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Integrating evolutionary prejudices in belief function theory*

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Abstract

This paper deals with belief change in the framework of Dempster-Shafer theory in the context where an agent has a prejudice, i.e., a priori knowledge about a situation. This situation is modeled as a sequence (p, m) where p reflects the prejudices of an agent and m is a mass function that represents the agent's uncertain beliefs. In contrast with the Latent Belief Structure introduced by Smets where a mass is decomposed into a pair of separable mass functions called respectively the confidence and diffidence, m can be any mass function (i.e., not necessarily separable) and p is not a mass. The aim of our study is to propose a framework in which the evolution of prejudices and beliefs are described through the arrival of new beliefs. Several cases of prejudice are described: the strong persistent prejudice (which never evolves and forbids beliefs to change), the prejudice that is slightly decreasing each time a belief contradicts it, etc.

1 Introduction

When dealing with information pervaded with uncertainty, several frameworks can be used: probabilities, possibilities, ... with their variants. The most general framework in which uncertainty can be expressed is belief-function theory. It is well suited to epistemic analysis in situations where there is little information to assess a probability, or where information is non-specific, ambiguous or contradictory. This theory makes it possible to express that there is evidence in favor of a set of events A without specifying the precise degree of certainty of each element of A, whereas in a probabilistic setting, the probability of each event of A should be known.

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Belief function theory also known as Dempster-Shafer theory was first introduced by Arthur P. Dempster in the context of statistical inference, then developed by Glenn Shafer into a formal framework for representing and reasoning with uncertain information [15]. G. Shafer viewed belief functions as the result of the conjunctive combination of pieces of evidence such as (more or less unreliable) testimonies from different sources, in order to form a representation of beliefs about certain aspects of the world.

This theory has been well studied and developped in order to reason with several sources of information. However one can be interested in combining positive evidence and personal a priori convictions (coming from moral values, tastes, and past experiences), that are called "prejudice". A prejudice can be defined as an a priori favorable or unfavorable[1] "opinion adopted without examination, imposed by an environment, an education" (Montaigne, Essais, II, 12, ed. P. Villey and V.-L. Saulnier, p.506). Unlike a belief, a prejudice can be disproved on the basis of facts. Prejudices can be more or less strong, the strongest they are the more difficult it is to disprove them and the more they will influence the reasoning. In his famous book *The Nature of prejudice* the American psychologist Gordon Allport [2] asserted that "prejudice is essentially a by product of the necessary mental shortcuts the human brain uses to process the vast amount of information it takes in".

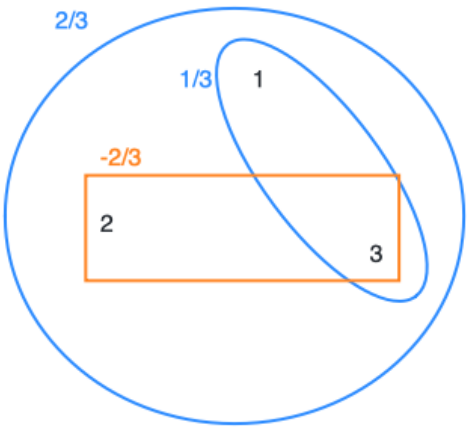
In the framework of Dempster-Shafer theory, there were attempts to encode the retraction of information (which turns out to be wrong): the operation of removal (or retraction) was proposed by many authors [7, 10, 16, 17] in order to decrease the degree of belief by retracting some piece of evidence. In particular, the model called "Latent Belief Structure" introduced by [17] then studied in [14] considers a pair of belief functions, one representing the confidence part, and the other the diffidence part playing the role of a moderator that can annihilate, via retraction, some information supplied by the former.

Recently, this model was reinterpreted by [5] in terms of *prejudice* of the receiver, and retraction was considered as a special kind of belief change. Its role is to weaken the support of some focal sets of a belief function, possibly stemming from the fusion of the incoming information. The authors suggest that prejudices are due to some prior knowledge that is more entrenched than incoming new pieces of uncertain evidence. They detail how such prior information can affect a belief function. They encode a prejudice by a negative mass function which should be combined with a positive mass function (representing a testimony). However the result of this combination must be a positive mass (because a negative mass is not interpretable in this framework). Moreover this approach is only able to combine some particular forms of prejudices and beliefs where the prejudices are against some part of a focal set of a simple mass function. In that case a revision is performed on the beliefs. The revision operation is a classical topic broadly studied in knowledge representation literature [8, 1], and also in the particular context of belief functions by [4].

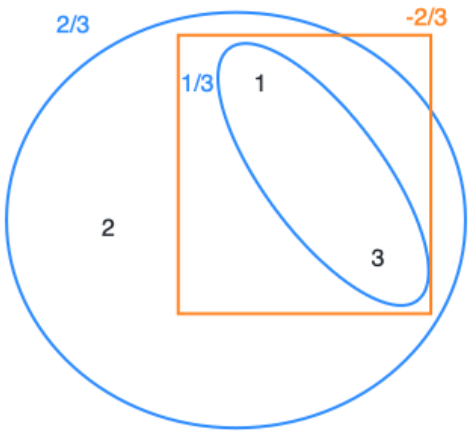
Example 1 *Let us consider the beliefs of a doctor about 3 diseases that can be encountered: pyelonephritis noted* 1*, urinary infection noted* 2 *and lumbago noted* 3*. Assume that the doctor has observed a stomach ache. In that case there is a prejudice against urinary infection (2) and*

[1] "(good or bad) opinion that one forms in advance" (Lanoue, Discours pol. et milit., 436 in Littré,1587)

lumbago (3) encoded by the orange rectangle. Suppose now that the doctor learns from health test results that the patient may have a disease among pyelonephritis (1) and lumbago (3) (blue oval). Then we may wonder how the prejudice can be taken into account for representing the final belief state of the doctor. The aim of this paper is to study how the beliefs and the prejudice may evolve, according to their strength and to their incompatibility.



