

Arbitrary stochastic transportation maps for data assimilation & the relations to physics-based stochastic transportation maps

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Job offer

Docteur en computer sciences H/F

CDI | Rennes (35) | 38k€ à 43k€

*Vous êtes un.e jeune docteur spécialisé.e en computer sciences, traitement du signal ou mathématiques appliquées avec des compétences en **informatique**, en **analyse numérique**, et éventuellement en **machine learning** ? Vous vous intéressez au spatial, au radar et à l'environnement si tel est le cas, alors vous avez votre place parmi nous !*

Au sein du LAB Scalian en charge du pilotage et de l'organisation de l'ensemble des travaux de R&D et d'innovation, vous participez à des travaux de R&D à l'interface entre HPC, simulations basées sur la physiques et méthodes d'apprentissage sur des données.

Vous mènerez des travaux de recherche et développement sur l'utilisation de techniques de simulation, de machine Learning, de traitement d'images et du signal, appliquées notamment au domaine du spatial, de la santé et de l'environnement. Vous analyserez et implémenterez les méthodes dernières méthodes de l'état de l'art en collaboration avec le CEN Simulation HPC RA/RV, leurs clients et un réseau de partenaires académiques et industriels.

*Avec le CEN Simulation HPC RA/RV, différentes solutions pourront être développés notamment pour la **simulation radar**, le **traitement d'images médicales**, la **super-résolution d'images satellites**, le **couplage modèle/données** pour l'éolien ou encore le **contrôle par IA** de lanceurs réutilisables pour le spatial.*

*Le poste comporte deux dimensions. La première est l'exécution de travaux de R&D avec le LAB en lien avec les activités du CEN Simulation HPC. La deuxième sera une contribution significative à l'**ANR RedLUM** (mots clefs : **SALT-LU, POD-Galerkin, assimilation de données, volumes finis, C++, CFD**)*

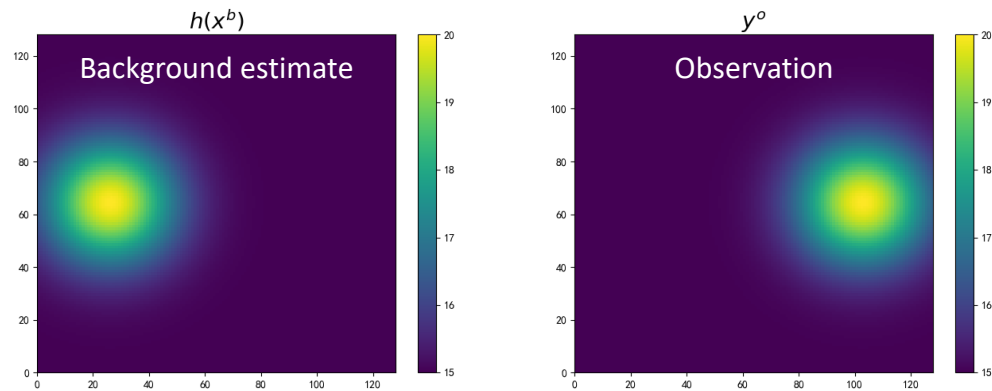
More details here : <https://www.valentinresseguier.com/jobs-offer>

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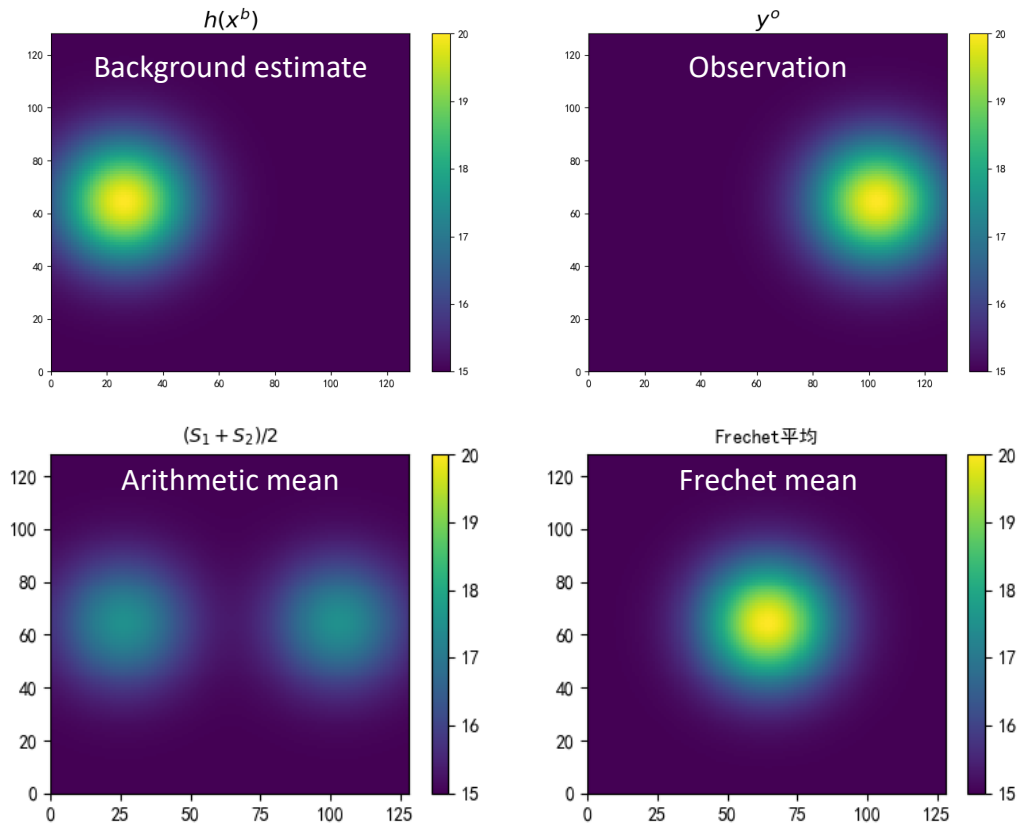
I. Context

