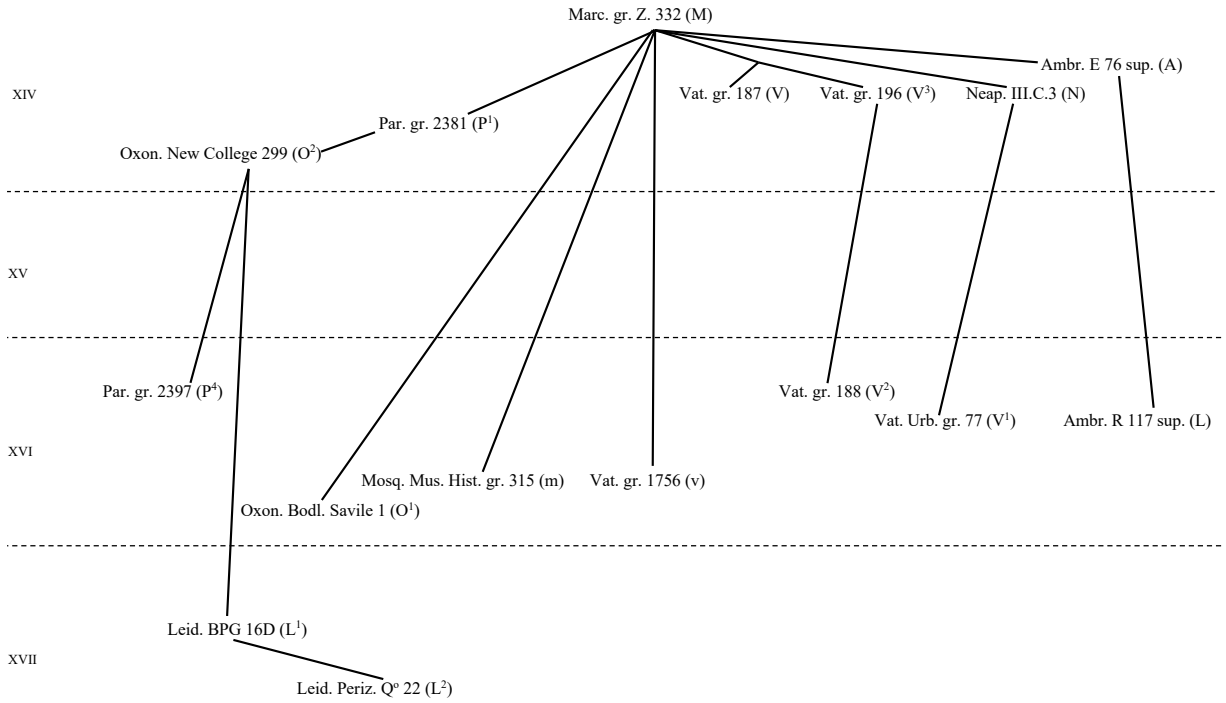
[88] See also MOGENET – TIHON – DONNET, Barlaam 45–46; FYRIGOS, Dalla controversia palamitica 37–64; ACERBI, Barlaam's Paraphrase 28. Fyrigos' discussion allows us to exclude the possibility that Vat. gr. 1756 is a copy of m.

τὰ μὲν δὴ τῆς ἀνασκευῆς, ἐνταῦθα πέρας ἐχέτω. καιρός δ' ἂν. εἴη λοιπὸν ἅπερ οἰόμεθ' αὐτοὶ ὀρθῶς ἔξειν, ἐντεῦθεν [[προ]] ἐκθέσθαι προδιορισαμένους τοσοῦτον, ὅτι παραβαλὼν ἐν τοῖς πρότερον Πτολεμαῖος τὰ ἐν ἁρμονικῇ διαστήματα τοῖς κατ' οὐρανὸν, ἐν τούτοις βούλεται αὐτὰ τὰ ποιοῦντα τὰ διαστήματα παραβαλεῖν· ἅπερ ἔστιν ἐκεῖ μὲν φθόγγοι· ἐνταῦθα δὲ, αἵ τε πρῶται τῶν ἐν τῷ κόσμῳ σφαῖραι, καὶ τὰ ἐν αὐταῖς φερόμενα σώματα· τριῶν δὲ τούτων περὶ ταῦτα ὄντων· ὄγκων τε καὶ κινήσεων καὶ δυνάμεων, ἐν μὲν τῷ ιδ$^{\omega}$, βούλεται παραδοῦναι τὴν κατὰ τοὺς ὄγκους παραβολήν· ἐν δὲ τῷ ιε$^{\omega}$, τὴν κατὰ τὰς κινήσεις· ἐν δὲ τῷ ις$^{\omega}$, τὴν κατὰ τὰς δυνάμεις· λέγομεν τοίνυν

There are dozens of other variant readings in m. The reader will forgive us for not listing them.

The fact that Vat. gr. 1756 does not share any of the characteristic readings of the other independent copies of Marc. gr. Z. 332, Lollino's description of the model, and the discussions in the editions of *De eclipsi* I and II, of the *Demonstratio*, and, most notably, of Barlaam's letters make it certain that the Vatican manuscript is a copy of Marc. gr. Z. 332[89]. This copy is independent of the contemporary copy in Mosq. Mus. Hist. gr. 315, for the omissions listed above prove that m is not a copy of v; conversely, m's readings 247 ἐοικότας 272 ΔΓΕ λάβομεν 322 ὀρθῶς show that v is not a copy of m.

The stemma of the manuscript tradition of the *Demonstratio* is depicted below. Accordingly, our edition reproduces the text of Marc. gr. Z. 332. The required corrections—all of which can be found in late witnesses—are made in the text and recorded in the critical apparatus.




[89] See MOGENET – TIHON – DONNET, Barlaam 44–45; ACERBI, Barlaam's Paraphrase 29; FYRIGOS, Dalla controversia palamitica 37–64. TIHON, Barlaam 367 shows that Vat. gr. 1756 contains only an abridged version of *De paschate*.

## PRELIMINARIES TO THE EDITION

*Edition*. We have reproduced the text of the *Refutatio* as it stands in Marc. gr. Z. 332, keeping the original punctuation and accents (even in cases like φᾶναι). In particular, the grave accent of oxytone words before a weak pause (marked by a comma) is retained. We have marked only upper and lower points; no attempt has been made to detect and reproduce middle points, if indeed any was marked by the copyist. When adscript iota is present in Marc. gr. Z. 332, it is transcribed as follows: τῷ˛. We chose to write the adscript iota because it is a noteworthy feature of the prototype; we chose to write it as we did because we regarded writing, for instance, τῶ˛, as even less reader-friendly. Paragraphs are inserted whenever the text is segmented by a conspicuous blank space. Misspellings and likely mistakes are corrected in the main text and recorded in the critical apparatus; words between back- and forward slashes were written above the line; the brackets are used according to the standard conventions. The critical apparatus is located at the end of the text; it is keyed to it by means of superscript Latin letters. The diagram that accompanies the text in Marc. gr. Z. 332 and in other manuscripts is reproduced in the "Iconographic Complement" at the end of this study. In the Greek text, we have marked where Marc. gr. Z. 332 goes to a new page. In the translation, we have marked the approximate beginning of every tenth line of our edition.

*Translation*. The paragraph structure of the translation is the same as the one of the Greek text. Different Greek terms are translated with different English terms, but the inverse is not always the case. All particles are translated, with intermittent exceptions for coordinant δέ. Specific choices of translation are motivated in the footnotes to the translation. We have strived to render the fast pace Barlaam's *Refutatio* obviously has in Greek. Most importantly, we have strived to render the lexical resonances and the wordplays that make the *Refutatio* a fine piece of technical prose.

## CRITICAL EDITION OF THE *REFUTATIO*

Ἀνασκευὴ εἰς τὰ προστεθέντα τρία κεφάλαια ταῖς τελευταίαις ἐπιγραφαῖς τοῦ τρίτου τῶν τοῦ Πτολεμαίου ἁρμονικῶν· Βαρλαὰμ μοναχοῦ

Ἐπεὶ δὲ καὶ τὰς ἐπιγραφὰς τῶν μὴ σῳζομένων κεφαλαίων ἀπαιτεῖς ἡμᾶς ἐξηγήσασθαι καὶ ἐκδοῦναί τίνα ἂν μάλιστα περὶ αὐτῶν Πτολεμαῖος εἶπεν, ἐκθέμενος πρῶτον τὰ μὴ καλῶς ὑπό τινων ἐναρμοσθέντα κεφάλαια ταῖς εἰρημέναις ἐπιγραφαῖς, καὶ ἐξελέγξας ὅπῃ ὀρθῶς οὐκ ἔχουσι, παρακολουθῶν ταῖς τοῦ Πτολεμαίου ἀστρονομικαῖς τε καὶ ἁρμονικαῖς ὑποθέσεσι, πειράσομαι αὐτὸς περὶ τούτων εἰπεῖν, ὅσα γέ μοι φανεῖται συμφώνως ἔχοντα τοῖς περὶ τῶν (οὐρανίων) σωμάτων καὶ τῶν ἁρμονικῶν συστημάτων ὑπ' ἐκείνου εἰρημένοις· ἔστι δὲ τά γε προστεθέντα κεφάλαια ταῦτα· ὧν τὰ μὲν πρότερα β, νέου τινὸς τῶν καθ' ἡμᾶς ἔοικεν εἶναι· τεκμαίρομαι δὲ τῷ μηδαμοῦ \ἐν/ τοῖς παλαιοῖς τῶν ἀντιγράφων φαίνεσθαι γεγραμμένα· τὸ δὲ τελευταῖον παλαιοῦ τινός ἐστιν· ἐν τοῖς παλαιοτάτοις γὰρ τῶν ἀντιγράφων εὑρήκαμεν αὐτὸ μέσον που |75r τοῦ τρίτου τῶν ἁρμονικῶν κείμενον. ὅτι δὲ οὐδ' αὐτό ἐστι Πτολεμαίου, οὐδὲ οἰκείως ἔχει πρὸς τὴν τελευταίαν ἐπιγραφήν, διὰ τῶν ἑπομένων ἔσται φανερόν· ποιήσομαι δ' ὅμως τὸν λόγον, διὰ τὸ προχειρότερον· ὡς τοῦ αὐτοῦ ὄντων καὶ τῶν τριῶν κεφαλαίων·

Ἀρχὴ οὖν ἔστω αὕτη· τὸν βουλόμενον ἐπιτυχεῖν ἐν τούτοις τῆς τοῦ Πτολεμαίου διανοίας ἕξ τινα δεῖ μάλιστα παραφυλάξαι·

Πρῶτον, μὴ ταυτὰ τοῖς ἤδη εἰρημένοις ἐν \τοῖς/ προλαβοῦσι κεφαλαίοις εἰπεῖν· οἷον ἐπεὶ ἐν τοῖς πρότερον τινὰ τῶν περὶ ἀστρονομίαν τισὶ τῶν κατὰ τὴν ἁρμονικὴν παραβέβληται, παραφυλάττειν δεῖ, ὅπως μὴ τῶν αὐτῶν πάλιν ἐν τούτοις γένηται ἡ παραβολή· δῆλον γὰρ ὡς ὁ Πτολεμαῖος ἕτερα τῶν πρότερον ἐνταῦθα, ἐκ τῶν τῆς ἀστρονομίας τοῖς τῆς ἁρμονικῆς ἐβούλετο παραβαλεῖν· δεύτερον, ὅπως τὰ ἐν τούτοις ῥηθησόμενα, μερικωτέρας ἔχηται θεωρίας, καὶ μὴ ἐξίσου τοῖς πρότερον καθόλου ὑπάρχῃ· διττῆς γὰρ οὔσης τῆς περὶ τούτων ἐφόδου, τῆς μὲν. κοινῆς πάντων ἢ τῶν πλείστων, τῆς δὲ. ἰδίας καθ' ἕκαστον τῶν μερικῶς λαμβανομένων, περὶ μὲν τῶν κοινῶν, μέχρι τοῦ τρισκαιδεκάτου διείλεκται· περὶ δὲ τῶν ἰδίων, ἐντεῦθεν ὑπισχνεῖται διαλέξεσθαι· φησὶ γὰρ

πρὸς τῷ τέλει τοῦ κεφαλαίου· “τὰ μὲν οὖν κοινῶς ἐφαρμοζόμενα ταῖς τῶν ἐμμελειῶν διαφοραῖς, καὶ ταῖς τῶν (οὐρανίων) κινήσεων, ἀπὸ τῶν τοιούτων ἐξομοιώσεων, μάλιστα ἂν κατανοήσαιμεν· λοιπὸν δὲ ἐπισκέψασθαι καὶ τὰ καθ’ ἕκαστον πιθανῶς ἐπιτηρηθέντα διὰ τῶν γενομένων”: δεῖ οὖν τὰ μετὰ ταῦτα ῥηθησόμενα πρὸς τὰ ἤδη εἰρημένα |75v ἔχειν, ὡς τὸ μερικὸν πρὸς τὸ καθόλου· τρίτον· ὅπως αἱ ῥηθησόμεναι τούτων ἐξομοιώσεις, ἐκ τῶν δι’ αἰσθήσεως πάλαι τηρηθέντων, ἔχωσι τὸ πιστόν· τοῦτο γὰρ αὐτὸς ὑπισχνεῖται Πτολεμαῖος, ἐπισκέψεσθαι τὰ διὰ τῶν πολλάκις γενομένων ἐν τῷ παντὶ συμπτωμάτων πιθανῶς ἐπιτηρηθέντα· ταῦτα δέ ἐστι δηλονότι, τὰ δι’ αἰσθήσεως τὴν ἀρχὴν τῆς περὶ αὐτὰ καταλήψεως εἰληφότα. δεῖ οὖν τοιούτων τινῶν τοὺς μετὰ ταῦτα ἠρτῆσθαι λόγους· τέταρτον, ὅπως οἰκεία ᾖ. ἡ παραβολὴ, καὶ μὴ συμβῇ τὰ ἀνόμοια τοῖς ἀνομοίοις. ἀλλ’ ὡς ἐνδέχεται μάλιστα τὰ ὅμοια τοῖς ὁμοίοις· καὶ οἷς τί ταυτὸ συμβέβηκε παραβληθῆναι· οὐ γὰρ τὰ τυχόντα τοῖς τυχοῦσι φᾶναι δεῖ ἐοικέναι· ἀλλ’ ὅσα φύσει ἴσχει τινὰ ὁμοιότητα, ταῦτα καὶ τῷ λόγῳ ἀλλήλοις παραβαλεῖν· πέμπτον, ὀρθῶς τῇ λέξει χρῆσθαι, καὶ οἰκείως τοῖς ὑποκειμένοις πράγμασιν· ἕκτον, ὅπως ἕκαστον τῶν κεφαλαίων οἰκείως ἔχῃ πρὸς τὴν ἰδίαν ἐπιγραφὴν, ὥστε δύνασθαί τινα καὶ μὴ οὔσης τῆς ἐπιγραφῆς, ἀνευρεῖν αὐτὴν ἐξ αὐτοῦ τοῦ λόγου·

Τούτων οὖν τῶν ἓξ ἀναγκαίως ὀφειλομένων τηρεῖσθαι, τῷ μὴ παντάπασιν ἐκπεσεῖν προαιρουμένῳ τοῦ περὶ ταῦτα τοῦ Πτολεμαίου σκοποῦ, φανεῖται ὁ τὰ παρόντα ἐκδοὺς κεφάλαια, οὐδενὸς αὐτῶν πεφροντικώς.