Prejudice of -2/3 against $\{2, 3\}$ and evidence of 1/3 for $\{1, 3\}$



Prejudice of -2/3 against $\{1, 3\}$ and evidence of 1/3 for $\{1, 3\}$

In this example, we consider that the prejudice has a strength of $-2/3$ (on a scale from 0 to $-\infty$) with a confidence in the new piece of information of $1/3$. It means that the prejudice is stronger than the beliefs hence the doctor should transfer its beliefs *towards pyelonephritis (1), the prejudice against urinary infection (2) and lumbago (3) can then either decrease or remain depending on its nature (stubborn prejudice will not be questioned by any new piece of information while open minded prejudice may decrease).*

Another more classic instance of this example would assume that the prejudice is against (1) and (3), and the piece of evidence is about the same set (1) and (3). When the prejudice is -2/3 and evidence is 1/3 then the prejudice being stronger it remains against (1) and (3) (maybe attenuated) but the evidence is canceled, while when the prejudice is weaker (-2/3) than the evidence (say 0.5), the prejudice is canceled but the evidence is only integrated with a strength of 1/6.

In classical belief function theory, two mass functions m and m' that are considered as two sources of evidence, are combined by using Dempster rule $m'' = m \textcircled{\cap} m'$. Dempster's rule aims at gathering the two sources, in a conjunctive way, but this raises several issues when prejudices are taken into account:

- what if we combine two sources that have both a prejudice and a mass: (p, m) and (p', m'): this kind of combination is considered as out of the scope of the paper because we reason only from the point of view of one agent who receives an incoming information. In this paper, the incoming information is delivered under the form of a mass function, with no

prejudice explicitly present (however it could be done with Latent belief structure). Note that, in this context, if we have no prejudice at start, i.e, $p = 0$, then the combination (p, m) with m' (i.e., the combination with $(0, m')$) should yield $(0, m \textcircled{\cap} m')$.

- what are the links between reasoning with prejudices and belief revision ?

In this paper we propose to extend the previous work of [5] with a general formalism that considers any prejudice and beliefs using a pair (p, m) containing an encoding of the prejudice p and an encoding of the beliefs by a mass m. We are interested in the evolution of this pair when new information arrives, this is why the study is related to Dempster's revision (recalled in Section 2.3). Some necessary background on belief functions is introduced in Section 2. After defining specific rules and constraints governing the evolution of beliefs (Section 3), we conclude with a comparison with the literature and some perspectives.

2 Background

2.1 Basics about belief functions

Let us consider a finite set $\Omega = \{\omega_1, \ldots, \omega_N\}$, called the frame of discernment, whose elements represent descriptions of possible situations, states of the world, one of which corresponding to the truth. In Dempster-Shafer theory [15], the uncertainty concerning an agent's state of belief on the real situation is represented by a *mass function* defined as a mapping $m : 2^\Omega \longrightarrow [0, 1]$ such that $m(\emptyset) = 0$ and verifying $\sum_{A \subseteq \Omega} m(A) = 1$. $m(A)$ expresses the proportion of evidence that the current state is in A. Each subset $A \subseteq \Omega$ such as $m(A) > 0$ is called a *focal set* of m.

An *elementary testimony* T with weight $(1 - \alpha)$ in favor of a non-contradictory and non-universal proposal $A \in 2^\Omega \setminus \{\Omega, \emptyset\}$ is represented by the simple mass function $m : 2^\Omega \longrightarrow [0, 1]$ denoted by $m = A^\alpha$ in [17, 3] such that

$$m = A^\alpha \quad \text{denotes} \quad \begin{cases} m(A) = 1 - \alpha \\ m(\Omega) = \alpha \end{cases}$$

Here, α evaluates the lack of confidence in the testimony T also called mistrust.

In presence of multiple sources of information or multiple uncertain testimonies, the result of the conjunctive combination of two mass functions m_1 and m_2, noted $m_1 \textcircled{\cap}_2$, is defined as follows:

$$m_1 \textcircled{\cap}_2(A) = \sum_{A_1 \cap A_2 = A} (m_1(A_1).m_2(A_2))$$

In particular, $m_1 \textcircled{\cap}_2(\emptyset)$ represents the conflict between the mass functions. We will denote $\oplus$ the normalized conjunctive combination rule called Dempster's rule (which assigns a zero mass to the empty set and divides all the masses of the focal elements by $1 - m_1 \textcircled{\cap}_2(\emptyset)$).

