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► **To cite this version:**

Renaud Chorlay. From the historical text to the classroom session: analysing the work of teachers-as-designers. ZDM, 2022, 54 (7), pp.1583-1596. 10.1007/s11858-022-01434-7. hal-04216281

**HAL Id: hal-04216281**

**<https://hal.science/hal-04216281>**

Submitted on 24 Sep 2023

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# From the historical text to the classroom session: Analysing the work of teachers-as-designers

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## Abstract

While classical studies have highlighted the many potential benefits of using original historical sources in the classroom, few studies have documented actual classroom practices outside of research contexts. In this case study, I aim to describe and explain how five French high school teachers autonomously designed and implemented classroom sessions starting from the same document, namely an excerpt from Euler’s *Elements of Algebra* presenting an algorithm for square root approximation. From a methodological viewpoint, it enables me show how two general frameworks for the study of teachers’ professional practices—the Documentational Approach to Didactics and the Didactic and Ergonomic Double Approach—can be tailored to fit the specific challenges of using historical sources. The empirical results provide fresh insights into the conditions for a mathematically rich use of historical sources in the classroom, and on the connections between this use and the integration of a historical perspective in the teaching of mathematics.

**Keywords** Cognitive demand · Didactic and Ergonomic Double Approach · Documentational approach · Primary historical sources · Task design · Teaching practices

## 1. Introduction

Since the 1970s, many voices in the HPM (History and Pedagogy of Mathematics) community and beyond have stressed the potential benefits of the use of historical sources in the mathematics classroom (Clark et al., 2016; Fauvel & van Maanen, 2000; Fried et al. 2014; Furinghetti et al., 2006; Jankvist, 2009). On this basis, resources for teaching were developed, either in the form of sourcebooks making selections of extracts from historical sources more easily available (Chabert, 1999; Katz, 2007; Stedall, 2008), or through accounts of teaching experiments designed and implemented by researchers or enthusiasts. However, few studies have addressed ‘ordinary’ teaching practices. I use the word ‘ordinary’ in a non-evaluative way, to describe teaching sessions that were not designed by researchers. It does not mean that the teachers involved are in any way representative of a hypothetical ‘average’ teacher, and interacted in no way with researchers, for instance in the context of teacher training (Clark, 2006); rather, this usage implies that they worked in ordinary teaching conditions (in terms of time, school audience

etc.), and were to a very significant extent responsible for the design and implementation of the session(s).

A recent reform of high school curricula in France gave the opportunity to design and carry out a research project on the use of original sources in the classroom. I selected an excerpt from Euler's *Elements of Algebra* (1828), whose mathematical content has many connections to the knowledge-to-be-taught as defined by the syllabus. This text also lends itself to activities that have been identified in the HPM literature as “specifically supported by reading mathematical sources”, in particular, with a focus on “the process of understanding” rather than on “results”; in which there is a need to “give reasons in support of interpretations” of a text whose meaning “may remain open” and may “lead to different readings” (adapted from Furinghetti et al., 2006, p. 1287). Five teachers agreed to design and implement teaching sessions based—in some way or another—on this text. They agreed to carry out this work with no interactions with me or with the four other teachers. This case study thus allows for a comparative analysis of the work of teachers-as-designers (Gueudet et al., 2017), in ‘ordinary’ conditions, starting from the kind of primary source that is typically found in sourcebooks.

This case study also allows me to further the research program on the use of original sources in the classroom presented at the 2016 HPM conference. I argued that, in order to “pave the way for a more intense collaboration with researchers in other fields of mathematics education”, one could select research terrains and questions which “are central for the HPM community, yet not completely specific to it” (Chorlay, 2016, p. 5). Consequently, my endeavour to study an HPM-specific form of the professional activity of teachers relied on two all-purpose theoretical approaches, namely, the Documentational Approach to Didactics (Gueudet & Trouche, 2009; Pepin et al., 2013), and the Didactic and Ergonomic Double Approach (Robert & Rogalski, 2005; Vandebrouck, 2013). Similarly, my research questions parallel those of Gueudet et al. (2016, p. 187), as follows:

- What are the choices made by teachers in their autonomous design and implementation of classroom sessions based on a historical source?
- Which are the factors shaping these choices?

In the first section, on the background of the two main theoretical tools, I specify two challenges facing sessions using historical sources. In the second section I present the specifics of the case study. In particular, the *a priori* content analysis of the source has the goal of mapping out potential uses, thus allowing for a principled description of teachers’ choices and actual practice. The empirical data are analysed in the final section.

## 2. Theoretical approaches

### 2.1 Tools for the study of teachers' design work

The Documentational Approach to Didactics—or DAD (Gueudet & Trouche, 2009; Gueudet et al., 2013) and the Didactic and Ergonomic Double Approach—or DEDA (Robert & Rogalski, 2005; Vandebrouck, 2013) overlap significantly: both are grounded in activity theory and aim to study teachers' activity—both in-school and out-of-school—as a professional activity. Teachers' work (*ergon*) is described as a goal-oriented activity taking place in an environment comprising institutions and work collectives; an environment whose constraints and affordances shape, but do not determine, the teaching activity. However, these two approaches have different foci.

DAD focuses on resources for teaching and their use. Drawing on the distinction between artefact and tool, the documental approach endeavours to distinguish between 'resources' for teaching—construed as *lato sensu* as encompassing not only printed or online resources such as textbooks or ICT tools, but also teaching experience (Adler, 2000)—and 'documents'. Just as a tool is seen as the combination of an artefact (e.g., a ruler, dynamic geometry software) along with schemes of use, a document (e.g., a lesson plan) is seen as the combination of a resource (e.g., a textbook, a curriculum) along with schemes of use. These schemes of use can be described in terms of rules (operational invariants) which form (partly) implicit knowledge derived from experience. These schemes can either be inferred by an observer or elicited through interviews.

To the best of my knowledge, DAD has not been used to study the use of original sources in the classroom. However, this facet of teaching seems particularly adapted to the study of the aspect of teachers' design work that Gueudet and Trouche call "documentation work", which consists in looking for resources, modifying them, "selecting/designing mathematical tasks, planning their succession and the associated time management, etc." (Gueudet & Trouche, 2009, p. 201). Moreover, both the notion of resource and the notion of scheme prove useful in a comparative study such as this one, which aims to describe and—to some extent—account for the (possibly different) choices made by five teachers, choices that may reflect different resources, or different schemes of use.