Πρῶτον μὲν γὰρ τοῦ Πτολεμαίου, ἐντελῶς ἤδη ἔν τε τῷ ἑνάτῳ καὶ δεκάτῳ τοῦ παρόντος συντάγματος παραδεδωκότος, τὰς τοῦ διὰ μέσων τῶν ζῳδίων κύκλου, πρὸς τὰ{ς} ἐν ἁρμονικῇ παραβολὰς, ὅπως τε αἱ κατ’ αὐτὸν |76r κατανενοη[[.]]μέναι, σύμφωνοι καὶ δραστικαὶ στάσεις, ὁμοίως ἔχουσι τοῖς τοῦ ἡρμοσμένου συμφώνοις, τῷ τε διὰ τῶν ἰσαρίθμων ταῖς συμφωνίαις τομῶν ἀπαρτίζεσθαι, καὶ τῷ περιέχειν τοὺς τοιούτους σχηματισμοὺς, τούς τε τῶν ὁμοφώνων καὶ συμφώνων λόγους πρὸς τῷ τονιαίῳ, καὶ τίνι τῶν συμφωνιῶν τίς τῶν σχηματισμῶν προσέοικεν, ὅτι ὁ μὲν κατὰ διάμετρον τῇ διὰ πασῶν συμφωνίᾳ, ὁ δὲ τριγωνικὸς τῇ διὰ πέντε· ὁ δὲ τετραγωνικὸς τῇ διὰ τεσσάρων, καὶ προσθέντος τὸ τῆς ὁμοιότητος αἴτιον, καὶ ἔτι ἀποθεμένου ὡς ἤδη ἀπηρτισμένον τὸν περὶ τούτων λόγον, ἐν οἷς λέγει, “τὰ μὲν οὖν παρ’ αὐτὴν τὴν ἐγκύκλιον κίνησιν θεωρούμενα κατ’ ἀμφοτέρας τὰς ἁρμονίας, καὶ τὰ κοινῶς καλούμενα σύμφωνά τε καὶ διάφωνα τῶν σχημάτων, ἐπὶ τοσοῦτον διωρίσθω[a]”, οὗτος ὡς μηδενὸς τοιούτου εἰρημένου, πάλιν τῶν αὐτῶν τούτων παραδίδωσιν ἡμῖν τὰς πρὸς τὰ ἁρμονικὰ παραβολάς· φάσκων τὴν μὲν διάμετρον στάσιν ἐοικέναι τῷ προσλαμβανομένῳ, ἣν ἐκεῖνος παρέβαλε τῇ˛ διὰ πασῶν συμφωνίᾳ. τὴν δὲ τρίγωνον, τῇ˛ ὑπάτῃ μέσων ἢ ὑπ’ ἐκείνου παραβέβληται τῇ διὰ πέντε συμφωνίᾳ. τὴν δὲ τετράγωνον, τῇ˛ νήτῃ διεζευγμένων· ἣν ἐκεῖνος τῇ διὰ τεσσάρων παραβέβληκεν. ἔτι ἐπεὶ οἱ τοιοῦτοι σχηματισμοὶ κατὰ τὰς ἀκριβεῖς τῶν πλανωμένων ἐποχὰς κρίνονται· αἱ δ’ ἀκριβεῖς ἐποχαὶ κατὰ τὸν διὰ μέσων τῶν ζῳδίων κύκλον θεωροῦνται, φανερὸν ὡς ἐνταῦθα τοῦ ζῳδιακοῦ κύκλου πεποίηται καὶ οὗτος τὴν |76v κατατομὴν, διὰ τοῦ τξ ἀριθμοῦ· ὥστε πάλιν συμβαίνει αὐτῷ ἐνταῦθα, περὶ τῆς ὁμοιότητος τοῦ τελείου συστήματος καὶ τοῦ ζῳδιακοῦ κύκλου διαλέγεσθαι· περὶ οὗ ἤδη ἐκεῖνος διείλεκται, τοῦ τξ ἀριθμοῦ μόνον ἐν τούτοις προσληφθέντος· καὶ τούτου μάτην καὶ ἀσυλλογίστως ὅσα γε πρὸς τὸν τῆς τούτων ὁμοιότητος λόγον· εἴπερ γὰρ ἡ κατὰ διάμετρον στάσις ἔοικε τῷ προσλαμβανομένῳ, ὡς οὗτός φησιν, ἄν τε ἄτμητόν τις τηρῇ˛ τὸ ἡμικύκλιον, ἄν τε εἰς ὁποσαδηποτοῦν τέμῃ, οὐδὲν ἧττον ἡ τούτων ὁμοιότης σωθήσεται· ἄλογον γὰρ τὸ φᾶναι, ὅτι εἰς μὲν ρπ ἴσα τμήματα τεμνομένου τοῦ ἡμικυκλίου ἔοικεν ἡ διάμετρος στάσις τῷ˛ προσλαμβανομένῳ˛, ἀτμήτου δὲ μένοντος, ἢ κατ’ ἄλλον ἀριθμὸν τεμνομένου, οὔ. ὁμοίως δὲ καὶ ἡ τρίγωνος στάσις οὐ διότι τὸ τριτημόριον τῆς ὅλης περιφερείας εἰς ρκ τέμνεται, ἔοικε τῇ ὑπάτῃ μέσων· τὸ γὰρ εἰς τόσα ἢ εἰς τόσα διαιρεῖν, ἢ τὴν ὅλην περιφέρειαν, ἢ τὸ τρίτον αὐτῆς μέρος, ἢ τὸ τέταρτον· ἢ ὁποιονοῦν ἄλλο, οὐκ ἀλλοιοῦν ἐστι τὴν τῶν κατ’ αὐτὰς στάσεων, ἢ τῆς τριγώνου ἢ τῆς τετραγώνου, ἢ ἄλλής τινος τῶν τοιούτων δύναμιν· ἀλλ’ εἰς ὅσ’ ἄν τις διέλῃ˛ τὴν ὅλην τοῦ κύκλου περιφέρειαν, ἡ αὐτὴ τοῖς τοιούτοις σχηματισμοῖς πρὸς τὰ ἐν ἁρμονικῇ ὁμοιότης πάντως σῴζεται· ἡ μὲν γὰρ ὁμοιότης, φύσει αὐτοῖς πρόσεστιν· ἡ δὲ εἰς τόσα κατατομὴ, λόγῳ μόνῳ ἐπινενόηται· ὥστε ὅ γε προσληφθεὶς ἐνταῦθα ἀριθμὸς, οὐδεμίαν ἀναγκαίαν χρείαν παρέχεται ταῖς ἐκκειμέναις ὁμοιότησι· δῆλον δὲ. καὶ ἐκ τοῦ τὰ αὐτὰ δυνατὸν

εἶναι δεικνύναι, καὶ ἐλάττονος ἀριθμοῦ μεταληφθέντος· τεμνομένου γὰρ τοῦ παντὸς |77r κύκλου καὶ εἰς δώδεκα ἴσα τμήματα, εἴ τις ἕκαστον τῶν τμημάτων μοῖραν ἐθελήσει ὀνομάσαι, ἐξέσται αὐτῷ διὰ τῶν αὐτῶν λέξεων, ἅπαντα τὰ ἐν τῷ πρώτῳ καὶ δευτέρῳ κεφαλαίῳ εἰρημένα, ἀνελλιπῶς δεικνύναι, τῶν ἀριθμῶν μόνον μεταληφθέντων, ἀντὶ μὲν τοῦ τξ, τοῦ ιβ· ἀντὶ δὲ τοῦ ρπ, τοῦ ϛ· ἀντὶ δὲ τοῦ ρκ, τοῦ δ· ἀντὶ δὲ τοῦ ρ, τοῦ γ· ἀντὶ δὲ τοῦ ξ, τοῦ β·

Φανερὸν οὖν ἐκ τῶν εἰρημένων, ὡς ἅπερ ἤδη τῶν περὶ ἀστρονομίαν ὑπὸ τοῦ Πτολεμαίου τοῖς ἐν ἁρμονίᾳ παραβέβληται, τῶν αὐτῶν[b] καὶ οὗτος πεποίηται ἐν τούτοις τὴν παραβολήν· ἔτι δὲ τῶν ἐν τῷ δευτέρῳ κεφαλαίῳ λεγομένων οὐδέν ἐστιν, ὃ οὐ καὶ πρότερον ἐλέχθη· τὸ μὲν γὰρ, τὴν μὲν διὰ τεσσάρων συμφωνίαν ἐν ἐπιτρίτῳ εἶναι λόγῳ, τὴν δὲ, διὰ ε ἐν ἡμιολίῳ. τὴν δὲ διὰ πασῶν ἐν διπλασίονι, τὴν δὲ δὶς διὰ πασῶν ἐν τετραπλασίονι, καὶ τὸ καθάπερ ὁ ἐπίτριτος λόγος συντεθεὶς τῷ ἡμιολίῳ, ποιεῖ τὸν διπλασίω λόγον, οὕτω καὶ τὴν διὰ τεσσάρων συμφωνίαν συντεθεῖσαν τῇ \διὰ/ πέντε ποιεῖν τὸ διὰ πασῶν, ταῦτα πάντα ἔν τε τῷ ζῳ καὶ ηῳ τοῦ πρώτου[c] τῶν ἁρμονικῶν ἱκανῶς εἴρηται· ὅπως δὲ αἱ τοιαῦται συμφωνίαι ὁμοίως ἔχουσι ταῖς ἐν τῷ ζῳδιακῷ κατατομαῖς, ἐν τῷ θῳ τοῦ τρίτου παραδέδοται· ὥστε ἔν γε τῷ δευτέρῳ τούτῳ κεφαλαίῳ, οὐδὲν πλέον τῶν πρότερον εἴρηκεν, ὅτι μὴ καὶ ἐλλιπέστερον, δύο παραλελοιπὼς συμφωνίας· τοῦ Πτολεμαίου τῶν λόγων πασῶν τῶν συμφωνιῶν, ἕκαστον ἑκάστης πολλαχῶς δείξαντος γινόμενον διὰ τῆς εἰς ιβ τοῦ ζῳδιακοῦ τομῆς· ἀλλὰ μὴν οὐδὲ μερικωτέρας ἔχεται θεωρίας παρὰ τὰ πρότερον κατὰ τὴν τοῦ Πτολεμαίου ὑπόσχεσιν τὰ ἐν τούτοις τοῖς |77v κεφαλαίοις λεγόμενα· ὁ γὰρ περὶ τῶν δραστικῶν σχηματισμῶν λόγος, τῶν κοινῶς λεγομένων περὶ τῶν πλανωμένων ἐστί· τέσσαρες γὰρ τῶν πλανωμένων ἀλλήλους τε καὶ τοὺς λοιποὺς τρεῖς, καὶ διαμετροῦσι· καὶ τριγωνίζουσι· καὶ τετραγωνίζουσι· καὶ ἑξαγωνίζουσι· διὸ καὶ Πτολεμαῖος περὶ τούτων διείλεκται, ἐν οἷς περὶ τῆς κοινῆς αὐτῶν ἐφόδου, διαλέγεσθαι ὑπέσχετο·

Ὅτι δὲ οὐδὲ ἐκ τῶν δι' αἰσθήσεως τηρηθέντων ἔχουσι τὰ ἐνταῦθα λεγόμενα τὸ πιστὸν, ὡς ἔφη τοῦτο καὶ Πτολεμαῖος, φανερόν· τὸ γὰρ εἰς τόσα· ἢ εἰς τόσα διαιρεῖν· ἢ τὸν ὅλον κύκλον· ἢ τὰ μέρη αὐτοῦ, καὶ τὸ φᾶναι τήνδε μὲν τὴν στάσιν, ἐοικέναι τῷδε τῷ φθόγγῳ, τήνδε δὲ τῷδε, καὶ τὸ[d] διεξιέναι τίνες μὲν ἀριθμοὶ περιέχουσι τὸν δεῖνα λόγον, τίνες δὲ τὸν δεῖνα, καὶ τίς λόγος ἐκ τίνων σύγκειται, ἐξ οὐδεμιᾶς τηρήσεως ἐστὶν εἰλημμένα οὐδ' αἰσθητικῆς· ἀλλὰ λογικῆς ἐστιν[e] ἐχόμενα ὑποθέσεως· διὸ οὐδ' αὐτὸς ὁ τὰ κεφάλαια ταῦτα ἐκθέμενος ἐπήγαγέ τι τοιοῦτον πρὸς τὴν τῶν λεγομένων πίστιν· ἔτι δὲ καὶ τὴν τῶν ἐκκειμένων παραβολὴν, ἀνάρμοστον παντάπασι πεποίηται· πρῶτον μὲν, οὐδεμίαν παρασχόμενος ἀπόδειξιν τῆς τῶν ὑποκειμένων ὁμοιότητος· οὐ γὰρ ἐξαρκεῖ τὸ φᾶναι τόδε τῷδε ἐοικέναι· ἀλλὰ δεῖ καὶ τὴν τῆς ὁμοιότητος προσθεῖναι αἰτίαν· ὅπερ Πτολεμαῖος πανταχοῦ φαίνεται ποιῶν, καὶ τὰς ἐξομοιώσεις λέγων, καὶ τὰς αὐτῶν πίστεις παρεχόμενος· ὡς ἐπὶ τῶν συμφωνιῶν· τὴν γὰρ διὰ πασῶν συμφωνίαν εἰπὼν ἐοικέναι τῇ διαμέτρῳ κατὰ τὸν κύκλον στάσει, τρία τεκμήρια |78r τούτου παρέσχε\το/· τό, τε τὸν διπλασίω λόγον ἐν ἑκατέρᾳ περιέχεσθαι, καὶ πλεῖστον ἰσότητος παρὰ τὰς ἄλλας στάσεις τε καὶ συμφωνίας, καὶ ἐνεργητικωτάτους εἶναι, καὶ τοὺς κατὰ διάμετρον τῶν ἀστέρων τοῦ ζῳδιακοῦ σχηματισμοὺς, καὶ τῶν φθόγγων τοὺς ποιοῦντας πρὸς ἀλλήλους τὸ διὰ πασῶν· ὁμοίως δὲ καὶ τῶν ἄλλων τὰς παραβολὰς, πρὸς ταῖς ἐγχωρούσαις παραστάσεσιν ἀποδέδωκεν· ἐπιστήμονος γὰρ ἔργον, οὐ τὸ ὅτι μόνον· ἀλλὰ καὶ τὸ διότι περὶ ἕκαστον δεικνύναι· οὗτος δὲ, φάσκων ἐοικέναι τοὺς ἑστῶτας τῶν φθόγγων ταῖς ἐκκειμέναις στάσεσιν, οὐδεμιᾷ χρῆται παραστάσει· τοῦτο τὲ οὖν ἥμαρτε, καὶ ὅτι τὰ ἀνόμοια τοῖς ἀνομοίοις παρέβαλε· δύο γὰρ ὄντων τούτων ἐν ἑκατέρᾳ τῶν ἐπιστημῶν· διαστημάτων τε καὶ τῶν ποιούντων τὰ διαστήματα, ἤτοι τῶν διισταμένων, εὔλογον τὰ μὲν διαστήματα τοῖς διαστήμασι παραβάλλειν· τὰ δὲ διιστάμενα τοῖς διισταμένοις· ὅπερ Πτολεμαίῳ ἐμμελέστατα πανταχοῦ τετήρηται· οὗτος δὲ, τὰ διαστήματα τοῖς διισταμένοις παραβάλλει· πρᾶγμα παραλογώτατον ποιῶν· τὸν γὰρ προσλαμβανόμενον[f] διιστάμενον καὶ ποιητικὸν διαστάσεως ἀλλ' οὐ διάστημα ὄντα, τῇ κατὰ διάμετρον ἀφομοιοῖ διαστάσει· ὁμοίως δὲ καὶ τοὺς ἄλλους· φέρε γὰρ, ἐπεὶ ταῦτα ἐοικέναι ἀλλήλοις λέγομεν οἷς τι συμβέβηκε ταυτὸ, τί ταυτὸ σὺ εὑρὼν, τῷ τε προσλαμβανομένῳ καὶ τῇ διαμέτρῳ στάσει, φῂς ταῦτα ἀλλήλοις παραβάλλεσθαι· ἡ μὲν γὰρ διάμετρος στάσις, μεγίστη καὶ ἐνεργητικωτάτη τῶν ἄλλων· ὁ δὲ προσλαμβανόμενος, ἐλάχιστός τε καὶ ἀσθενέστατος |78v τῶν ἐν

τῷ τελείῳ συστήματι φθόγγων, ἅτε βαρύτατος πάντων ὤν· καὶ ἐν μὲν τῇ διαμέτρῳ στάσει, περιέχεται ὁ διπλασίων λόγος τοῦ τε ὅλου κύκλου πρὸς τὸ ἡμικύκλιον, καὶ τῆς διαμέτρου πρὸς τὴν ἐκ τοῦ κέντρου· ἐν δὲ τῷ προσλαμβανομένῳ, ἢ ἐν ἄλλῳ τινὶ φθόγγῳ, οὐδεὶς περιέχεται λόγος· "μόνος γάρ φησιν ἕκαστος τῶν φθόγγων ἄλογος· καὶ πρὸς ἑαυτὸν ἀδιάφορος· ὁ γὰρ λόγος τῶν πρός τι, καὶ ἐν δυσὶ τοῖς πρώτοις"· καὶ ἡ μὲν διάμετρος στάσις, δύο τινῶν ἐστι μεταξὺ τῶν κατὰ διάμετρον ἀλλήλων διισταμένων· ὁ δὲ προσλαμβανόμενος, οὐ πέπονθε τοῦτο· βαρύτερος γὰρ αὐτοῦ φθόγγος ἐν τῷ τελείῳ συστήματι οὐκ ἔστι· πῶς οὖν ὁ προσλαμβανόμενος τῇ διαμέτρῳ στάσει παραβληθήσεται, μηδενὸς αὐτοῖς τοῦ αὐτοῦ συμβεβηκότος· εἶτα εἰ ὁ προσλαμβανόμενος ἔοικε τῇ διαμέτρῳ στάσει· ἐπεὶ κατὰ τὸν Πτολεμαῖον τῇ τοιαύτῃ στάσει ἔοικεν ἡ διὰ πασῶν συμφωνία· τὰ δὲ τῷ αὐτῷ ὅμοια καὶ ἀλλήλοις ἐστὶν ὅμοια, ἔσται ἄρα ὁ προσλαμβανόμενος, ὅμοιος τῇ διὰ πασῶν συμφωνίᾳ· τὸ διιστάμενον τῇ διαστάσει· τὸ ἁπλοῦν τῷ συνθέτῳ· τὸ πέρας τῷ πεπερασμένῳ· τὸ ἄλογον τῷ ἐν λόγῳ· ἀλλὰ μὴν καὶ εἴ τις ἀναστρέψας εἴποι τὴν μὲν νήτην τῶν ὑπερβολαίων ἐοικέναι τῇ διαμέτρῳ στάσει· τὴν δὲ νήτην τῶν διεζευγμένων τῇ τριγώνῳ, τὴν δὲ ὑπάτην μέσων τῇ τετραγώνῳ, τὸν δὲ προσλαμβανόμενον τῇ ἑξαγώνῳ, εἴ τις οὖν οὕτω ποιήσειε τὴν παραβολὴν, οὐδὲν χεῖρον δόξει εἰρηκέναι, ὅτι μὴ καὶ βέλτιον· ὅσῳ τὰ τιμιώτερα τοῖς τιμιωτέροις ἔγνωκε παραβαλεῖν· |79r τί οὖν ἡ διάμετρος στάσις τῷ προσλαμβανομένῳ μᾶλλον, καὶ οὐ τῇ νήτῃ τῶν ὑπερβολαίων παραβληθήσεται· συνίσταται δὲ τὸ παράλογον τῆς ἐκκειμένης παραβολῆς, καὶ ἐκ τοῦ μὴ ταῖς ἀνάλογον στάσεσι παραβαλεῖν τοὺς φθόγγους· ἡ μὲν γὰρ νήτη τῶν ὑπερβολαίων πρὸς τὸν προσλαμβανόμενον, ἐν τετραπλασίονι ἐστὶ λόγῳ· ἡ δὲ διάμετρος στάσις, πρὸς τὴν ἑξάγωνον ἐν τριπλασίονι· τριπλασίων γὰρ ὁ ρπ τοῦ ξ· καὶ πάλιν ἡ μὲν νήτη τῶν διεζευγμένων πρὸς τὸν προσλαμβανόμενον, ἐν τριπλασίονι ἐστὶ λόγῳ, κατὰ τὴν διὰ πασῶν καὶ διὰ πέντε συμφωνίαν· ἡ δὲ διάμετρος στάσις πρὸς τὴν τετράγωνον, αἷς τοὺς τοιούτους φθόγγους παραβέβληκεν, ἐν διπλασίονι· δὶς γὰρ ὁ ρ, ποιεῖ τὸν ρπ· ὡσαύτως δὲ καὶ ἡ μὲν νήτη τῶν ὑπερβολαίων πρὸς μὲν τὴν νήτην τῶν διεζευγμένων, ἐν ἐπιτρίτῳ ἐστὶ λόγῳ, κατὰ τὸ διὰ τεσσάρων· πρὸς δὲ τὴν ὑπάτην μέσων, ἐν διπλασιεπιδιτρίτῳ[g] κατὰ τὸ διὰ πασῶν καὶ διὰ τεσσάρων· ἡ δὲ ἑξάγωνος στάσις πρὸς μὲν τὴν τετράγωνον, ἐν ἡμιολίῳ, ἐστὶ λόγῳ· πρὸς δὲ τὴν τρίγωνον, ἐν διπλασίονι· ὁμοίως δὲ τῆς νήτης διεζευγμένων πρὸς ὑπάτην μέσων, διπλασίονα περιεχούσης λόγον, οὗτος παραβάλλει τοὺς τοιούτους φθόγγους στάσεσι περιεχούσαις τὸν ἐπίτριτον λόγον· ἡ γὰρ τρίγωνος στάσις πρὸς τὴν τετράγωνον, τοῦτον ἔχει τὸν λόγον· οὐ μὴν, ἀλλὰ καὶ τοὺς περιέχοντας τὸν διαζευκτικὸν τόνον δύο ἑστῶτας φθόγγους, ἀνομοιότατα παραβέβληκεν· ἐχρήσατό \τε/ γὰρ αὐτοῖς[h] ὡς ἑνὶ, δυσὶ καὶ διαφόροις οὖσι κατὰ τὴν δύναμιν, καὶ τοῖς |79v αὐτοῖς πάλιν καὶ αὐτοὺς παραβάλλει· ἐπεὶ γὰρ παραβάλλεσθαι αὐτούς φησι τῇ στάσει, ἀφ' ἧς ἐστιν ἡ ἀρχὴ τῶν εἰρημένων διαστάσεων, τὰ δὲ διιστάμενα ἀντιστρέφει πρὸς ἄλληλα· ὅτε γὰρ \ὁ/ Ἄρης διαμετρεῖ τὸν Κρόνον. τότε καὶ ὁ Κρόνος διαμετρεῖ τὸν Ἄρην, καὶ ἐξαγωνίζει ὅ τε Κρόνος τὸν Δία καὶ ὁ Ζεὺς τὸν Κρόνον, ὁμοίως καὶ ἐπὶ τῶν ἄλλων, φανερὸν ὡς καὶ τούτους τοὺς φθόγγους τοῖς τέσσαρσι σχηματισμοῖς παραβάλλει· καὶ περὶ μὲν τοῦ, ἀνάρμοστον πεποιηκέναι αὐτὸν τὴν παραβολὴν, τοσαῦτα εἰρήσθω·