A belief function $Bel(A)$ is a non-additive set function which represents the total quantity of pieces of evidence supporting the proposition $A \subseteq \Omega$ and is defined by

$$Bel(A) = \sum_{\emptyset \neq E \subseteq A} m(E)$$

The plausibility $Pl(A)$ is the dual set-function of $Bel(A)$ where $Pl(A) = 1 - Bel(\overline{A})$, i.e., : $Pl(A) = \sum_{E \cap A \neq \emptyset} m(E)$. A mass function m can be equivalently represented by its associated commonality function defined for all $A \subseteq \Omega$ by $Q(A) = \sum_{B \supseteq A} m(B)$ The commonality function $Q(A)$ represents the total quantity of incomplete evidence that makes *all* elements of A possible. [15] calls *a separable support function*, a belief function $m = \oplus_{i=1}^{k} A_i^{d_i}$ resulting from Dempster rule combination of simple mass functions $A_i^{d_i}$, with $A_i \neq \Omega, 0 < d_i < 1, i = 1, \ldots, k$. Each single mass represents an independent testimony.

2.2 Defiance, retraction and latent structures

In 1995, [17] extended the range of defiance functions δ initially defined on $[0, 1]$ to the interval $]0, +\infty)$ and has defined the notion of retraction for $\delta > 1$. The retraction of a simple mass function $B^y, y < 1$ supporting B from a simple support function $A^x, x < 1$, denoted by $\circledcirc$ is defined such that :

$$A^x \circledcirc B^y = A^x \bigcirc\!\!\!\!\cap B^{1/y}$$

and yields the diffidence function $\delta = \delta_A / \delta_B$ with $\delta_A(E) = x$ if $E = A, 1/x$ for $E = \Omega$ and 1 otherwise and $\delta_B(E) = y$ if $E = B, 1/y$ for $E = \Omega$ and 1 otherwise. However $A^x \circledcirc B^y$ is a belief function if only if $A = B$ and $x/y < 1$. Indeed the result $A^x \bigcirc\!\!\!\!\cap B^y$ is NOT a belief function in general as the mass function induced by δ may fail to be positive. Retraction also fails if the set to retract is not focal. It is not possible neither when the focal set A to be retracted intersects some other focal set B without being included in it (i.e., $A \cap B \neq A$). In other words retraction is only possible on a set $(A \cap B)$ that is an intersection of two focal sets (A and B).

Hence, a necessary and sufficient condition required for retraction is that the set of focal sets of m should be closed under intersection. Note that it is a necessary but not sufficient condition of separability. Indeed retracting a focal set E_J from a separable mass function m affects and may delete all focal sets $E_I \subset E_J$ as well, namely all combinations between the merging of information E_J from sources indexed in J, with information from other sources.

Based on the canonical decomposition of belief functions and the retraction operation, the concept of *latent belief structure (LBS)* [17, 14] was defined as a pair of separable non dogmatic[2] masses m^c and m^d called respectively the confidence and diffidence components such that $m = m^c \circledcirc m^d$ with $m^c = \bigcirc\!\!\!\!\cap_{A \in C} A^{w(A)}$ and $m^d = \bigcirc\!\!\!\!\cap_{A \in D} A^{\frac{1}{w(A)}}$. The disjoint subsets C and D come from a partition of 2^Ω such that $C = \{A : A \subset \Omega, w(A) \in (0, 1]\}$ and $D = \{A : A \subset \Omega, w(A) \in (1, \infty)\}$. The diffidence component may be interpreted as a prejudice against the subset D. However, only a few particular cases of prejudice can be modeled by LBS because the constraints linked to the retraction operation detailed above are very restrictive.

[2]A mass is dogmatic when $m(\Omega) = 0$.

2.3 Conditioning in Dempster and revision

2.3.1 Revising by a sure observation C

When dealing with statistical data, [4] differentiates revision and prediction with respect to the new piece of information. The initial information corresponds to a belief function encoded by a mass function (the assignment of non-negative weights $m(E)$ to subsets E of Ω). This mass m is modified by taking the new observation saying that the states in C are observed. When this new piece of information is totally certain then it is a *revision* and the Dempster conditioning is used for handling it. [4] defines this operation as the revision $m(.||C)$ of a mass function m by a totally certain new piece of information C, as follows:

$$m(B||C) = \sum_{E:B=C\cap E\neq\emptyset} m(E)$$

In other words $m(B||C) = Pl(E\cap C)$. Moreover [4] defines $Pl(B||C) = Pl(B\cap C)/Pl(C)$ and $Bel(B||C) = 1 - Pl(\overline{B}||C)$.

Example 2 (Ellsberg's paradox) *We consider an urn with three kinds of balls: white, black and red. We know that 1/3 are reds, the universe is all the possible outputs obtained after the event to draw a ball from the urn: $\Omega = \{\omega_1, \omega_2, \omega_3\}$ where ω_1 (resp. ω_2, ω_3) represents the fact that the ball is white (resp. black and red). The mass representing the initial information is named m in Table 1. We don't know the proportions of balls of each kind let us call α the proportion of white among the white and black balls. We learn that the ball that is extracted is not black, $C = \{\omega_1, \omega_3\}$. It yields the results presented in Table 1 column 2. Note that in this example, in both revision and prediction cases $Pl = Bl$, which translate the fact that focal elements after revision/prediction are singletons.*

In the case called "revision" by [4], Dempster conditioning transfers the full mass of each focal set E to $E \cap C \neq \emptyset$ (followed by a renormalisation). This means that the new information C modifies the initial mass function in such a way that $Pl(\overline{C}) = 0$: situations where C is false are considered as impossible. In the "prediction" case, only a proportion of the mass of E is transferred to $E \cap C$, but after normalization the rest is distributed over the new focal elements (the ones in $E \cap C$).