DEDA is an all-encompassing framework for the study of teachers' professional activity. From a descriptive viewpoint, it endeavours to describe specific engagements in teaching, such as a given session, in terms of the following five "components":

- The *personal* component encompasses knowledge, orientations, and beliefs about mathematics and its teaching; it reflects a teacher's personal and professional history.

- The *institutional* component sums up the features of a given school system (e.g., curricula, standard textbooks).
- The *social* component describes the social groups that shape the teacher's work, from school audience to teachers' unions or networks of textbook authors.
- The *mediative* component describes in-the-moment classroom management, including how students' work is organized (time, tools, pedagogical organization, interactions). "Agency" and "accountability" are key issues here: "To what extent do students have the opportunity to make mathematical conjectures, explanations and arguments, developing 'voice' (agency and authority) while adhering to mathematical norms (accountability)?" (Schoenfeld, 2013, p. 616)
- The *cognitive* component describes the intended cognitive path of students, along with the means for its actualization, including sequencing of topics at several scales (from school year to micro-episode), selection and sequencing of tasks, level of scaffolding, expected difficulties, etc.

Beyond their descriptive use, the first three components also act as 'determinants', enabling the researcher to make hypotheses on what explains or determines the observed practices, usually in a comparative perspective.

## 2.2 Specific challenges, specific observables

These approaches provide all-purpose descriptive and explanatory tools, but they need to be tailored to reflect the challenges of specific forms of engagement in teaching. The purpose of this section is to show that the research literature on mathematics education or on history of mathematics provides theoretical constructs that prove relevant for this tailoring. Here, the adjective 'specific' does not mean that these constructs are relevant only for the study of original sources in a classroom, nor that they are relevant for any kind of use of history of mathematics in the classroom. They are tailored to fit challenges emerging from the specifics of this case study, in which an algorithmic text expounding a calculation method is used in the context of a curriculum with a definite "history as a tool" flavour (Jankvist, 2009).

In the wake of Schoenfeld's work on problem-based, mathematically rich teaching of mathematics, a series of papers by Stein (Grover et al., 1996; Henningsen & Stein, 1997) contributed to the understanding of the connections between resources for teaching which are *potentially* cognitively demanding, and the teaching practices which lead—or fail to lead—to the actualisation of this potential. Although the notions of 'doing mathematics' and 'cognitive demand' are not crisply delineated, the overlap with some of the expected benefits of using historical sources is significant, namely,

(...) capacity to engage in *the process* of mathematical thinking; in essence do what makers and users of mathematics do: framing and solving problems, looking for patterns, making conjectures, examining constraints, making

inferences from data, abstracting, inventing, explaining, justifying and so on. (Grover et al., 1996, p. 456; emphasis in the original)

Henningsen and Stein (1997) showed how the mediative and cognitive components interfere, and usually lead to the lowering of the cognitive demand in actual classroom implementations of potentially mathematically rich tasks. Two of the three “patterns of decline” which they identified are relevant in the context of this paper. The first one is “decline into using procedures without connections to concepts, meaning, and understanding”, where teachers change the focus of the activity on the results rather than the process, and provide so much scaffolding that only procedural and unconnected tasks are entrusted to students. The main factors leading to this type of decline are lack of time, and the perception by teachers that high-level tasks are “ambiguous, risky, or both, [a perception which leads to a] “pull” toward reducing their complexity so as to manage the accompanying anxiety” (Henningsen & Stein, 1997, p. 535). The second pattern is “decline into unsystematic exploration”, where students go round in unproductive circles. The main factors leading to this type of decline are the inappropriateness of the task (too difficult, requiring knowledge that is not available to students), and excessive time.

Another line of investigation questions the schemes of use of an original source in the classroom. It draws on Rheinberger’s analysis of the functions of “things” (*Dinge*) which play a part in science, be they “genes”, “cyclotrons”, or “general relativity” (Epple, 1999; Rheinberger, 1997). Rheinberger distinguished between, on the one hand, “epistemic objects” which are the object of scientific investigation, and which, by their very nature (of partially unknown objects) are open-ended and constantly subject to redefinition, and, on the other hand, “technological objects” which make up the machinery of science and serve as tools for the study of epistemic objects. The difference is not one of nature, but one of function: epistemic objects are *question-generating* objects, whereas technological objects are *answer-generating*. Forms of engagement with a given object can make it play either function: these forms of engagement are context dependent and reflect schemes of use. Although this distinction is not to be used in the same way by historians and by researchers in education, I considered that it could be adapted to this case study to capture one of its key issues. The original source on which the study is based expounds an algorithmic method. As such, it can be regarded a technological object, which, if used correctly, should yield numerical answers to mathematical questions. Since this technological object is new to students, these ‘instructions for use’ need to be understood and implemented, an endeavour that generates a wide range of routine micro-tasks (reformulate, calculate, expand brackets). However, the specific original source I selected for this study leaves room for interpretation in several respects, and should elicit rather open-ended questions as to its genre, its scope, its correctness, the intentions and the knowledge of its author etc. These questions do not call for routine mathematical answers, or for answers of a mathematical nature. I regard the intended and actual schemes of use of the artefact as *both* a technological and an epistemic object as a key observable for this study, since I consider that regarding it as an epistemic object is a necessary condition for engagement of students at the level of ‘doing mathematics’.

### **3. Context and methods**

#### **3.1 Institutional context: A new high school curriculum**

In 2019, in France, new national high school syllabi were implemented in all subjects (grades 10 to 12). As far as history of mathematics is concerned, the introduction of the 2019 syllabus gives fairly general guidelines, as follows:

It can be judicious to enlighten the content of the mathematics course by a historical, epistemological or cultural contextualization. Indeed, history can be seen as a source of problems which clarify the meaning of some notions. The passages labelled ‘History of mathematics’ point to some possibilities along this line. Teachers can implement them by relying on the study of historical documents. (MEN 2019; author translation)

Here, history of mathematics is seen as a tool rather than a goal (Jankvist, 2009); historical activities are not mandatory, and they should be woven into the general fabric of the course. Although this tool is versatile, the main goal is to foster conceptual understanding and meaning-making, rather than, for instance, motivating students, finding real-life or extra-mathematical applications of mathematics, or showing that school mathematics reflects the cultural heritage of humanity as a whole (Jankvist, 2009). The 2019 syllabus is lavishly peppered with paragraphs explicitly labelled “history of mathematics”, which makes this topic one of the three guiding threads of the whole syllabus (along with “proof” and “algorithmic thinking and programming”). While some of these paragraphs are carefully worded and give specific suggestions, most are rather pithy and ambiguous. No recommendations are made as to the amount of time that could or should be allotted to “historical” activities.