Misplacement issue in data assimilation



I. Context

Misplacement issue in data assimilation



PART I

OPTICAL FLOW CORRECTIONS

1. Optical flow for large displacement & differential forms
2. Data assimilation procedure
3. Numerical results

Optical flow for large displacement and differential forms

- ▶ For **differential forms** θ

$$\operatorname{argmin}_T \|\mathbf{T}^* \theta - \theta^{obs}\|_{L^2}^2$$

Resolution :

While (not converged)

- $v = \operatorname{argmax}_v \langle \theta - \theta^{obs}, \mathcal{L}_v \theta \rangle_{L^2} + \|\nabla v\|_{L^2}^2$
s.t. $v|_{\partial\Omega} = v_0$
- $\theta = \theta + \mathcal{L}_v \theta \epsilon$
- $T(x) = T(x) + v(T(x)) \epsilon$

- ▶ For **functions** (0-form) θ

$$\operatorname{argmin}_T \int_{\Omega} \|\theta(\mathbf{T}(x)) - \theta^{obs}(x)\|_2^2 dx$$

Resolution :

While (not converged)

- $v = \operatorname{argmax}_v \langle \theta - \theta^{obs}, v \cdot \nabla \theta \rangle_{L^2} + \|\nabla v\|_{L^2}^2$
s.t. $v|_{\partial\Omega} = v_0$
- $\theta = \theta + v \cdot \nabla \theta \epsilon$
- $T(x) = T(x) + v(T(x)) \epsilon$

Data assimilation procedure

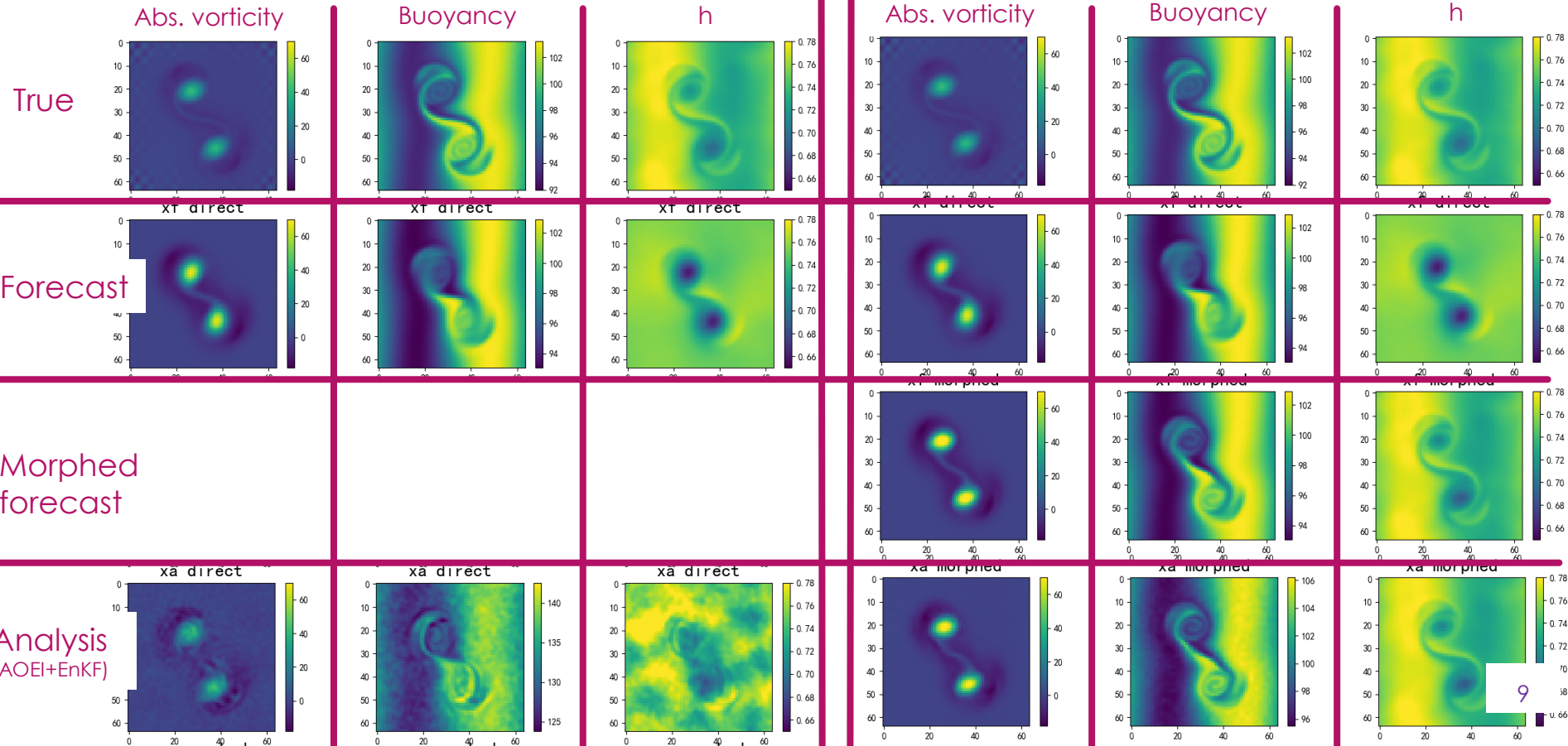
- ▶ Forecast
- ▶ Optical flow (OF) correction
- ▶ Adaptative Observation Error Inflation (AOEI)
- ▶ EnKF (analysis step)