2.3.2 Revising by a new mass function m_I:

[12] introduced a "revision operator" $\circ$ s.t. given two mass functions m and m_I over Ω defined by:

$$\text{for any } E \neq \emptyset, m \circ m_I(E) = \sum_{A\cap B=E} \sigma(A,B) m_I(B) \tag{1}$$

where $\sigma(A,B)$, called *specialization matrix*, is s.t. $\sigma(A,B) = 0$ when $A \cap B = \emptyset$ and otherwise:

$$\sigma(A,B) = \begin{cases} \frac{m(A)}{Pl(B)} & \text{for } Pl(B) > 0 \\ 0 & \text{for } Pl(B) = 0 \text{ and } A \neq B \\ 1 & \text{for } Pl(B) = 0 \text{ and } A = B \end{cases}$$

E	m	Pl	$m_{\|\|C}$	m_I	$m \circ m_I$	$m_{I'}$	$m \circ m_{I'}$
$\emptyset$	0	0	0	0	0	0	0
$\{1\}$	0	$\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{2}{3}$	0.5	0.5+$\frac{0.8}{3}$
$\{2\}$	0	$\frac{2}{3}$	0	0	0	0.1	0.1
$\{3\}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{0.4}{3}$
$\{1,2\}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0	0	0	0
$\{1,3\}$	0	1	0	1	0	0.4	0
$\{2,3\}$	0	1	0	0	0	0	0
Ω	0	1	0	0	0	0	0

$\sigma(A,B)$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	Ω
$\{1\}$	0	0	0	0	0	0	0
$\{2\}$	0	0	0	0	0	0	0
$\{3\}$	0	0	1	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
$\{1,2\}$	1	1	0	1	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
$\{1,3\}$	0	0	0	0	0	0	0
$\{2,3\}$	0	0	0	0	0	0	0
Ω	0	0	0	0	0	0	0

Table 1: Computing the revision $m_{||C}$ by $C = \{\omega_1, \omega_3\}$ of the mass functions m in the Ellsberg paradox example, where ω_1 means white, ω_2 is black and the revision $m \circ m_I$ (resp. $m \circ m_{I'}$) of a given mass m by the simple mass m_I (resp. with the more complex mass $m_{I'}$). The table on the right gives the specialization matrix for m. In this table ω_i is abbreviated i for any $i \in \{1, 2, 3\}$.

In other words, it flows down a portion of $m_I(B)$ to $A \cap B$, making the revision result a "specialization"[3]. An example of specialization matrix is depicted on the right of Table 1, it is the one of the mass m representing the Ellsberg's paradox (Example 2).

As we can see in Table 1, the two revision operators are equivalent when the new piece of information can be represented by a simple mass function. However if the new piece of information is more complex, only $\circ$ can be applied, hence $\circ$ is a refinement of the revision operator based on Dempster conditioning.

3 Formalizing prejudices

In this section we propose to model prejudices against a piece of evidence. We propose to define what happens in the situation where the receiver already has some prejudices and some knowledge, the prejudice being characterized by a strength and a tenacity. We study how new incoming information can modify both the levels of prejudice and beliefs. We do not address the case of the integration of a new prejudice or the reinforcement of an existing one. More precisely

[3]Specialization was introduced in [6], m specializes m' iff there exists a square matrix Σ with general term $\sigma(A, B)$ being a proportion (i.e., verifying $\sum_A \sigma(A, B) = 1$, for any B. $\sigma(A, B) > 0$ implies $A \subseteq B$ for any A, B) such that $m(A) = \sum_B \sigma(A, B)m'(B)$ for all A. In [12], the definition of specialization matrix is taken in a broader sense: only imposing that $\sigma(A, B) > 0$ implies $A \cap B \neq \emptyset$ for any A, B.

the incoming information is only a testimony that can decrease some prejudice (or not affecting it at all) but cannot increase any prejudice or create a new one. The *creation* of prejudice is left for further studies.

In order to both encode prejudice and knowledge, we propose to consider the couple (p, m) where m is a mass function and p is a prejudice against some set A such that $p(A) \leq 0$. This is done to be consistent with Smets retraction operation (recalled in Section 2.2).

Definition 1 (belief state) *A* belief state *is a pair (p, m) where p, representing a* prejudice against *some piece(s) of evidence, is a prejudice function $p : 2^{\Omega} \setminus \{\Omega, \emptyset\} \to (-\infty, 0]$ and m is a mass function. Intuitively $\forall\, \emptyset \subset A \subset \Omega$, $p(A)$ represents the threshold of evidence required to change one's mind about $A \neq \emptyset$:*

- $p(A) = 0$ *indicates the absence of prejudice against A*
- $p(A) = -\infty$ *means an unshakable prejudice against A.*

p is extended to 2^{Ω} by setting $p(\Omega) = 1 - \sum_{X \subset \Omega} p(X)$ and $p(\emptyset) = 0$ (normalization).

In the following sections we are going to study the cases where there is only one focal set for the beliefs (called A), and only one prejudice that focuses on a set (called B). We describes all the situations where A and B intersects, namely $A = B$ (case 1), $A \subset B$ (case 2), $B \subset A$ (case 3), and $A \setminus B \neq \emptyset$ and $B \setminus A \neq \emptyset$ (case 4). The prejudice is characterized by a threshold of evidence (under which the evidence is not affected) and a tenacity function that describes how this threshold evolves when new information contradicts it (this tenacity function is decreasing, because in this paper we restrict our study to the case where no piece of evidence can increase a prejudice).