Moreover, the traditional resources on which teachers usually draw to meet the requirements of a new curriculum (Chongyang et al., 2019) are wanting: First, up until the 2010 reform of teacher training, very few teachers had any academic background in the history of mathematics, nor any experience of its inclusion in the classroom. Second, for lack of time and expertise, many publishers failed to take this feature of the curriculum into account in their new textbooks (there are no official textbooks in France). Third, when a new syllabus is published, it is customary for the Ministry of Education to publish guidelines for its implementations along with online teaching resources. In 2019, such resources were published to scaffold the implementation of the “proof” and “algorithmic thinking and programming” strands, but none regarding the historical suggestions.

#### **3.2 Research protocol and participants**

Since the 1980s, both in the French national context of the Institutes for Research on Mathematics Education (IREM) network, and in the international context of the HPM Study Group, the “Epistemology and History” subcommission of the IREM network has been developing teaching resources along the general lines mentioned

in the syllabi, arguing that many historical documents provide opportunities for a genuine engagement in mathematics (Barbin, 2018; Chorlay, 2016; Fauvel, 1990). In 2019, this subcommission launched the collective development of a book meant to provide high school teachers with a range of classroom activities based on historical documents and compatible with the new curriculum.

This collective project is the context of this study, which involves two types of participants, namely, five high school teachers and a researcher in history of mathematics and mathematics education (the author). This study weaves together two distinct projects: one is the development of classroom sessions, with a view to contributing a chapter to the new IREM book; one is a research project bearing on the work of the five teachers as they engaged in the resource development project. It was agreed from the outset between each teacher and me that both projects were distinct but compatible. The following protocol was agreed upon:

1. I would select one historical document that I considered suitable for the book development project. All the teachers would be given the same document.
2. I would meet each teacher, individually. The historical document would be read together; its mathematical content would be discussed; possible connections to the curriculum would be discussed, in a ‘brainstorming’ mode. A detailed account of this phase is given in Sections 3.3 and 3.4. The goal of this session is to generate a shared understanding of the didactical potential of the document, an understanding that is shared between me and each of the teachers, and is similar for all teachers, even though they did not communicate with one another. No specific choices of implementation would be made or even discussed.
3. Each teacher would work independently from the other teachers and the author in order to design some teaching session(s) compatible with the resource development project. For research purposes, teachers were asked to endeavour to keep a record of their work, including personal notes, draft versions of the final documents, etc.
4. Each teacher would implement the session(s) she/he designed.
5. Two short interviews would take place: (1) shortly before the actual session(s), I would carry out a semi-guided interview bearing on (a) the teacher’s self-recollection of the design process, (b) the choices which the teacher made along the way, (c) the goal(s) of the session(s), (d) the expected or possible difficulties, to be experienced either by the students or by the teacher. (2) Shortly after the session(s), an informal debrief would focus on issues (c) and (d).
6. This would be the end of the research project. Collaborative work would then begin in the context of the book development project.

It is important to underline some specific features of this research protocol. First, the nature of the teacher/researcher interaction was of the “clinical partnership” type (Wagner, 1997), hence not an instance of teachers and researchers working as

“partners in task design” (Jones & Pepin, 2016). Second, in the research phase (phases 1 to 5), there was no communication among teachers, hence no collaborative task design. This methodological choice reflects the goals of the study. Since the field of teacher task design with historical sources is virtually unexplored, gathering as much data as possible was key in order to document schemes of use. To compare the choices made by the five teachers, I collected the recordings of all pre- and post-session interviews, all the final student worksheets, and the recordings of the sessions (3 video- and 2 audio-recordings, ranging from 1 to 3 hours per teacher). Four of the five participants also provided draft versions of the student worksheet. The data were systematically compared in terms of similarities and dissimilarities. The points of comparison were defined *a priori*, on two bases: one was the set of observables identified in Section 2.2 (indicators of an engagement at the level of ‘doing mathematics’, a question generating or an answer generating function of the document); the other one stemmed from the *a priori* analysis of the didactical potential of the original source (Section 3.3).

Five teachers, whom I call T1, ..., T5, took part in this twofold project. Table 1 provides an overview based on their own description:

**Table 1.** Some features of the participants

	T1	T2	T3	T4	T5
Teaching experience (years)	30	30	12	22	12
Experience as teacher educator	X				
Former participation in design research	X	X	X		
Participation in a history-oriented IREM group	X		X	X	
Participation in another IREM group	X		X	X	
Personal experience with historical sources in the classroom (a Lot, Some, None)	L	S	S	S	S

These data show that the teachers who engaged in the book development project were by no means a random sample. In this table, “experience with historical sources” refers to the use of original sources as found in sourcebooks or other non-didacticized publications, as opposed to activities found in textbooks. Also, apart from teacher T5, all the teachers had some experience either in collaborative resource design (in the IREM context) or in collaborative design research on non-historical topics (with the author). While T5 stated she had some experience with historical sources in the classroom, it was to a lesser extent than the others. Teachers T4 and T5 worked in the same school, and T5 implemented only *one* session using a historical document, a session designed by T4 the year before.

The reading of the source in partnership with the researcher (phase 2) is pivotal since the content of the original source and the way it was presented and discussed in phase 2 set the stage for teachers’ design choices.

### 3.3 A focus on phase 2: *A priori* analysis of the historical source

I selected a three-page excerpt from the 1774 French edition of the first volume of Euler's *Elements of Algebra*. The teachers were told that this book is not a research treatise but rather a didactic work, covering algebraic topics ranging from the very elementary (operations with fractions and directed numbers) to the rather advanced (solutions of algebraic equations up to the fourth degree). The extracts were taken from the final chapter. The teachers were given a three-page document, but I explained that we would focus on the first part consisting of paragraphs 784 and 786, the rest being provided mainly for context.