Περὶ δὲ τοῦ μὴ ὀρθῶς κεχρῆσθαι τῇ λέξει, ἐντεῦθεν ῥητέον·

Πρῶτον μὲν οὖν ἡμάρτηται αὐτῷ, τὸ, "τούτοις οὖν παραβαλλομένου τοῦ τελείου τῆς μουσικῆς συστήματος"· οὐ γὰρ ὅλου τοῦ τοιούτου συστήματος ἐστὶν ἡ παραβολὴ, οὐδὲ τοῦθ' ὑπισχνεῖται ἡ τοῦ κεφαλαίου ἐπιγραφή· ἀλλὰ τῶν ἑστώτων μόνον φθόγγων, καὶ τὸ, "τῆς μουσικῆς", παρελκόντως καὶ παρὰ τὴν τοῦ Πτολεμαίου χρῆσιν προσείληπται· τὸ μὲν, ὅτι ἤρκει φάναι "τοῦ τελείου συστήματος"· δῆλον γάρ ἐστι ποίας ἐπιστήμης ἐστὶ τὸ τοιοῦτον σύστημα· τὸ δὲ, ὅτι Πτολεμαῖος μουσικὴν καὶ μουσικὸν ἐπὶ τῆς χειρουργικῆς μόνον χρήσεως ἐκλαμβάνει, τήνδε δὲ τὴν πραγματείαν, οὐδαμοῦ "μουσικὴν" προσεῖπεν· ἀλλ' "ἁρμονικήν"· ἔδει οὖν "τοῦ τελείου τῆς ἁρμονικῆς συστήματος" εἰπεῖν· εἶτα καὶ τὸ. "τῇ στάσει τουτωνὶ τῶν ἀριθμητικῶν διαστημάτων", οὐ καλῶς ἔχει· ἡ γὰρ στάσις, οὐ τῶν διαστημάτων ἐστὶ στάσις· ἀλλὰ τῶν διισταμένων· ἵσταται μὲν γὰρ τὰ διιστάμενα· |80r οὐ τὰ διαστήματα· ἡ δὲ στάσις τῶν ἱσταμένων ἐστὶ στάσις· ἀλλὰ μὴν καὶ τὰ διαστήματα οὐ καλῶς προσεῖπεν "ἀριθμητικά"· "ἀριθμητὰ" γὰρ ὡς "ἀριθμούμενα" καὶ "μετρούμε-

να" λεγέσθω· "ἀριθμητικὰ" δὲ οὔ· ἐπεὶ γὰρ πᾶν διάστημα διισταμένων ἐστὶ διάστημα, τότε μόνον τὸ διάστημα καλῶς ἔχει "ἀριθμητικὸν" ὀνομάσαι, ὅτε τὰ διιστάμενα ἀριθμοί εἰσιν· ἐνταῦθα δὲ διιστάμενά εἰσι, τὰ τῶν ἀστέρων μεγέθη· ὥστε οὐκ ἔδει "ἀριθμητικὰ" τὰ τοιαῦτα διστήματα προσαγορεῦσαι· ἔτι δὲ καὶ τὸ. "τῇ˛ ὅθεν ἡ ἀρχὴ τῶν εἰρημένων διαστάσεων, ἤτοι ἐν ᾧ˛ τόπῳ˛ νοεῖται ἡ στάσις τοῦ ἡλίου", ἀμούσως εἴρηται· ἑτέρου γὰρ ὄντος τοῦ τόπου παρὰ τὴν τῶν ἐν αὐτῷ ἱσταμένων στάσιν, εἰ μὲν πρὸς τὴν στάσιν ἐβούλετο ποιῆσαι τὴν παραβολὴν, ἔδει μετὰ τὸ εἰπεῖν "τῇ ὅθεν ἡ ἀρχὴ τῶν εἰρημένων διαστάσεων", ἐπαγαγεῖν "καθ' ἣν νοεῖται ὁ ἥλιος ἱστάμενος"· ἢ ἄλλός τις· εἰ δὲ πρὸς τὸν τόπον, ὤφειλε γράφειν "τῷ τόπῳ, ἐξ οὗ ἡ ἀρχὴ τῶν εἰρημένων διαστάσεων· ἐν ᾧ νοεῖται ἡ στάσις τοῦ ἡλίου". τὸ δὲ "τῇ στάσει" εἰπόντα, ἐπενεγκεῖν τὸ, "ἐν ᾧ τόπῳ", ἄμουσον ὡς εἰπεῖν καὶ ἀσύντακτον· ὁμοίως δὲ ἡμαρτημένως τὸν διπλασίονα λόγον, ἀνάλογον ἔφη τῷ διὰ πασῶν· καὶ τὸν τετραπλασίω ἀνάλογον τῷ˛ δὶς διὰ πασῶν· τὸ γὰρ ἀνάλογον, οὐ τῶν λόγων κατηγορεῖται· ἀλλὰ τῶν ὅρων τῶν τοὺς λόγους περιεχόντων· οὐδαμοῦ γὰρ οἱ μαθηματικοὶ τοὺς λόγους αὐτοὺς, ἀνάλογόν τῳ˛ φασὶν εἶναι· ἀλλὰ τοὺς ὅρους τοὺς τὸν αὐτὸν ἔχοντας λόγον· αἴτιον οὖν τῆς διὰ πασῶν συμφωνίας λεγέσθω ὁ διπλασίων |$_{80v}$ λόγος· ἀνάλογον δὲ αὐτῇ οὐδαμῶς·

Ἔτι ἐπεὶ τὸ τέλειον σύστημα "δὶς μὲν διὰ πασῶν" ὀνομάζεται. κατὰ τοὺς ἄκρους φθόγγους, σύστημα δὲ τέλειον κατὰ τὴν σύνταξιν τὴν τοιάνδε τῶν μεταξὺ τῶν ἄκρων φθόγγων, ἀνάγεται δὲ ὑπὸ τὸν τετραπλασίω λόγον, οὐ κατὰ τοὺς μεταξὺ φθόγγους· ἀλλὰ κατὰ τοὺς ἄκρους, ἔδει μὴ λέγειν ἀνάλογον ἔχειν τὸν τετραπλασίω λόγον, τῷ˛ δὶς διὰ πασῶν τελείῳ συστήματι· ἀλλὰ μόνον τῷ˛ δὶς διὰ πασῶν ὁμοφώνῳ· οὐ γὰρ καθὸ τέλειόν ἐστι σύστημα, ἀνάγεται ὑπὸ τὸν τοιοῦτον λόγον· ἀλλὰ κατὰ τοὺς ἄκρους φθόγγους· ὥστε ὡς ἁπλῶς διαστήματος, καὶ οὐχ ὡς συστήματος ἔδει ἐνταῦθα ποιεῖσθαι τὸν περὶ αὐτοῦ λόγον·

Ἃ μὲν οὖν καὶ περὶ τὴν λέξιν αὐτῷ ἡμάρτηται, ταῦτά ἐστιν·

Ὅτι δὲ οὐκ οἰκείως ἔχουσι τὰ κεφάλαια πρὸς τὰς ἰδίας ἐπιγραφὰς, κατανοῆσαι οὐ χαλεπόν·

Πρῶτον μὲν γὰρ ἐν τῇ πρώτῃ τῶν ἐπιγραφῶν τοῦ Πτολεμαίου διὰ πρώτων ἀριθμῶν ἐπαγγελλομένου δείξειν τὰ τῆς παραβολῆς, οὗτος οὐ πρώτοις ἐχρήσατο ἀριθμοῖς· ἔστι δὲ πρῶτος ἀριθμὸς κατὰ τοὺς μαθηματικοὺς, τριχῶς· καθ' ἑαυτὸν, καὶ πρὸς ἕτερον, καὶ τάξει· καθ' ἑαυτὸν μὲν, ὁ μονάδι μόνῃ˛ μετρούμενος· οἷον ὁ ε, ἢ ὁ ζ· πρὸς ἕτερον δὲ, πρὸς ὃν οὐ χρῆται κοινῷ μέτρῳ˛ ἀριθμῷ˛ τινι, οἷον πέπονθεν ὁ η πρὸς τὸν ιε· ὑπ' οὐδενὸς γὰρ τοῦ αὐτοῦ ἀριθμοῦ μετροῦνται· τάξει δὲ, ὥσπερ λέγομεν ὅτι τῶν ἐχόντων ἥμισυ καὶ τρίτον |$_{81r}$ μέρος, πρῶτος ἐστὶν ὁ ς· ἐπεὶ ἐλάττων αὐτοῦ ἀριθμὸς, οὐκ ἔχει τὰ τοιαῦτα μέρη· καὶ τῶν ἐχόντων ἥμισυ καὶ τέταρτον, πρῶτος ἐστὶν ὁ δ· ὁμοίως καὶ ἐπὶ τῶν ἄλλων· τριχῶς οὖν λεγομένου ἐν ἀριθμοῖς τοῦ πρώτου, κατ' οὐδένα τῶν τρόπων πρῶτοι εἰσὶν οἱ παραληφθέντες ὑπὸ τούτου ἀριθμοί· ὁ γὰρ τξ. καὶ ὁ ρπ· καὶ ὁ ρκ· καὶ ὁ ϙ· καὶ ὁ ξ· οὗτοι οἱ ἀριθμοὶ, οὔτε καθ' ἑαυτούς εἰσι πρῶτοι, οὔτε πρὸς ἀλλήλους· οὔτε πρῶτοι εἰσὶ τῶν ἐχόντων αὐτοῖς τοὺς αὐτοὺς λόγους· ἐπεὶ γὰρ ὑπὸ τῶν στάσεων, περὶ ὧν ποιεῖται οὗτος τὸν λόγον, συμβαίνει τὸν κύκλον τέμνεσθαι, εἵς τε δύο ἴσα· καὶ εἰς τρία· καὶ εἰς τέσσαρα· καὶ εἰς ς, ἀριθμοῦ μόνον ἐδεῖτο, ὅστις ἔχει ἥμισυ· καὶ τρίτον· καὶ τέταρτον· καὶ ἕκτον μέρος· πρῶτος δὲ τῶν ἐχόντων τὰ τοιαῦτα μέρη ἐστὶν ὁ ιβ· οὐχ ὁ τξ· φανερὸν οὖν ὡς εἴπερ Πτολεμαῖος διὰ τούτων τῶν ἀριθμῶν ἐβούλετο παραδοῦναι τὰς παραβολὰς, ἀνοήτως καθάπαξ εἶχε πρὸς τὰ ἀριθμητικά· ἀγνοῶν τί ποτέ ἐστι τὸ ἐν ἀριθμοῖς πρῶτον· περιέχει δὲ πρῶτος ἀριθμὸς, τοὺς τῶν ἑστώτων φθόγγων τοῦ τελείου συστήματος λόγους, κατὰ μὲν τὴν ὕλην ἤτοι τὰς χορδὰς, ὅταν τῷ μήκει μόνῳ διαφέρωσιν, ὁ λς· κατὰ δὲ τὴν αὐτῶν τῶν φθόγγων [[.]] δύναμιν, ὁ λβ· ταῖς μὲν γὰρ χορδαῖς ταττομένου τοῦ προσλαμβανομένου κατὰ τὸν λς, ἡ μὲν ὑπάτη ὑπατῶν, ἔσται τοιούτων λβ· ἡ δὲ ὑπάτη μέσων, κδ· ἡ δὲ˙ μέση, ιη· ἡ δὲ παραμέση, ις· ἡ δὲ νήτη, διεζευγμένων ιβ· ἡ δὲ νήτη ὑπερβολαίων, θ· πάλιν δὲ |$_{81v}$ κατὰ τὸ εἶδος τασσομένου τοῦ προσλαμβανομένου κατὰ τὸν η, ἡ μὲν ὑπάτη ὑπατῶν, τοιούτων ἔσται θ· ἡ δὲ μέσων ὑπάτη, ιβ· ἡ δὲ μέση ις· ἡ δὲ παραμέση ιη· ἡ δὲ νήτη, διεζευγμένων κδ· ἡ δὲ νήτη ὑπερβολαίων λβ· ἐλάττονες δὲ τούτων ἀριθμοὶ, τοὺς τοιούτους οὐ περιέχουσι λόγους· ὁ δὲ τούτου τξ ἀριθμὸς, περιέχει μὲν τοὺς τῶν ἑστώτων φθόγγων λόγους, τούς γε κατὰ τὴν ὕλην· ἀλλ' οὐ πρῶτος· εἶτα δέον διελεῖν τὸν τξ ἀριθμὸν, εἰς τοὺς ἀριθμοὺς τοὺς περιέξοντας αὐτοὺς τοὺς τῶν

ἑστώτων φθόγγων λόγους, καὶ ἀπονεῖμαι ἑκάστῳ τῶν φθόγγων, τὸν ἀνάλογον ἀριθμὸν, οὐδὲν τοιοῦτο πεποίηκεν, ἐπ' ἄλλ' ἄττα μὴ προσήκοντα ἐκτραπείς·

Τὸ μὲν δὴ πρῶτον τῶν ἐν τῷ πρώτῳ˛ κεφαλαίῳ˛ ἁμαρτημάτων, περὶ τὸ ἐν ἀριθμοῖς συμβεβηκὸς πρῶτον, τοιοῦτόν ἐστι· δεύτερον δὲ, ὅτι τοῦ Πτολεμαίου παραλαμβάνοντος ἐνταῦθα τοὺς ἀριθμοὺς, ὡς ἀναγκαίους πρὸς τὸν τῆς παραβολῆς λόγον, καὶ ὡς ἀδυνάτου ὄντος, ἄνευ τῶν ἀριθμῶν τὰ τῆς παραβολῆς δειχθῆναι, οὗτος ἡμῖν τοιαύτας παραδίδωσι παραβολὰς, οἵας μηδενὸς δεῖσθαι ἀριθμοῦ, πλὴν τοῦ τοῖς σχηματισμοῖς συστοίχου· δῆλον δὲ, εἴ τις τὸν ἀριθμὸν ἀφελόμενος καὶ πάντα τὰ περιττὰ, οὐτωσὶ μεταβάλοι τὸν λόγον, τεμνομένου γὰρ τοῦ παντὸς κύκλου εἵς τε δύο ἴσα· καὶ τρία· καὶ δ· καὶ ς, τὸν μὲν κατὰ διάμετρον σχηματισμὸν, (σελήνης) πρὸς ἥλιον, ἢ ἄλλού τινος τῶν πλανωμένων, κατὰ τὸ ἥμισυ φαμὲν εἶναι τῆς ὅλης περιφερείας, τὸν δὲ τρίγωνον, κατὰ τὸ τρίτον· τὸν δὲ |82r τετράγωνον, κατὰ τὸ τέταρτον· τὸν δὲ ἑξάγωνον κατὰ τὸ ς$^{ov}$· τούτοις δὲ παραβαλλομένου τοῦ τελείου συστήματος, ὁ μὲν προσλαμβανόμενος παραβληθήσεται τῇ \κατὰ/ τὸ ἡμικύκλιον στάσει· ἡ δὲ τῶν μέσων ὑπάτη, τῇ κατὰ τὸ τριτημόριον· ἡ δὲ νήτη διεζευγμένων, τῇ κατὰ τὸ τέταρτον· ἡ δὲ νήτη ὑπερβολαίων, τῇ κατὰ τὸ ἕκτον·