3.1 Case 1 : Information and prejudice focused on the same subset $A \neq \emptyset$ and $A \neq \Omega$

The simplest situation occurs when the prejudice and the information (testimony) are concerning the same set of pieces of evidence.

Definition 2 *A simple belief state about $A \neq \emptyset$ has the form $(p = A^{\beta}, m = A^{\alpha})$ with $\alpha \in [0, 1]$, $\beta \in [1, +\infty)$ where p is a prejudice against A, called* simple prejudice*, and m is a simple mass function on A. It can be* simplified *according to the following rules:*

- $\alpha\beta \leq 1$*: the prejudice is canceled and the confidence in A decreases so the pair becomes $(A^1, A^{\alpha\beta})$,*
- $\alpha\beta > 1$ *the prejudice decreases but the informative mass $m(A)$ is deleted : the pair becomes $(A^{\alpha\beta}, A^1)$.*

Note that, $p = A^\beta$ is a shortcut for $p(A) = 1 - \beta$ and $p(\Omega) = \beta$, hence due to $\beta \geq 1$, $p(A)$ is negative in accordance to Definition 1. In other words, a simple mass function m about A, $m = A^\alpha$ in presence of a prejudice p against A, $p = A^\beta$, can give three situations according to the incoming information A^α:

1. The prejudice is deleted if the incoming information is sufficiently convincing. In other words, the threshold of persuasiveness required to change one's mind is overtaken, the prejudice was low compared to the strength of the evidence A^α.

2. The prejudice remains but is possibly attenuated and the piece of evidence is rejected. So the attenuation of the prejudice is a function (called f now on) dependent of the strength β of the prejudice against A.

3. The prejudice is preserved whatever the incoming information (which is canceled).

Definition 3 (Evolving prejudice) *Given $A \subset \Omega$ with $A \neq \emptyset$, we denote by A^β the prejudice of strength β which evolves according to f, where $\beta \in [1, +\infty]$ and $f : [0, 1] \times [1, +\infty] \to [1, +\infty]$ is a function s.t. $f(\alpha, \beta)$ represents a new threshold of prejudice: $f(\alpha, \beta)$ replaces β when the prejudice is attenuated*

- $\alpha\beta \leq 1$ *the prejudice is canceled and the level of evidence decreases so the pair becomes* $(A^1, A^{\alpha\beta})$,

- $\alpha\beta > 1$ *the prejudice changes but $m(A)$ is canceled: the pair becomes* $(A^{f(\alpha,\beta)}, A^1)$.

In this paper, due to the fact that we assume that prejudices can only decrease or stay still, f is a decreasing or constant function. Here are some examples of special cases for f:

- $f(\alpha, \beta) = \max(1, \beta - \varepsilon)$ decreases by $\varepsilon \in [0, +\infty[$ after receiving each new evidence.

- $f(\alpha, \beta) = \max(1, \alpha\beta)$ decreases in function of the strength of the certainty on the incoming information, at most the prejudice is removed.

- $f(\alpha, \beta) = \beta$ leads to the conservation of the prejudice in the case of a narrow-minded person yielding (A^β, A^1),

If there is no prejudice against A, whatever the new piece of evidence that may arrive about A, it cannot create a new prejudice on A. So this amounts to having a simple mass function as shown in the following proposition:

Proposition 1 *The pair (A^1, A^α) is equivalent to the simple mass function $m = A^\alpha$*

Proof : Here $\beta = 1$, hence $\alpha\beta \leq 1$, according to definition 3, the pair becomes (A^1, A^α). □

Proposition 2 *When $\alpha\beta \leq 1$, the pair (A^β, A^α) is equivalent to a revision of the simple mass function $m = A^\beta$ by the simple mass function A^α yielding a simple mass m' such that $m' = A^\beta \circ A^\alpha = A^{\alpha\beta}$*

E	p_b	m_a	(p'_b, m'_a) any f $\alpha\beta = 5/6 \leq 1$	m_c	(p'_b, m'_c) $f = \max(1, \beta - 0.2)$	(p''_b, m''_c) $f = \max(1, \alpha\beta)$	(p'''_b, m'''_c) $f(\alpha, \beta) = \beta$
					$\alpha\beta = 10/9 > 1$		
$E \in 2^\Omega \setminus \{\{1,3\}, \Omega\}$	0	0	(0, 0)	0	(0, 0)	(0,0)	(0,0)
$\{1,3\}$	-2/3	0.5	(0,1/6)	1/3	(-7/15,0)	(-1/9,0)	(-2/3,0)
Ω	5/3	0.5	(1,5/6)	2/3	(22/15,1)	(10/9,1)	(5/3,1)

Table 2: Kidney ache (a), stomach ache (b) and back ache (c) (p_b is a prejudice against pyelonephritis and lumbago)

Proof : Due to Equation 1, $A^\beta \circ A^\alpha(\Omega) = \sigma(\Omega, \Omega)\alpha$ and $\sigma(\Omega, \Omega) = \beta$. □

Example 3 *Let us consider a universe $\Omega = \{\omega_1, \omega_2, \omega_3\}$ where three diseases can be encountered:* pyelonephritis *noted ω_1,* urinary infection *noted ω_2 and* lumbago *noted ω_3. Three symptoms can be observed* Kidney ache *(a),* stomach ache *(b) and* back ache *(c). When* stomach ache *is observed there is a prejudice against* pyelonephritis *and* lumbago *encoded by p_b.*

