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An hybrid solver to compute a turbulent compressible model

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Abstract. In this paper, we propose a numerical strategy to compute a turbulence model for compressible flows, including a dynamic estimation of the jump in turbulent kinetic energy across shock waves. The model is taken from [4] and the Finite Volume method applies to a specific Riemann solver interface. The entire procedure requires the detection of shock waves in unsteady flows.

Keywords: Riemann solver, compressible turbulence, shock detection.

1 Introduction

Several compressible turbulence models have been proposed in the literature during the last thirty years, most of them relying on Favre's formalism. One classical difficulty in that framework concerns the correct definition of shock waves, due to the possible occurrence on first-order non-conservative terms in the PDE formulation. Another one is linked with the correct prediction of the turbulent kinetic energy when shock patterns occur in the flow. For that purpose, a specific turbulence model has been proposed in 2006 ([4]), that aims at predicting the turbulent entropy production through the discontinuity in a dynamical way, still using the standard Euler set of equations grounded on mass balance, momentum and energy balances.

Nonetheless, this model is indeed more complex than the simple one proposed in [2], and has never been implemented in CFD codes up to the knowledge of authors. We thus propose in this note a first numerical strategy based on Riemann solvers and the Finite Volume method, in order to handle approximate solutions of this turbulent compressible model. We will first briefly present the model and its main properties. Next we will turn to the numerical procedure. The latter requires defining a suitable shock detection, and it then makes use of a specific interface approximate Riemann solver when some shock wave occurs.

Some numerical results will eventually be presented. More details can be found in [3].

2 Governing equations

The system of PDEs governing the time-space evolution of the variables is:

$$\left\{ \begin{array}{l} \partial_t(\rho) + \partial_x(\rho u) = 0 \\ \partial_t(\rho u) + \partial_x\left(\rho u^2 + P + \frac{2K}{3}\right) = 0 \\ \partial_t(\rho E) + \partial_x\left(u\left(\rho E + P + \frac{2K}{3}\right)\right) = 0 \\ \partial_t(\rho \xi) + \partial_x(\rho u \xi) = -M(x, t)\delta_{(x-\sigma t=0)} \end{array} \right. \quad (1)$$

The quantities ρ , u , P , K , E and ξ respectively represent the mean density, the mean velocity, the mean pressure, the turbulent kinetic energy, the mean total energy and the turbulent entropy. We define the mean total energy, the modified pressure and the turbulent entropy, as follows:

$$\rho E = \rho e(P, \rho) + \frac{\rho u^2}{2} + K, \quad P^* = P + \frac{2}{3}K \quad \text{and} \quad \xi = K\rho^{-5/3}.$$

The turbulent speed of density waves \tilde{c} is given by:

$$\tilde{c}^2 = c^2 + \frac{10K}{9\rho} = c^2 + \frac{10}{9}\xi\rho^{2/3}, \quad \text{where} \quad c^2 = \left(\frac{\partial e}{\partial P}\Big|_{\rho}\right)^{-1}\left\{\frac{P}{\rho^2} - \frac{\partial e}{\partial \rho}\Big|_P\right\}.$$

The Dirac mass is $\delta_{(x-\sigma t=0)}$ located in $x = \sigma t$, $M(x, t)$ is the source term located at the shock position given by formula introduced in [4] and detailed in [3], and σ is the velocity of the shock wave. The properties of system (1) in a general framework with respect to the **EOS** are as follows:

Proposition 1. *The system is hyperbolic. It admits four real eigenvalues:*

$$\lambda_1(W) = u - \tilde{c}, \quad \lambda_{2,3}(W) = u, \quad \lambda_4(W) = u + \tilde{c}. \quad (2)$$

Proposition 2. *We introduce the entropy-entropy flux pair (η, f_η) with:*

$$\eta(w) = -\rho s, \quad \text{and} \quad f_\eta(w) = -\rho u s. \quad (3)$$

Then the following inequality holds for smooth solutions:

$$\partial_t \eta + \partial_x f_\eta \leq 0. \quad (4)$$

Proposition 3. *The Rankine-Hugoniot relations associated with model (1) are:*

$$\left\{ \begin{array}{l} -\sigma[\rho] + [\rho u] = 0, \\ -\sigma[\rho u] + \left[\rho u^2 + P + \frac{2K}{3}\right] = 0, \\ -\sigma[\rho E] + \left[u\left(\rho E + P + \frac{2K}{3}\right)\right] = 0 \\ -\sigma[\rho \xi] + [\rho u \xi] = -M \end{array} \right. \quad (5)$$

By noting the jump:

$$[f] = f_R - f_L,$$

between the left and right states on each side of a discontinuity travelling at speed σ .

