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### Optimization on frequency constraints with FFT using Automatic Differentiation on hybrid ODE applications

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Abstract. Optimizing electrical systems represented by ODE and events, using their frequency spectrum is an important issue for designers. This paper presents a methodology to answer to this issue. Using gradient-based optimization algorithm, the paper proposes to simulate the electrical system according time, and then to compute its frequency spectrum. To optimize it by SQP, Automatic Differentiation is mainly used to compute the model gradients.

Keywords: Automatic Differentiation, Dynamic systems, Frequency analysis, SQP Optimization

#### INTRODUCTION

Sizing applications (e.g.: power electronics (PE) systems) described by hybrid Ordinary Differential Equations (ODE) by constraining their frequency spectrum is an important issue for designers. Computing the Fast-Fourier Transform (FFT) on a changing-configuration system is a real challenge. Indeed, most of existing techniques [1][2] prefer to impose pre-requisites on the operating model, e.g. by calculating harmonics with analytical formula, using measured spectrum as a part of the system. FFT seems a very good solution to study harmonics from time simulators. However, for sizing by optimization, it is limited. Indeed, in one way, stochastic optimization on time simulation consumes time; in another way, using gradient-based optimization on the time simulators requires the use of finite differences, which also increases computation time with possible numerical problems. The paper proposes to use Automatic Differentiation [3] to compute derivatives which is an exact calculation method, on both the time simulation and the FFT computation, which needs an access to the time simulation code.

#### METHODOLOGY

#### General principle of the methodology

The paper proposes to use AD to carry out optimization on dynamics systems based on hybrid ODE. Such ODE consist in ODE changing according to the values of state variables or observable variables during the dynamic-simulation process. Tests on simulated variables are made to detect changes of ODE. In the optimization loop, a time simulation of a changing-configuration system is run, until the steady-state is reached. Then the simulation is followed by one operating period to compute the FFT of some variables before to be stopped (so no useless simulation). From this, the harmonics of some simulated variables can be extracted to constraint them. The full paper will develop the structure of the time solver, to allow AD on hybrid ODE with FFT.

#### Implementation

The approach has been implemented in the open-source Python package NoLOAD [4]. It solves gradient-based optimization problems. To solve the ODE, an adaptive ODE solver (Dormand-Prince 45) [5] was implemented into NoLOAD framework, to be able to use AD. The derivatives of the time simulation according to any optimization input I are computed by Automatic Differentiation (AD). The derivatives of the FFT can be computed with AD, but also with the analytical formula given in expression (1). The two methods are compared.

$$\forall k \in [0, M], \frac{d(FFT(y))}{dI}(k) = \frac{d}{dI} \left( \sum_{n=0}^{N-1} \left( y(n) * e^{-2i\pi * k * \frac{n}{M}} \right) \right) = \left( \sum_{n=0}^{N-1} \left( \frac{dy(n)}{dI} * e^{-2i\pi * k * \frac{n}{M}} \right) \right) = FFT\left( \frac{dy}{dI} \right) (k) \quad (1)$$

The two approaches will be detailed in the full paper.

#### Specifications of the optimization problem

The paper proposes to illustrate the approach with the sizing of a single-phase rectifier (see Figure 1). Such a structure seems simple; but as there are only soft switchings, its behaviour cannot be predicted, which greatly makes complex its study. The model of the circuit is a hybrid ODE, which consists in several ODE solved one after the other, according to the state of the diodes (not defined a priori). Over an operating period (the period of the AC source), an ODE may appear several times. The optimization inputs are summarized up in Table 1.

Table 1: Inputs of the optimization problem		
Name of the optimization input	Type of input	Fixed value / Gap
18	variable	[10 <sup>-6</sup> ; 10 <sup>-3</sup> ] H
rs	fixed	0.01 Ω
С	variable	[10 <sup>-4</sup> ;3*10 <sup>-3</sup> ] F
L	variable	[10 <sup>-6</sup> ; 10 <sup>-3</sup> ] H
R	fixed	10 Ω
Frequency f of Vs	fixed	50 Hz

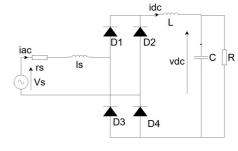


Figure 1: The single-phase rectifier

In this optimization problem, the objective function is to minimize the volume of inductors and capacitors. The fundamental and harmonics (until 15000Hz) of the inputs current *iac* and output voltage *vdc* are constrained. The Total Harmonic Distorsion (THD) for *iac* is constrained between 0,1 and 1,1. The harmonics of *vdc* have be lower than 40V, and the one corresponding to 100Hz frequency must be closed to 10% of the fundamental (a compromise between the value of the component and the quality of the DC voltage *vdc*). The optimization is run twice : first with AD to compute FFT derivatives, and second by using analytical formula in (1).

#### **Results of the optimization problem**

In both cases, the optimization converged in 30s on the following optimum point : C=2.9mF, ls=0.3mH, L=33.5  $\mu$ H. The volume of inductors and capacitor equals 3.24 mm<sup>3</sup>. The analytical derivatives of the FFT give the same result than AD. The full paper will develop the computation cost and the implementation of the optimization loop using AD.

#### CONCLUSION

In the paper, a methodology to solve optimization problems on changing-configuration ODE with frequency constraints is presented. It is illustrated with the single-phase rectifier model. The derivatives according to optimization inputs are computed by Automatic Differentiation of the time simulation, and by using a symbolic expression of the FFT derivatives, or AD in the FFT.

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