The two first columns of Table 2 shows a belief state (p_b, m_a) where the initial belief is $m_a = \{\omega_1, \omega_3\}^{0.5}$ (Kidney ache which is translated by an evidence for ω_1 or ω_3, meaning the presence of pyelo-nephritis *or* urinary infection*) and there is a prejudice against those two diseases because of a* stomach ache *$p_b = \{\omega_1, \omega_3\}^{5/3}$.*

The third column shows the results of conjunctive combination of m_a and p_b as described in Section 2.1, i.e., the prejudice disappear *(hence the belief state is a simple mass) and the mass on A decreases: $p'_b(\{\omega_1, \omega_3\}) = 0$ and $m'_a(\{\omega_1, \omega_3\}) = 1 - \alpha\beta = 1/6$. In the case where the belief is described by m_c then the prejudice decreases (depending of $f(\alpha, \beta)$) and the evidences are canceled. Column 6 and 7 respectively show the behavior of the prejudice for $f = max(1, \beta - \varepsilon)$ (with $\varepsilon = 0.2$) and $f = max(1, \alpha\beta)$. The last column presents the case of a narrow minded agent with $f(\alpha, \beta) = \beta$.*

3.2 Case 2: Evidence on A and prejudice against B with $\emptyset \subset A \subset B \subset \Omega$

In this case, we have $A \setminus B = \emptyset$ and $B \setminus A \neq \emptyset$. Note that there is a discontinuity: when $A = B$, there was an inconsistency between the information and the prejudice while in the current case $A \subset B$, the set $B \setminus A$ is non-empty, hence the transfer is possible.

- $\alpha\beta \leq 1$ the prejudice on A is deleted, the prejudice on $B \setminus A$ is maintained but the evidence on A decreases.
- $\alpha\beta > 1$ the mass on A is canceled and the prejudice against A is possibly decreased by $f(\alpha, \beta)$ while the prejudice on $B \setminus A$ is maintained

Definition 4 *Given a belief state (p, m) such that $p = B^\beta$ and $m = A^\alpha$, (with $\emptyset \subset A \subset B \subset \Omega$) this state is simplified into (p', m') such that:*

- $\alpha\beta \leq 1$ *(the prejudice is weaker than the information): $(p', m') = ((B \setminus A)^\beta, A^{\alpha\beta})$*

- $\alpha\beta > 1$ *(the prejudice is stronger than the information): information is canceled and the prejudice can be affected on the set A: (p', A^1) with p' such that:*
  - $p'(A) = 1 - f(\alpha, \beta)$ *(as in the Case 1 Section 3.1)*
  - $p'(B \setminus A) = 1 - \beta$
  - $p'(\Omega) = 1 + f(\alpha, \beta) + \beta$

3.3 Case 3: Evidence on A and prejudice against B with $\emptyset \subset B \subset A \subset \Omega$

In this case $A \setminus B \neq \emptyset$ and $B \setminus A = \emptyset$. In the current case there is no contradiction between the target of the prejudice and the information, information can be transferred to $A \setminus B$, concerning the prejudice it can either decrease or remain the same depending on its strength.

- $\alpha\beta \leq 1$: the prejudice on B is deleted, but the belief is transferred to $A \setminus B$. In this case, there is no evidence for B hence the mass on B remains 0 it cannot increase.

- $\alpha\beta > 1$: the mass on B is canceled and the prejudice about B is possibly decreased by $f(\alpha, \beta)$ while the prejudice on $A \setminus B$ is maintained

Definition 5 *Given a belief state (p, m) such that $p = B^\beta$ and $m = A^\alpha$ with $\emptyset \subset B \subset A \subset \Omega$, this state is simplified into (p', m') such that:*

- $\alpha\beta \leq 1$ *(weak prejudice): $(p', m') = (B^1, (A \setminus B)^\alpha)$*

- $\alpha\beta > 1$ *(strong prejudice): $(p', m') = (B^{f(\alpha,\beta)}, (A \setminus B)^\alpha)$*

Note that in the Definition 5, we could have considered that after removing the prejudice against B, the mass would remain on the entire set A, i.e., $(p', m') = (A^1, A^\alpha)$. However we think that this is a less cautious attitude, it would amount to forget completely the old prejudice. In this case, there is no precise information about B hence whatever the strength of the prejudice, the information is transferred from A to $(A \setminus B)$.

3.4 Case 4: Evidence on $A \neq \Omega$ and prejudice against $B \neq \Omega$ with $(A \setminus B) \neq \emptyset$ and $(B \setminus A) \neq \emptyset$

In all cases where $A \neq B$ (cases 2, 3 and 4) there is no frontal contradiction between the beliefs and the prejudice, here $A \setminus B \neq \emptyset$ means that information can be transferred to $A \setminus B$, and $B \setminus A \neq \emptyset$ means that a prejudice can remain on $B \setminus A$.

- $\alpha\beta \leq 1$ the prejudice on $A \setminus B$ is deleted, but remains on $B \setminus A$, the belief is transferred to $A \setminus B$.