3 Numerical method

The method requires the definition of an interface hybrid solver to calculate approximate solutions for the problem. This hybrid solver is made up of two solvers: a classical approximate Riemann solver when no shock wave is detected, and a specific interface solver dedicated to the interface where a shock wave has been detected. We will refer to this as the Modified Approximate Riemann Solver "MARS". For this we use a classical Finite Volume formulation and we focus on an one-dimensional domain.

In the sequel, we define a practical technique to detect shock waves, then the modified solver "MARS" (the classical approximate Riemann solver is detailed in [1]) and finally the numerical cell scheme.

a- Shock detector: For an interface ' $i + \frac{1}{2}$ ' separating the two cells i and $i + 1$, we define the quantity $g_{i,i+1}$ as follows:

$$g_{i,i+1}^n = -\sigma_{i+1/2}[\eta^n]_i^{i+1} + [f_\eta^n]_i^{i+1}, \quad (6)$$

where (η, f_η) denotes the entropy-entropy flux pair, with $\eta = -\rho s$ and $f_\eta = u\eta$. The estimation of the shock velocity is made on the basis of the mass balance:

$$\sigma_{i+1/2} = [(\rho u)^n]_i^{i+1} / [\rho^n]_i^{i+1}. \quad (7)$$

For a discontinuity between cells 'i' and 'i+1' travelling at speed $\sigma_{i+\frac{1}{2}}$, we have the following inequality: $g_{i,i+1}^n < 0$. Though redundant, we also enforce the test: $[u^n]_i^{i+1} < 0$. Using the Lax inequalities (see [5]), we can deduce if it is a 1-shock or a 3-shock.

b- Modified Approximate Riemann Solver when a 4-shock occurs: The wave configuration retained for the hybrid solver at the interface is depicted in figure 1, when a 4-shock occurs. We calculate the modified solver in three steps which are:

- use of the jump relation through 4-wave,
- exact connection through the double contact wave $\lambda_{2,3} = u$,
- connection between state 'i' and '1' through linearized 1-wave

Step 1: The jump relations associated with the 4-wave are as follows :

$$\begin{cases} \rho_{i+1}(u_{i+1} - \sigma) = \rho_2(u_2 - \sigma) = m \\ m^2[\tau]_2^{i+1} + [P^*]_2^{i+1} = 0 \\ [e + K\tau]_2^{i+1} + \bar{P}_{i+1,2}^*[\tau]_2^{i+1} = 0 \\ m[\xi]_2^{i+1} + M_0 = 0 \end{cases} \quad (8)$$

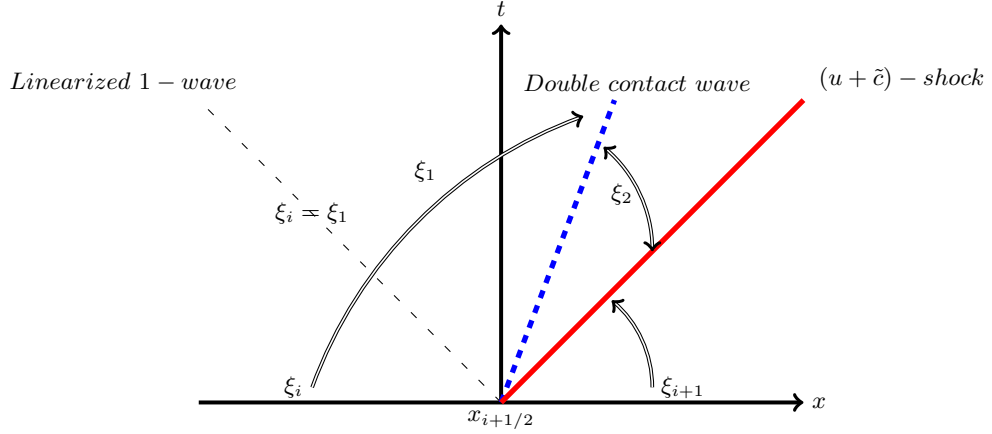


Fig. 1. Approximate solution of the Riemann problem in terms of ξ , when a 4-shock occurs. Intermediate states are noted '1', '2'.

The shock speed $\sigma_{i+1/2}$ is known numerically on the interface thanks to the detection stage (7). Using relation (8a), we have $m = \rho_{i+1}(u_{i+1} - \sigma_{i+1/2})$, with M_0 given by [3] and (8d), we find $\xi_2 = \xi_{i+1} + \frac{M_0}{m}$. We solve (8b)-(8c) to get τ_2 , and thus $P_2^* = P_{i+1}^* + m^2[\tau]_2^{i+1}$ and $u_2 = \frac{m}{\rho_2} + \sigma_{i+1/2}$.