Οὕτως οὖν τοῦ λόγου μεταβληθέντος, ἆρ' ἔστι τι ἔλαττον ἔχουσα ἡ τῆς ὁμοιότητος ἀπόδοσις; οὐδαμῶς· οὐκοῦν εἰ ὄντος τὲ καὶ μὴ τοῦ ἀριθμοῦ, οἱ αὐτοὶ τῇ παραβολῇ σῴζονται λόγοι, περιέργως ὁ ἀριθμὸς εἴληπται· ἔτι δὲ φανερόν ἐστι τοῦτο μᾶλλον καὶ ἐκ τοῦ δυνατὸν εἶναι τοῖς τοιούτοις ἀριθμοῖς χρῆσθαι πρὸς παράστασιν τῆς τῶν φθόγγων πρὸς τὰς στάσεις ἀνομοιότητος· σκόπει γάρ· ἐπεὶ ἡ μὲν κατὰ διάμετρον στάσις μοιρῶν ἐστιν ρπ· ἡ δὲ καθ' ἑξάγωνον ξ, ὧν τριπλασίων ἐστὶν ὁ λόγος, ἀδύνατον τὴν μὲν, τῷ˛ προσλαμβανομένῳ ἐοικέναι· τὴν δὲ, τῇ˛ νήτῃ τῶν ὑπερβολαίων· ὧν τετραπλασίων ἐστὶν ὁ λόγος· καὶ πάλιν ἐπεὶ ἡ μὲν τρίγωνος στάσις μοιρῶν ἐστιν ρκ, ἡ δὲ τετράγωνος ρ, ὧν ἐπίτριτός ἐστιν ὁ λόγος, ἀδύνατον τῇ μὲν, τὴν ὑπάτην μέσων, τῇ δὲ, τὴν νήτην διεζευγμένων ἐοικέναι, ὧν διπλασίων ἐστὶν ὁ λόγος· καὶ ἁπλῶς ῥᾴστά τις τοῖς ἀριθμοῖς τούτοις παρακολουθῶν ἀποδείξει, μηδὲν μετεῖναι ταῖς στάσεσι πρὸς τοὺς φθόγγους ὁμοιότητος· ὥστε πρός γε τὴν ὁμοιότητα, οὐδὲν χρησιμεύουσιν· ἀδύνατον γὰρ ἐκ τῶν αὐτῶν τἀναντία συμπεραίνεσθαι·

Ἀλλὰ μὴν καὶ τοῦ Πτολεμαίου ἐπαγγελ|82vλομένου δείξειν ἐοικότας τοὺς ἑστῶτας τῶν φθόγγων ταῖς πρώταις σφαίραις τῶν ἐν τῷ κόσμῳ, οὗτος οὐ ταύταις, οὐ τοῖς ἐν αὐταῖς$^{j}$ σώμασιν· ἀλλὰ τοῖς τούτων διαστήμασι τοὺς φθόγγους παραβάλλει· σφαίρας δὲ πρώτας ὀνομάζει Πτολεμαῖος, οὐ τοὺς ἀστέρας αὐτούς· ἀλλ' ἐν αἷς αὐτοὶ φέρονται· οἷον σεληνιακὴ μὲν σφαῖρα ἐστὶν, ἐν ᾗ ἡ σελήνη φέρεται· αὕτη δέ ἐστιν, ἧς κέντρον μὲν τὸ αὐτὸ τῷ παντὶ διάστημα δὲ, τὸ μέγιστον τῆς σελήνης ἀπόστημα· ὡσαύτως δὲ καὶ ἐπὶ τοῦ ἡλίου· ἡλιακὴ λέγεται σφαῖρα, ἧς κέντρον μὲν, τὸ εἰρημένον. διάστημα δὲ, τὸ μέγιστον αὐτοῦ ἀπόστημα· καὶ ἐπὶ τῶν ἄλλων ὁμοίως· ἑπτὰ δὲ οὐσῶν τῶν τοιούτων σφαιρῶν, ἰσαρίθμων δὲ καὶ τῶν ἑστώτων φθόγγων τοῦ τελείου συστήματος, βούλεται ταύταις τούτους κατὰ βάθος παραβαλεῖν· δι' ἣν δ' αἰτίαν, ἐν τοῖς ἑπομένοις ἔσται δῆλον· οὗτος δὲ, οὐδένα ἐν τούτοις περὶ τῶν τοιούτων σφαιρῶν πεποίηται λόγον·

Φανερὸν οὖν ἐκ τούτων ὅτι ἀνοικείως ἔχει πρὸς τὴν ἐπιγραφὴν τὸ τοιοῦτον κεφάλαιον· ἥμαρτε δέ τι παρὰ ταῦτα καὶ ἕτερον ἐν τῷδε τῷ κεφαλαίῳ, τὸ τὰς τῶν διισταμένων διαστάσεις κατὰ τὰς περιφερείας ἐκλαμβάνειν· δέον κατὰ τὴν ἐπιζευγνυμένην ἀπὸ τοῦ ἑτέρου ἐπὶ τὸ ἕτερον εὐθεῖαν· δύο γὰρ δοθέντων σημείων ἀπείρων μὲν δι' αὐτῶν δυναμένων γράφεσθαι περιφερειῶν, εὐθείας δὲ μόνον μιᾶς, κατ' οὐδεμίαν τῶν περιφερειῶν φαμὲν αὐτὰ διίστασθαι |83r ἀπ' ἀλλήλων· ἄπειροι γὰρ ἂν. οὕτω γε τῶν αὐτῶν αἱ διαστάσεις εἶεν· ἀλλὰ κατὰ μόνην τὴν εὐθείαν· αὕτη γὰρ μία καὶ ὡρισμένη, ἅτε ἐλαχίστη οὖσα τῶν ἀπὸ τοῦ αὐτοῦ σημείου ἐπὶ τὸ αὐτὸ ἀγομένων γραμμῶν· οἷον ἔστωσαν δύο σημεῖα, τὰ ΑΒ· καὶ ἐπεζεύχθω ἡ ΑΒ εὐθεῖα· καὶ τετμήσθω δίχα κατὰ τὸ Γ· καὶ ἀνεστάτω ἀπὸ τοῦ Γ τῇ ΑΒ πρὸς ὀρθὰς ἐφ' ἑκάτερα τὰ μέρη, ἡ ΔΓΕ$^{k}$· ἀπείρων δὴ σημείων δυναμένων λαμβάνεσθαι ἐν ἑκατέρᾳ τῶν ΔΓ ΓΕ, καὶ ἑκάστου τῶν γραφομένων κύκλων κέντρῳ˛ ἑκάστῳ τῶν σημείων καὶ διαστήματι τῷ ἀπὸ τοῦ σημείου μέχρις ὁποτερουοῦν τῶν ΑΒ σημείων ἥκοντος, καὶ διὰ τοῦ ἑτέρου, ἀπείρους συμβήσεται ἀπὸ τοῦ Α ἐπὶ τὸ Β περιφερείας γράφεσθαι· εἰλήφθω γὰρ τυχὸν σημεῖον ἐπὶ τῆς ΔΓ, τὸ Ζ· καὶ ἐπεζεύχθωσαν αἱ ΖΑ, ΖΒ· καὶ ἐπεὶ ἴση ἐστὶν ἡ

ΑΓ τῇ ΓΒ, κοινὴ δὲ καὶ πρὸς ὀρθὰς ἡ ΓΖ, ἴση ἄρα ἐστὶν ἡ ΖΑ τῇ ΖΒ· ὁ ἄρα κέντρῳ μὲν τῷ Ζ· διαστήματι δὲ τῷ ΖΑ κύκλος γραφόμενος, ἥξει καὶ διὰ τοῦ Β σημείου· γεγράφθω· καὶ ἔστω ὁ ΑΗΒ· ὁμοίως δὲ κἂν ἄλλο τι σημεῖον ἐπὶ τῆς ΔΓΕ[l] λάβωμεν, ὡς τὸ Θ, καὶ κέντρῳ μὲν αὐτῷ, διαστήματι δὲ τῷ ΘΑ γράψωμεν κύκλον, ἥξει καὶ αὐτὸς καὶ διὰ τοῦ Β σημείου· ἔστω οὖν ὡς ὁ ΑΚΒ[m]· ἀπειραχῶς οὖν τούτου γινομένου, εἰ μὲν κατὰ τὰς περιφερείας αἱ διαστάσεις τῶν διισταμένων νοοῦνται, ἄπειροι ἔσονται τοῦ Β ἀπὸ τοῦ Α διαστάσεις· |83v ἄπειροι γὰρ αἱ περιφέρειαι· ὅπερ ἄτοπον· εἰ δὲ ἀνάγκη μίαν εἶναι καὶ ὡρισμένην τὴν τοῦ Β, ἀπὸ τοῦ Α διάστασιν, μία δὲ καὶ ὡρισμένη ἐστὶν ἡ ΑΓΒ[n] εὐθεῖα, ἐλάττων γὰρ αὐτῆς ἢ μείζων εὐθεῖα, ἀπὸ τοῦ Α ἐπὶ τὸ Β οὐκ ἀχθήσεται, ἀνάγκη τὴν ΑΓΒ[o] εὐθεῖαν, τὴν διάστασιν εἶναι, ἣν φαμὲν τὰ ΑΒ ἀπ' ἀλλήλων διίστασθαι, καὶ οὐ τὴν ΑΗΒ περιφέρειαν· ἢ τὴν ΑΚΒ, ἢ ἄλλην τινά· οὕτως οὖν καὶ ἐπὶ τοῦ ζῳδιακοῦ τὰ διαμετροῦντα ἄλληλα σώματα, τὴν διάμετρον αὐτὰ [[δια]] διεστηκέναι φαμὲν, εὐθεῖαν οὖσαν καὶ οὐ τὴν περιφέρειαν· σημεῖον δὲ, πλεῖστον αὐτὰ διεστᾶναι λέγομεν, οὐ τῷ τὴν ἡμίσειαν τῆς ὅλης περιφερείας μεγίστην εἶναι· ἔστι γὰρ αὐτῆς καὶ μείζονα λαβεῖν· ἀλλὰ τῷ τὴν διάμετρον μεγίστην εἶναι τῶν ἐν τῷ κύκλῳ εὐθειῶν· ὁμοίως δὲ καὶ τὰ τριγωνίζοντα ἄλληλα, φαμὲν διεστᾶναι τὴν τοῦ ἰσοπλεύρου τριγώνου πλευρὰν, τοῦ ἐγγραφομένου εἰς τὸν κύκλον. ἥτις ὑποτείνει τὸ τρίτον τῆς ὅλης περιφερείας· τὸν αὐτὸν δὲ τρόπον καὶ ἐπὶ τῶν ἄλλων στάσεων· φανερὸν δὲ καὶ ἐξ αὐτῶν τῶν ὀνομάτων· τρίγωνος γὰρ ὀνομάζεται σχηματισμὸς, οὐκ ἀπὸ τοῦ τρίτου τῆς περιφερείας· ἀλλ' ἀπὸ τῆς ὑποτεινούσης αὐτὸ εὐθείας. ὁμοίως δὲ καὶ τετράγωνος, ἀπὸ τῆς \τοῦ/ τετραγώνου πλευρᾶς, εὐθείας οὔσης· ὥστε οὐκ ὀρθῶς τῶν τοιούτων σχηματισμῶν τὰς διαστάσεις, κατὰ τὰς περιφερείας εἴληφεν· ἰστέον μέντοι ὡς οἱ ἀστρονόμοι χρῶνται μὲν τῷ τῆς διαστάσεως ὀνόματι, καὶ ἐπὶ τῶν περιφερειῶν· |84r ὡς ὁπόταν ἐπὶ τοῦ ζῳδιακοῦ κύκλου τὴν ἀπὸ ἐποχῆς τινὸς τῶν ἀστέρων, μέχρις ἐποχῆς ἄλλου τινὸς ἐπισκέπτωνται περιφέρειαν· πλὴν οὐ κυρίως· ἀλλ' ὁμωνύμως· δηλῶσαι μὲν γὰρ βούλονται μόνον τὴν τῆς μεταξὺ περιφερείας πηλικότητα· δι' οὗ δ' ἂν. ὀνόματος τοῦτο ποιήσωσιν, οὐδὲν αὐτοῖς διαφέρει·

Ἀλλὰ μὲν οὐδὲ ἐν τῷ δευτέρῳ κεφαλαίῳ, ἀψάμενος φαίνεται ὅλως τῆς ἐπιγραφῆς· ἡ μὲν γὰρ, τὰς κινήσεις βούλεται τῶν πρώτων ἐν τῷ κόσμῳ, σφαιρῶν ἀναγαγεῖν, κατὰ τὰ τάχη, ὑπολόγους ἀριθμητικούς· τούτῳ δὲ, οὔτε περὶ τῶν κινήσεων τῶν σφαιρῶν, οὔτε περὶ τῶν λόγων τῶν κινήσεων ἔστι τις λόγος· ἀλλὰ πάλιν τὰ ἐπὶ τοῦ ζῳδιακοῦ διαστήματα, ὑπόκειται αὐτῷ, τῷ λόγῳ·

Περὶ δὲ τοῦ τρίτου κεφαλαίου, ὅτι μὲν οὐκ ἔστι Πτολεμαίου, δῆλον ἐκ τοῦ τὸν τοῦ Ἄρεος ἀναλόγως ἔχειν τῇ νήτῃ συνημμένων λαμβάνειν· ἐκβαλὼν γὰρ τὸ λεγόμενον συνημμένον σύστημα τοῦ τέλειον εἶναι, ἀποδείξας μόνον εἶναι τοιοῦτον τὸ δὶς διὰ πασῶν, οὐδένα ἐν τοῖς ἑξῆς περὶ τοῦ συνημμένου συστήματος πεποίηται λόγον· οὔτε ἐν ταῖς τοῦ κανόνος κατατομαῖς οὔτε ἐν ταῖς πρὸς τὰς ψυχὰς καὶ τὰ (οὐράνια) παραβολαῖς· ἀλλ' ἐν πᾶσι τὸ τέλειον αὐτῷ ὑπόκειται σύστημα· εἶτα εἰ τὰς πρώτας τῶν ἐν τῷ κόσμῳ σφαίρας τοῖς ἑστῶσι φθόγγοις τοῦ τελείου συστήματος ἐν τῷ ιδ[ω] παρέβαλε, δῆλον ὡς καὶ ἡ τοῦ Ἄρεος, τινὶ τῶν τοιούτων φθόγγων ὑπ' ἐκείνου παρεβλήθη. καὶ οὐ τῇ νήτῃ συνημμένων· ὃς οὐκ ἔστι φθόγγος τοῦ τελείου συστήματος· |84v ὅτι δὲ οὐ καλῶς τῇ ἐπιγραφῇ ἥρμοσται, φανερὸν ἔκ τε τοῦ. τὸν Ἑρμῆν παραλελεῖφθαι πλανώμενον ὄντα, καὶ τοῦ. τοὺς ἑστῶτας μόνον τῶν φθόγγων παρειλῆφθαι, τῆς ἐπιγραφῆς μήτε τινῶν πλανωμένων λεγούσης, μήθ' ἑστώτων φθόγγων· ἀλλ' ἁπλῶς καὶ πλανωμένων καὶ φθόγγων·

Ὃ δὲ μάλιστα θαυμάζω, τοῦ συντεταχότος τὸ τελευταῖον τοῦτο κεφάλαιον, τοῖς πρὸ αὐτοῦ, ὅτι τοῦ "μή τις δὲ οἰέσθω"· ἀναφορὰν βουλομένου ἔχειν πρὸς τὰ προσεχῶς πρὸ αὐτοῦ, καὶ διὰ ταῦτα δέον, τὰ ἐν τῷ δευτέρῳ κεφαλαίῳ εἰρημένα τοιαῦτ' εἶναι, ἐφ' οἷς ἂν. ἁρμοζόντως ἐπήγετο τὰ ἐν τούτῳ λεγόμενα, οὐδὲν φροντίσας εἴ τις αὐτοῦ ἐπιλάβοιτο, τὰ ἀνάρμοστα συναρμόσαι ᾠήθη δεῖν· οὐδὲν γάρ ἐστι τῶν ἐν τῷ δευτέρῳ, τοιοῦτον, πρὸς ὃ ἂν. ἀνάγοιτο τὰ ἐν τῷ, τρίτῳ·

