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► **To cite this version:**

Olivier Allix, Pierre Ladevèze, Daniel Gilletta, Roger Ohayon. A damage prediction method for composite structures. *International Journal for Numerical Methods in Engineering*, 1989, 27 (2), pp.271-283. 10.1002/nme.1620270205 . hal-04215255

HAL Id: hal-04215255

<https://hal.science/hal-04215255>

Submitted on 22 Sep 2023

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A DAMAGE PREDICTION METHOD FOR COMPOSITE STRUCTURES

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SUMMARY

Damage generally refers to the more or less gradual development of micro-voids and micro-cracks. Damage mechanics is the modelling of these phenomena on a structural analysis scale. In this paper we first recall the non-linear behaviour models we have developed to model composite laminates. Then we present two examples of implementations of such models in a structural analysis code in order to simulate the inner-failure of a structure, or to study delamination initiation.

INTRODUCTION

Damage is maybe the main mechanical phenomenon in composite materials. Therefore it has given rise to many studies.¹⁻³ The classical theory of isotropic damage⁴ is not sufficient to deal with composite materials because for such materials damage is generally of a highly complex nature. For example, fibrous composite laminates are prone to a wide range of damage which may significantly reduce their residual stiffness and strength. In carbon-epoxy laminates such as T300-914 or IM6-914 the main types of damage which can occur are (i) a progressive transverse matrix cracking, (ii) brittle fracture of fibres, (iii) debonding of fibre-matrix interface and (iv) debonding of adjacent layer (delamination). In order to model such phenomena it is possible to describe the microstructure of the composite in detail and to apply calculation techniques such as homogenization.^{5,6} Nevertheless, for composite structures submitted to complex loadings it is not possible to keep the calculation cost of such techniques within reasonable limits.

A Damage Mechanics of Composite Materials is proposed in References 7-10 which enables us to model these phenomena on a structural analysis scale and thus to apply this modelling to simulate the failure of a structure or more generally to estimate the state of its damage compared to one or several ultimate damage states. The aim of this paper is to recall the main features of this modelling, which has been proposed by Ladevèze,^{11,12} and to explain how it is now applied to predict through calculation the state of damage of complex composite structures.

The mechanical behaviour study of any composite laminate is reduced to the modelling of two constituents: the elementary single layer and the interface. The latter is a zero thickness entity which depends on the angle between two adjacent layers. The general idea we use to model the deterioration is that proposed by Kachanov¹³ and Rabotnov:¹⁴ variations in the elastic characteristics of a material are indicators of its deterioration. The opening and shutting effects of

the micro-defects are schematized by splitting up the strain energy into 'tension'-energy and 'compression'-energy. The layer model includes anisotropic unilateral damage and elasto-plastic behaviour.

At the constituents level we get some qualitative information on the nature of the damage. Simple homogenization techniques allow us to transfer this information at the single layer level in order to build comprehensive models. The classical laminate theory enables us to identify the single layer by means of tensile tests on different stacking sequences. The identification is achieved by means of three tensile tests on $[+45, -45]_{2S}$, $[0, 90]_{2S}$, $[45]_8$ specimens. The modelling has been checked out on other stacking sequences and is supported by works that have been carried out in joint collaboration with Aerospatiale.

The interface is a two-dimensional entity which ensures displacements and traction transfer from one layer to another. It depends on the angle between the fibre directions of adjacent layers. The interface influence is located near edges or defects, where a tridimensional stress state may occur and lead to delamination. In the first instance the interface is assumed to be elastic and damageable. Thus delamination, which has been studied in many different manner,¹⁵⁻¹⁷ is here modelled as a damage mode. This modelling will be completed through comparisons between computation and experimental tests; this presents further difficulties. It is to be noted that Damage Mechanics allows us to predict delamination initiation and growth; nevertheless we restrict the delamination analysis to small perturbations and so we do not yet take into account the interaction between buckling and delamination as in References 18-20. Inner-layer effects such as transverse cracking have to be taken into account in delamination analysis since in the first instance it can considerably affect delamination initiation and secondly, it explains the fact that delamination may 'jump' from one interface to another.¹⁵ This approach to delamination is compared to the more classical approach of Fracture Mechanics in the case of elastic layers for an existing delaminated area.

