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Wave propagation of acoustic waves in porous media is considered. The medium is assumed to have a rigid frame, so that the propagation takes place in the air which fills the material. The Euler equation and the constitutive relation are generalized to take into account the dispersive nature of these media. It is shown that the connection between the fractional calculus and the behavior of materials with memory allows time-domain wave equations, the coefficients of which are no longer frequency dependent, to be worked out. These equations are suited for direct and inverse scattering problems, and lead to the complete determination of the porous medium parameters. © *2000 Acoustical Society of America.* [S0001-4966(00)02701-6]

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INTRODUCTION

During the last two decades, there has been a reborn interest in sound propagation in porous materials. The main reason is not only the necessity of noise control in architectural acoustics or in transport vehicles, but also a better theoretical understanding of wave propagation in complex media. Special attention has been paid to wave propagation in porous media having a rigid frame, and nowadays the monochromatic wave propagation in these media is well understood. Some of the recent progress in this area is reviewed in Refs. 1 and 2. If many applications like medical imaging or inverse scattering problems[3] require the study of the behavior of pulses traveling in porous media, it is only recently that the response of these media to such excitations has been addressed.[4] To efficiently cope with the specific problems appearing in the transient acoustic field propagation, suited methods not related to fixed frequency formulation must be applied. Among them, one can cite the hereditary mechanics[5] or the time-domain approaches. The time-domain response of a material is described by an instantaneous response and a ''susceptibility'' kernel responsible for the memory effects. Evidently, the Fourier transformation translates the fixed frequency results into the time domain. However, in the analysis of the transient behavior of the fields, especially the short time behavior near the wave front, the investigations of the problem as a time-domain problem are more appropriate. A time-domain approach differs from the frequency analysis in that the susceptibility functions of the problem are convolution operators acting on the velocity and the pressure fields and therefore a different algebraic formalism has to be applied to resolve the wave equation. In the past, many authors have used the fractional calculus as an empirical method of describing the properties of viscoelastic materials.[6,7] The observation that the asymptotic expressions of stiffness and damping in porous materials are proportional to fractional powers of frequency suggests the fact that time derivatives of fractional order might describe the behavior of sound waves in this kind of material. In addition to that, fractional-order time derivatives simultaneously model relaxation and frequency dependence.

The outline of the paper is as follows. In Sec. I the model of equivalent fluid is presented and the basic equations in the frequency domain are given. Section II is devoted to the connection between fractional derivative and wave propagation in rigid porous media. Section III contains the asymptotic analyses for the low and high frequencies; the generalized Euler equation and constitutive relation and the time-domain wave equations are worked out.

I. MODEL OF THE EQUIVALENT FLUID

Let a homogeneous isotropic porous material with porosity ϕ be saturated with a compressible and viscous fluid of density ρ_f and viscosity η. It is assumed that the frame of this porous solid is not deformable when it is subjected to an acoustic wave. It is the case, for example, for a porous medium which has a large skeleton density or very large elastic modulus or weak fluid-structure couplings. To apply the results of linear elasticity it is required that the wavelength of sound waves should be much larger than the sizes of pores or grains in the medium.

In these porous materials, acoustic waves propagate only in the fluid. They can be seen as an equivalent fluid, the density and the bulk modulus of which are ''renormalized'' by the fluid-structure interactions. Viscous and thermal effects are well described by the Kirchhoff theory of the sound propagation in cylindrical tubes. Unfortunately, the fundamental equations of acoustics that are used in this theory can be very difficult to solve in the case of tubes with a noncircular cross section. Zwikker and Kosten[8] have worked out a simplified model of the sound propagation in the case of a circular cross section where viscous and thermal effects are treated separately. They derived the important result that the correction factor of bulk modulus of the fluid only contains the thermal effect and that of density only the viscous one. The validity of this model has been justified later for a large

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range of radii at acoustical frequencies. Taking this result into consideration, it is assumed that the basic equations of the model of the equivalent fluid are the Euler equation (E) and the law of the mass conservation (M) associated with the behavior (or adiabatic) equations, and we will henceforth refer to them as EM equations,

$$\rho_f \alpha(\omega) \frac{\partial v_i}{\partial t} = -\nabla_i p, \quad \frac{\beta(\omega)}{K_a} \frac{\partial p}{\partial t} = -\nabla \cdot \mathbf{v}. \tag{1}$$