#### CHAP. XVI. Of the Resolution of Equations by Approximation.

784. When the roots of an equation are not rational, and can only be expressed by radical quantities, or when we have not even that resource, as is the case with equations which exceed the fourth degree, we must be satisfied with determining their values by approximation; that is to say, by methods which are continually bringing us nearer to the true value, till at last the error being very small, it may be neglected. Different methods of this kind have been proposed, the chief of which we shall explain.

(...)

786. We shall illustrate this method first by an easy example, requiring by approximation the root of the equation  $xx = 20$ .

[Footnote by J. Bernoulli: This is the method given by Sir Is. Newton at the beginning of his *Method of Fluxions*.]

Here we perceive, that  $x$  is greater than 4, and less than 5; making, therefore,  $x = 4 + p$ , we shall have  $xx = 16 + 8p + pp = 20$ ; but as  $pp$  must be very small, we shall neglect it, in order that we may have only the equation  $16 + 8p = 20$ , or  $8p = 4$ . This gives  $p = \frac{1}{2}$ , and  $x = 4\frac{1}{2}$ , which already approaches nearer the true root.

If, therefore, we now suppose  $x = 4\frac{1}{2} + p$ ; we are sure that  $p$  expresses a fraction much smaller than before, and that we may neglect  $pp$  with greater propriety. We have, therefore,  $xx = 20\frac{1}{4} + 9p = 20$ , or  $9p = -\frac{1}{4}$ ; and consequently,  $p = -\frac{1}{36}$ ; therefore  $x = 4\frac{1}{2} - \frac{1}{36} = 4\frac{17}{36}$ .

And if we wished to approximate still nearer to the true value, we must make  $x = 4\frac{17}{36} + p$ , and should thus have  $xx = 20\frac{1}{1296} + 8\frac{34}{36}p = 20$ ; so that  $8\frac{34}{36}p = -\frac{1}{1296}$ ,  $322p = -\frac{36}{1296} = -\frac{1}{36}$  and  $p = -\frac{1}{36 \times 322} = -\frac{1}{11592}$ ; therefore  $x = 4\frac{17}{36} - \frac{1}{11592} = 4\frac{4473}{11592}$ , a value which is so near the truth, that we may consider the error as of no importance.

**Fig. 1.** Excerpts from Euler’s *Elements of algebra*<sup>1</sup>

Two complementary excerpts were briefly discussed with the teachers. In the first one, Euler provides what can be read as a recursive formula for sequences approximating any root: if  $n$  is a given approximation of  $\sqrt{a}$ , then the next approximation is  $\frac{nn+a}{2n}$ . In the second one, the same method leads to a similar formula for any third-degree equation ( $x^3 + axx + bx + c = 0$ ).

The goal of phase 2 was to reach a shared understanding regarding some features of the text. The features listed below were all discussed with the teachers in phase 2, including the connections with curriculum assigned goals. In this list, I bracketed and italicized the grade(s) in which these goals are mentioned (except for the transversal “algorithmic thinking and programming” and “proof” strands).

- This text provides opportunities to carry out routine calculation [*grades 8–10*]: expansion of  $(4+p)^2$ , calculation with fractions, solving first degree equations, solving equation  $x^2 = 20$ . The procedures are routine, but the numerical values quickly become difficult to operate upon using pen and paper. Several digital tools could be considered [*grades 10–12*].
- This provides opportunities to carry out comparisons between numbers [*grades 8–10*], either by comparing the successive approximations  $4, 4\frac{1}{2}, 4\frac{17}{36}$  ... to a numerical approximation of  $\sqrt{20}$ ; or by comparing the squares of  $4, 4\frac{1}{2}, 4\frac{17}{36}$  ... with 20.
- The meaning of the main warrant (“ $pp$  must be very small, we shall neglect it”) is ambiguous, and the text does not provide any proof-type justifications for it. Several interpretations are possible. A static interpretation is that  $|p|$  being less than one (a claim which also calls for warrants!),  $p^2$  is less than  $|p|$  and  $|8p|$  [*grade 10*] etc. But Euler wrote that, as the algorithm unfolds, one is *ever more* justified in neglecting  $p^2$ , thus possibly pointing to an asymptotic interpretation such as: when  $p$  tends to 0,  $p^2$  becomes infinitely less than  $|p|$  since the ratio  $|p|/p^2$  tends to  $+\infty$ .
- The text weaves together two *genres* of mathematical texts, namely, the exposition of an algorithm, and a heuristic argumentation providing some warrants for claims regarding key steps of the calculation ( $p^2$  will be crossed out *because* it is “very small”). By contrast, it does not seem to belong to the genre “proof”: it deals with an example, no background theory is clearly used, the warrants are not backed by theorems.
- The text mentions or points to several topics in number theory. The introductory paragraph mentions a classification of numbers (some are “rational” while some other necessarily involve “radical quantities”), and, implicitly,  $\sqrt{20}$  belongs to the second category [*grade 10*]. Also, number

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<sup>1</sup> The French 1774 edition was used with the teachers. Figure 1 is based on the version (Euler, 1828, pp. 289–290), which I altered slightly to give a more faithful account of the source used in the experiment.

theory provides an answer to a key algorithmic question: starting from rational inputs (e.g., 4), the algorithm will never yield a number with square 20, so it will *not* terminate. A proof by induction can be carried out in *grade 12*.