- $\alpha\beta > 1$ the mass on B is canceled and the prejudice about $B \cap A$ is possibly decreased by $f(\alpha, \beta)$ while the prejudice on $B \setminus A$ is maintained

E	p_b	m_a	(p'_b, m'_a) any f $\alpha\beta = 5/6 \leq 1$	m_c	(p'_b, m'_c) $f = \max(1, \beta - 0.2)$	(p''_b, m''_c) $f = \max(1, \alpha\beta)$	(p'''_b, m'''_c) $f(\alpha, \beta) = \beta$
					$\alpha\beta = 10/9 > 1$		
$\{\emptyset, \{1,2\}\}$	0	0	(0, 0)	0	(0, 0)	(0,0)	(0,0)
$\{1\}$	0	0	(0,0.5)	0	(0,1/3)	(0,1/3)	(0,1/3)
$\{2\}$	0	0	(-2/3,0)	0	(-2/3,0)	(-2/3,0)	(0,0)
$\{3\}$	0	0	(0,0)	0	(-7/15,0)	(-1/9,0)	(0,0)
$\{1,3\}$	0	0.5	(0,0)	1/3	(0,0)	(0,0)	(0,0)
$\{2,3\}$	-2/3	0	(0,0)	0	(0,0)	(0,0)	(-2/3,0)
Ω	5/3	0.5	(5/3,0.5)	2/3	(32/15,2/3)	(16/9,2/3)	(5/3,2/3)

Table 3: Kidney ache (a), stomach ache (b) and back ache (c) (p_b is a prejudice against pyelonephritis and urinary infection)

Definition 6 *Given a belief state (p, m) such that $p = B^\beta$ and $m = A^\alpha$ with $A, B \in 2^\Omega \setminus \{\Omega\}$ and $(A \setminus B) \neq \emptyset$ and $(B \setminus A) \neq \emptyset$, this state is simplified into (p', m') such that:*

- $A \cap B = \emptyset$ *:* $(p', m') = (p, m)$ *: no change,*
- $\alpha\beta \leq 1$ *(weak prejudice):* $(p', m') = ((B \setminus A)^\beta, (A \setminus B)^\alpha)$*,*
- $\alpha\beta > 1$ *(strong prejudice):* $(p', (A \setminus B)^\alpha)$ *with p' such that:*
  - *if $f(\alpha, \beta) = \beta$ then $p' = p$ (strong and persistent prejudice)*
  - *else*
    - $p'(A \cap B) = 1 - f(\alpha, \beta)$ *: the prejudice decreases as in the Case 1 and 2*
    - $p'(B \setminus A) = 1 - \beta$
    - $p'(\Omega) = 1 + f(\alpha, \beta) + \beta$

Example 3 (continued): *Let us now consider a prejudice against* urinary infection *and* lumbago *encoded by $p_b = \{\omega_2, \omega_3\}^{5/3}$. The fourth columns shows the results of the conjunctive combination of m_a and p_b as described in Section 3.4, i.e., the* prejudice disappears *on ω_3 but remains on ω_2 ($p'_b(\{\omega_2\}) = -2/3$). The mass $m_a(\{\omega_1, \omega_3\} = 0.5$ is transferred to $m'_a(\{\omega_1\})$. In the case where the belief is described by m_c, the prejudice against $\{\omega_3\}$ decreases (depending on $f(\alpha, \beta)$) and is transferred to $\{\omega_2\}$. The evidence for $\{\omega_3\}$ is canceled but transferred to $\{\omega_1\}(m'_c(\{\omega_1\}) = 0.5)$. The last column presents the case of a narrow minded agent with $f(\alpha, \beta) = \beta$.*

4 Properties

In this section, we establish two propositions that are concerning the belief part of the cognitive state of the agent: it appears that when beliefs are contradicted by prejudice but not radically, i.e. there are sets of beliefs that can remain uncontradicted, then a transfer can be made towards the more specialized set of uncontradicted beliefs. However, when there is a radical opposition

between beliefs and prejudices, i.e. when prejudices are against a set of worlds that contains the set of worlds we believe in, then a revision should be performed (Proposition 4).

We start by showing that our definition of belief state evolution under prejudices is conserving the belief masses.

Proposition 3 *Given a belief state (p, m) such that $p = B^\beta$ and $m = A^\alpha$, in the cases (3 and 4) where $A \setminus B \neq \emptyset$ there is conservation of masses without addition of further contradiction :*

$$m'(A) + m'(A \setminus B) = m(A)$$

Proof : For the cases 3 and 4: in both cases $\alpha\beta \leq 1$ and $\alpha\beta > 1$, m' is $(A \setminus B)^\alpha$ (due to Definitions 5 and 6). Hence $m'(A) = 0$ and $m'(A \setminus B) = 1 - \alpha$. □

When the prejudice is weak, i.e., $\alpha\beta \leq 1$, we recover Dubois-Denøeux revision:

Proposition 4 *Given $\emptyset \subset A \subset \Omega$ and $\emptyset \subset B \subset \Omega$ and a belief state (p, m) such that $p = B^\beta$ and $m = A^\alpha$, according to the respective position of the two sets A and B, when $\alpha\beta \leq 1$ then*

- *$A \setminus B = \emptyset$ (cases 1 and 2): $m' = B^\beta \circ A^\alpha$ (revision)*
- *$A \setminus B \neq \emptyset$ (cases 3 and 4): $m' = (A \setminus B)^\alpha$ (transfer)*

Proof : Due to Equation (1), $p \circ m(\Omega) = \sigma_p(\Omega, \Omega)m(\Omega)$ where $\sigma_p(\Omega, \Omega) = \frac{p(\Omega)}{Pl_p(\Omega)}$ hence $\sigma_p(\Omega, \Omega) = \frac{\beta}{1} = \beta$. Moreover, $m(\Omega) = \alpha$ this $p \circ m(\Omega) = \alpha\beta$. Now, except from Ω, $m(E) \neq \emptyset$ only for $E = A$, hence $p \circ m(A) = 1 - \alpha\beta$ thus $p \circ m = A^{\alpha\beta}$, it corresponds exactly to Definitions 2 and 4. The second item corresponds directly to Definitions 5 and 6. □

5 Discussion and Related work

In this section, we first recall the approaches that deal with information deletion, namely retraction and updating. Indeed, belief function theory is made to add new pieces of evidence through Dempster combination rule, but the issue of deleting or modifying the agent's belief states when some evidence is invalidated or modified has deserved some attention.