In these relations, $\mathbf{v}$ and p are the particle velocity and the acoustic pressure, $K_a = \gamma P_0$ is the compressibility modulus of the fluid, $\alpha(\omega)$ and $\beta(\omega)$ are the dynamic tortuosity of the medium and the dynamic compressibility of the air included in the porous material. These two response factors are complex functions which heavily depend on the frequency $f = \omega/2\pi$. Their theoretical expressions are given by Allard[1] and Lafarge,[2]

$$\alpha(\omega) = \alpha_\infty \left(1 + \frac{\eta\phi}{j\omega\alpha_\infty\rho_f k_0} \sqrt{1 + j\frac{4\alpha_\infty^2 k_0^2 \rho_f \omega}{\eta\Lambda^2\phi^2}} \right), \tag{2}$$

$$\beta(\omega) = \gamma - (\gamma - 1) \times \left(1 + \frac{\eta\phi}{j\omega\rho_f k_0' \,\mathrm{Pr}} \sqrt{1 + j\frac{4k_0'^2 \rho_f \omega \,\mathrm{Pr}}{\eta\phi^2\Lambda'^2}} \right)^{-1}, \tag{3}$$

where $j^2 = -1$, γ represents the adiabatic constant, Pr the Prandtl number, α_∞ the tortuosity, k_0 the static permeability, k_0' the thermal permeability and Λ and Λ' the viscous and thermal characteristic lengths.[9] This model was initially developed by Johnson, Koplik and Dashen,[10] and completed by Allard and Champoux by adding the description of thermal effects.[11] Later on, Lafarge introduced the parameter k_0' which describes the additional damping of sound waves due to the thermal exchanges between fluid and structure at the surface of the pores.[2]

The functions $\alpha(\omega)$ and $\beta(\omega)$ express the viscous and thermal exchanges between the air and the structure which are responsible for the sound damping in acoustic materials. These exchanges are due on the one hand to the fluid-structure relative motion and on the other hand to the air compressions-dilatations produced by the wave motion. The parts of the fluid affected by these exchanges can be estimated by the ratio of a microscopic characteristic length of the media, as, for example, the sizes of the pores, to the viscous and thermal skin depth thickness $\delta = (2\eta/\omega\rho_0)^{1/2}$ and $\delta' = (2\eta/\omega\rho_0 P_r)^{1/2}$. For the viscous effects this domain corresponds to the region of the fluid in which the velocity distribution is perturbed by the frictional forces at the interface between the viscous fluid and the motionless structure. For the thermal effects, it is the fluid volume affected by the heat exchanges between the two phases of the porous medium. In this model, the sound propagation is completely determined by the five following parameters: ϕ, α_∞, $\sigma = \eta/k_0$, Λ and Λ'. In the next section, we will show that the values of these parameters are given by the low- and high-frequency wave equations.

The sound velocity in the porous material is derived from Eqs. (2) and (3) and yields the usual equation,

$$c(\omega) = \sqrt{\frac{K_a}{\rho_f \alpha(\omega)\beta(\omega)}}. \tag{4}$$

In this expression, the velocity is a complex function of the frequency, which is not very convenient to investigate the propagation of ultrasonic short pulses or to deduce the values of the parameters of the medium. This is due to the fact that the EM equations are neither expressed in time domain nor in frequency domain: they are correct only for monochromatic waves. To restore their validity for transient signals, we need to write them in the time domain.

II. FRACTIONAL DERIVATIVE AND BEHAVIOR OF MATERIALS

The constitutive relation between the strain $\epsilon(t)$ at time t and the driving stress $\sigma(t)$ is at the heart of the conventional description of the theory of elasticity. In the ''classical'' theory, the stresses and strains are related by constants. Accordingly, the time histories of these values are similar and the deformation process is completely reversible. It was found out, however, that most elastic materials exhibit an explicit departure from this type of behavior due to the fact that they partially absorb energy. These deviations from pure elasticity may be taken into account by replacing the elastic constants by integral or differential time operators.

A. Fractional derivative and viscoelasticity

The fractional calculus model of viscoelastic behavior employs derivatives of fractional orders to relate stress fields to strain fields in viscoelastic materials and it has been shown that constitutive equations employing such derivatives are linked to the microscopic theories describing the macroscopic behavior of the media. Bagley and Torvik[7] have explored the models of the form

$$\sigma(t) + bD^\lambda[\sigma(t)] = G_0\epsilon(t) + G_1 D^\nu[\epsilon(t)],$$

where b, G_0, G_1, λ and ν are five parameters to be determined by least-squares fit to experimental data. $D^\nu[x(t)]$ is the fractional derivative of order ν defined by

$$D^\nu[x(t)] = = \frac{1}{\Gamma(-\nu)} \int_0^t (t-u)^{-\nu-1} x(u)\,du, \tag{5}$$

where ν is a real number and $\Gamma(x)$ is the gamma function. A fractional derivative no longer represents the local variations of the function but on the contrary, it acts as a convolution integral operator. More details about the properties of fractional derivatives and about fractional calculus are given in Ref. 12.