Numerical results

Thermal shallow water
1 assimilation step

Without OF correction

With OF correction

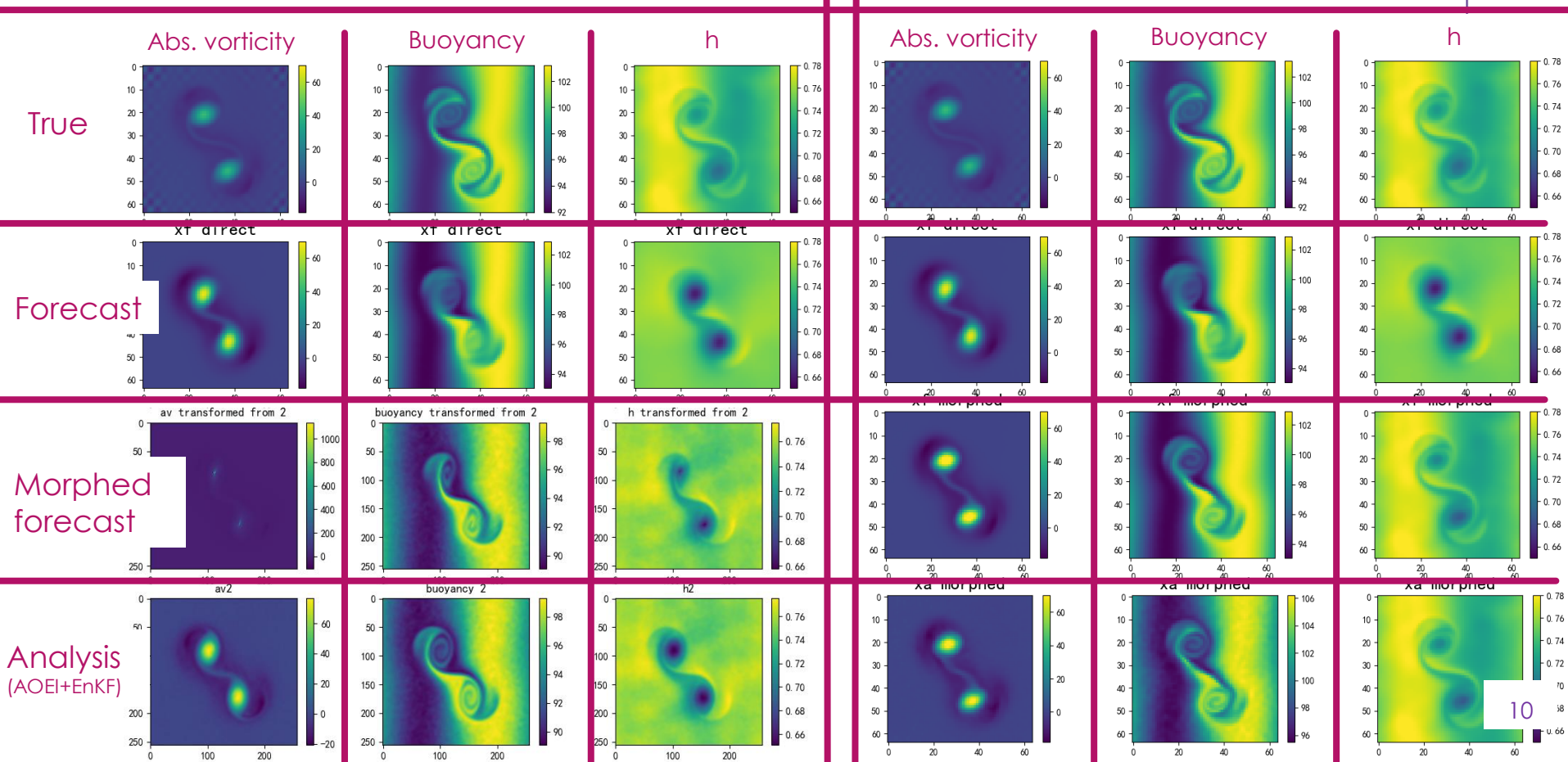


Numerical results

OF without differential geometry ?