In probability and possibility theory, the retraction operation consists of a division followed by a normalization. Since 1984, Ginsberg [7] proposed a special case of retraction applied to belief functions in the simple case of a frame of discernment with only two elements. In the valuation-based system framework, Shenoy [16] defined removal as point-wise division followed by normalization (if normalization is possible). Kramosil [9] generalized the notion of belief functions with basic signed measure assignment (BSMA) and proposed an operation inverse to Dempster's rule. He introduced the notion of q-invertibility that may be seen as generalizing non-dogmaticism. Pichon [13] pursued Kramosil's seminal work by defining the so-called *conjunctive signed weight function*. But the absence of a semantic, the lack of intuitive interpretation of such generalized belief functions and the fact that only the conjunctive rule is used to combine

BSMAs (normalization cannot be applied) are obstacles to the potential use of this approach. Smets [17] generalized the concept of simple support function, allowing the diffidence values to range on the positive reals and introduced the retraction operation defined by the division of commonality functions. Smets defined then the concept of *latent belief structure* for non dogmatic mass functions. This concept is studied in more details by Pichon and Denoeux [14]. Lukaszewski [11] proposes an algorithm for what he calls *updating* which consists in removing or changing some pieces of evidence without carrying out all the combinations again except for the ones that have been deleted or modified. Dubois, Faux et Prade [5] consider retraction as a special symmetric belief change operation that avoids the explicit use of negative mass functions.

Table 4 presents an abstract example which deal with the three different approaches (negation viewed as a conjunctive combination with the complementary, revision by the complementary and retraction) that can be used to remove a piece of evidence on a set B and the same example dealt with a strong persistent prejudice on B. It is important to note that retraction of a focal set B differs (fifth column) from conjunctive combination with the complementary of this focal set $\overline{B}$ (third column). Indeed retraction allows us to focus and reduce or delete the mass on B (it is possible to find values for x, y, z and u such that $m \circledcirc B^u(B) = 0$ with $y, u \neq 0$) while negation never allows us to cancel beliefs on B (since $yu \neq 0$ as soon as $y, u \neq 0$). In other words, integrating a piece of evidence on the complementary of B ($\overline{B}^u$) is different from canceling an evidence for B (by integrating $B^{1/u}$). Revision gives priority to the new piece of information, hence revising by $\overline{B}$ amounts to transfer pieces of evidence from A, B and $A \cap B$ to $A \setminus B$ (since $A \setminus B \subset \overline{B}$). Our approach takes a complementary point of view relatively to revision and retraction since it allows us to make evolve the beliefs either by transfer or by attenuation. The important difference is the introduction of a new dimension for prejudices allowing us to distinguish them from negative evidence and to handle their evolution independently.

	m	$\overline{B}^u$	Negation $m \bigcirc\!\!\!\!\cap \overline{B}^u$	Revision $m \circ \overline{B}^u$	Retraction $\beta = \frac{1}{u}$ $m \circledcirc B^u = m \bigcirc\!\!\!\!\cap B^\beta$	Strong and persistent Prejudice $\beta = \frac{1}{u} > y$ $(B^{1/u}, m)$
$\emptyset$	0	0	$(y+z)(1-u)$	0	0	(0,0)
A	x	0	xu	xu	x/u	(0, 0)
B	y	0	yu	yu	$1-x-z-$ $(1-x-y-z)/u$	$(1-\beta, 0)$
$A \cap B$	z	0	zu	zu	$x+z-x/u$	$(0,0)$
$A \setminus B$	0	0	$x(1-u)$	$1-u$	0	$(0,x)$
$\overline{B}$	0	$1-u$	$(1-x-y-z)(1-u)$	0	0	$(0,0)$
Ω	$1-x-y-z$	u	$(1-x-y-z)u$	$(1-x-y-z)u$	$(1-x-y-z)/u$	$(\beta, (1-x))$

Table 4: Four different views of "negative information": negation (i.e., combination with $\overline{B}$), revision, retraction and strong persistent prejudice

To sum up this paper presents a preliminary study about the integration of evidence in a belief state where the agent has some prejudices. We propose a bipolar model considering prejudices

and uncertain beliefs. Prejudices have a strength and a tenacity. We study how new incoming information can modify both the levels of prejudice and belief. This model is compatible with revision and retraction operations.

Note that in a more general case, it may be impossible to represent masses and prejudices with a simple belief state (i.e., a simple prejudice and a simple mass). In that case, i.e., with complex masses and prejudices, in order to know if the prejudice is stronger than the beliefs, we could either consider a veto approach that takes into account only the focal sets with highest evidence that are concerned by the prejudice or a cumulative approach that would consider the sum of the beliefs on focal sets that are concerned by it. It could be interesting to study whether it is possible to recover a purely Dempster-Shafer framework from a general belief state made of complex masses and prejudices.

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