Implementation of this refined mechanical behaviour in a computer program is quite complex, but it allows us to simulate the state of a structure after loading. The computational analysis of laminate failure is split into two parts: (i) inner-layer degradation and (ii) edge effects damage such as delamination.

The first aspect has been studied and numerically set in References 21-23. A Finite Element program using isoparametric finite elements and including a non-linear analysis of laminates has been realized. The failure of laminates is simulated through a global instability condition which means that the structure cannot support any increase in prescribed loadings. The calculation of the corresponding limit points is made by using the Riks algorithm.²⁴ In order to decrease the calculation cost an automatic process of an arc-length-parameter determination has been implemented.

At a structural level, the computation of delamination initiation and growth leads to a non-linear three-dimensional evolution problem. Nevertheless, this analysis can be restricted to the vicinity of edges where the three-dimensional effects are located. We have restricted the analysis to composite laminates with initially circular holes. The computation is kept within reasonable limits by using (i) a new approach for non-linear structural computation, i.e. the 'large time increment method' proposed in References 25 and 26, and (ii) a semi-analytical method which allows us to solve only two-dimensional problems, owing to the particular geometry of the edge.^{27,28} The latter method combines the gradient conjugate method and Fourier expansions using Fast Fourier Transforms. An axially symmetrical pre-conditioned operator is used to solve non-axially symmetrical problems. The large time increment method is totally different from the step-by-step time process of all previous computation techniques. It proceeds by a single global iterative procedure on the whole loading history. This considerably reduces first, the number of transfers

between local and global levels and second, the number of global resolutions and hence the numerical cost of calculation.^{29,30} The initial results are presented below showing the practicability and efficiency of this approach.

LAMINATE MODELLING:

At the level we called meso-level the composite may be schematized by (see Figure 1)

- a homogeneous single layer in thickness
- an interface which is a surface entity connecting two adjacent layers and which depends on the relative directions of their fibres.

These entities being modelled and identified, the mechanical behaviour reconstitution of any laminate is then a relatively easy task. It is to be noticed that we limit ourselves to single layers with only one reinforced direction. The single layer is also analysed at a smaller level, the level of its constituents: fibres, matrix, interfaces. Some more or less qualitative information is thus transferred at the single-layer level by means of a homogenization process. Homogenization techniques have become classical, and therefore we shall not emphasize them.^{5,6} We have noticed and verified that some approximations work remarkably well:

- the local shear stress distributions are uniform
- the local strain distributions in the fibre direction are uniform too.

This leads to very simple homogenization techniques even in the non-linear field. As regards laminate plates and shells, the homogenization consists in writing the basic assumption of C.L.T. This allows us (i) to identify the single layer behaviour from tests on different stacking sequences and (ii) to construct the laminate inner behaviour from our knowledge of the layer behaviour.

A DAMAGE AND PLASTICITY MODEL OF THE SINGLE LAYER^{8-10,12}

The kinematics of the single-layer damage

The identification and modelling of the single layer is made with the assumption of in-plane stresses. In what follows subscripts 1, 2 and 3 designate respectively the fibre direction, the transverse direction inside the layer, and the normal direction to the plate.

Beside brittle fractures in the fibre direction, the matrix and the fibre-matrix interfaces inside the layer are deteriorated in a very particular manner the microcracks are parallel to the fibre direction. A homogenization calculation shows that the only moduli which are modified are the transverse modulus E_2 and the shear modulus G_{12} . The other independent elastic characteristics E_1 and ν_{12} remain constant. These properties are confirmed by experimental observations.

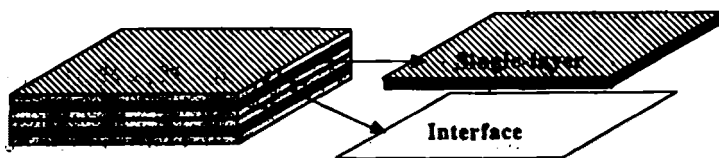


Figure 1. Laminate modelling

The undamaged material strain energy, is written in the following form, reached by splitting up the energy into 'tension'-energy and 'compression'-energy:

$$E_D = \frac{1}{2} \left[\frac{\sigma_{11}^2}{E_1^0} - \left(\frac{\nu_{12}^0}{E_1^0} + \frac{\nu_{21}^0}{E_2^0} \right) \sigma_{11} \sigma_{22} + \frac{\langle \sigma_{22} \rangle_+^2}{E_2^0} + \frac{\langle -\sigma_{22} \rangle_+^2}{E_2^0} + \frac{\sigma_{12}^2}{G_{12}^0} \right] \quad (1)$$