0-form OF correction

Diff. form OF correction



Part II

ARBITRARY STOCHASTIC TRANSPORTATION MAPS

1. Motivation
2. Stochastic transportation maps
3. Induced SPDE
4. A simple case : 0-forms
5. Relations to physics-based stochastic closures

Motivation

**Numerical
Simulation
(ROM)**

→ erroneous

**On-line
measurements**

→ incomplete
→ possibly noisy

Velocity



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$3 \text{ m} \cdot \text{s}^{-1}$

Velocity

Motivation

**Numerical
Simulation
(ROM)**

→ erroneous

**On-line
measurements**

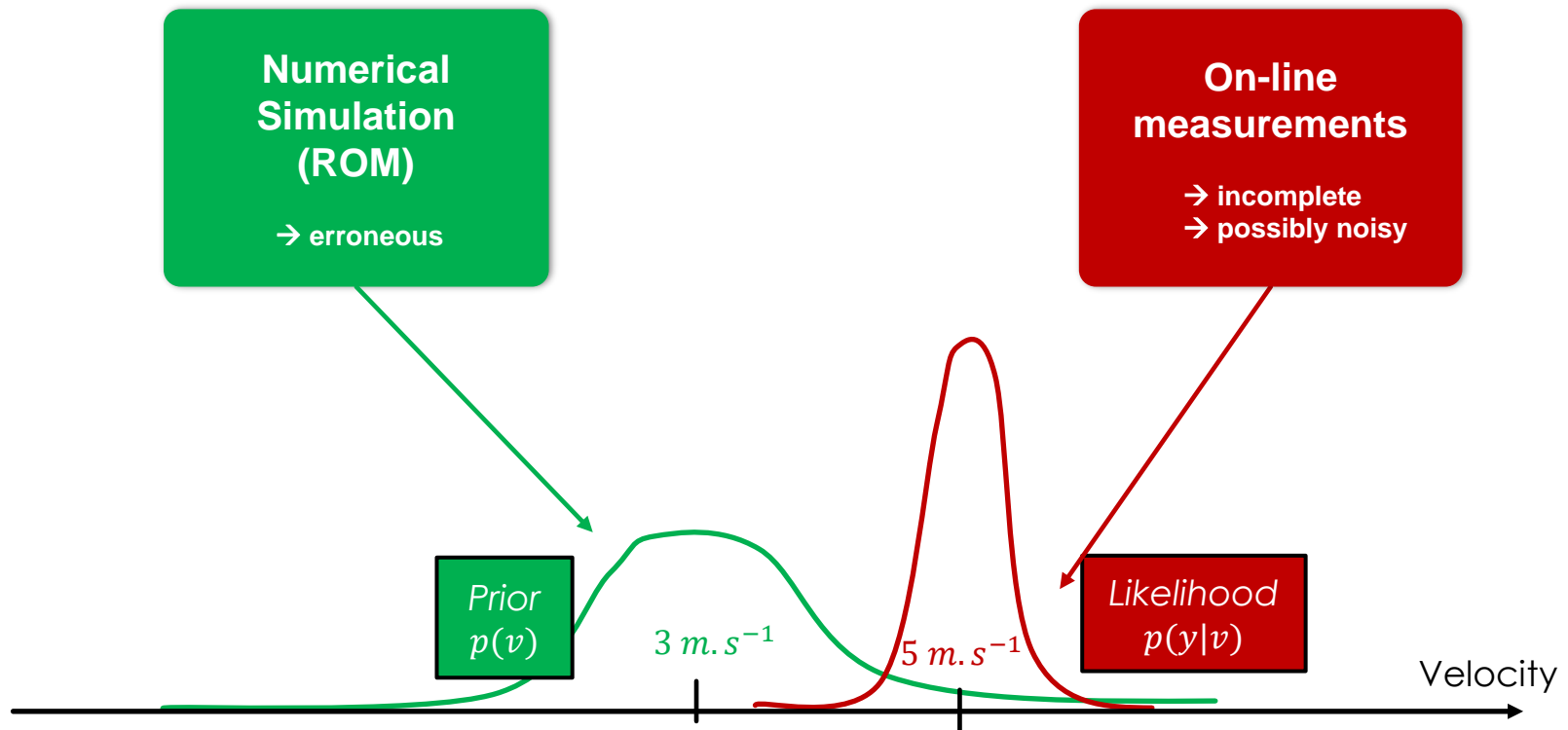