In view of the success obtained with these models, one is inclined to ask if they can be generalized to the case of wave motion in rigid porous materials.

B. Fractional derivative relationship for a porous medium

To write the basic equations in the time domain we use a quite different method than the one described in Ref. 7.

As seen above, the complex susceptibilities $\alpha(\omega)$ and $\beta(\omega)$ have been worked out in the framework of the equivalent fluid model. They must be mapped into the time domain. For that, the trick is to expand these functions into series of ω,

$$\alpha(\omega)=\sum_{\nu} a_{\nu}(j\omega)^{\nu}, \quad \beta(\omega)=\sum_{\lambda} b_{\lambda}(j\omega)^{\lambda}, \tag{6}$$

where ν and λ may be fractional numbers. Then we use the rules of the fractional calculus and the Fourier transform

$$F[g(t)]=\int_{-\infty}^{\infty} g(t)e^{j\omega t}dt, \tag{7}$$

to write the useful relationship

$$F[D^{\nu}g(t)]=(j\omega)^{\nu}F[g(t)]. \tag{8}$$

At last, we substitute the fractional derivatives for the powers of ω,

$$(j\omega)^{\nu} \overset{t}{\rightarrow} D^{\nu}[\cdot].$$

Unfortunately, this method leads to very complicated equations if valid expressions are required for all values of ω and exact solutions for general conditions are not easily obtained. For example, in this framework the EM equations have the following general form:

$$\rho_f \sum_{\nu} a_{\nu} D^{\nu}\left[\frac{\partial v_i}{\partial t}\right]=-\nabla_i p, \quad \frac{1}{K_a}\sum_{\lambda} b_{\lambda} D^{\lambda}\left[\frac{\partial p}{\partial t}\right]=-\nabla\cdot\mathbf{v}. \tag{9}$$

Another way to tackle this problem is to consider the asymptotic expressions of $\alpha(\omega)$ and $\beta(\omega)$, when ω tends to zero or to infinity, which are generally very simple functions of ω.

III. WAVE EQUATIONS IN THE TIME DOMAIN

The fact that more than one time-domain equation must be considered, one equation for each particular range of frequencies, appears to be the drawback of this approach. Of course, it would be better to have only one model to describe the sound propagation in the whole time domain. However, we know that the relative importance of the effects responsible for the behavior of a fluid in a porous media is significantly modified when the frequency goes from zero to high values. In this case it seems reasonable that the physics laws which govern the wave propagation may be quite different from one domain to the other. We now consider the cases of low- and high-frequency responses.

A. Low-frequency approximations

The range of frequencies such that viscous skin thickness $\delta=(2\eta/\omega\rho_0)^{1/2}$ is much larger than the radius of the pores r,

$$\frac{\delta}{r}\gg 1, \tag{10}$$

is called the low-frequency range. For these frequencies, the viscous forces are important everywhere in the fluid. At the same time, the compression-dilatation cycle in the porous material is slow enough to favor the thermal exchanges between fluid and structure. At the same time, the temperature of the frame is practically unchanged by the passage of the sound wave because of the high value of its specific heat: the frame acts as a thermostat. In this case the isothermal compressibility is directly applicable.

We consider the low-frequency approximations of the response factors $\alpha(\omega)$ and $\beta(\omega)$. When $\omega\rightarrow 0$, Eqs. (2) and (3), respectively, become

$$\alpha(\omega)\approx\alpha_0\left(1+\frac{\eta\phi}{j\omega\alpha_0\rho_f k_0}\right), \tag{11}$$

$$\beta(\omega)\approx\gamma. \tag{12}$$

α_0 is the the low-frequency approximation of the tortuosity given by Lafarge in Ref. 2 and also obtained by Norris[13] from homogenization theory,

$$\alpha_0=\frac{\langle\mathbf{v}(\mathbf{r})^2\rangle}{\langle\mathbf{v}(\mathbf{r})\rangle^2}, \tag{13}$$

where $\langle\mathbf{v}(\mathbf{r}\rangle$ is the average velocity of the viscous fluid for direct current flow within a volume element small compared to the relevant wavelength, but large compared to the individual grains/pores of the solid.