The transverse rigidity in compression being supposed equal to E_2^0 , one obtains the following energy for the damaged material:

$$E_D = \frac{1}{2} \left[\frac{\sigma_{11}^2}{E_1^0} - \left(\frac{\nu_{12}^0}{E_1^0} + \frac{\nu_{21}^0}{E_2^0} \right) \sigma_{11} \sigma_{22} + \frac{\langle -\sigma_{22} \rangle_+^2}{E_2^0} + \frac{\langle \sigma_{22} \rangle_+^2}{(1-d')E_2^0} + \frac{\sigma_{12}^2}{(1-d)G_{12}^0} \right] \quad (2)$$

where d and d' are two scalar damage variables. The variables associated with dissipation are

$$Y_d = \frac{\partial E_D}{\partial d|_\sigma} = \frac{1}{2} \frac{\sigma_{12}^2}{G_{12}^0 (1-d)^2} \quad (3)$$

$$Y_{d'} = \frac{\partial E_D}{\partial d'|_\sigma} = \frac{1}{2} \frac{\langle \sigma_{22} \rangle_+^2}{E_2^0 (1-d')^2}$$

Modelling of the damage evolution

From experimental results it follows that the governing quantities of damage evolution are

$$Y|_t = \sup_{|t' \leq t} [Y_d + b Y_{d'}]^{1/2}; \quad Y'|_t = \sup_{|t' \leq t} [Y_{d'} + b' Y_d]^{1/2} \quad (4)$$

b, b' are characteristic constants of the material which balances the transverse energy influence and the shear energy influence. In fact, experimentally we obtain very simple evolution laws for damage:

$$d = \frac{1}{Y_c} \langle Y - Y_0 \rangle_+ \quad \text{if } d < 1; d = 1 \text{ otherwise} \quad (5)$$

$$d' = \frac{1}{Y'_c} \langle Y' - Y_0 \rangle_+ \quad \text{if } d' < 1; d' = 1 \text{ otherwise} \quad (6)$$

where Y_0, Y_c are characteristic constants of the material. In practice, damage equal to 1 is rarely encountered: the rupture appears far below that level and is associated with instability. Hence Y, Y' appears quantities which govern the damage evolution and then the transverse fracture of the single layer. Near the edges it is necessary to take out-of-plane stresses into account. In order to simplify, Young's modulus E_3 and the shear moduli G_{13} and G_{23} remain constant and thus damage effects of out-of-plane stresses are assumed to affect interface behaviour only.

Plasticity modelling and damage-plasticity coupling

ϵ^p denotes the inelastic part of the strain and the dot designates the derivative with respect to time. We define effective quantities $\tilde{\sigma}$ and $\tilde{\epsilon}^p$ as

$$\tilde{\sigma}_{12} = \frac{\sigma_{12}}{(1-d)}, \quad \tilde{\sigma}_{22} = \frac{\sigma_{22}}{(1-d)}, \quad \dot{\tilde{\epsilon}}_{12}^p = \dot{\epsilon}_{12}^p (1-d), \quad \dot{\tilde{\epsilon}}_{22}^p = \dot{\epsilon}_{22}^p (1-d') \quad (7)$$

The plasticity modelling and damage-plasticity coupling are build upon these quantities because experimentally the hardening curves which appear to be very different in 'tension' and in 'compression' are, in fact, the same if expressed in effective quantities. From a homogenization

calculation, and assuming that only the matrix plastifies we have

$$\varepsilon_{11}^p = 0 \quad (8)$$

In order to make the incremental problem symmetrical we use the following description of the elasticity domain:

$$f = \left\langle \int_0^T 2 \left[\bar{\sigma}_{12} \frac{\dot{\bar{\sigma}}_{12}}{(1-d)} + a^2 \bar{\sigma}_{22} \frac{\dot{\bar{\sigma}}_{22}}{(1-d')} \right] dt \right\rangle_+^{1/2} - R - R_0 \leq 0 \quad (9)$$