- The text displays the first steps of a method, but—at least in this extract—the claims as to the scope of this method are implicit. Is Euler claiming only that the four values from (4 to  $4\frac{4473}{11592}$ ) are increasingly better approximations of  $\sqrt{20}$ ? Or that an iterative interpretation of the algorithm leads to a sequence of numbers with limit  $\sqrt{20}$  [*grades 11–12*]? That this “method” works for *all* square roots (Heron’s sequence)? Or even for all polynomial equations?
- Bernoulli’s footnote reminds the reader that Euler is merely expounding the Newton-Raphson method (Chabert, 1999). To the expert reader, this should bring to mind tangents and derivatives; topics which, on the face of it, do not play any part in the text. However, in a polynomial context and from a contemporary perspective, the change of variables ( $x = 4 + p$ ) and the neglect of  $p^2$  amounts to working out derivatives. Euler’s text can be seen as presenting a very useful special (polynomial) case of a key concept (the derivative as provider of local linear approximations) [*grades 11–12*].
- The text illustrates the first steps of what is clearly an iterative algorithm. Identifying this text as presenting an iterative algorithm, extracting the algorithm by editing out the heuristic parts, and implementing it in a programming language, are all high school tasks.
- Other algorithms for the approximation of the solutions of numerical equations are to be studied and implemented [*grades 10–12*], possibly calling for a comparison in terms of efficiency. Also, implementing Newton’s method requires a first rough approximation of a solution (here  $\sqrt{20} \approx 4$ ), which, in itself, may call for another algorithm. The dependence of the algorithm on the initial value can be questioned: would the “method” also work starting from 5, or 20, or 0?
- Euler used the same letter  $x$  to denote different numbers (same for  $p$ ), which can be questioned as to rigour. At least two reactions could be mathematically and didactically relevant: one could either realise the fact that this iterative method generates recursively defined sequences, and introduce notations such as  $x_{n+1} = \frac{1}{2}\left(x_n + \frac{20}{x_n}\right)$  [*grade 11*]; or consider that the letters represent programming variables and not mathematical variables [*grade 10*]. In this second context, some of the “=” symbols should be read as value-change operators and not as mathematical equalities.

### 3.3 A focus on phase 2: historical source in the classroom or History in the classroom?

The features of the text discussed with the teachers were mainly ‘mathematical’ (*lato sensu*), bearing not only on the mathematical notions and techniques at stake, but also on the nature of the underlying mathematical endeavour (scope, argumentative strength). By contrast, historical features were mentioned to a much lesser extent. I first mention those that were, and then explain this choice.

At the beginning of phase 2, each teacher and I went through the table of content of the first volume of Euler’s *Elements of Algebra*, so as to provide some context for the excerpt to be studied. It showed that this text is neither a short research publication nor an advanced treatise. The fact that it starts by explaining how to calculate with different types of numbers (rational, negative, imaginary) then moves on to explain how to solve equations—starting from the 1<sup>st</sup> degree—led to the hypothesis that it is a rather comprehensive but elementary publication, probably written with a didactical purpose. I mentioned the following two additional elements: (1) the text is available online, should the teacher want or feel the need to dig more deeply into it; (2) as the footnote explains, the excerpt studied does not reflect Euler’s personal contribution but is an account of Newton’s method. I confirmed that Newton’s work contains a very similar technique—a technique that involves neither derivatives nor tangents—albeit a slightly more complicated one, expounded on a more sophisticated example (an equation of the 3<sup>rd</sup> degree). Euler’s text was selected for this project for its simpler exposition of the method (in §786), and its more general formulae (in §787–789).

Arguably, the notion of ‘historical content’ of the text is much more open-ended, and even elusive, than that of mathematical content. In the context of historical research, several lines of investigations could be pursued: what was the role of Euler’s textbook in the landscape of mathematics teaching in the 18<sup>th</sup> century? What are the connections between this textbook and Euler’s research work? Since this excerpt bears on a method devised by Newton, one could also investigate questions such as the following: how did Newton originate this method? Why did he not word it in terms of derivatives and tangents, and who did? Who used numerical methods of approximation in the early modern period (mathematicians, astronomers, engineers, etc.), and was Newton’s method actually used? The answers to these historical research questions are not generally known. At any rate, they would not be straightforward and would require a significant level of historical expertise. They could be considered for a project of a different scale and a different nature than this one.

The fact that the answers to these historical questions are not within the scope of this project does not imply that the questions themselves were not worth asking. Within the context of this project, I hypothesized that the little information imparted to the teachers during phase 2 provided some opportunities to regard it as a question-generating artefact.

## 4. Results

The five teachers independently designed and implemented teaching sessions in the 2020–21 school year. The project was originally meant to be implemented in 2019–20, but due to the COVID-19 pandemic there was a long span of time between phase 2 and the implementation of the sessions (Table 2).

**Table 2.** Overview of the five sessions

Teacher	Phase 2	Classroom implementation	Grade	Planned duration	Actual duration
T1	Jan. 2020	June 2021	Grade 10	2h	2½ h
T2	Jan. 2020	June 2021	Grade 10	2h	3h
T3	Feb. 2020	March 2021	Grade 11	2h	2½ h
T4	Oct. 2020	Feb. 2021	Grade 10	2h	2h
T5	Oct. 2020	June 2021	Grade 11	1h	1h

### 4.1 Similar choices as to the insertion of the session at the scale of the school year

Since the text touches on a great variety of mathematical topics, the teachers had considerable leeway concerning its insertion into the general fabric of the mathematics course at the scale of the school year. In terms of the cognitive component, this insertion testifies to the goals that the teachers assigned to the session and to their reading of its connections to specific concepts or methods studied throughout the year.

Each teacher planned the sequencing of topics on a yearly scale in terms of ‘chapters’. Several schemes of use of the text could have been considered: (1) forward-looking use, using the text as an introduction to a topic-specific chapter; (2) backward-looking use with respect to a specific topic, with a mathematically rich problem studied at the end of a specific chapter, providing an opportunity to use the recently discovered concept in non-routine tasks; and (3) non-specific backward-looking use, giving students opportunities to review several concepts studied in different chapters earlier in the school year, and to make connections.

All five teachers opted for a non-specific backward-looking use. The two teachers who worked with grade 11 classes did not explicitly connect this session to previous ones, which is consistent with the fact that, for them, no specific grade 11 content was at stake. By contrast, the teachers working in grade 10 used the introductory paragraph of the text (§784) to connect this session to the topic of equations, which is transversal to most of the chapters of grade 10 (algebraic calculation, equations, functions and graphs, number sets). Teachers T2 and T4 focused on vocabulary and definitions: “Explain the meaning of root of an equation / rational root / radical

quantity / equation exceeding the 4<sup>th</sup> degree” (first questions in T2’s worksheet). Along the same lines, T1 devised more challenging tasks: “Read paragraph 784. Give an example of an equation whose roots are not rational but can be expressed with radicals.”