The time-domain expression for α becomes

$$\alpha(\omega)\overset{t}{\rightarrow}\alpha_0\left(I+\frac{\eta\phi}{\alpha_0\rho_f k_0}\partial_t^{-1}\right), \tag{14}$$

where I is the unit operator and $\partial_t^{-1}g(t)=\int_0^t g(t')dt'$. For a wave traveling along the direction $0x$, the generalized forms of EM equations in the time domain are now

$$\rho_f\alpha_0\frac{\partial v}{\partial t}+\frac{\eta\phi}{k_0}v=-\frac{\partial p}{\partial x} \quad \text{and} \quad \frac{\gamma}{K_a}\frac{\partial p}{\partial t}=-\frac{\partial v}{\partial x}. \tag{15}$$

In this approximation, the Euler equation expresses the balance between the driving forces of the wave, the drag forces $\eta\phi v/k_0$ due to the flow resistance of the material and the inertial forces.

The wave equation is derived from these two relations by elementary manipulations,

$$\frac{\partial^2 v}{\partial x^2}-a\frac{\partial^2 v}{\partial t^2}-d\frac{\partial v}{\partial t}=0. \tag{16}$$

The first coefficient of this equation,

$$a=\frac{\rho_f\alpha_0\gamma}{K_a}, \tag{17}$$

leads to the sound velocity $c=a^{-1/2}$ in the air filling the structure of the material. This result shows that the viscous forces and the shape of the pores increase the fluid density by the factor $\alpha_0\geqslant 1$. The second coefficient,

$$d=\frac{\eta\phi\gamma}{k_0K_a}, \tag{18}$$

is the damping-distortion term due to viscous and thermal effects which take place in the porous material. From these equations it is possible to estimate α_0 and the flow resistivity $\sigma=\eta/k_0$. At very low frequency, the asymptotic expressions for $\alpha(\omega)$ and $\beta(\omega)$ are

$$\alpha(\omega)\approx\frac{\eta\phi}{j\omega\rho_f k_0}, \quad \beta(\omega)\approx\gamma. \tag{19}$$

In this range of frequencies, EM equations become

$$\frac{\eta\phi}{k_0}v=-\frac{\partial p}{\partial x} \quad \text{and} \quad \frac{\gamma}{K_a}\frac{\partial p}{\partial t}=-\frac{\partial v}{\partial x}, \tag{20}$$

where the Euler equation is reduced to the Darcy's law which defines the static flow resistivity $\sigma=\eta/k_0$. The fields which are varying in time, the pressure, the acoustic velocity, etc., follow a diffusion equation with the diffusion constant

$$D=\frac{K_a k_0}{\eta\phi\gamma}.$$

A quite similar result is given by Johnson in Ref. 14. However, the adiabatic constant γ does not appear in Johnson's model in which the thermal expansion is neglected.

B. High-frequency approximation

When the frequency increases, the skin thickness becomes narrower and the viscous effects are concentrated in a small volume near the frame $\delta/r\ll 1$. In this case, the viscous effects in the fluid can be neglected: the fluid behaves almost like a perfect fluid (without viscosity).

At high frequencies the compression/dilatation cycle is much faster than the heat transfer between the air and the structure, and in this case, it is a good approximation to consider that the compression is adiabatic.

The high-frequency approximations of the response factors $\alpha(\omega)$ and $\beta(\omega)$ when $\omega\to\infty$ are given by the relations

$$\alpha(\omega)\approx\alpha_\infty\left(1+\frac{2}{\Lambda}\left(\frac{\eta}{j\omega\rho_f}\right)^{1/2}\right), \tag{21}$$

$$\beta(\omega)\approx 1+\frac{2(\gamma-1)}{\Lambda'}\left(\frac{\eta}{j\omega\,\mathrm{Pr}\,\rho_f}\right)^{1/2}. \tag{22}$$