The hardening is assumed to be isotropic, which means that the threshold R is a function of the cumulated strain p ; $p \rightarrow R(p)$ is a characteristic function of the material. The yield conditions are written as follows:

$$\dot{\varepsilon}_{12}^p = \frac{1}{2} \dot{p} \frac{\bar{\sigma}_{12}}{R + R_0}, \quad \dot{\varepsilon}_{22}^p = a^2 \dot{p} \frac{\bar{\sigma}_{22}}{R + R_0} \quad (\dot{p} \geq 0) \quad (10)$$

Identification and checking

The model described above, apart from the elastic characteristics, depends on:

- the rupture tension and compression strains of the fibre: ε_T and ε_c
- the hardening curve $p \rightarrow R(p)$, the constant a^2 . Assuming an isotropic matrix and a perfect fibre-matrix bonding, one is led to attribute to a the value of 0.33, which is in good agreement with experimental results
- the Y_0 , Y_c , Y'_c constants which define the damage evolution.

This model had been identified on T300-914 laminates. The hardening curve and the b , b' , Y_0 , Y_c , Y'_c constants have been obtained from a tensile test on a $[+45, -45]_{2S}$ specimen and a $[+67, 5; 67, 5]_{2S}$ specimen. For single layers constituted with the same matrix but with IM6 fibres, the previous characteristics have also been identified. For laminates constituted with the same matrix, it appears that the plasticity and the damage curves are very close. These results confirm the idea that the matrix and the fibre-matrix bond play a prominent part in this phenomenon. This modelling has been checked out on numerous experimental tests.

INTERFACE MODELLING

Interface definition

The interface (see Figure 2) is a surface entity (of zero thickness) which ensures stress and displacement transfers from one ply to another. In elasticity the interface is schematized by means

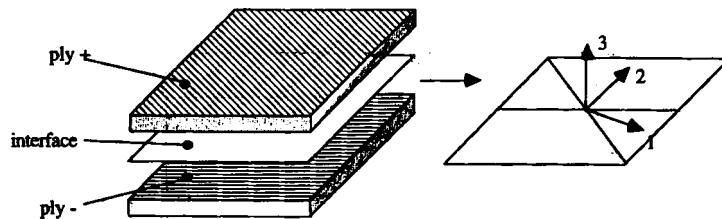


Figure 2. Interface modelling

of a model which has been used in order to model the fibre–matrix interface.³¹ It depends on the relative orientation of the upper and lower plies. The displacement discontinuities are denoted by

$$[[U]] = U^+ - U^- = [[U]]_1 \mathbf{N}_1 + [[U]]_2 \mathbf{N}_2 + [[W]] \mathbf{N}_3 \quad (11)$$

The $(\mathbf{N}_1, \mathbf{N}_2)$ axes are associated with the bisectrix of directions the fibres. The undamaged energy of the interface is

$$E_D = \frac{1}{2} [k^0 [[W]]^2 + k_1^0 [[U_1]]^2 + k_2^0 [[U_2]]^2] = \frac{1}{2} \left[\frac{\sigma_{33}^2}{k^0} + \frac{\sigma_{33}^2}{k_2^0} + \frac{\sigma_{31}^2}{k_1^0} \right] \quad (12)$$

where k^0, k_1^0, k_2^0 are the initial elastic characteristics.

Kinematics and evolution of the damages

With a restriction to one scalar damage variable, the deterioration can be described by

$$E_D = \frac{1}{2} \left[\frac{\langle -\sigma_{33} \rangle_+^2}{k^0} + \frac{\langle \sigma_{33} \rangle_+^2}{k^0(1-d)} + \frac{\sigma_{32}^2}{k_2^0(1-\gamma_2 d)} + \frac{\sigma_{31}^2}{k_1^0(1-\gamma_1 d)} \right] \quad (13)$$

where γ_1 and γ_2 are constants. The variable associated with dissipation is

$$Y_d = \frac{\partial E_D}{\partial d} = \frac{1}{2} \left[\frac{\langle \tilde{\sigma}_{33} \rangle_+^2}{k^0} + \gamma_2 \frac{\tilde{\sigma}_{32}^2}{k_2^0} + \gamma_1 \frac{\tilde{\sigma}_{31}^2}{k_1^0} \right] \quad (14)$$