Step 2: Knowing that the Riemann invariants associated with the contact wave are (u, P^*) , we set: $u_1 = u_2$ and $P_1^* = P_2^*$.

Step 3: u_1 and P_1^* are now known, thus using VFRoe-ncv we obtain: $\rho_1 = \rho_i + \frac{[P^*]_i^1}{2c_{i,i+1}^2} - \bar{\rho} \frac{[u]_i^1}{2c_{i,i+1}}$.

Numerical cell scheme: The numerical scheme reads :

$$\Delta x_i (Z_i^{n+1} - Z_i^n) + \Delta t (\mathcal{F}_{i+\frac{1}{2}}^n - \mathcal{F}_{i-\frac{1}{2}}^n) + \Delta t B_i^n = 0, \quad (9)$$

with $Z = (\rho, \rho u, \rho E, \rho \xi)$, $B_i^n = (0, 0, 0, M_i^n)$, and:

$$\mathcal{F}_{i+\frac{1}{2}}^n = \begin{cases} \mathcal{F}_{i+\frac{1}{2}}^{lnd} = F(Z(W_{lnd}^*(W_i^n, W_{i+1}^n))) & \text{when no shock wave has been detected,} \\ \mathcal{F}_{i+\frac{1}{2}}^{MARS} = F(Z(W_{MARS}^*(W_i^n, W_{i+1}^n))) & \text{if a shock wave has been detected,} \end{cases} \quad (10)$$

where:

$$F(Z) = (\rho u, \rho u^2 + P^*, u(\rho E + P^*), \rho u \xi). \quad (11)$$

The solution $W_{lnd}^*(W_L, W_R)$ of the approximate Riemann problem, is found with the linearized solver VFRoe-ncv (see [1, 2]) and the solution $W_{MARS}^*(W_L, W_R)$ of the approximate Riemann problem, is found with the modified interface solver

'MARS'. M_i^n is a contribution in cell i to the global jump of turbulent entropy M_0 . The calculation of M_i is done as follows:

We define: $M_i^n = 0$, except:

$$\begin{cases} M_{i_0}^n = \frac{1}{2} M_0, \\ M_{i_0+1}^n = \frac{1}{2} M_0. \end{cases} \quad (12)$$

where $(i_0 + 1/2)$ is the interface where the shock wave has been detected at time t^n .

4 Numerical results

We consider a single 4-shock wave. We assume that $M_0 \neq 0$ is given by the user using [4]. We consider a perfect gas EOS: $P=(\gamma-1)\rho e$ where $\gamma = 7/5$. The exact solution is shown in figure 2. The initial condition of the Riemann problem is

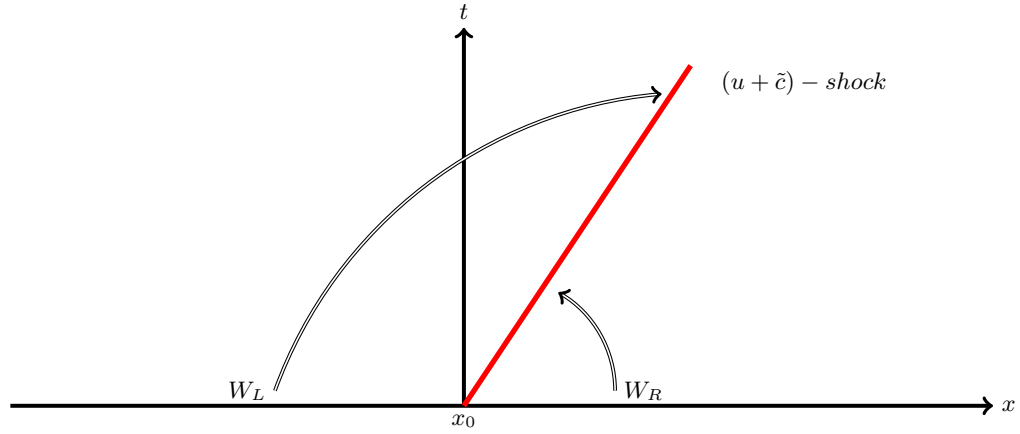


Fig. 2. The exact solution of the Riemann problem for the 4-shock wave.

given by:

$$\begin{aligned} \sigma &= 580, \text{ and } M_0 = -50000, \\ (\rho_L, u_L, P_L, \xi_L) &= (1.936, 280.451, 261265.856, 1000) \\ (\rho_R, u_R, P_R, \xi_R) &= (1, 650, 10^5, 913.793) \end{aligned}$$

where W_R , σ , M_0 are given, and W_L calculated with (8). Figure 3 shows the behavior of the density ρ , velocity u , modified pressure P^* and turbulent entropy

ξ at a given time $T_f = 3 \times 10^{-3}$, on different meshes with 100 cells, 1000 cells and 10000 cells. Figure 4 shows the convergence curve, for the set of variables $\{\rho, u, P^*, \xi\}$. The error varies as $\approx h^1$ for all variables (ρ, u, P^*, ξ), hence the convergence of all the variables.