Ἃ μὲν οὖν ἔχοι τις ἂν. εἰπεῖν καὶ περὶ τοῦ μὴ ἀκολούθως ταῖς ἐπιγραφαῖς τὰ κεφάλαια αὐτὸν ἐκθέσθαι, ταῦτά ἐστιν· ὅτι δὲ τοπαράπαν οὐχ ἥψατο τῆς περὶ ταῦτα τοῦ Πτολεμαίου διανοίας, οὐ χαλεπὸν ἐκ τῶν εἰρημένων συμπεράνασθαι· ἓξ γὰρ ὄντων ἀναγκαίων αὐτῷ, τηρεῖσθαι, τοῦ θ' ἕτερα τῶν πρότερον λέγειν καὶ μερικώτερα· καὶ τηρήσεων ἐχόμενα· καὶ οἰκείως παραβαλλόμενα· καὶ ὑγιῶς ταῖς λέξεσι δηλούμενα· καὶ οἰκείως ἔχοντα πρὸς τὰς ἐπιγραφὰς, κατὰ πάντα φαίνεται

ἡμαρτηκὼς. ταυτά τε τοῖς πρότερον λέγων, καὶ ἐπίσης καθόλου μήτε τηρήσεσι προσχρώμενος, |85r
μήτε οἰκείαν ποιούμενος τὴν παραβολήν, μεθ' ὑγιῶς ἀπαγγέλλων, μήτε περὶ ὧν εἰσιν αἱ ἐπιγραφαὶ τὸν λόγον ποιούμενος·

Τὰ μὲν δὴ τῆς ἀνασκευῆς, ἐνταῦθα πέρας ἐχέτω. καιρός δ' ἂν. εἴη λοιπὸν ἅπερ οἰόμεθ' αὐτοὶ ὀρθῶς ἔξειν, ἐντεῦθεν ἐκθέσθαι προδιορισαμένους τοσοῦτον, ὅτι παραβαλὼν ἐν τοῖς πρότερον Πτολεμαῖος τὰ ἐν ἁρμονικῇ¸ διαστήματα τοῖς κατ' οὐρανὸν, ἐν τούτοις βούλεται αὐτὰ τὰ ποιοῦντα τὰ διαστήματα παραβαλεῖν· ἅπερ ἔστιν ἐκεῖ μὲν φθόγγοι· ἐνταῦθα δὲ, αἵ τε πρῶται τῶν ἐν τῷ¸ κόσμῳ σφαῖραι, καὶ τὰ ἐν αὐταῖς φερόμενα σώματα· τριῶν δὲ τούτων περὶ ταῦτα ὄντων· ὄγκων τε καὶ κινήσεων καὶ δυνάμεων, ἐν μὲν τῷ ιδ$^{ῳ}$, βούλεται παραδοῦναι τὴν κατὰ τοὺς ὄγκους παραβολήν· ἐν δὲ τῷ ιε$^{ῳ}$, τὴν κατὰ τὰς κινήσεις· ἐν δὲ τῷ ις$^{ῳ}$, τὴν κατὰ τὰς δυνάμεις· λέγωμεν τοίνυν

[a] διωρίσθω mNP$^{1}$V : διορίσθω M [b] τῶν αὐτῶν e τὸν αὐτὸν fecit m.1 M [c] τοῦ πρώτου e τῷ πρότῳ fecit m.1 M [d] τὸ e corr. M [e] λογικῆς ἐστιν e corr. M [f] προσλαμβανόμενον e corr. M [g] –διτρίτῳ e corr. M [h] αὐτοῖς corr. e του– M [i] δὲ e με fecit m.1 M [j] αὐταῖς NV : αὐτοῖς M [k] ΔΓΕ e corr. M [l] ΔΓΕ λάβωμεν e corr. M [m] ΑΚΒ NV : ΑΗΒ M [n] ΑΓΒ N : ΑΒ V : ΑΓ M [o] ΑΓΒ N : ΑΒ V : ΑΒΓ M

## TRANSLATION OF THE *REFUTATIO*

Refutation of the three chapters added to the last titles of the third <book> of Ptolemy's harmonics; by the monk Barlaam

Since you[90] ask us to explain the titles of the unpreserved chapters too[91], and to publish what exactly Ptolemy said about them, I shall try myself—after setting out first the chapters some authors[92] did not finely tune[93] to the titles mentioned, and after refuting, by closely following Ptolemy's astronomic and harmonic models, the points where they are not correct—to say about them all that will seem *to me*[94] to be in consonance with what has been said by him about the heavenly bodies and the harmonic systems. The added chapters are indeed as above[95]. The first 2 of them appear to have been recently authored by someone among our contemporaries[96]; I infer this |10 from the fact that <these chapters> do not show up witnessed anywhere in the oldest copies[97]. The last <chapter> was authored by someone of old, for in the oldest copies we have found it as part of the text some-

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[90] The addressee is unknown, and it might well be fictitious.

[91] This adverbial καί "too" does not entail that Barlaam explained other portions of the *Harmonica*. Adverbial καί is highly idiomatic of ancient Greek; its abundance is a notorious headache for any translator. For adverbial καί in Greek mathematics see F. ACERBI, The Logical Syntax of Greek Mathematics (*Sources and Studies in the History of Mathematics and the Physical Sciences*). Heidelberg – New York 2021, 297–299.

[92] We use "author(s)" (and "opponent(s)" in these footnotes), but Barlaam refers to them by means of pronouns. Here, as intermittently elsewhere, we have turned the construct from passive to active. Passive constructs are much more idiomatic in Greek than in English; their frequency in the *Refutatio* is not higher than one would expect though.

[93] We take it that Barlaam makes deliberate use of technical terms of harmony in their generic sense. Two technical ranges are involved: the one that includes the verb ἐναρμόζω "to tune" and the adjective ἀνάρμοστος "out of tune", and the one that includes συμφώνως "in consonance", as at line 7 below.

[94] Italics translates the particle γε.

[95] This sentence proves that the presence of *Harm*. III 14–16 before the *Refutatio* is an original, and essential, feature of the *Refutatio* itself.

[96] This "someone" (and our "opponent") is Nikephoros Gregoras. Barlaam, a sharp polemicist, never mentions the polemical targets of his technical writings; for the *Demonstratio* being aimed at George Pachymeres, see ACERBI, Barlaam's Paraphrase 10.

[97] The oldest witness—and the only independent one—of the texts located as the last three chapters of the *Harmonica* is Par. Coislin 173, the blueprint of Gregoras' exegetic work on Ptolemy's treatise, annotated by Gregoras himself. See the section "A Background to the *Refutatio*" for a fuller discussion.

where in the middle of the third <book> of the harmonics[98]. That this is not Ptolemy's either, nor does it fit[99] the last title, will be clear in what follows[100]. Nevertheless, for the sake of a greater simplicity I shall frame my argument as if the three chapters were authored by the same person.

Now, let the beginning be this: anyone willing to be, in these <chapters>, in keeping with Ptolemy's thought must carefully meet six requirements.

First, never say the same as what has already been said in the foregoing chapters. For instance, since in the previous <chapters> something related to astronomy has been mapped[101] into something related to harmonics[102], one must be careful about there not coming about again, in these matters, a mapping between the same items; for it is clear that here[103] Ptolemy wanted |20 to map items pertaining to astronomy[104] into items pertaining to harmonics that are different from the previous ones. Second, <one must be careful>[105] about that which will be said in these matters being encompassed by a more particular outlook, and not holding in general, on an equal level to what precedes. For as the approach to these matters is double—a general one whenever all or most of the items are taken into account, a specific one for each of the items taken particularly—<Ptolemy> has discussed general matters up to the thirteenth <chapter>, and undertakes to discuss specific matters here; for at the end of the chapter he states[106]: "Now, from such relations of similarity as these we can grasp most clearly the general concordances between the features distinguishing melodic intervals and those distinguishing the heavenly motions. It remains to investigate, in each particular case too, what has been reliably detected by observation through what has come about"[107]. Then, what will be said after this must relate to what has already been said as the particular to the general. Third, <one must be careful> about the relations of similarity that will be stated between these items gaining |30 their warrant from what has long been detected by factual observation. For Ptolemy himself undertakes to do that, namely, to investigate what has been reliably detected by observation through events that have frequently come about in the Universe[108]: clearly, these are the

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[98] The oldest witness of "*Harm*. III 16" in the position specified by Barlaam is Monac. gr. 361a, f. 40v. The *Harmonica* in this manuscript was annotated by Gregoras, and collated by Gregoras' pupil Philotheos of Selymbria against a witness carrying Gregoras' recension. See again the section "A Background to the *Refutatio*" for a fuller discussion.

[99] The lexical range of οἰκεῖος "fitting" is crucial to the *Refutatio*. It figures in three of the six requirements the additional chapters should meet to be regarded as originally Ptolemaic, see lines 33–38.

[100] We shall keep in translation almost all proleptic sentences. These are typical of the Greek language, and an obvious rhetorical tool Barlaam uses to convey saliency to his opponent's mistakes.

[101] We translate παραβάλλω by "to map" and παραβολή by "mapping". The point is that the παραβολή here is really a "mapping" in a mathematical sense, namely, a one-to-one correspondence between two sets that preserves some relevant relations between the elements of the sets (we hesitated over "isomorphism"). Using a technical term of modern mathematics is validated by the fact that παραβάλλω "to apply" (within the so-called "theory of application of areas"; see *Elem*. II 14 and VI 28–29) and παραβολή "application", hence "parabola" (see most famously Apollonius, *Con*. I 11), are also technical terms of Greek geometry. The last meaning gave rise to παραβολή "division" as a technical term of Greco-Byzantine logistic.

[102] Ptolemy does this from *Harm*. III 8 on.

[103] That is, in the missing chapters whose titles have been preserved. Barlaam takes it for granted that Ptolemy did write these lost chapters, despite a Byzantine debate, witnessed by scholia stemming from Gregoras' recension, about Ptolemy's untimely death leaving the *Harmonica* incomplete (see DÜRING, Die Harmonielehre LXXXI–LXXXIII).

[104] Barlaam frequently uses the standard locutions τὰ τῆς X and τὰ περὶ X to denote the items related to discipline X, and ultimately discipline X itself. We always translate with periphrases, like "items pertaining to" here or "items related to" just above. We normally nominalise Greek gender neuter in the plural by adding "items".

[105] The syntax of the list of requirements is complex, and the scope of the ruling verb wide.

[106] Contrary to Düring's edition, Barlaam locates the quote at the end of a chapter, which is indeed the case in all witnesses of the text prior to Barlaam. Some of these manuscripts do not carry a coherent partition into chapters though.

[107] *Harm*. III 14, 109, 8–11 Düring, which echoes the opening statement at III 8, 100, 26–28 Düring. We adapt the translation in BARKER, Greek Musical Writings II 388. The quotes from Ptolemy are part of Barlaam's strategy of refutation.

[108] Barlaam takes up Ptolemy's sentence, which he has just quoted, and he completes it with πολλάκις "frequently" and ἐν τῷ παντί "in the Universe"; he also specifies the συμπτώματα "events" as the object of the dangling participle γενομένων.

events that turn out to take on the principle of their understanding through factual observation. Then, the subsequent arguments must be made to depend on some such events. Fourth, <one must be careful> about the mapping being fit, and not such that dissimilar items[109] came to agree with one another, but such that, as far as possible, similar items did that[110], and in such a way that some shared essential property[111] was mapped; for one should not assert that random items are alike; on the contrary, in an argument, one should map into one another such items as also keep some similarity as a matter of fact[112]. Fifth, using a phrasing which is correct and fits the underlying objects. Sixth, <one must be careful> about each chapter fitting its own title, so that, even if the title is missing, anyone can find it out starting from the argument itself.

Now, these six requirements necessarily ought to be observed by anyone willing not to fall short altogether |40 of Ptolemy's goals about these matters; however, it will appear that the author of the present chapters took no heed of any of them.

As for the first requirement, in the ninth and tenth <chapter> of the present treatise[113] Ptolemy has already expounded completely a mapping of the circle of the zodiac into items in harmonics[114]—in such a way that the aspects[115] he came to regard as concordant and active are in a relation of similarity to the concords of the attunement[116], both because the former are made up of sections exactly equal in number to the concords and because such configurations[117] encompass the ratios of homophones and concords[118] in addition to the tone—and <he has expounded> what configurations seem like what concords—namely, an opposition, an octave concord, a trine, a fifth, a quartile, a fourth—and he added the cause of the similarity, and further he set aside a discussion about such things insofar |50 as already completed, when he says: "then, let that be a sufficient account of circular motion itself, considered in respect of both kinds of harmony, and of the configurations that are generally called 'concordant' and 'discordant'"[119]. As if nothing of the sort were said, our author expounds again to us a mapping of these same <heavenly items> to harmonic items[120], by

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[109] Writing "dissimilar items come to agree with dissimilar items", as Greek idiomatically does, is opaque in English. We modify the expression, here as elsewhere.

[110] In clauses that contrast the approach of the opponent and what Barlaam claims he should have done, the former comes usually first, and within a negative clause. Giving saliency to the opponent's wrong approach is part of Barlaam's strategy of refutation.

[111] Our translation of Barlaam's syntagm nominalises the verb συμβαίνω "to occur" to the standard Aristotelian term τὸ συμβεβηκός "essential property" (see H. BONITZ, Index Aristotelicus. Berolini 1870 s.v. συμβαίνειν 3*b*), a term that will occur—within an obvious reference to the clause at line 120 (which is in its turn nearly the same as the one here)—at line 130 below (see also line 223). A less connoted translation is simply "some shared property was mapped", litt. "something identical has happened to be mapped", but Barlaam's barring, in the subsequent clause, random items from being mapped into one another makes us believe that he wanted to bar random shared properties too. To clarify our point, items in harmonics and aspects share the property of being named by a syntagm that contains more than two Greek letters, but this cannot count as a property relevant for their being mapped into one another. Finding properties that allow to establish even the weirdest mappings is easy as well.

[112] Here and at line 71, "as a matter of fact" translates φύσει.

[113] See again the section "A Background to the *Refutatio*" for a fuller discussion. Writing τοῦ παρόντος συντάγματος "of the present treatise" is not entirely congruous; read "the treatise under present examination". One also expects to find the indication of the book.

[114] The clause is asyntactic as it stands. No ancient copyist corrected it though. We did that hesitatingly.

[115] The στάσεις are the astrological "aspects", for which see the classical in account in A. BOUCHÉ-LECLERCQ, L'astrologie grecque. Paris 1899, 165–179. We sometimes omit the noun when it is qualified, as in ἡ διάμετρος στάσις "an opposition".

[116] Here, and to a lesser degree just below, Barlaam's sentences are a patchwork of cut-and-pasted clauses, syntagms, and words from the title of *Harm*. III 9 and from the beginning of this chapter, most notably 102, 2–3 and 10–11 Düring.

[117] Barlaam takes the word "configuration" (σχηματισμός, and σχῆμα at line 51, within Ptolemy's citation) to be a synonym of "aspect", but in *Harm*. III 10 Ptolemy uses it with the meaning of "phase" (see also Ptolemy, *Tetr*. I, 8).

[118] The distinction between homophones and concords is introduced in *Harm*. I 7, 15, 10–14 Düring.

[119] *Harm*. III 10, 104, 18–20 Düring. We adapt the translation in BARKER, Greek Musical Writings II 384.

[120] This is done in *Harm*. III 14, 109, 25–28 Düring.

claiming that an opposition, which <Ptolemy> mapped into an octave concord, is like *proslambanomenos*, that a trine, which has been mapped by <Ptolemy> into a fifth concord, is like *hypatē mesōn*, that a quartile, which <Ptolemy> had mapped into a fourth, is like *nētē diezeugmenōn*. Further, since such configurations are determined according to the exact positions of the planets, and the exact positions are observed along the circle of the zodiac, it is manifest that in this case the author also turns out to carry out, by means of number 360, a partition of the zodiacal circle. Consequently, it occurs to him to discuss here again the similarity of the |60 perfect system and of the zodiacal circle—something <Ptolemy> had already discussed[121]—number 360 being just brought to bear in addition in these <chapters>—and this, to no purpose and on the grounds of bad logic[122], as of course regards the argument about their similarity. For if an opposition is really like *proslambanomenos*, as the author claims, whether one keeps the semicircle uncut or cuts it in as many <parts> as one pleases, nevertheless their similarity will stand. For it is unreasonable to claim that an opposition is like *proslambanomenos* the semicircle being cut into 180 equal segments, whereas, when staying uncut or being cut according to another number, <an opposition> is not <like *proslambanomenos*>[123]. Similarly, too, a trine is like *hypatē mesōn* not because the third part of the whole circumference is cut into 120 <parts>: for dividing the whole circumference, or its third part, or its quarter, or whatever else, into so many or so many <parts> is unable to change the power of the associated aspects, either trine, or quartile, or any such other of them. |70 On the contrary, the same similarity of such configurations to items in harmonics will in any case stand, into however many <parts> one cuts the whole circumference of the circle: for this similarity belongs to them as a matter of fact, whereas any partition into so many <parts> turns out to be conceived for the sake of the argument only. Consequently, the number actually taken in this case does not convey any necessary support to the similarities set out[124]. This is also clear from the fact that it is possible to prove the same things by taking a smaller number too: for the entire circle being cut into twelve equal segments too, if one wished to call each of the segments "degree", it will be possible for him to prove without any loss, by means of the same words, everything that has been said in the first and in the second chapter, just by taking other numbers, 12 instead of 360, 6 instead of 180, 4 instead of 120, 3 instead of 90, 2 instead of 60[125].

So, it is manifest from what has been said that, in these <chapters>, the author also turns out to map into harmonic items |80 exactly such astronomical items as have been mapped by Ptolemy. Further, there is nothing of what is said in the second chapter that was not said before either: for that the concord of a fourth is in a sesquitertian ratio, a fifth in a sesquialter ratio, an octave in a double ratio, a double octave in a quadruple ratio, and that, exactly as a sesquitertian ratio compounded with a sesquialter gives a double ratio, so the concord of a fourth compounded with a fifth also gives an octave—all of this has been abundantly said in the 7th and in the 8th <chapter> of the

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121 Ptolemy does this in *Harm*. III 9.