where $\tilde{\sigma}_{i3}$ denotes the effective normal stress. The γ_1 and γ_2 constants introduce a coupling between the energy connected with the normal tension stress and the shear energy. In terms of delamination modes, the first term is associated with the first opening mode, the two others with the second and third modes. The damage evolution is described by the following standard model:

$$d = w(Y) \quad \text{if } d < 1; \quad d = 1 \text{ otherwise} \quad (15)$$

Connections between two approaches of delamination: Damage Mechanics and Fracture Mechanics

The comparison has been carried out on a D.C.B. specimen constituted with two elastic layers loaded with a pure-mode-I loading (see Figure 3). We designate respectively by e and b the

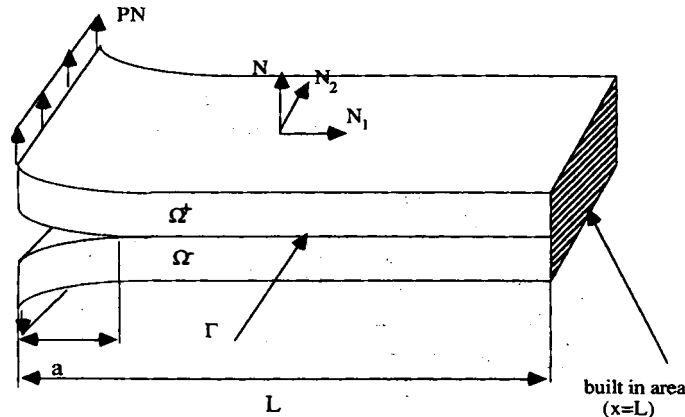


Figure 3. Schematization of the specimen and of the loading mode

thickness and the width of the specimen. The layers are connected with an interface Γ , whose elastic damageable behaviour is characterized by k_0 and Y_c ; d denotes the damage variable. The specimen has initially been separated along the interface. We denote by 'a' the delaminated area length. The specimen is loaded in the N_3 direction with uniform tensile forces whose resultants are denoted by P and $-P$; moreover, the specimen is clamped at $x = L$. The computation of the energy release rate could be achieved by different computation methods.³² In order to derive analytical results, the specimen calculation is achieved using Reissner's plate theory, under the assumption of a plane strain state in the (N_1, N_3) plane. The admissible displacement fields are restricted to

$$U_s = \{ (U, v, w) \text{ regular in } \Omega_+ \text{ and } \Omega_- \text{ respecting the symmetry} \\ \text{and the displacement conditions} \} \quad (16)$$

In order to simplify, we assume that, before instability, the load is increasing monotonically. So the problem is to find (σ, U) satisfying

— displacement conditions $U \in U_s$

$$\int_{\Omega} \text{Tr}[\sigma \cdot \varepsilon(U^*)] d\Omega + \int_{\Gamma} \sigma_{33}[[w^*]] d\Gamma = 2Pw^*(0) \quad (17)$$

$$\sigma = K_c \varepsilon(U) \text{ in } \Omega, \text{ and: } \sigma_{33} = k_0(1-d)[[w]] \text{ on } \Gamma (w(Y) = Y/Y_c) \quad (18)$$

If the crack is long enough, it is possible to obtain, for the energy release rate, the following approximate expression:

$$G = \left. \frac{\partial(2w(0))}{\partial a} \right|_p \approx \frac{4P^2}{E_{11}be^3} \left(3a^2 + \frac{3E_{11}e^2}{10G_{12}} \right) \quad (19)$$

The delamination propagation condition is obtained by means of an instability condition, associated with the incremental variational problem. In this particular case, the instability is reached for $d = 1$ on the delamination front. Denoting by P_c the value of P at the instability we get

$$Y_c \approx \frac{2P_c^2}{E_{11}be^3} \left(3a^2 + \frac{3E_{11}e^2}{10G_{12}} \right) \text{ and } G_c \approx 2Y_c \quad (20)$$

For an established delamination, the critical energy release rate G_c is independent of the crack length. Fracture mechanics appear as a simplified tool for the delamination study in the case of an established front and elastic layers. Moreover, the previous result enables us to identify behaviours of different materials from experimental results obtained by Laksimi.^{33, 34} Such an approach is made with the assumption that layers are elastic; thus it can constitute a first approach only.