Teachers T1 and T4 also intended to connect this session to several others in which other algorithms for the numerical approximation of roots of equations (trial and improvement, bisection) were studied.

#### 4.2 A focus on the algebraic-algorithmic content

Predictably, all the sessions included calculations with fractions, expanding  $(\dots + \dots)^2$ , and solving first degree equations. The choices teachers made were reflected in what *else* was entrusted to students, and what topics they chose *not* to cover, although phase 2 showed there was potential.

Except for T4—who felt her class had not enough experience with programming in March 2021—the study of an *algorithm* was a key component of the sessions. In terms of the institutional component, it enabled the teachers to ‘kill two birds with one stone’, since algorithmic thinking and using history in the classroom are two of the three guiding threads of the whole high school curriculum. They used the text as a tool to entrust students with two standard, curriculum-assigned tasks, as follows: to apply, by pen and paper calculation, with various input values, an algorithm worded in everyday language (e.g., “Can you use Euler’s method to find an approximation of  $\sqrt{5}$ ?” in T5’s worksheet); to translate an algorithm worded in everyday language into a programming language (Python). The pre-session interviews showed that this *selection* of topics—among a larger range of possibilities—was a conscious choice. T2 put it very clearly:

First you ask yourself ‘what can be done with this text?’; then ‘needs’ come into the picture. This calls for pragmatic answers: it so happens that, this year, I need to do more work on algorithms, I didn’t do enough with my grade 10 students. I call this part of the work ‘refocusing’: at the beginning lots of ideas pop up, then, little by little, you get to focus on one or two ideas for using the text.

The tasks were entrusted at a rather routine level, since no teacher chose to seize some of the more cognitively demanding opportunities. First, the (mathematical) fact that the algorithm will not stop because  $\sqrt{20}$  is irrational was mentioned by only one of the teachers (T5), in passing. Second, the issue of the nature of the loop (either a count-controlled ‘for’-loop or a condition-controlled ‘while’-loop) was never discussed. This could have given an opportunity to consider the text as an epistemic object and question the meaning of Euler’s confident but unjustified coda: the third approximation “is so near the truth, that we may consider the error as of no importance” (of no importance for whom? In what context?).

The notion of derivative is central in grade 11 in France. Moreover, the curriculum requires that Newton’s method of tangents be studied in this class. However, the two teachers who designed sessions for grade 11 students made no connections to

these topics. Institutional factors can account for this fact. The notion of derivative is multifaceted, and curricula can either highlight or downplay one of these facets. Since the turn of the 21<sup>st</sup> century, French curricula require that derivatives be introduced in the geometric context of tangents to curves; they also mention the ‘instantaneous rate of change’ aspect; they do not mention the ‘local linearization’ aspect. The absence of this third aspect in the curricula as well as in textbooks has an impact on the mathematical knowledge of teachers (personal component). In phase 2, none of them were able spontaneously to connect the ‘crossing out of  $p^2$ ’ to derivatives, although, based on Bernoulli’s footnote, they all assumed the text bore on Newton’s method of tangents. Overall, except for a marginal use of the recursive notations for sequences (which is new in grade 11) and a marginal informal use of the word ‘limit’, the two grade 11 sessions bore on exactly the same mathematics as the three grade 10 sessions.

#### **4.3 The artefact as an epistemic object: zoom-in and zoom-out schemes of use**

The two aspects of the sessions analysed so far show great similarities in the teachers’ design choices. Moreover, both the analysis in terms of insertion of the session in the scope of the school year (backward-looking rather than forward-looking), and the analysis in terms of cognitive demand of the tasks, suggest the teachers wanted to keep the overall demand of the sessions under control. Even so, the meaning and potential impact of these choices remain to be determined. The apparent neglect of part of the potential of the text may reflect a ‘decline into procedures without connection’ and a use of the artefact *only* as a technological object (to be used correctly in order to generate numerical approximations). However, since lack of time and the inappropriateness of too demanding tasks also represent major decline factors, these choices can be made to secure necessary conditions for the devolution of *some* mathematically rich tasks, in order to let students explore the epistemic function of the object. Furthering the analysis of the teachers’ design work shows that, except for T5, the second alternative holds. To this end, several aspects of this work must be considered. Part of this work is reflected in students’ worksheets, with its sequencing of tasks and its way of weaving together two texts, namely Euler’s text and the teachers’ text. Beyond these textual aspects, key issues such as time allocation and the nature of teacher scaffolding can be studied through the analysis of the implemented sequences, complemented by the teachers’ personal notes and interviews.

All the worksheets show that some high-level tasks were to be entrusted to students. However, the nature and the sequencing of these tasks show a variety of practices.

From a purely descriptive viewpoint, two schemes of use of the original source can be distinguished. The choice made by T3 illustrates the first scheme (Figure 2). The worksheet she designed has four parts, the first of which bears on the text as a whole, considered as an epistemic object:

Part 1: Introduction—Whole text

1. Present the text (author, title, topic) and number its lines.
2. Do you think this text is rigorous? That it is correct? Why?
3. In the text, underline the passages which you do not understand. Explain why in the margin.

**Fig. 2.** First part of the students' worksheet designed by T3

The second part calls for a line-by-line checking of the calculations and claims. T3 was the only teacher to use this 'zoom-in' (i.e., from whole to parts) approach. The 'zoom-out' approach can be illustrated by T5's worksheet: in the first part, the student worksheet is divided into two columns, a left column entitled "in the 18<sup>th</sup> century" which displays Euler's text (divided in short sections), and a right column entitled "in 2021", with a blank space facing each section; students are to engage in step-by-step reformulations, calculations etc. The second part of the worksheet lists several open-ended questions, triggering a *gestalt switch* from the technological to the epistemic object:

Part II: Critical analysis

- Do you think Euler's method is efficient?
- How can we check whether the final approximation is sufficient?
- How can we calculate the approximation error?
- What are the simplifications carried out by Euler?

**Fig. 3.** Part 2 of T5's worksheet

T2's worksheet ends with a larger range of similarly open-ended questions:

- Is Euler right when he says that the final value "is so near the truth, that we may consider the error as of no importance"?
- Is Euler's text a proof-text? If so, what does it prove? If not, how would you describe this text?
- In which type of mathematical texts can a single variable  $x$  successively take on different values?