Other schemes such as those detailed in [3] may lead to some consistency error (see figure 5), in connection with the definition of the mass integration, which is different from (12).

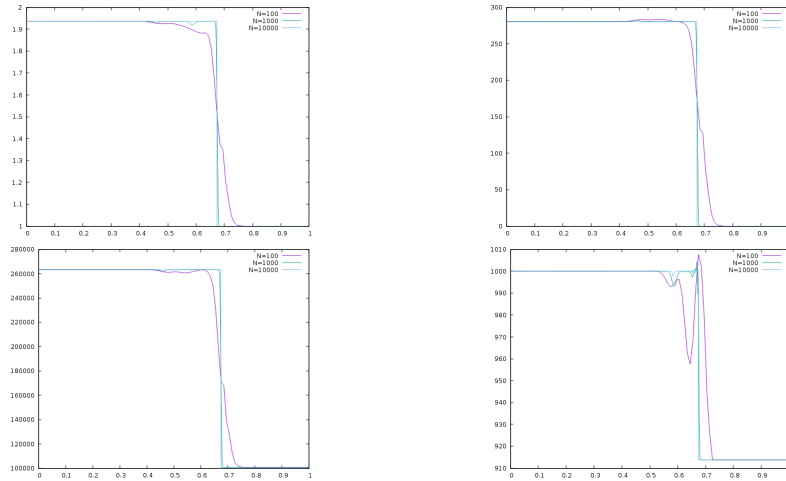


Fig. 3. Density (top left), velocity (top right), P^* (bottom left) and ξ (bottom right). Profile of the approximate solution for different meshes =100 cells, 1000 cells, 10000 cells at $t = 3 \times 10^{-3}$ s, $CFL = 0.5$.

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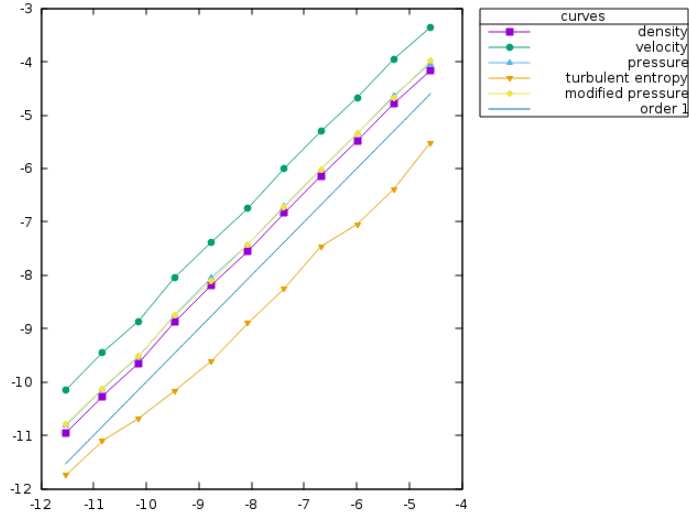


Fig. 4. Simple 4-shock wave test case 1, with modified scheme. Convergence curves: logarithm of the relative L1-error versus the logarithm of the mesh size with uniform meshes containing from 200 to 200000 cells. The error is plotted for variables, ρ , u , P , P^* and ξ .

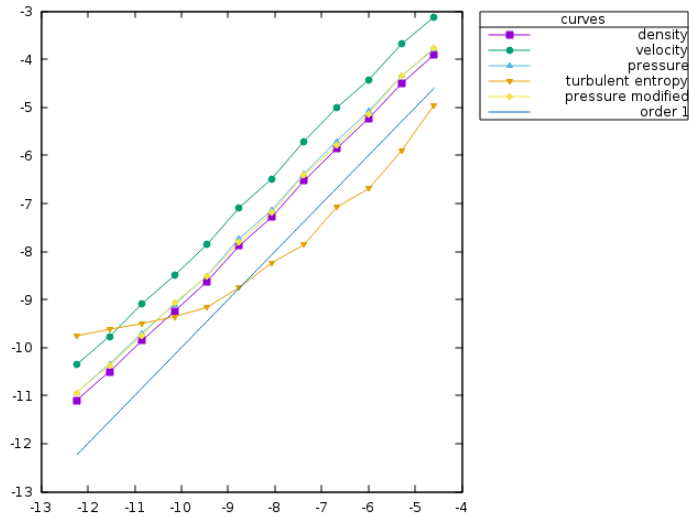


Fig. 5. Simple 4-shock wave test case 1, with another scheme (see [3]).