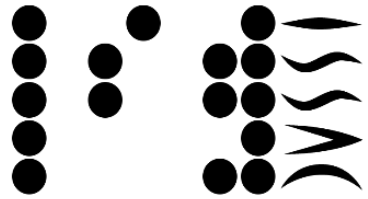
# ISSAC 2023



July 24-27, 2023 - Tromsø, Norway

Proceedings of the 2023
International Symposium on
Symbolic and Algebraic Computation

# Proceedings of the

## International Symposium on Symbolic & Algebraic Computation (ISSAC 2023)

### July 24-27, 2023 | Tromsø, Norway

General Chair:
***Alicia Dickenstein (University of Buenos Aires, Argentina)***

Program Committee Chair:
***Elias Tsigaridas (INRIA Paris and Sorbonne University, France)***

Proceedings Editor:
***Gabriela Jeronimo (University of Buenos Aires, Argentina)***

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# Foreword

The International Symposium on Symbolic and Algebraic Computation (ISSAC) is the premier conference for research in symbolic computation and computer algebra. ISSAC is the continuation of conferences with the names SYMSAC, EUROCAL, EUROSAM, and EUROCAM, which have alternated between North America and Europe before merging into ISSAC. The whole series started in 1966 with the first ACM Symposium on Symbolic and Algebraic Manipulation, and it has been held annually since 1981. The conference typically presents a range of invited talks, tutorials, poster sessions, software demonstrations, and vendor exhibits, with its center-piece being peer-reviewed contributed research papers.

ISSAC 2023 is the 48th meeting in the series. The meeting was held from the 24th to the 27th of July 2023 in Tromsø, Norway. It was sponsored by the ACM Special Interest Group on Symbolic Manipulation (ACM SIGSAM) of the Association for Computer Machinery (ACM), and also generously supported by

- Maplesoft,
- the Fachgruppe Computeralgebra
- UiT The ARtic University of Norway,
- the Lie-Størmer Center for Fundamental Structers in Computational and Pure Mathematics,
- the Trømso Research Foundation (TFS),
- the Trond Mohn Foundation (TMS),
- the Research Council of Norway.

As ever, the ISSAC meeting is a showcase for original research contributions on all aspects of computer algebra and symbolic mathematical computation, including:

Algorithmic aspects:

- Exact and symbolic linear, polynomial and differential algebra
- Symbolic-numeric, homotopy, perturbation and series methods
- Computational algebraic geometry, group theory and number theory
- Quantifier elimination and logic
- Computer arithmetic
- Summation, recurrence equations, integration, solution of ODEs and PDEs
- Symbolic methods in other areas of pure and applied mathematics
- Complexity of algebraic algorithms and algebraic complexity

Software aspects:

- Design of symbolic computation packages and systems
- Language design and type systems for symbolic computation
- Data representation
- Consideration for modern hardware
- Algorithm implementation and performance tuning
- Mathematical user interfaces
- Use with systems for digital libraries, courseware, simulation and optimization, theorem proving, computer-aided design, and automatic differentiation, among others.

Application aspects:

- Applications that stretch the current limits of computer algebra algorithms or systems, use computer algebra in new areas or new ways, or apply it in situations with broad impact.

The ISSAC 2023 Program Committee adhered to the highest standards and practices in the evaluation of the submitted research papers, and we are very pleased with the quality of the papers appearing at the conference. The 103 papers submitted to ISSAC 2023 were judged, and accepted or rejected, solely according to their scientific novelty and excellence. The total number of obtained reviews is 246, thus an average of 2.39 referee reports for each submission. A first selection of the submissions occurred, and a number of papers were given the opportunity to be reconsidered after a rebuttal phase. Ultimately 56 contributed papers were accepted for presentation and publication in these Proceedings.

In addition to the contributed research presentations, the program of ISSAC 2023 features three invited talks, three tutorials, a software presentation session, and a poster session. These Proceedings contain all accepted contributed papers, as well as abstracts of the invited talks and tutorials, some of which in extended form.

We gratefully acknowledge the thorough and important work of the Program Committee and the 160 external reviewers, as well as all the authors and speakers for their contributions.

In addition to the dedicated effort of the local organizers, running a large conference such as ISSAC requires the dedicated work of many volunteers. We extend our thanks to them. Without their contributions, the conference would not have been possible.

**Alicia Dickenstein** *(General Chair)*
**Elias Tsigaridas** *(Program Committee Chair)*
**Cordian Riener** *(Local Arrangements Chair)*
**Gabriela Jeronimo** *(Proceedings Editor)*

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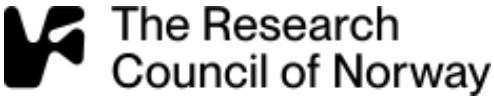
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