**Fig. 4.** Three questions from the final section of T2's worksheet

Whether or not these two schemes of use reflect operational invariants—such as “Students cannot engage in a reflective analysis of a text until all the details have been clarified” (for T5 and T2)—cannot be determined based on the data. By contrast, T3 explicitly explained to her students how to engage with the text:

Take 10 minutes. I advise you to take a scrap sheet of paper and a pen, it helps to understand the maths. Write down whatever you want. My advice: do not dive into calculations on first reading, try to forget them and take them for granted.

T3’s singular zoom-in choice can be accounted for by considering the personal and social components. For several years she had been taking part in an IREM group focusing on “language and mathematics” which aims to help teachers find ways to teach students how to read and write in mathematics. This gave her opportunities to work with teachers of French, and to gain insights coming from the didactics of reading comprehension (Chorlay, 2019). T3’s unusual third question (Figure 2) bears direct witness to this experience.

#### **4.4 The artefact as an epistemic object: questions answered, questions put to students, and questions raised by students**

However useful, the analysis of the worksheets paints a partial and somewhat distorted picture of the teachers’ designs. In-session observables such as time allocation and scaffolding provide additional insight into their reading of the situation.

The analysis of time allocation and scaffolding helps back the claim that T5 used the text mostly as a technological object. Admittedly, her worksheet *did* mention several ‘critical’ questions (Figure 3), and she is the only teacher who mentioned the connection between the irrationality of  $\sqrt{20}$  and the non-termination of the iterative algorithm. But in the 55-minute session, only 5 minutes were devoted to an informal exchange of ideas about these questions. Students had not been given any time to reflect on these ‘critical’ questions, and hardly contributed. With respect to the mediative component, in terms of agency, one could argue that the students were never really *asked* the ‘critical’ question, even if these questions were printed in their worksheet; and even if T5 answered them. At the other end of the spectrum, T3 inaugurated the session by giving her students 12 minutes to read through the text and reflect on the open-ended questions mentioned in part 1 of her worksheet (Figure 2). On the whole, students were to work autonomously (in small groups) for almost half the time of the two consecutive 55-minute periods. The remaining half was devoted to several phases of collective investigation into the various questions: 20 minutes were allotted to the low-level tasks (calculations), while 30 minutes were spent on the high-level questions, with a focus on the polysemy of the word ‘rigour’.

The pre- and post-session interviews with the teachers show that the analysis of the worksheets paints a biased picture of the design and can downplay the epistemic function they assign to the text. For instance, teacher T3 explained that she regarded the text as a great opportunity to make students discover the importance of the notion of limit of a sequence. When asked why she did not include questions on this aspect in the worksheet, she explained: “It’s not a question I would put in print. It should emerge in the dialogue, since it’s what questions such as ‘does the method work? How can we be sure?’ boil down to.” Teacher T1 was even more explicit and explained what can be seen as a general operational invariant in session design. She provided two draft versions of the worksheet, along with the final version, and I asked her why some questions disappeared along the way. Her answer shows that she removed these questions for two reasons, the main one being that she *does* value them:

I: Would you like students to ask these questions?

T1: Sure! (...) The more you write down questions, the more you lead what they’re going to do and say. (...) For me, the less you ask, the better; so as to let them move forward and ask questions themselves.

In terms of mediative component, it reflects how much T1 values students’ agency. It also testifies to a genuine intention to trigger a use of the artefact as an epistemic object, i.e., as a truly ‘question-generating’ object. For T1, students engage with the text at a mathematically higher level if *they* raise questions than if they only answer the teacher’s questions (whether printed or oral). She also mentioned a second reason why several questions were removed from the final worksheet: “If I can’t answer a question myself, I try not to ask students!” This does not mean that she did not value these questions, but that no specific written form captured them adequately for her. However, she did raise these questions orally during the sessions; one concerned the algorithmic nature of the text, and one addressed the possible justifications of the claim that  $p^2$  can be neglected.

#### **4.5 Routine non-routine sessions**

Stein’s work (1996, 1997) suggested that a major factor for the decline into routine, low-level engagements with mathematics is teachers’ anxiety. This anxiety may stem from the fact that the session departs from familiar “activity formats” (Gueudet et al., 2013, p. 934), or because they think the session might be too demanding for students.

In the pre-session interviews, the five teachers said their students might find the session a bit unsettling at first, but that, all things considered, it should not be particularly difficult for them. This echoes the analysis of the choices that enabled the teachers to keep the general cognitive demand of the sessions under control. They were also asked to explain what their motivation was for designing and implementing this session (leaving aside the requirements of the curriculum), and whether they felt this session departed from their usual way of teaching. Apart from T5, they said it did not. This does not mean that they used historical sources in the

classroom on a regular basis, but that they regarded this use as one of the many ways to teach non-routine mathematics, or to teach mathematics in a non-routine way. When asked if this session was in keeping with her general way of teaching, T4 answered:

Yes. Of course, there are times when we deal with more ... down-to-earth things ... specific notions, exercises ... still, even in textbooks we now have some variety of things to use, coming from historical contexts, or from economics, physics ... things which connect maths with real life.

T2 and T3 also emphasized the importance of giving students the opportunity to *read* in mathematics: “The [approximation] method itself is not what interests me here. What interests me here is to show them a mathematical text, to do this reading work ... and to review algorithms” (T2, pre-session interview). When pressed to explain what “reading work” meant for him, T2 explained:

I mean reading a text, whatever the text. To come across expressions, notations ... the way mathematics is written ... for instance, all the business with equalities, I’m looking forward to seeing what the students have to say about it. About rigour too, whose meaning changes. (...) To me, reading the text connects you to the spirit [sic].

The text is seen as a sample of more genuine mathematics than what standard textbook exercises offer. To him, the meaningful distinction is not ‘contemporary vs historical’ but ‘routine vs non-routine’. T2 clearly anticipated that the session would be unsettling for students, just like other types of non-routine sessions, which he valued highly and jointly described as sessions with a “research dimension”. When asked to illustrate this notion, he mentioned “the sessions in which we attempt to construct a definition for monotonic functions.” T3’s discourse was very similar, emphasizing how much she values “different” (i.e., non-routine) ways of doing mathematics, as with “open ended problems, or research narrations”.