122 Our "on the grounds of bad logic" is Barlaam's ἀσυλλογίστως. He easily shows that number 360 can (and should) be replaced by a smaller number, but he is unable to prove that the opponent's argument is flawed. The main drawback in Barlaam's refutation here is ascribing the numerical sequence based on 360 a character of necessity that is not borne out by the opponent's argument. Barlaam insists on the number sequence based on 360 because its presence could be perceived as an element of originality in the opponent's exposition with respect to Ptolemy's.

123 The opponent never states the second clause of this sentence.

124 The gist of the argument is this: as no specific number sequence conveys any necessary support to the similarities set out, any sequence is used to no purpose. As pointed out in the next-to-last footnote, Barlaam's actual argument purports to (dis)prove more, for he (tendentiously) has his opponent claim that the numerical sequence based on 360 is necessary for mapping aspects into items in harmonics. This argument will be refined at lines 235–246.

125 This sequence of numbers will be (implicitly) used more than once by Barlaam because they are the smallest numbers that have the same ratios as the sequence based on 360; Ptolemy uses these numbers in *Harm*. III 9, and asserts that we should reasonably expect that Nature made the zodiac a twelve-part item because the perfect system spans almost exactly twelve tones (103, 12–104, 2). This is one of the key technical points of Barlaam's refutation.

first <book> of the harmonics, whereas the way such concords stand in a relation of similarity to the partitions of the zodiac, this has been expounded in the 9th <chapter> of the third <book>. Consequently, in this second chapter the author did not really say anything more than before—on the contrary, the author even did so quite defectively, for he has neglected two concords[126], whereas Ptolemy has repeatedly discussed the ratios of all concords, each of each, by referring to a partition of the zodiac |90 into 12 <parts>. Moreover[127], that which is said in these chapters is not really encompassed by a more particular outlook beyond what has been said before, as it should be according to Ptolemy's undertaking. For the argument about the active configurations is part of a general discourse about the planets: for four of the planets[128] can be in opposition and in trine and in quartile and in sextile both to one another and to the remaining three; this is also why Ptolemy discussed these issues in those <chapters> in which he undertook to discuss a general approach to them.

That what is said here does not gain warrant from what has been detected by factual observation either, as it should be according to what Ptolemy itself said, this is manifest. For dividing the whole circle or its parts into so many or so many <parts>, and to claim that this aspect is like this note, and this one is like this one, and spending so much detail about what numbers contain such-and-such a ratio and what such-and-such a one, and about what ratio is compounded |100 of what <ratios>[129]—all of this could not be grasped through any observation, not does it pertain to sense-perception, but to abstract modelling[130]. This is why the author of these chapters did not adduce anything of the sort to lend credibility to his arguments either. Further, he has also carried out the mapping of the items set out in a way that is altogether out of tune. First, he did not adduce any proof of the similarity of the items set out[131]. For it is not enough to claim that this is like that, but one must also add the reason for the similarity, something Ptolemy appears to do everywhere, both when he spells out the relations of similarity and when he adduces elements of credibility for them. Take the concords: when he says that the concord of an octave is like an opposition in a circle, he adduced three pieces of evidence[132] for this[133]: (1) that the double ratio is contained in both of them and (2) that <this ratio> pertains more to equality than the other aspects and concords[134], and (3) that both the oppositions of the heavenly bodies on the zodiac and the notes that make |110 to one another an octave are the most active ones[135]. Similarly, he also related the mappings of the others to corroborating evidence[136]. For the job of a scientist about each object of study is not only to

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126 The neglected concords are an octave plus a fifth and an octave plus a fourth, see *Harm*. III 9, 102, 27–29 and 102, 31–103, 2 Düring, respectively.

127 Checking the second of Barlaam's requirements takes a handful of lines, from here to the end of the paragraph.

128 This must be Barlaam's slip, for the correct numbers appear to be five (Sun, Moon, Mars, Jupiter, and Saturn) and two (Mercury and Venus, neither of which can form any of the four aspects with the Sun).

129 On compounded ratios see F. ACERBI, Composition and Removal of Ratios in Geometric and Logistic Texts from the Hellenistic to the Byzantine Period, in: Revolutions and Continuity in Greek Mathematics, ed. M. Sialaros. Berlin 2018, 131–188.

130 The copying mistake λογιστικῆς "computational" for λογικῆς "abstract", found in some witnesses and very likely in M before correction, was certainly induced by the list of actions it refers to.

131 Barlaam is widening the scope of his third requirement, something he is not entitled to do in a dialectical game, once he has clearly set out the grounds of his refutation.

132 The term τεκμήριον "piece of evidence", allowing to establish a demonstrative argument, is standard Aristotelian terminology: see *Rh*. II 25, 1402b13–20 and 1403a10–16.

133 *Harm*. III 8, 101, 18–26 Düring. Here again, Barlaam uses only words that can be read in the *Harmonica*.

134 This is explained in *Harm*. I 7, 15, 24–25 Düring.

135 Of course, Ptolemy's—and Barlaam's—ἐνεργητικώτατοι "most active" is an astrological term, even if there are no occurrences in Ptolemy's *Apotelesmatica*, in Paul of Alexandria, or in Hephaestion, all of which use ἐνεργής. The term occurs in Vettius Valens.

136 See *Harm*. III 9, 103, 5–12 Düring, but Berlaam is slightly cheating here.

prove the fact, but also the reason why[137]. The author, on the contrary, while claiming that the standing notes are like the aspects set out, does not adduce any evidence. Well, this was a mistake[138], and also the fact that he mapped dissimilar items into one another[139]. For there being two kinds of item in both sciences, the distances[140] and the things that make the distances, namely, what is set at a distance, it is reasonable to map distances into distances and what is set at a distance into what is set at a distance, a requirement which is everywhere met with the greatest care by Ptolemy. The author, on the contrary, maps distances into what is set at a distance, thereby doing something utterly absurd. For he makes *proslambanomenos*, which is set at a distance and productive of a distancing but is not a distance, similar to the opposition distancing[141], and similarly for the others too. Now, since we say that such items are like |120 to one another as share some essential property, what on earth did you[142] find shared by *proslambanomenos* and an opposition enabling you to claim that they can be mapped into one another? For opposition is the widest and most active aspect as compared to the others[143], whereas *proslambanomenos* is the smallest and feeblest note in the perfect system because it is the lowest of all. And, the double ratio of the whole circle to a semicircle and of the diameter to the radius is contained in an opposition, whereas in *proslambanomenos*, or in any other note, no ratio is contained, for <Ptolemy> says "taken in isolation, no note can be in a ratio, and each of them is undifferentiated with respect to itself, for a ratio is a relation and occurs first in two <terms>"[144], and an opposition lies between any two items that are set at a distance in opposition to one another, whereas *proslambanomenos* does not turn out to have this property, for in the perfect system there is no note lower than it. Then, how will *proslambanomenos* be possibly |130 mapped into an opposition, if they do not share any essential property[145]? Next, if *proslambanomenos* is like an opposition, since according to Ptolemy the concord of an octave is like such an aspect[146], and items similar to the same are also similar to one another[147], therefore *proslambanomenos* will be similar to the concord of an octave, what is set at a distance to a distancing, a simple to a composite, a boundary to what is bounded, what cannot be in a ratio to what

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137 Litt. "to prove 'that', but also 'because'". This is a standard Aristotelian distinction: see *APo*. I 13. We adopt the translation in J. BARNES, Aristotle, Posterior Analytics. 2nd ed. Oxford 1993.

138 The transition to the discussion of the fourth requirement takes place here.

139 This is the most obvious blunder in the chapters added by the opponent, for (1) the mapping associates relations with terms and (2) this move is patently at variance with Ptolemy's discussion in *Harm*. III 9 (for whose drawbacks, however, see the notes in BARKER, Greek Musical Writings II 381–385). Barlaam rightly, and mercilessly, insists on this point (which BARKER, Greek Musical Writings II 389 concealed in translation). Here as elsewhere, our translation does not use the same construct as the Greek expression.

140 This is the first occurrence of a diabolical wordplay by Barlaam, namely, the one involving the lexical range of διὰ + ἵστημι, which denotes items set apart and the interval that separates them. We keep the wordplay and translate διάστημα by "distance" (usually, "interval"), διίσταμαι by "to be set at a distance", διάστασις by the atrocious "distancing", and, when it does not mean "aspect", στάσις by "standing". The only exception to our translation rule is rendering διάστημα by "radius" in the formulaic expression for tracing a circle with a given centre and radius (first occurrence at line 267).

141 One would expect to read στάσει "to the aspect". We do not correct because we assume that Barlaam wants to keep his linguistic lightshow on.

142 The presence of "you" is the *akmē* of Barlaam's polemical tone; this pronoun makes Barlaam's outrage manifest.

143 Ptolemy says that at *Harm*. III 8, 101, 24–26 Düring.

144 *Harm*. I 4, 10, 19–21 Düring. We adapt the translation in BARKER, Greek Musical Writings II 284. For a discussion of the meaning of ἄλογος here required, see F. ACERBI, Unaccountable Numbers. *GRBS* 55 (2015) 902–926: 921–924.

145 This argument shows that the fundamental mapping in *Harm*. III 14–15 is ill-conceived, for it can be applied only to a bounding note of a system. In the rest of the *Refutatio*, Barlaam more than once assumes such a fundamental mapping as a supposition, and he infers unacceptable conclusions from it.

146 This is argued at *Harm*. III 8, 101, 12–26 Düring. Barlaam sets out here a standard transitivity argument.

147 This is the canonical formulation of transitivity rules in Greek mathematics: see the archetypal occurrence in *Elem*. I nc 1; Barlaam generalises here the enunciation of *Elem*. VI 21. See ACERBI, The Logical Syntax 180–184, for a discussion of this kind of statement.

is in a ratio. And moreover: even if, by converting[148], someone were to claim that *nētē hyperbolaiōn* is like an opposition, *nētē diezeugmenōn* is like a trine, *hypatē mesōn* is like a quartile, *proslambanomenos* is like a sextile—well, if someone were to set out the mapping in such terms, he will not seem to have said anything worse, and possibly even something better, to the extent that he knew to map the most valuable items to one another[149]. Why then will an opposition be more properly mapped into *proslambanomenos* and not into *nētē hyperbolaiōn*[150]? The absurdity of the mapping |140 set out also consists in not comparing the notes to proportionally related aspects[151]. For *nētē hyperbolaiōn* is in a quadruple ratio to *proslambanomenos*, whereas an opposition is in a triple <ratio> to a sextile—for 180 is triple 60. And again, *nētē diezeugmenōn* is in a triple ratio to *proslambanomenos* according to the concord of an octave and a fifth, whereas an opposition is in a double <ratio> to a quartile (the author mapped the above notes into these)—for twice 90 makes 180. Likewise, too, *nētē hyperbolaiōn* is in a sesquitertian ratio to *nētē diezeugmenōn* according to the concord of a fourth, and in a double-sesquiditertian ratio to *hypatē mesōn* according to an octave and a fourth, whereas a sextile is in a sesquialter ratio to a quartile and in a double ratio to a trine. Similarly, while *nētē diezeugmenōn* contains a double ratio to *hypatē mesōn*, |150 the author maps such notes into aspects that contain a sesquitertian ratio, for a trine has this ratio to a quartile. And there is more, as he has mapped in the most dissimilar way the two standing notes that contain the disjunctive tone: for he both used them as one, even if they are two and differentiated as to function[152], and maps them again into the same items <as before>: for, since he claims that these are mapped into the standing from which is the starting point of the distancings mentioned[153], and items that are set at a distance convert with one another—for when Mars is opposite Saturn, then Saturn is opposite Mars too, and both Saturn is in sextile with Jupiter and Jupiter with Saturn, similarly for the others too—it is manifest that he also maps these notes into the four configurations[154]. And let so much be said about the fact that he has carried out his mappings out of tune.

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148 The verb ἀναστρέφω "to convert" is standard for denoting the conversion of a ratio (see *Elem*. V def 16 and V 19 por). The technical verb for converting relations or premises is ἀντιστρέφω (see Aristotle, *Cat*. 7, 6b28–7b14, and *APr*. I 2–3, respectively), which Barlaam will use later (line 155). Barlaam is here inverting the order of one of the two lists before mapping them into one another, so he correctly uses the mathematical term.

149 Opposition is the most active aspect, *nētē hyperbolaiōn* is the higher note of the perfect system.

150 Barlaam will identify this typical refutation argument—namely, drawing mutually exclusive conclusions (both are false, according to Barlaam!) from the same premise, thereby proving that this premise is false—at lines 244–245, after using it again. In this case, the false premise is that notes are like aspects (*Harm*. III 14, 109, 25–28 Düring). Barlaam spends his third rhetorical question here.

151 This argument will be also presented below, see lines 239–243. Barlaam is here conceding two assumptions to his opponent, for he argues on the supposition that notes are like aspects (something he has proved untenable at lines 113–127) and by using the numerical sequence based on 360 (something he has proved to be not necessary at lines 56–78), in order to deduce a παράλογον "absurdity". Of course, and as Barlaam has already pointed out, the ratios between the aspects do not depend on the specific numerical sequence used to "measure" them.

152 See *Harm*. III 14, 109, 28–32 Düring; the opponent maps the two notes into one and the same position on the zodiac. The notion of δύναμις "function" of a note is explained in *Harm*. II 5, where Ptolemy also deals with the disjunctive tone.

153 *Harm*. III 14, 109, 29 Düring.

154 Barlaam is exceedingly subtle here. His opponent has mapped notes into aspects, with one exception: the two notes that define the disjunctive tone, which are made as it were to coalesce (a move Barlaam disposes of in seven words) and are thereby mapped into a single position on the zodiac (the στάσις in its meaning of "standing"). Barlaam's counterargument runs as follows. Suppose that any of the notes that define the disjunctive tone be mapped into one of the endpoints of an aspect. But an aspect is a symmetric relation (it ἀντιστρέφει "converts"; see the note for line 134 above), and it is identified by a line segment or by a zodiacal arc, which go into themselves if their endpoints are interchanged. Therefore, a note (in fact, anything) attached to an endpoint must also be attached to the other. Consequently, any of the notes that define the disjunctive tone is mapped into both endpoints of an aspect. But an aspect is identified by the position of its endpoints. Therefore, any of these notes is mapped into that aspect itself. But the argument applies to any aspect. Therefore, any of the notes that define the disjunctive tone is mapped into any aspect. Barlaam's argument assumes his opponent's thesis as a supposition and deduces a statement that is logically incompatible with it. This is a kind of self-refuting argument falling under the category the sceptical tradition called περιτροπή. On self-refuting arguments in the Greek philosophical tradition

|160 One must speak here about the fact that the author has not used correct phrasing.

First, it occurs to the author that "now, the perfect system of music being mapped into these"[155] be mistaken. For the mapping does not concern such a system as a whole—nor does the title of the chapter undertakes to do that—but the standing notes only, and "of music" is added superfluously and at variance with Ptolemy's usage. As for the former, <it is a mistake> insofar as saying "of the perfect system" would have sufficed—for it is clear to what science such a system relates—; as for the latter, <it is a mistake> insofar as Ptolemy chooses "music" and "musical" when dealing with execution only[156], and he nowhere calls his own treatise "music", but "harmonics". Then, one should have said "of the perfect system of harmonics". Next, "to the standings of these very arithmetical distances"[157] does not sound correct either. For a standing is not a standing of distances, but a standing of what is set at a distance, |170 for what is set at a distance, not the distances, stands. On the contrary, a standing of what is standing is a standing. Moreover, the author is not correct in calling the distances "arithmetical": for let them be called "computable", like "computed" and "measured", and not "arithmetical". For since every distance is a distance of what is set at a distance, it is correct to call a distance "arithmetical" when and only when what is set at a distance are numbers, whereas what is set at a distance here are the magnitudes of the heavenly bodies. Consequently, such distances should not have been called "arithmetical". Further, "to where is the starting point of the distancings mentioned, namely, in the location in which the standing of the Sun is conceived"[158] is so incongruously[159] phrased. For, as a location is something different from the standing of what stands in it, if the author wanted to carry out the mapping with respect to the standing, after saying "to where is the starting point of the distancings mentioned" he should have added "at which the Sun, or another <heavenly body>, is conceived to stand"; if <he wanted to carry out the mapping> with respect to location instead, he should have written "to the location, in which the position of the Sun is conceived, |180 from which is the starting point of the distancings mentioned": yet, while saying "to the standing", he made it referred to by "in the location", which is so to speak incongruous and asyntactic. Similarly, the author mistakenly claimed that the double ratio is proportional to the octave and the quadruple <ratio> to the double octave[160]. For "proportional" is not predicated[161] of ratios, but of the terms that contain the ratios: for the mathematicians nowhere claim that the ratios themselves are proportional to anything, but that the terms that are in a same ratio are[162]. So, let the double ratio be called the "cause" of an octave concord[163], but in no way "proportional" to it.

Further, since the perfect system is called "double octave" in virtue of the extremal notes and "perfect system" in virtue of such-and-such an ordering of the notes between the extremal ones, and it is reduced to the quadruple ratio not in virtue of the intervening notes, but in virtue of the extremal ones, one should not |190 say that the quadruple ratio is proportional to the double octave perfect system[164], but only to the homophone of the double octave: for this system is not reduced to

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see L. CASTAGNOLI, Ancient Self-Refutation. The Logic and History of the Self-Refutation Argument from Democritus to Augustine. Cambridge 2010.

155 *Harm*. III 14, 109, 23–24 Düring.

156 This is not entirely true, as a generic meaning of μουσική "music" is required at 20, 2 and (with reference to Archytas' achievements) 30, 10 Düring, and Didymos is called μουσικός "music theorist" at 41, 19 Düring.