→ incomplete
→ possibly noisy

3 m.s^{-1}

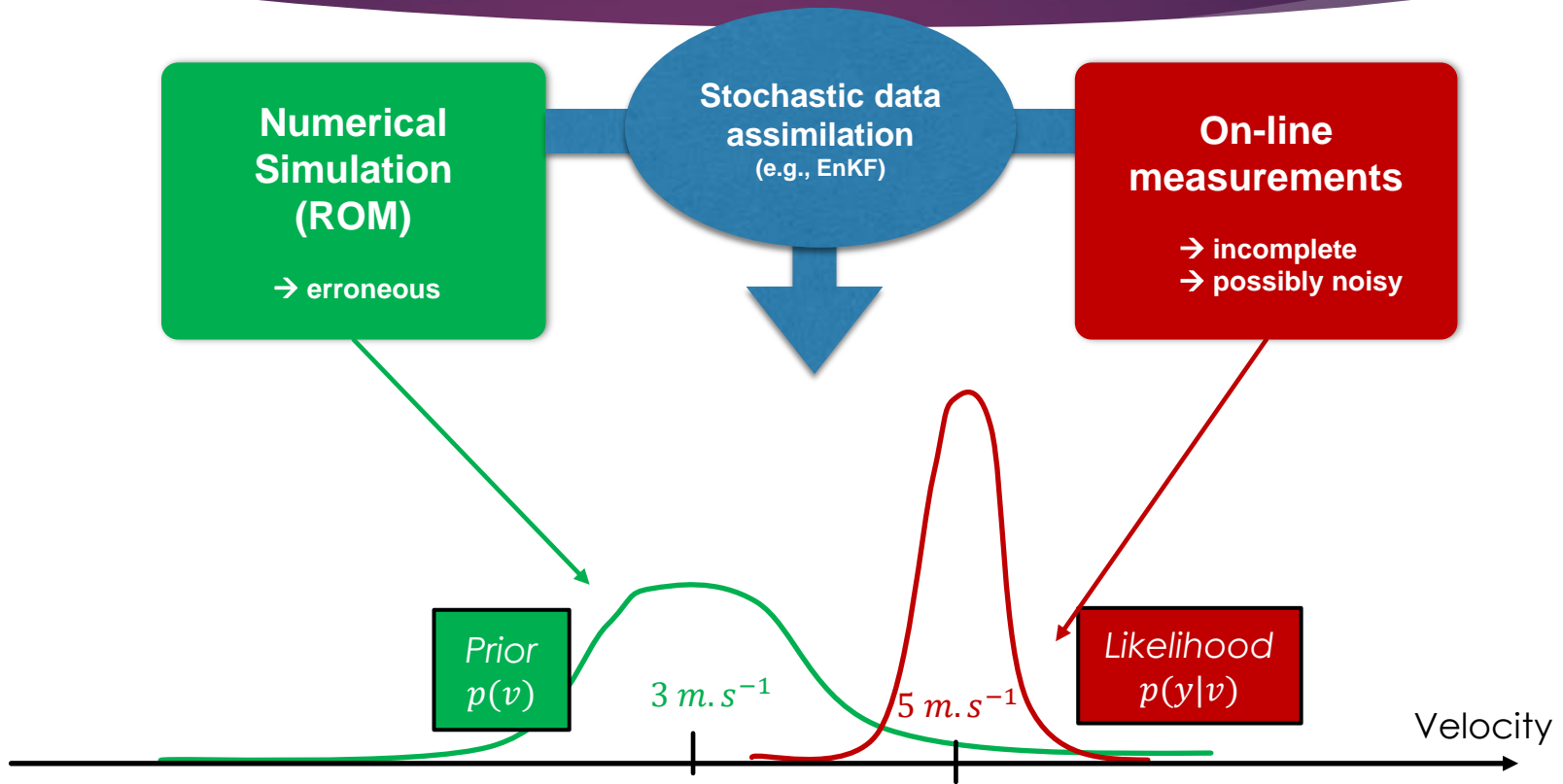
5 m.s^{-1}

Velocity

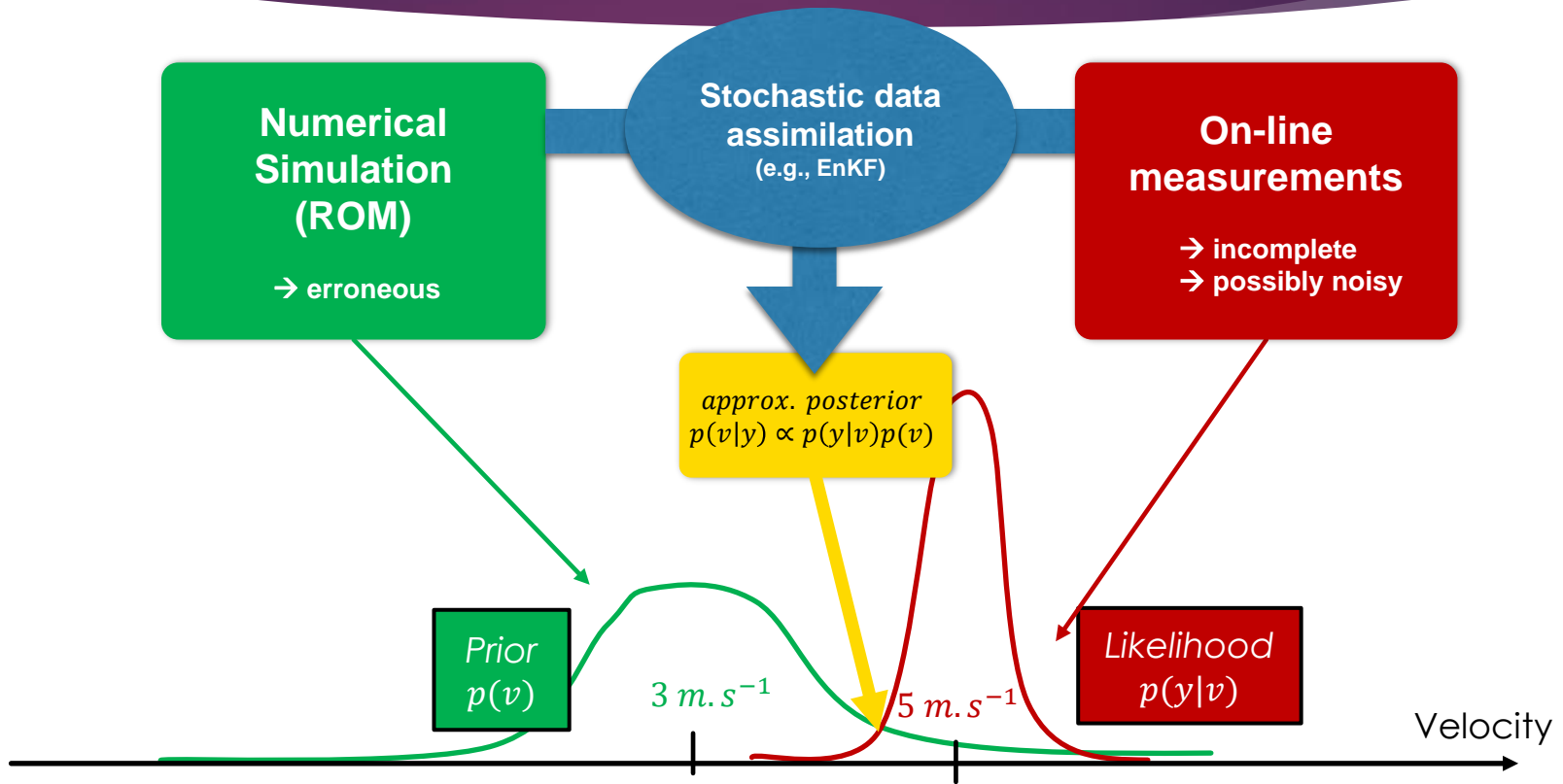
Motivation



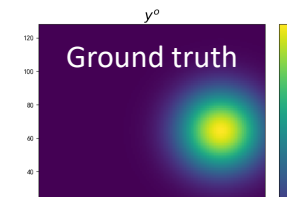
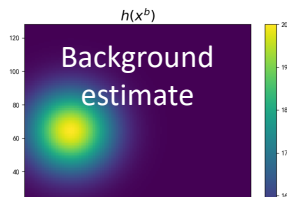
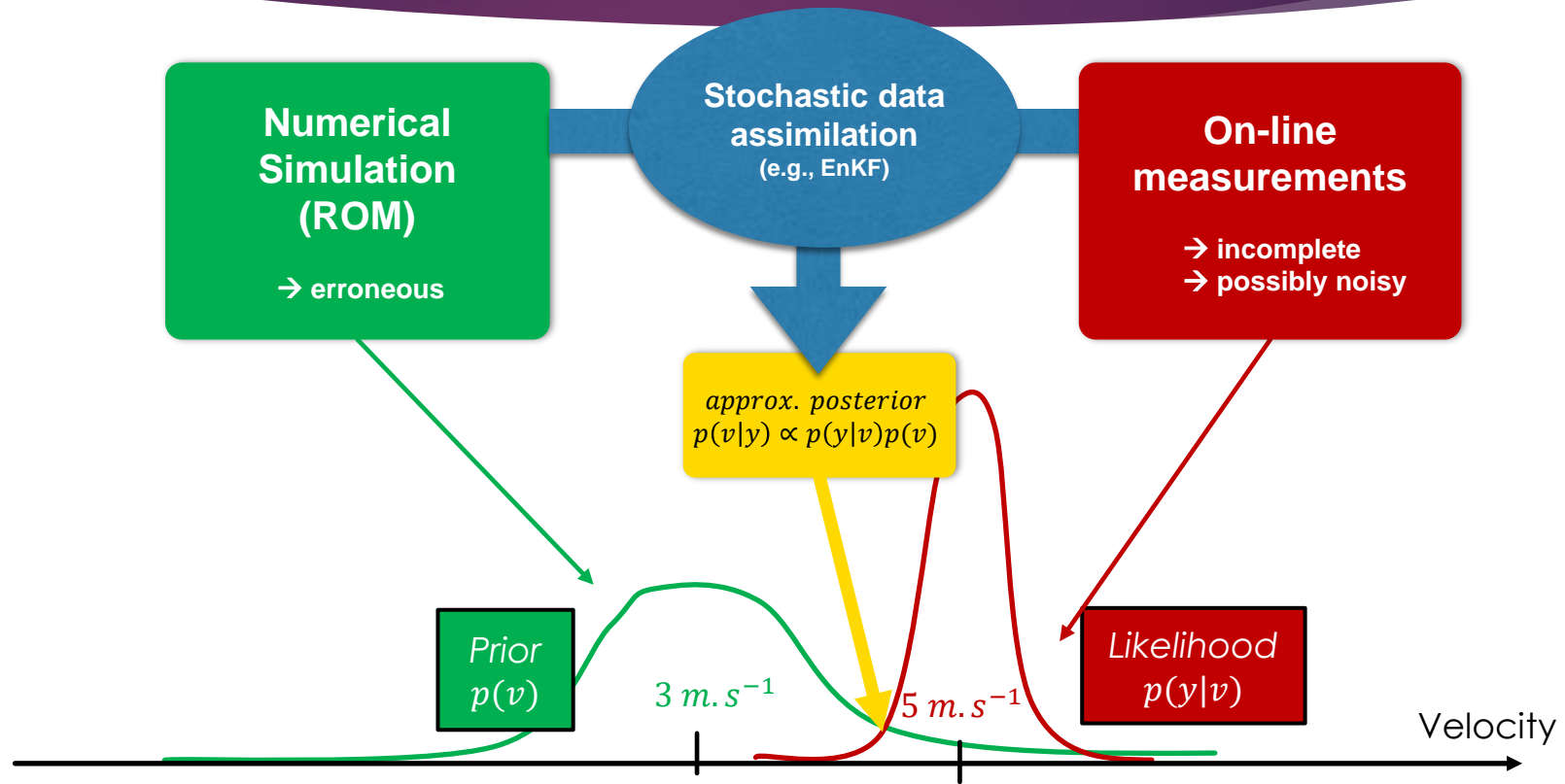
Motivation