#### **4.6 The text as a generator of *historical* questions?**

Until now, the survey of schemes of use on the one hand, and the focus on observables bearing on the level and nature (answer-oriented or question-oriented) of the cognitive demand on the other hand, left almost no room for historical knowledge or questions. In the five implementations, the teachers imparted minimal historical information, usually saying a few words about Euler when introducing the session and barely mentioning Newton. This omission substantiates Fried’s analysis of the leeway for history of mathematics in ordinary teaching conditions (2014). A closer investigation of the data gathered outside the classroom brought some additional elements, but only a few.

The exploration of Euler’s text raised some historical questions for some of the teachers. In phase 2, all of them noted that the equation  $x^2 = 20$  has two real solutions, while Euler repeatedly wrote “the” root, which raised the question of his knowledge of negative numbers. Since the table of contents mentions negative

numbers, it was assumed that he just did not consider the case of  $-\sqrt{20}$  worth mentioning. None of the teachers planned to explicitly address the historical issue in the classrooms. For teacher T1, Euler's text raised another question. She knew Newton's original text and wondered why Newton used decimal numbers while Euler did not. Digging more deeply into this question would lead to the conclusion that, while the expounded methods are similar, Newton's and Euler's intent differed significantly, since, for Newton, the decimal expansion of the solutions of equations with one unknown was just a preliminary step towards a general method for the power series expansion of algebraic functions. Teacher T1 did not consider this relevant for the classroom.

Oddly enough, teacher T5 is the only one who, when asked to explain her motivation(s) for this session, seriously took into account the fact that the artefact comes from the 18<sup>th</sup> century. She mentioned two motivations: "Indeed, every time we teach square roots students ask: how did people calculate them before, without calculators?" When asked why she decided to use the original source rather than write an exercise based on it (mentioning Newton and Euler in passing) she explained that her exercise would have been "simplified, guided". She valued the fact that students would struggle to make sense of the original source, thereby coming to the conclusion that, in the 18<sup>th</sup> century, only scholars could engage in mathematics—as opposed to the more streamlined and 'accessible' school mathematics of today. This attitude contrasts with that of the other teachers. Not only is T5 the only one who mentioned some 'historical' motivation in the pre-session interview. Also, this 'historical' aspect was not formulated as a question ("what about negatives?", "what about decimal expansions?") but as an oddly worded and makeshift view of 18<sup>th</sup> century mathematics.

## 5. Conclusion

The five teachers' design work was deeply shaped by the two textual resources which set the stage for their professional activity, namely the 2019 curriculum and Euler's text. The interactions between the two resources were manifold. First, the curriculum acted as an incentive for the use of original sources, a fact that played a key role for T5, who had almost no prior experience of this type of work. Second, the curriculum gave very general guidelines—with a focus on problem-solving and meaning-making—and the interviews showed that the teachers designed sessions which they regarded as perfectly in keeping with these guidelines. Third, the curriculum provided constraints and affordances, since, beyond its unusual format, the sessions had to rely on curriculum assigned content-to-be-taught.

These shared constraints and affordances led to seemingly similar choices, which enabled the teachers to keep the general cognitive demand of the sessions under control: studying the text did not serve the purpose of discovering new and deep concepts (derivative, limits of sequences); some mathematical subtleties were hardly mentioned, such as the connection between the irrationality of  $\sqrt{20}$  and the

non-termination of the iterative algorithm. In this context, the text was used as a technological object, to be used for a very specific purpose, namely, to generate a computer program which yields approximations of  $\sqrt{20}$ . Making use of this technological object generated numerous micro-tasks. While some of them do not reflect the fact that it is an 18<sup>th</sup>-century text (e.g., expand brackets), some of them do and rely on “competencies of translations and switching between (...) languages” (Furinghetti et al., 2006, p. 1287).

Beyond these commonalities, a closer look at the mediative and cognitive components of the sessions showed a larger range of practices. From a methodological viewpoint, the description of these practices gave the opportunity to specify the following observables and schemes of use, which are of general relevance: backward- or forward-looking uses, zoom-in and zoom-out uses, scientific and technological functions of the artefact, factors affecting the selection of questions to ask (or not to ask) and ways of asking them.

The fine-grained analysis shows that for all the teachers—except for T5—the sessions were designed in order to trigger an engagement of students at the level of ‘doing mathematics’, to prevent the symmetrical perils of decline into ‘procedures without connection to meaning’ and ‘decline into unsystematic explorations’, and to leave room for the epistemic function of the artefact. In terms of personal and social components, the four teachers who designed these sessions were more experienced and knowledgeable, yet not necessarily in the specific field of history of mathematics: an interest in the didactical (for T4, with a recent master’s degree in didactics) or linguistic (for T3) aspects of mathematics also provided resources. They did not regard the use of this historical source as a means of introducing a historical perspective and highlighting the “historical dimension” of human knowledge (Furinghetti et al., 2007), but as one of the many ways to enjoy what they (in terms of the personal component) consider to be a genuine mathematical experience. For them, other teaching activities can provide a similar experience, for instance, open-ended problems, situations of definition construction, non-trivial modelling activities (for T1, 2, 3, 4), and reading-assessing-rewriting proof-texts (for T3, and, to some extent, for T2).

These findings suggest that the dissemination of non-didacticized resources such as those found in sourcebooks can trigger the design of mathematically rich tasks in ordinary contexts, and that it does not require that teachers be specifically knowledgeable in the history of mathematics. This study also shows that the use of original sources can serve other purposes than changing students’ image of mathematics or imparting historical knowledge. More empirical studies are needed to investigate which factors shape this selection among the many possible goals for using original sources, including nature of the source, curriculum, teachers’ personal and social components.

T5’s narrower resource base led to symmetrical results: in the implemented session, the artefact hardly played any epistemic function; while in her justification(s) for designing this session, she was the only one who mentioned reasons that specifically reflected the fact that the artefact is a mathematical text from two centuries ago. Her case calls for further investigation on the impact of this first

experience of using an original source—then collaborating in a collective book development project—in terms of professional development.

**Disclaimer:** Parts of Section 3 were presented at the CERME 12 conference.

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