157 *Harm*. III 14, 109, 25 Düring.

158 *Harm*. III 14, 109, 29–30 Düring.

159 The adverb "incongruously" is ἀμούσως, which is obviously a wordplay.

160 *Harm*. III 15, 110, 10–11 and 12–13 Düring, respectively.

161 The verb form translated by "it is predicated" is κατηγορεῖται, which is perfectly Aristotelian jargon (the κατηγορίαι are the fundamental predicates, and *Cat*. derives its title from its subject-matter).

162 This is the gist of *Elem*. V def 6.

163 This αἴτιον "cause" can only be an Aristotelian formal cause (causes are discussed in *Ph*. II 3).

164 *Harm*. III 15, 110, 11–12 and 19–20 Düring.

such a ratio insofar as it is perfect, but in virtue of the extremal notes[165]. Consequently, in this case one should have framed the argument about it simply <regarded> as a distance, and not as a system.

So, these are the author's mistakes as to phrasing too.

It is not difficult to see that the chapters do not fit their own titles.

For first, in the first title, Ptolemy announces that he shall substantiate what pertains to his mappings by means of prime numbers, but the author did not use prime numbers. According to the mathematicians, a number can be prime in three ways: in itself, to another, and in order[166]. In itself, it is the <number> that can be measured by a unit only, like 5 or 7; to another, whenever there is no room for any number as a common measure with it, like 8 happens to be to 15, for they are not measured by any and the same number; in order, exactly as when we say that 6 is the first among the <numbers> that have a half |200 and a third—since a number less than it does not have such parts—and that 4 is the first among the <numbers> that have a half and a quarter, and similarly for the others. Now, "prime" for numbers being said in three ways, the numbers picked up by the author are not prime in any of these ways, for 360, 180, 120, 90, and 60—these numbers are prime neither in themselves nor to one another, nor are they the first among those that have the same ratios as them. For, since the circle happens to be cut into two, three, four, and 6 equal <parts> by the aspects about which the author is talking, he needed only a number that has a half, a third, a quarter, and a sixth part, and the first <number> among those that have such parts is 12[167], not 360. Now, it is manifest that, were Ptolemy |210 really willing to expound his mappings by means of these numbers, he would have been totally ignorant of arithmetic[168], for he would have ignored what prime in numbers ever is[169]—and that 36 is the first number that encompasses the ratios of the standing notes of the perfect system according to the substrate, namely, to the chords[170], whenever they differ in length only, whereas according to the function of the notes themselves, it is 32. For setting *proslambanomenos* at 36 in chords, *hypatē hypatōn* will be 32 of these, *hypatē mesōn* 24, *mesē* 18, *paramesē* 16, *nētē diezeugmenōn* 12, *nētē hyperbolaiōn* 9; inversely, setting *proslamba-*

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165 The opponent commits a logical mistake here. The point is that the feature that makes the extension of the double octave a perfect system—namely, the ordering of the notes between the extremes—is irrelevant to its being mapped into a quadruple ratio. So, the opponent's argument proves too much.

166 See *Elem*. VII def 12 (quoted verbatim) and 13, respectively, for the former two. The third is Barlaam's elaboration on the Euclidean notion of οἱ ἐλάχιστοι ἀριθμοὶ τῶν τὸν αὐτὸν λόγον ἐχόντων "the least numbers of those which have the same ratio as them"; his move is forced by Ptolemy's use of πρῶτος at *Harm*. III 9, 102, 12. Barlaam's third category rests on sound bases, for *Elem*. VII 21–22 prove that two numbers are mutually prime if and only if they are the least of those that have the same ratio as them. Still, if this is true, it is also true that Barlaam's "the mathematicians" is unwarranted, for the *Elements* appears to make a point of not multiplying the meanings attached to πρῶτος "first". The three qualifiers καθ' ἑαυτόν, πρὸς ἕτερον, and τάξει are Barlaam's. Relevant to Barlaam's argument are also the problems *Elem*. VII 33 and 39.

167 Compare *Harm*. III 9, 102, 11–13 Düring: Ptolemy picks up number 12 exactly for the same reason, and, as just seen, uses the qualifier πρῶτος in Barlaam's third sense.

168 Of course, this amounts to saying that the opponent is totally ignorant of arithmetic.

169 In Aristotelian jargon, the syntagm τί ἐστι denotes the primary sense of "being" (read the beginning of *Metaph*. Z) and is a synonym of "definition"; the entire Book II of *APo*. is devoted to this. Barlaam inserts πότε "ever" to put emphasis, very much in Aristotelian style (see Bonitz, Index Aristotelicus, s.v. ποτέ).

170 The difference is between notes as represented on a *kanōn*—which conventionally locates the lowest note (here *proslambanomenos*) at an extreme of the *kanōn* and identifies the others as section points of the *kanōn* itself, so that, a common extreme for all segments being kept fixed, the ratios between the resulting segments are the same as the intervals between the corresponding notes—and notes as represented in a sequence from lowest to highest, which is in fact a cyclic arrangement and constitutes the basis on which Ptolemy's mapping between intervals and aspects rests (see III 8, 101, 6–15 Düring). As there are seven standing notes in the perfect system (namely, the ones listed just below in the text), and as the six intervals between any two adjacent notes are assigned and are asymmetrically distributed (these are $^9/_8$, $^4/_3$, $^4/_3$, $^9/_8$, $^4/_3$, $^4/_3$), two different numerical sequences in least numbers (that is, in Barlaam's third meaning) can be assigned to these seven notes, according to whether we assign the largest ("according to the chords") or the smallest ("according to function") number to *proslambanomenos*. The two sequences are those set out by Barlaam.

*nomenos* at 8 according to form[171], *hypatē hypatōn* will be 9 of these, *hypatē mesōn* 12, *mesē* 16, *paramesē* 18, *nētē diezeugmenōn* 24, *nētē hyperbolaiōn* 32[172]. Numbers less than these do not encompass such ratios; the author's number 360 does encompass the ratios of the standing notes—of course according to the substrate—|220but not as a prime <number>. Next, even if the author must divide number 360 into the numbers that will encompass the ratios themselves of the standing notes and assign proportionally a number to each note, he has done nothing of the sort, turning to some other, and unrelated, matters[173].

Thus, the one about what happens to be prime in numbers, such is the first mistake in the first chapter. The second is this[174]: even if Ptolemy selects here numbers as a necessary feature of the mapping argument, and as it is impossible to carry out anything that pertains to the mapping without numbers, nevertheless the author hands down to us such mappings as not needing any numbers except those associated with the configurations. This is clear if one modifies the argument as follows, by eliminating numbers and all the embellishments. For the entire circle being cut into two, three, four, and 6 equal <parts>, we claim that an opposition of the Moon—or of some |230 other planet—to the Sun occurs at half the whole circumference, a trine, at one-third, a quartile, at one-quarter, a sextile, at ⅙. Once the perfect system is mapped into these, *proslambanomenos* will be mapped into the aspect associated with a semicircle, *hypatē mesōn* into the one associated with a third part, *nētē diezeugmenōn* into the one associated with a quarter, *nētē hyperbolaiōn* into the one associated with a sixth.

Now, the argument being thus modified, is the explanation of the similarity being thereby diminished? Not at all. Consequently, if the same arguments in favour of the mapping apply whether numbers are there or not, numbers are introduced to no effect[175]. Further, this is even clearer from the fact that such numbers can be used to corroborate the view that the notes are dissimilar to the aspects. For consider this. Since an opposition is 180 degrees and a sextile 60, whose ratio is |240 triple, it is impossible that the former be like *proslambanomenos* and the latter be like *nētē hyperbolaiōn*, whose ratio is quadruple[176]. And again, since a trine is 120 degrees and a quartile 90,

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[171] For δύναμις "function" and εἶδος "form", which Barlaam treats as synonyms, see *Harm*. II 5–6.

[172] This appears to be Barlaam's elaboration. The complete sequence of numbers—namely, from 8 to 36, with 21⅓ in addition—is the third column of the συστήματος κοσμικοῦ φθόγγοι ἑστῶτες "standing notes of the cosmic system" in Ptolemy's *Inscriptio Canobi*, which lists, side by side in facing columns, the following items: nine cosmic entities (the seven planets, preceded by the sphere of the fixed stars, and followed by the four elements, paired fire-air and water-earth; Venus and Mercury are paired too); the seven standing notes of the great perfect system, plus the *nētē synēmmenōn*, all of them preceded by †μέση† (μετὰ Vincent) ὑπερβολαίων "after the *hyperbolaiōn*"; and the above-mentioned nine-token numerical sequence, from greater to lesser. This table is followed by short texts listing how many numerical means and how many concords are contained in the numerical sequence (editions in HEIBERG, Claudii Ptolemaei II 154; A. JONES, Ptolemy's *Canobic Inscription* and Heliodorus' Observation reports. *SCIAMVS* 6 [2005] 53–97: 74–76). The table was excerpted (sects. 24–25) and simplified (sects. 1–2) in the so-called *Excerpta Neapolitana* (edition in VON JAN, Musici 411–423), whose earliest witness is Vat. gr. 2338, once one and the same manuscript with the oldest portion of Monac. gr. 361a, the earliest witness of "*Harm*. III 16" as included in III 9 (see the section "A Background to the *Refutatio*"): see ACERBI – PANTERI, Eratosthenes and ACERBI – GIOFFREDA, Harmonica Membra. The table and its witnesses are studied in K. VON JAN, Die Harmonie der Sphären. *Philologus* 52 (1894) 13–37, and SWERDLOW, Ptolemy's *Harmonics* 165–176. See also the scholium with associated diagram (ascribed to Gregoras in other witnesses) in Par. Coislin 173, f. 31v, edited in DÜRING, Die Harmonielehre XCIX and in ACERBI, I problemi aritmetici 137–138 n. 21 and Testo 6, first transcript.

[173] The opponent turns to other matters in the second part of *Harm*. III 15, from 110, 17 Düring on.

[174] This mistake should not be discussed under the heading "contents that do not fit the title".

[175] This argument refines the one at lines 59–78, by showing that any partition of the zodiac other than that induced by the aspects themselves is redundant, if the goal is to map notes into aspects. When Barlaam says "for the entire circle being cut into two, three, four, and 6 equal <parts>, […] occurs at half the whole circumference, a trine, at one-third, a quartile, at one-quarter, a sextile, at ⅙," numbers are really eliminated from the argument, for what is provided in the sentence are ratios between magnitudes, not numbers. As Barlaam anticipated at line 75 (where the focus was on the arbitrariness of the arc attached to a "degree"), the point is that no "degree" needs to be introduced in the argument.

[176] This argument was already presented above, see lines 140–151.

whose ratio is sesquitertian, it is impossible that *hypatē mesōn* be like the former and *nētē diezeugmenōn* be like the latter, whose ratio is double. And in general, by sticking to these numbers, anyone will most easily prove that the aspects do not bear any similarities with the notes—consequently, <these specific numbers> do not corroborate at all the similarity, for it is impossible that opposite conclusions be deduced from the same <premises>[177].

Now indeed, while Ptolemy also announces that he will show that the standing notes are like the first heavenly spheres[178], the author does not map the notes into these nor into the bodies in them, but into the latter's distances. Now, Ptolemy does not call "first spheres" the heavenly bodies themselves, but <the spheres> in which they move: for instance, the lunar sphere is the one in which the Moon |250 moves; this is the <sphere> whose centre is the same as the one of the Universe and whose radius is the maximum elongation[179] of the Moon. Likewise for the Sun too, the solar sphere is said to be the one whose centre is the mentioned one and whose radius is the maximum elongation of it, and similarly for the others. As there are seven such spheres and the standing notes of the perfect system are also equal in number <to them>, he wants to map the latter into the former through and through—for what reason, it will be clear in what follows. The author, on the contrary, does not frame any argument about such spheres in these <chapters>.

So, it is manifest from this that such a chapter does not fit its title. Apart from this, something else in this chapter[180] is also mistaken, namely, determining the distancings of what is set at a distance along arcs[181], whereas one must do that along the straight line joined from either <object> to the other. |260 For two points being given, as infinitely many arcs can be traced through them but one straight line only, we do not claim that the distance of <the points> from one another is set along any of the arcs—for there would be infinitely many <of them>, and of course their distancings would also be so many—but along the straight line only. For this is unique and well-defined because it is the shortest of the lines drawn from one and the same point to one and the same <point>[182]. For instance, let there be two points, AB, and let straight line AB be joined, and let it be bisected at Γ, and from Γ let ΔΓΕ be erected on both sides at right <angles> with AB[183]. Thus, as infinitely many points can be taken in both of ΔΓ ΓΕ, and as each of the circles traced with centre each of the points and radius the one from that point as far as either of points AB[184] also reaches the other <point>, it will result that infinitely many arcs are traced from A to B. For let a

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[177] The opposite conclusions are that the notes are similar to the aspects and that the notes are dissimilar to the aspects. The former conclusion is drawn by the opponent in *Harm*. III 14, the latter by Barlaam here. Both conclusions are deduced from the premise, stated at *Harm*. III 14, 109, 25–28 Düring, that *proslambanomenos* is like opposition, *nētē hyperbolaiōn* is like sextile, *hypatē mesōn* is like trine, and *nētē diezeugmenōn* is like quartile, and from the fact that assigning numerical "degrees" to the aspects entails that they necessarily have ratios to one another.

[178] This is announced in the title of *Harm*. III 14.

[179] The noun ἀπόστημα "elongation" is a technical term in astronomy, as is confirmed by the over 200 occurrences in the *Almagest*. In the *Harmonica*, Ptolemy employs it exactly where it is needed, namely, in III 11, 105, 27, 106, 6 and 106, 9 Düring. Barlaam takes the heavenly spheres to be closely packed. As the spherical shell in which a planet moves has a size because of its motion κατὰ βάθος "in depth" (cf. *Harm*. III 11), Barlaam takes the greatest radius of the shell as defining the sphere itself.

[180] The opponent does this in both chapters *Harm*. III 14 and 15. This mistake should not be discussed under the heading "contents that do not fit the title" either.

[181] Ptolemy's does exactly this in *Harm*. III 9. Barlaam's argument is weak; he himself perceived this, for otherwise he would not have added the final remark about the astronomers measuring distances along arcs too.

[182] This is the first assumption in Archimedes, *Sph*. *Cyl*. I (which Barlaam is likely not to have known), commented on in Eutocius' commentary (which Barlaam is likely to, and where an argument different from Barlaam's is put forward corroborating Archimedes' assumption): see J. L. HEIBERG, Archimedis opera omnia cum commentariis Eutocii. I–III. Lipsiae 1910–15 I 8, 3–4 and III 6, 4–8, 2, respectively.

[183] Barlaam applies *Elem*. I post 1, I 10, and I 11, in this order. His argument is semi-formal but the Greek demonstrative code is correctly adhered to.

[184] *Elem*. I post 3.

random point, Z, be taken on ΔΓ, and let ZA, ZB be joined[185]. And since AΓ is equal |$_{270}$ to ΓB, and ΓZ is common and at right <angles>, therefore ZA is equal to ZB[186]; therefore, the circle traced with centre Z and radius ZA will also pass through point B[187]. Let it be traced, and let it be AHB. Similarly, if we also take some other point on ΔΓE, as Θ, and we trace a circle with centre the same <point> and radius ΘA, it will also pass through point B. Then, let it be as AKB. Now, as this comes about in infinitely many ways, if the distancings of what is set at a distance are conceived along arcs, there will be infinitely many distancings from B to A—for there are infinitely many arcs—which is absurd. If, on the contrary, the distancing of B from A is necessarily unique and well-defined, and straight line AΓB is unique and well-defined—for no straight line less or greater than it will be drawn from A to B—necessarily straight line AΓB, and not arc AHB or AKB or any other, is the distancing by which we say AB are set at a distance from one another. In this way, then, the bodies in mutual opposition on the zodiac too|$_{280}$, we claim that the diameter—which is a straight line and not an arc—turns out to distantiate them. A sign[188] of this is that we do not say that they are at a maximum distance on account of the fact that half the whole circumference is the greatest <arc>—for it is also possible to take <an arc> greater than it—but on account of the fact that the diameter is the greatest chord in a circle. Similarly too, we claim that the side of an equilateral triangle inscribed in the circle—which subtends a third of the whole circumference—distantiates the <bodies> in trine to one another, and in the same way for the other aspects too. This is also clear from the names themselves[189]: for it is not called "trine" from a third of the circumference, but from the straight line that subtends <such a third>. Similarly, a quartile <is so called> from the side of a square, which is a straight line. Consequently, the author has been incorrect in taking the distancings of such configurations along arcs. One must know, however, that the astronomers also make use of the |$_{290}$ noun "distancing" when referring to arcs[190]; they do that whenever they investigate the arc on the zodiacal circle going from the position of some heavenly body to the position of another one, except that they do not do that in the proper sense, but by homonymy[191], for they simply want to designate the numerical value[192] of the intervening arc—by means of whatever noun they do that, this does not make any difference to them.

Well, in the second chapter the author does not appear to be in full contact with the title either. For the latter wants to reduce the motions of the first heavenly spheres, as regards their speeds, under arithmetical ratios, whereas in the former there is hardly any discussion of the motions of the spheres or of their ratios; on the contrary, the distances along the zodiac underlie again the discussion itself.

As for the third chapter, that this is not Ptolemy's is clear from its taking Mars to be related to *nētē synēmmenōn*[193]. For, after depriving the so-called *synēmmenon* system from being perfect[194],

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185 *Elem*. I post 1.

186 Barlaam applies *Elem*. I 4. AΓ is equal to ΓB because AB has been bisected at Γ.

187 This is a consequence of the definition of a circle in *Elem*. I def 15.

188 The term σημεῖον "sign" (here obviously a wordplay) with this meaning is standard Aristotelian terminology: see *APr*. II 27; *Rh*. II 25, 1402b13–20 and 1403a2–5; and read the first sentence of the *Metaphysics*.