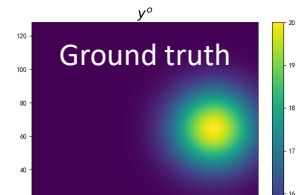
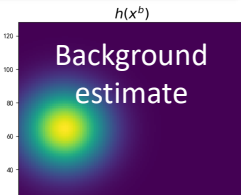
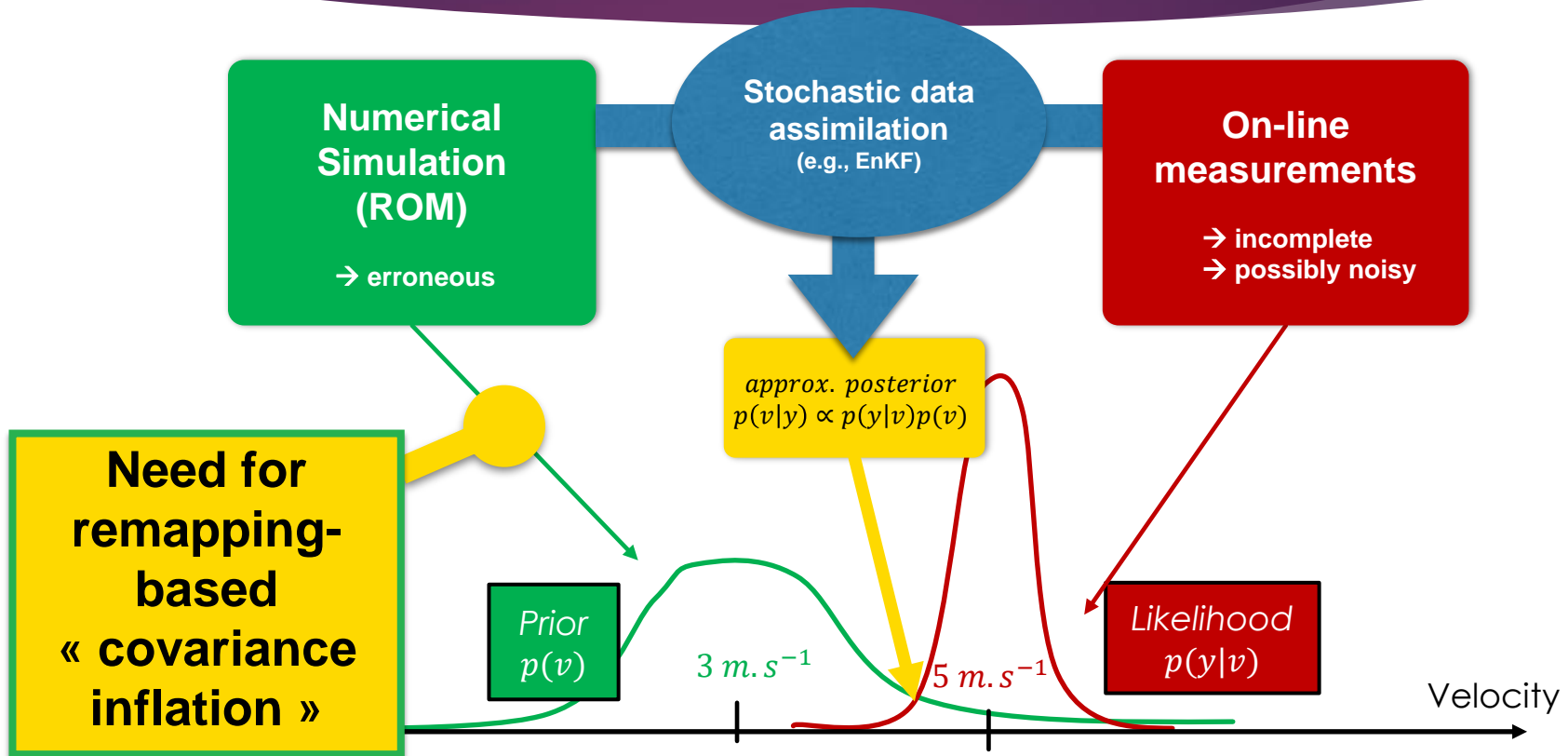
Motivation