In the time domain, the expressions of the responses α and β become

$$\alpha(\omega)\overset{t}{\to}\alpha_\infty\left(\delta(t)+\frac{2}{\Lambda}\left(\frac{\eta}{\rho_f}\right)^{1/2}t^{-1/2}\right)*, \tag{23}$$

$$\beta(\omega)\overset{t}{\to}\left(\delta(t)+\frac{2(\gamma-1)}{\Lambda'}\left(\frac{\eta}{\mathrm{Pr}\,\rho_f}\right)^{1/2}t^{-1/2}\right)*, \tag{24}$$

where * denotes the time convolution and $\delta(t)$ is the Dirac function. The tortuosity α_∞ is real-valued and defined as in Eq. (13) but from the field of the microscopic velocity $\mathbf{v}(\mathbf{r})$ in a perfect fluid. Brown[15] has shown that this quantity is related to the electrical formation factor F by $\alpha_\infty=F\phi$. When the wave propagates along the coordinate axis $0x$, the EM equations are generalized as follows in the time domain:

$$\rho_f\alpha_\infty\frac{\partial v}{\partial t}+2\frac{\rho_f\alpha_\infty}{\Lambda}\left(\frac{\eta}{\pi\rho_f}\right)^{1/2}\int_{-\infty}^{t}\frac{\partial v/\partial t'}{\sqrt{t-t'}}dt'=-\frac{\partial p}{\partial x}, \tag{25}$$

$$\frac{1}{K_a}\frac{\partial p}{\partial t}+2\frac{\gamma-1}{K_a\Lambda'}\left(\frac{\eta}{\pi\,\mathrm{Pr}\,\rho_f}\right)^{1/2}\int_{-\infty}^{t}\frac{\partial p/\partial t'}{\sqrt{t-t'}}dt'=-\frac{\partial v}{\partial x}. \tag{26}$$

A consequence of this is a surprising result: the retarding force is no longer proportional to the time derivative, it is found to be proportional to a fractional derivative of order 1/2 of the acoustic velocity. This occurs because the volume of fluid participating in the motion is not the same for all motion, as it is for a fully developed steady flow. The phenomena may be understood by considering such a volume of fluid in a pore to be in harmonic motion. At a higher frequency, only a thin boundary layer is excited: the average shear stress is high. At a lower frequency, the same amplitude of the fluid motion allows a thicker layer of fluid to participate in the motion, and consequently the shear stress is less. The penetration distances of the viscous forces and therefore the excitation of the fluid depend on the frequency. In the time domain, such a dependence is associated with a fractional derivative. The generalized mass conservation equation is interpreted in the same way. In these equations, the convolutions express the dispersive nature of the porous material. They take into account the memory effects due to the fact that the response of the medium to the wave excitation is not instantaneous but needs some time to take place.

The wave equation is derived from these two relations by elementary calculation and can be written as

$$\frac{\partial^2 v}{\partial x^2}-A\frac{\partial^2 v}{\partial t^2}-B\int_{-\infty}^{t}\frac{\partial^2 v/\partial t'^2}{\sqrt{t-t'}}dt'-C\frac{\partial v}{\partial t}=0, \tag{27}$$

where the coefficients are given by

$$A=\frac{\rho_f\alpha_\infty}{K_a}, \quad B=\frac{2\alpha_\infty}{K_a}\sqrt{\frac{\rho_f\eta}{\pi}}\left(\frac{1}{\Lambda}+\frac{\gamma-1}{\sqrt{\mathrm{Pr}}\,\Lambda'}\right),$$

$$C=\frac{4\alpha_\infty(\gamma-1)\eta}{K_a\Lambda\Lambda'\sqrt{\mathrm{Pr}}}.$$

The first one gives the velocity $c=A^{-1/2}$ of the wave in the air included in the porous material. In this case the fluid density is modified by the factor α_∞. The other coefficients are essentially dependent on the characteristic lengths Λ and Λ', and express the viscous and thermal interactions between the fluid and the structure. The knowledge of these three coefficients allows the determination of the three parameters α_∞, Λ and Λ'. This can be achieved from the measurements of the damping and broadening of ultrasonic pulses during the propagation in the porous medium.

With this model, one can hope to get an easier estimation of the characteristic lengths than for example Ref. 16. In this paper, the characteristic lengths are deduced from the high-frequency asymptotic behavior of either the velocity or