189 The Greek qualifiers that single out the στάσεις "aspects" coincide with those that single out the geometric figures: τρίγωνος "trine" but also "triangular", τετράγωνος "quartile" but also "square", ἑξάγωνος "sextile" but also "hexagonal".

190 Barlaam's statement is confirmed by dozens of occurrences of διάστασις "distancing" measured along arcs in Ptolemy's *Almagest* and *Geography*. In the latter, distances are measured along great circles on the surface of the Earth.

191 The adverbs κυρίως "in the proper sense" and ὁμωνύμως "by homonymy" can be taken to refer to standard Aristotelian doctrines (see BONITZ, Index Aristotelicus, s.v. κύριος 2 and ὁμώνυμος). The Aristotelian corpus famously begins with the word ὁμώνυμα.

192 The "numerical value" is the πηλικότης, a key notion in Barlaam's *Logistikē* V, and which archetypally occurs in *Elem*. VI def 5. See ACERBI, Composition and Removal of Ratios 161–163.

193 *Harm*. III 16, 111, 4–5 Düring.

194 Ptolemy does this in *Harm*. II 6.

|300 proving that only the double octave is of such a kind, in what follows <Ptolemy> has not carried out any discussion of the *synēmmenon* system, either in the sections of the canon or in the mappings related to the soul and to heavenly matters[195]; rather, he presupposes the perfect system everywhere. Accordingly, if in the 14th <chapter> <Ptolemy> had to map the first heavenly spheres into the standing notes of the perfect system, it is clear that Mars ought also to have been mapped by him into some of such notes, and not into *nētē synēmmenōn*, which is not a note of the perfect system. That <the chapter> has not been finely tuned to the title is manifest from the facts that Mars has been passed over[196], while being a planet, and that only the standing notes have been selected, whereas the title does not speak of specific planets nor of standing notes, but simply of planets and notes[197].

Yet, what bewilders me most about the person who put this chapter as last in order together with those preceding it, is that, as |310 "let no one think"[198] wants to be a reference to the <chapters> immediately preceding it[199], and as, for this reason, what has been said in the second chapter must be such that what is said in this <chapter> should be made to stand in tune with it, he deemed—without caring about the possibility that someone might attack him—that items out of tune should be in tune. For there is nothing in the second <chapter> such that what is in the third could possibly refer back to it[200].

So, this is what one might also have to say about the fact that the author did not set out the chapters in a way consequent with the titles. Now, it is not difficult to conclude from what has been said that, in these matters, the author lost altogether contact with Ptolemy's thought. For there being six necessary requirements that the author should meet—namely, to say something different and more particular than before, and congruous to the observations, and fittingly mapped, and clearly expounded by means of sound phrasing, and fitting the titles—he appears to be mistaken on all counts, for he said the same things as before and on an equal level of generality, and he did not use observations, nor did he make his mapping fitting, nor did he use sound formulations, |320 nor did he frame his arguments about what the titles indicate.

Let what pertains to the refutation stop here. It would be time, finally, to set out here what we ourselves think these things should really be, by setting as much as a preliminary constraint, that, as in what precedes Ptolemy mapped the harmonic distances into the heavenly ones, in these <chapters> he wants to map the items themselves that realise the distances, which, in the former domain, are the notes, in the latter, the first heavenly spheres and the bodies moving in them. As there are three items related to these, namely, masses, motions, and powers, in the 14th <chapter> he wants to expound the mapping as regards the masses, in the 15th, as regards the motions, in the 16th, as regards the powers. Now then, let us say …

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195 After the preliminary chapter *Harm*. III 4, Ptolemy does this in *Harm*. III 5–7 and III 8–13, respectively.

196 Mars is omitted in the initial clause of the chapter, at 110, 25–26 Düring.

197 This part of the argument is rather weak.

198 *Harm*. III 16, 110, 25 Düring.

199 As seen, this opening—as well as the rest of the text that makes III 16—is out of place even if it is inserted in the middle of *Harm*. III 9.

200 It should be clear that this is not an argument against the author of "III 16", but against the opponent, who did not harmonise III 15 with the text of "III 16", and who arranged the restored chapters in their present order. Barlaam states here for the first time that, besides writing III 14–15, the opponent *also* moved "III 16" from its position in the middle of III 9 to the very end of the *Harmonica*.

## APPENDIX. A COLLATION OF *HARM*. III 14–16 IN MARC. GR. Z. 332

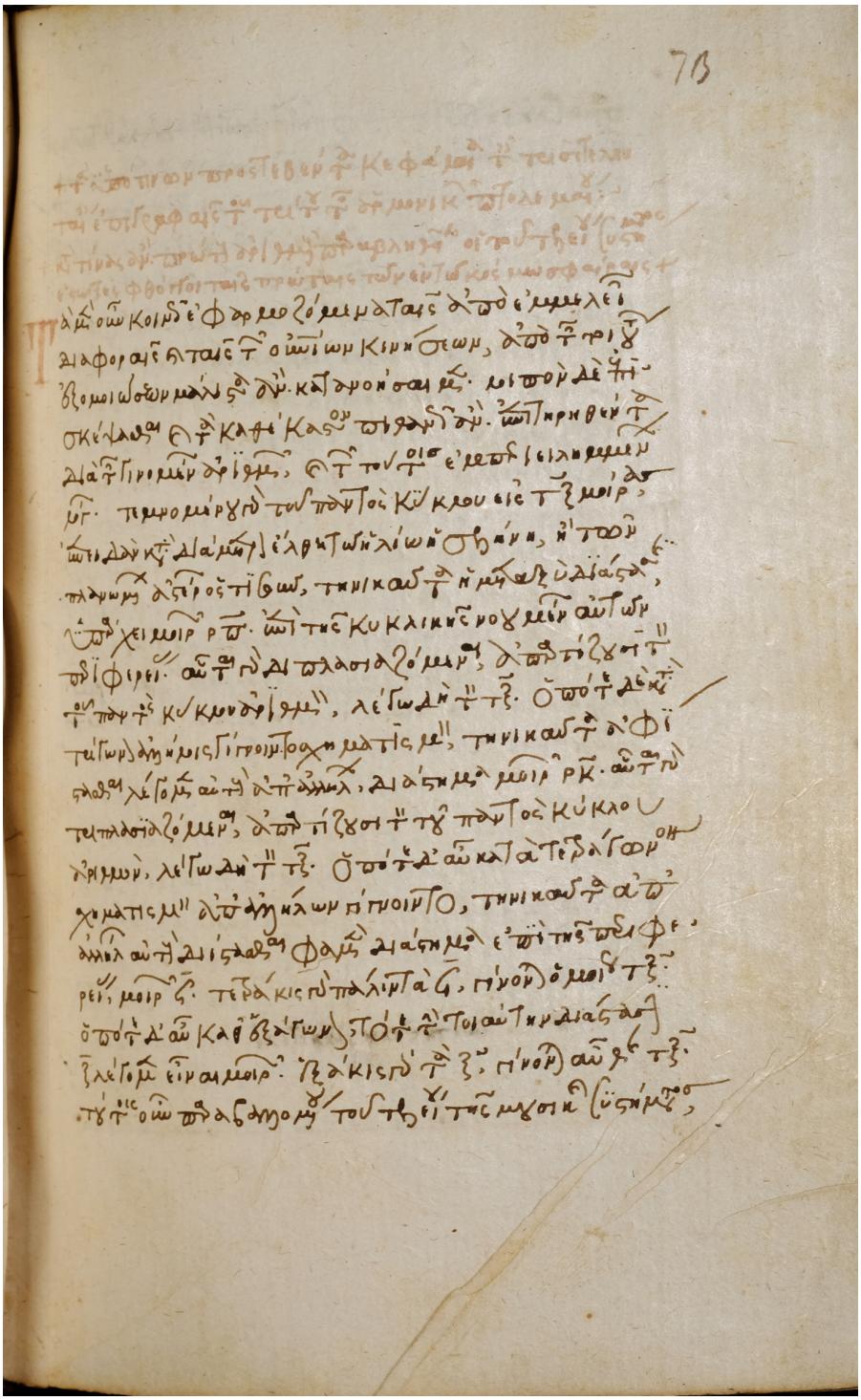
The present collation records the handful of variant readings in *Harm*. III 14–15 (the text is During's, who omitted a sequence by *saut du même au même*), and the entire chapter 16 as it is witnessed in Marc. gr. Z. 332. See the section "A Background to the *Refutatio*" for a discussion of the variant readings.

109, 9 (οὐρανίων) 29 ποιεῖται] νοεῖται 110, 6 πέντε] ε 7 τεσσάρων] δ 8 πέντε] ε τεσσάρων] δ 12–13 διὰ πασῶν ἐπὶ τῆς μουσικῆς τελείῳ in ras. 15 δύο] β 16 ποιεῖ ἐπίτριτον λόγον, πρὸς δὲ τὸν β πρόλογος γίνομενος ποιεῖ τὸν 19 om. τὸν$^2$ 20 om. τοῦ

Πῶς ἂν αἱ τῶν πλανωμένων οἰκειώσεις παραβάλλοιντο ταῖς τῶν φθόγγων
Μή τις δὲ οἰέσθω τὸν μὲν τοῦ Διὸς φθόγγον ἑκατέρῳ τῶν φώτων διὰ συμφωνίας εἶναι, τὸν δὲ τῆς Ἀφροδίτης μόνῳ τῷ τῆς σελήνης, ἐπειδὴ ὁ τόνος οὐκ ἔστιν ἐν λόγῳ συμφωνίας· οὗτος μὲν γὰρ τῆς σεληνιακῆς γέγονεν αἱρέσεως, ὁ δὲ τοῦ Διὸς καταλέλειπται τῆς ἡλιακῆς. Κατὰ ταῦτα ἐπεὶ καὶ τῶν φθοροποιῶν [om. ἑκάτερος φθόγγος ἑκατέρῳ τῶν ἀγαθοποιῶν] τὴν διὰ τεσσάρων ποιεῖ συμφωνίαν, ὁ μὲν τῆς νήτης τῶν ὑπερβολαίων τοῦ Κρόνου πρὸς τὸν τῆς νήτης τῶν διεζευγμένων τοῦ Διός, ὁ δὲ τῆς νήτης τῶν συνημμένων τοῦ Ἄρεος πρὸς τὸν τῆς μέσης τῆς Ἀφροδίτης, ἠκολούθησε τὸ καὶ τὸν μὲν τοῦ Κρόνου τῆς ἡλιακῆς μᾶλλον αἱρέσεως γενέσθαι, τὸν δὲ τοῦ Ἄρεος τῆς σεληνιακῆς. διὸ καὶ τῶν σχηματισμῶν τοὺς μὲν τοῦ Κρόνου πρὸς Δία πάντας ἀγαθοποιοὺς καθίστασθαι συμβέβηκεν, τῶν δὲ Κρόνου πρὸς ἥλιον μόνους τοὺς τριγώνους, ὡς τῶν λοιπῶν συμφωνοτέρους. ὁμοίως δὲ καὶ τοὺς τοῦ Ἄρεος πρός τε τὴν Ἀφροδίτην καὶ τὴν σελήνην μὴ πάντας πάλιν, ἀλλὰ μόνους τοὺς τριγώνους, τὸ δ' ἐναντίον τοὺς μὲν Κρόνου πρὸς σελήνην καὶ Ἀφροδίτην πάντας φαύλους, τοὺς δ' Ἄρεος πρὸς ἥλιον καὶ Δία πάντας ἐπισφαλεῖς.

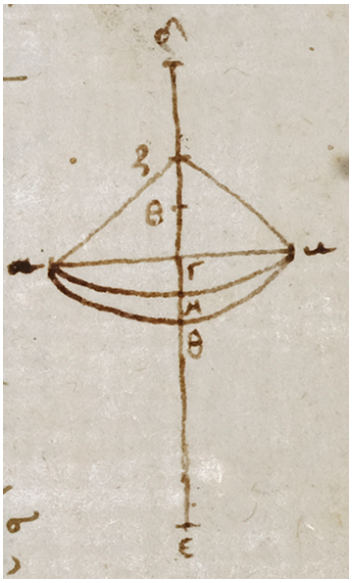
## ICONOGRAPHIC COMPLEMENT

The opening page of the *Refutatio* in the prototype of the tradition

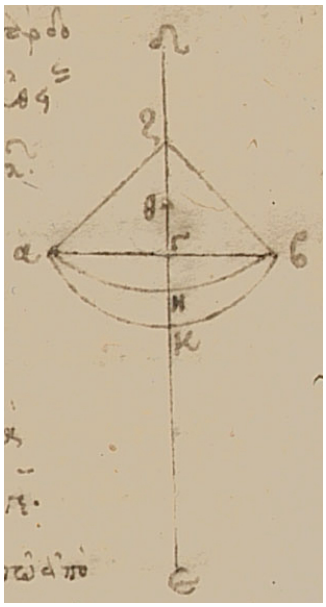


1. Marc. gr. Z. 332, f. 73r

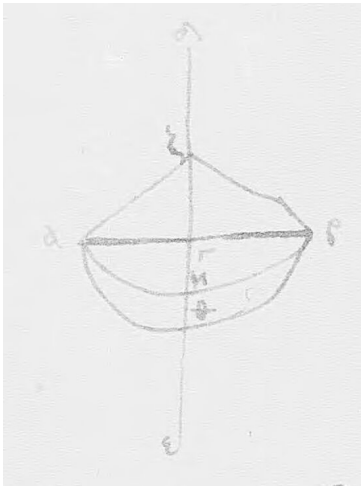
The diagram of Barlaam's *Refutatio* in some manuscripts



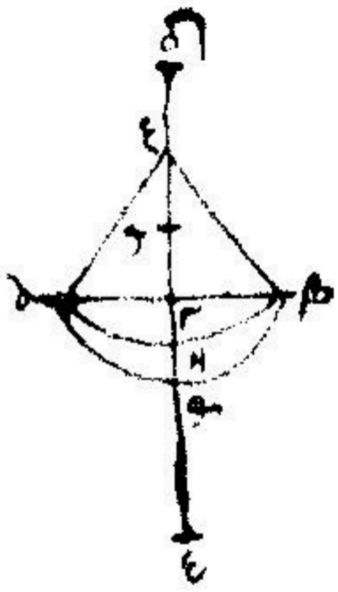
2a. Marc. gr. Z. 332, f. 83r



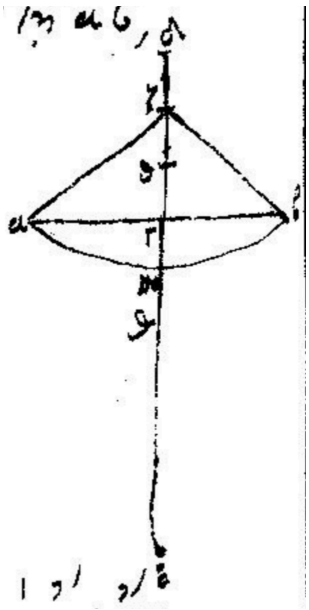
2b. Neap. III.C.3, f. 105r



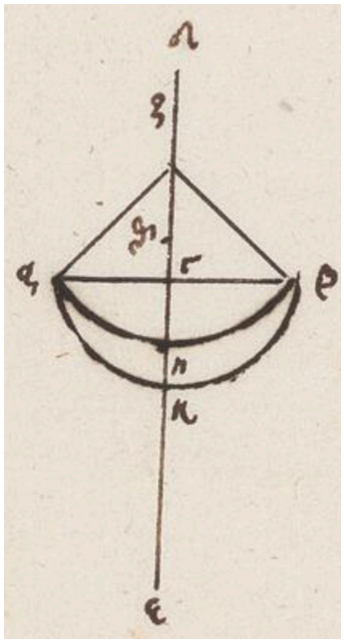
2c. Par. gr. 2381, f. 34v



2d. Vat. gr. 187, f. 79v



2e. Vat. gr. 1756, f. 166v



2f. Vat. Urb. gr. 77, f. 183v

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## PLATES

1 Venezia, Biblioteca Nazionale Marciana, gr. Z. 332 (coll. 643; *Diktyon* 69803), f. 73r, the beginning of Barlaam's *Refutatio*. Su concessione del MiC – Biblioteca Nazionale Marciana. Divieto di riproduzione.

2a Venezia, Biblioteca Nazionale Marciana, gr. Z. 332 (coll. 643; *Diktyon* 69803), f. 83r, the diagram of Barlaam's *Refutatio*. Su concessione del MiC – Biblioteca Nazionale Marciana. Divieto di riproduzione.

2b Napoli, Biblioteca Nazionale Centrale Vittorio Emanuele III, III.C.3 (*Diktyon* 46279), f. 105r, the diagram of Barlaam's *Refutatio*. Su concessione del MiC – Biblioteca Nazionale Centrale Vittorio Emanuele III. Divieto di riproduzione.

2c Paris, Bibliothèque Nationale de France, grec 2381 (*Diktyon* 52013), f. 34v, the diagram of Barlaam's *Refutatio*. @ Paris, Bibliothèque Nationale de France

2d Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. gr. 187 (*Diktyon* 66818), f. 79v, the diagram of Barlaam's *Refutatio*. @ Città del Vaticano, Biblioteca Apostolica Vaticana

2e Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. gr. 1756 (*Diktyon* 68385), f. 166v, the diagram of Barlaam's *Refutatio*. @ Città del Vaticano, Biblioteca Apostolica Vaticana

2f Città del Vaticano, Biblioteca Apostolica Vaticana, Vat. Urb. gr. 77 (*Diktyon* 66544), f. 183v, the diagram of Barlaam's *Refutatio*. @ Città del Vaticano, Biblioteca Apostolica Vaticana