INNER-FRACTURE ANALYSIS OF LAMINATE STRUCTURES²¹⁻²³

According to experiment the multilayered coupon's fracture is well defined by the first ply failure. However, the use of internal damage variables enable us to translate the instability calculation from the layer to more global domains. In a first step, instability has been calculated on the whole stacking sequence of each element. The following examples (Figure 4) show that this approach often leads to very close fracture-load levels compared to those determined by first ply failure.

In a next step, the limit point of the structure's equilibrium path is determined. On the example which corresponds to Figure 5 this point and first ply failure occurring in one of the elements of the discrete structure are reached for nearly the same load level.

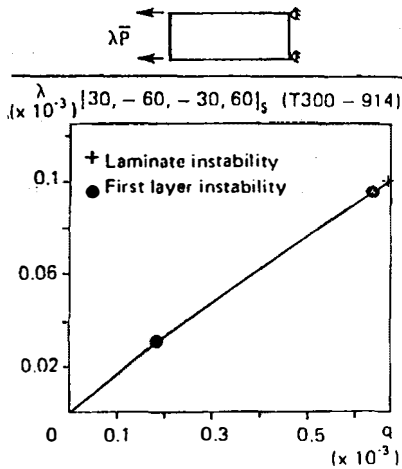


Figure 4. Laminate and first ply instabilities

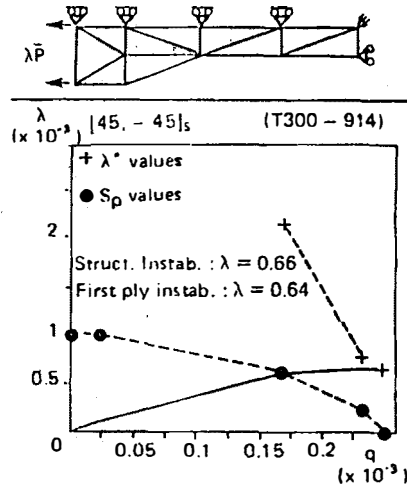


Figure 5. Structure and first ply instabilities

The point of the equilibrium path is calculated for given value of the arc length parameter Δs according to Reference 24. The values of the arc-length parameter Δs , which fix the different load increments, may be calculated automatically through extrapolation methods.³⁵ The aim of such methods is to reach the limit as fast as possible. The current stiffness parameter S_p is the quantity on which the extrapolation is performed. In the elastic range S_p is equal to one and near the limit point it is equal to zero. By a linearization method this last condition allows us to predict a value Δs^* for which S_p is supposed to be zero. In order to ensure the determination of the limit point, a safety factor β is introduced such that the value of Δs which is used is

$$\Delta s = \beta \Delta s^* \quad (21)$$

DAMAGE COMPUTATION FOR DELAMINATION ANALYSIS^{27,28}

Implementation of the above mechanical behaviour in a computer program to study the initiation and propagation of delamination leads us to solve a three-dimensional (because of edge effects) non-linear evolution problem. To study edge effects in elasticity it is possible to use mixed techniques to ensure traction continuities from one ply to another as in Reference 36, or to use boundary layer techniques³⁷ in the case of a quasi-straight edge. Owing to the interface behaviour the traction continuities are ensured by means of a displacements approach. The study of the full non-linear three-dimensional evolution problem in the case of an initially circular hole has been achieved in References 27 and 28 and is presented below:

To solve with a reasonable computational cost this non-linear three-dimensional evolution problem we use, (i) the 'Large Time Increment Method' and (ii) a semi-analytical method which requires the solution of two-dimensional problems only.