Motivation



Motivation



Stochastic transportation maps

- ▶ Here, the (correction) transportation maps $T_t: \mathbb{R}^m \rightarrow \mathbb{R}^m$ is **random** and **given**

$$T_t(x) = x + a(t, x) \Delta t + \sum_i \xi_i(t, x) \Delta W_i(t)$$

- ▶ For a differential k-form $\theta = \sum_{i_1 < \dots < i_k} f^{i_1, \dots, i_k} dx^{i_1} \wedge \dots \wedge dx^{i_k}$

$$\begin{aligned} T_t^* \theta &= \sum_{i_1 < \dots < i_k} f^{i_1, \dots, i_k}(T_t(x)) T_t^*(dx^{i_1} \wedge \dots \wedge dx^{i_k}) \\ &= \theta + \mathcal{M}(\theta) \Delta t + \mathcal{N}_i(\theta) \Delta W_i \end{aligned}$$

Eulerian view : Induced SPDE

- ▶ Let f be a collection of component of the state variables, all contained in S .

$$df = g^f(S)dt \quad (\text{physical model}) \quad (1)$$

- ▶ Depending on the conservation we want to enforce, we associate f to a specific differential form θ .
- ▶ Then, applying the stochastic transportation maps T_t at each time step is equivalent to replace (1) by

$$df = g^f(S)dt + \mathcal{M}^f(f)dt + \sum_i \mathcal{N}_i^f(f)dW_i \quad (2)$$

A simple case : 0-forms

0-form (function) f

Considering

$$df = -w \cdot \nabla f dt \quad (\text{physical model}) \quad (1)$$

► Then, applying the stochastic correction map

$$T_t(x) = x + a(t, x) \Delta t + \sum_i \xi_i(t, x) \Delta W_i(t) \text{ at each time step}$$

is equivalent to replace (1) by

$$df = \left((-w + a)dt + \sum_i \xi_i dW_i \right) \cdot \nabla f + \frac{1}{2} \sum_i \xi_i \xi_i^T : \nabla \nabla^T f dt \quad (2)$$

Relations to physics-based stochastic closures

(Itô) **velocity w as a vector of 0-form** (function)

Considering

$$dw = -w \cdot \nabla w dt + F dt \quad (\text{Navier - Stokes}) \quad (1)$$

- ▶ Then, applying the stochastic correction map T with $T_t^{-1}(x) = x + \sigma \Delta B_t(x, t)$ at each time step is equivalent to use **LU**-like closure

$$\mathbb{D}_t w \doteq dw + ((w + w_S^c) dt + \sigma \circ dB_t) \cdot \nabla w = F dt \quad (2)$$

Mémin (2014). *Geophys. Astro. Fluid.*, 108(2), 119-146.

Resseguier et al. (2020). *Nonlin. Processes Geophys.*, 27(2), 209-234.

Zhen et al. (2023). *Nonlin. Processes Geophys.*, 30, 237-251

Relations to physics-based stochastic closures

(Itô) **momentum** ρw as a vector of **n-form** (function)

Considering

$$d(\rho w_j) = -\nabla \cdot (\rho w_j w) dt + \rho F_j dt \quad (\text{Navier - Stokes}) \quad (1)$$

► Then, applying the stochastic correction map T with

$$T_t^{-1}(x) = x + \sigma \Delta B_t(x, t) \text{ at each time step}$$

is equivalent to use **LU**-like closure

$$d(\rho w_j) + \nabla \cdot \left(\rho w_j \left((w + w_S^c) dt + \sigma \circ dB_t \right) \right) dt = \rho F_j dt$$

Mémin (2014). *Geophys. Astro. Fluid.*, 108(2), 119-146.

Tissot et al. (Submitted). [arXiv:2309.12077](https://arxiv.org/abs/2309.12077)

Zhen et al. (2023). *Nonlin. Processes Geophys.*, 30, 237-251

Relations to physics-based stochastic closures

(Stratonovich) **velocity** $u(= w + w_S^c)$ **as 1-form** (function)

Considering

$$du = -u \cdot \nabla u dt + F dt \quad (\text{Navier - Stokes}) \quad (1)$$

► Then, applying the stochastic correction map T with

$$T_t^{-1}(x) = x + w_S^c(x, t)\Delta t + \sum_i \xi_i(x, t)\Delta W_i(t)$$

is equivalent to use **SALT**-like closure

$$du + \left(u dt + \sum_i \xi_i \circ dW_i \right) \cdot \nabla u + \nabla \left(u dt + \sum_i \xi_i \circ dW_i \right) \cdot u = F dt \quad (2)$$

Holm (2015). Proc. R. Soc. A., 471(2176), 20140963.

Resseguier et al. (2020). Nonlin. Processes Geophys., 27(2), 209-234.

Zhen et al. (2023). Nonlin. Processes Geophys., 30, 237-251

CONCLUSION

Conclusion

- ▶ For better data assimilation procedures
 - ▶ Structure-preserving optical-flow correction before analysis
 - ▶ Space-morphing-based covariance inflation for better *prior*
Arbitrary and/or physics-based (SALT/LU)
- ▶ Next step:

Learning stochastic morphing statistics ξ_i / σ
from estimated structure-preserving optical-flow corrections

Bonus slides

Optical flow on velocity without differential geometry ?