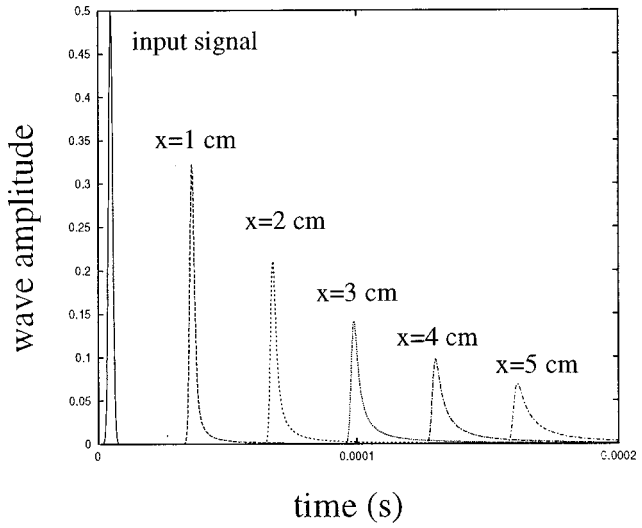

FIG. 1. Space evolution of a Gaussian pulse in a porous medium. The medium parameters are those of the polyrethan foam Bulpren R60, manufactured by Recticel. $\alpha_\infty = 1.04$, $\phi = 0.98$, $\Lambda = 2\times10^{-4}$ m, $\Lambda' = 5\times10^{-4}$ m, $\sigma = 2850$ Nm^{-4} s.

the attenuation curves which allow only the determination of the length $L = [\Lambda^{-1} + (\gamma - 1)(\sqrt{\mathrm{Pr}}\,\Lambda'^{-1}]^{-1}$. In this case, the estimation of the characteristic lengths needs the data obtained from the dispersion curves in air- and helium-filled materials. On the contrary, by using inverse scattering methods, the time-domain approach provides the viscous Λ and the thermal Λ' characteristic lengths from the coefficients B and C of Eq. (27). On the other hand, Eq. (27) is well suited for analytical solutions and numerical applications. Figure 1 shows the evolution of a Gaussian pulse in an air-filled porous medium as a time function. These results are obtained by convolution of the Green's function of Eq. (27) with the pulse. During the propagation, the pulse gets wider while its amplitude decreases; the result is a warped Gaussian pulse due to the combined effects of dispersion and attenuation where high frequencies are faster but more attenuated than the low frequencies. Figure 2 presents numerical simulations for two values of the B coefficient of Eq. (27). Warping of the input signal is mainly due to this effect. [The contribution of the other term of Eq. (27) is not significant.] Other numerical applications and comparisons with experimental results will be given in Ref. 17. Accordingly, it seems possible to get the specific contribution of viscous and thermal effects to the spreading and attenuation of ultrasonic pulses by standard inverse scattering methods.

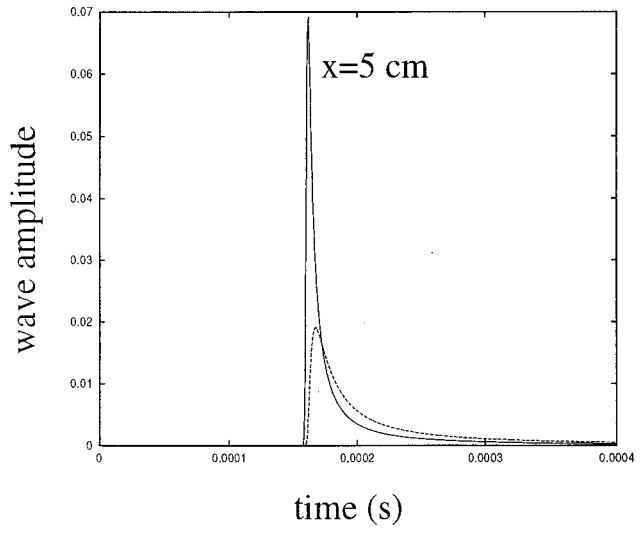

FIG. 2. Numerical simulations of warped Gaussian pulse for $x = 5$ cm for two values of B coefficient [Eq. (27)]; solid line: $B = 2.41\times10^{-4}$ and broken line: $B = 4.41\times10^{-4}$.

IV. CONCLUSION

In this paper the time-domain equations for the transient wave propagation in a porous medium have been derived in the low- and high-frequency ranges. In each range, the coefficients of the wave equation are no more frequency dependent, and viscous and thermal effects can be distinguished. At very low frequency, the viscous forces and the thermal exchanges are strong enough to prevent the waves from propagating in these materials: the field's evolution follows a diffusion equation. At low frequency the damping is weaker and the fields can propagate. In the high-frequency range, the wave propagation is described by hereditary mechanics. As a consequence of the relaxation phenomenon, the fluid density and the bulk modulus are no longer constant numbers but they are signal dependent.

The results of preliminary numerical simulations tend to indicate that studies of the propagation of transient waves may yield useful data. Ultrasonic impulses seem to be an efficient tool to probe the properties of sound absorbing materials. Last, the time-domain wave equations have a well adapted form for the analysis of the direct and inverse scattering problems.

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