Formulations of the mechanical problem

The analysis is restricted to the vicinity of the edges in the domain Ω , where the three-dimensional effects are significant. A link with the solution obtained by a 'shell' computation is

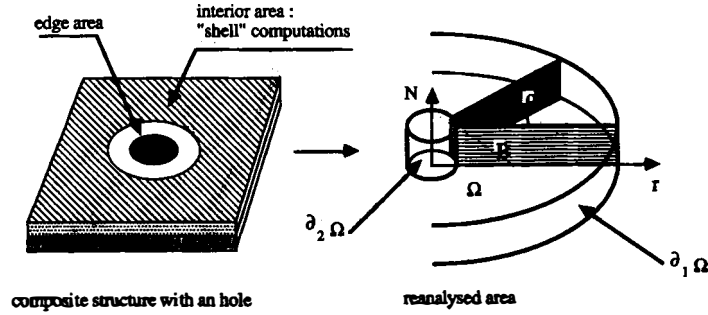


Figure 6

made on the $\partial_1\Omega$ area through given displacement U_d . On the complementary part of the boundary of Ω either displacements or forces may be prescribed. We assume, for example, that a load F_d is prescribed on $\partial_2\Omega$ (Figure 6).

The problem to be solved is: Find $(\sigma, \varepsilon(U))$ such that: U kinematically admissible (K.A.), which means:

$$U \text{ regular and } U|_{\partial_1\Omega} = U_d(t) \quad \forall t \in [0, T]$$

σ statically admissible (S.A.), which means that σ is in equilibrium with the given data:

$$\forall U^* \in U = \{U/U \text{ regular and } U|_{\partial_1\Omega} = 0\}, \forall t \in [0, T]$$

$$\int_{\Omega} \text{Tr}(\sigma \varepsilon(U^*)) d\Omega + \sum_{i=1}^{n-1} \int_{\Gamma} \sigma N[U^*] d\Gamma = \int_{\partial_2\Omega} F_d(t) U^* dS$$

and (σ, ε) satisfies the constitutive relations $\forall t \in [0, T]$ (22)

In these equations data as well as unknowns are functions of time and space. In order to be clearly distinguished such function are called processes. We denote by \mathbb{A}_d the (linear) subspace of couples of processes $s = (\sigma, \varepsilon)$ such that U is K.A. and σ is S.A. We denote by \mathbb{F} the (non-linear) subspace of couples of processes $s = (\sigma, \varepsilon)$ such that (σ, ε) satisfies the constitutive relations $\forall t \in [0, T]$. So the problem may then be written:

$$\text{find } s \text{ such as: } s = \mathbb{A}_d \cap \mathbb{F} \quad (23)$$

Scheme of the large time increment method^{25,26,29,30}

The 'large time increment method' breaks with the step by step process of all previous computation techniques. This method allows the treatment of the whole loading using only one increment. The intersection of \mathbb{A}_d and \mathbb{F} is obtained by an iterative procedure on the whole loading history by means of two stages. From a couple of processes $s_n \in \mathbb{A}_d$ we search a couple ξ which belongs to \mathbb{F} , this stage being non-linear but local. From ξ we build a couple $s_{n+1} \in \mathbb{A}_d$ 'better' than s_n ; this stage is global but linear.

An important point is to define how to build the couples ξ and s_{n+1} from the couple s_n . For this two search directions E^+ and E^- are defined (Figure 7).

There are many different choices which are possible for the direction E^+ and E^- . Because of the damage constitutive law there may be local and global instability. We use the simplest version

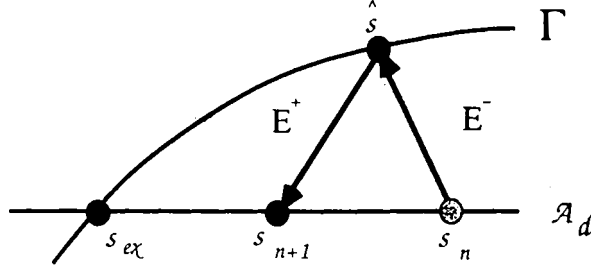


Figure 7. Schematic representation of the two steps

which is also the safer; we denote $\Delta \hat{\epsilon}_n = \hat{\epsilon}_{n+1} - \hat{\epsilon}_n$ and $\Delta \hat{\sigma}_n = \hat{\sigma}_{n+1} - \hat{\sigma}_n$.

E^- (local stage): $\hat{\epsilon} = \hat{\epsilon}_n$

E^+ (global stage): $\Delta \hat{\sigma}_n = K_\epsilon \Delta \hat{\epsilon}_n + \hat{\sigma} - \hat{\sigma}_n$ (K_ϵ is the elastic operator) (24)

The first step is a local but non-linear problem. This leads to solve a non-linear problem at each Gauss point with the Finite Element Method.

This last problem is an elastic problem where the time is a parameter. The stresses and strains corrections are searches of the following form:

$$\Delta \hat{\epsilon}_n = \sum_{i=1}^m \alpha_i(t) A_i(M) \quad (A_i \text{ K.A. to zero}) \quad (25)$$

$$\Delta \hat{\sigma}_n = \sum_{j=1}^n \beta_j(t) B_j(M) \quad (B_j \text{ S.A. to zero}) \quad (26)$$

In other words, the corrections are searched as superposition of proportional loadings. Such loadings are known to be quite good approximations of non-linear small perturbation problems. This reduces considerably the number of global resolutions. With a kinematics approach the global problem leads us to find $\Delta \hat{\epsilon}_n$ K.A. to zero such that

$$\forall \epsilon^* \text{ K.A. to zero: } \int_0^T \int_{\Omega} \epsilon^* [K_\epsilon(\Delta \hat{\epsilon}_n) + (\hat{\sigma} - \hat{\sigma}_n)] d\Omega dt = 0 \quad (27)$$

In order to simplify, we suppose that the number m of time functions in the approximation (33) is equal to 1. The problem (35) is resolved iteratively in time and space. This process is initiated by the choice of a time function $\alpha_0(t)$ (which is normalized); we then search again the best space function $A_0(M)$ associated with $\alpha_0(t)$ that is:

$$\forall A^* \text{ K.A. to zero: } \int_{\Omega} A^* K_\epsilon(A_0) d\Omega = \int_{\Omega} A^* \int_0^T \alpha_0(t) (\hat{\sigma} - \hat{\sigma}_n) dt d\Omega \quad (28)$$

This problem is a classical elastic problem and leads to a solution $A_0(M)$. We then search the best time function $\alpha_1(t)$ associated with $A_0(M)$; this lead to a very small problem. This process is stopped when the time functions $\alpha_i(t)$ and $\alpha_{i+1}(t)$ are sufficiently close together. In practice this process need two or three iterations. So the determination of the correction $\Delta \hat{\epsilon}_n$ on $[0, T]$ needs only two or three elastic resolutions. Since the determination of the solution needed a number of global stages which do not exceed five or six, the resolution of a non-linear problem on the whole loading often needed only about ten resolutions of classical elastic problems. Nevertheless, the problem we have to solve is three-dimensional. So, in order to decrease its numerical cost, we

present a semi-analytical method which allows the treatment of this three-dimensional problem by means of two-dimensional problems in a strip, which is made by using (i) the conjugate gradient method and (ii) the Fast Fourier Transform.

Scheme of the semi-analytical method

An axially symmetric conditioned operator is used to solve non-axially symmetric problems, so the elastic problem to be solved is replaced by a series of intermediate problems:

$$K_0 X = F \text{ where } K_0 = \frac{1}{2\pi} \int_0^{2\pi} K_\epsilon d\theta \quad (K_\epsilon: \text{elasticity operator}) \quad (29)$$

By means of the Fast Fourier Transform the displacement is developed in Fourier series, and each of its components is computed as the solution of two-dimensional elastic problems associated with K_0 which are solved in a finite strip. The displacement is then optimized by the conjugate gradient method.

An example of a structure $(0, 45, -45, 90)_S$ which is loaded by a radial pressure on the hole is presented. These results were obtained with the above semi-analytical method. This computation was made in 64 strips which represented about 100 000 degrees of freedom for the whole problem. It needed only 25 iterations. Figure 8 shows the displacements in strip B at 0° . Figure 9 shows the peeling stress in this strip at the $(90^\circ, 90^\circ)$ interface.

A first non-linear example computation of a laminate $(0, 90)$ loaded in pure mode I is given. A normal displacement is prescribed on the edge of the hole (radius r_0) in the following form: $U(r_0, t) = \lambda(t)U_0$, where $\lambda(t) = t/T$. Figure 10 shows the work of the outside forces divided by λ , with respect to λ . Through a global instability condition, this curve allows us to predict the delamination initiation and the Gauss point at which it happens. Figure 11 shows the evolution of the peeling stress at this Gauss point.

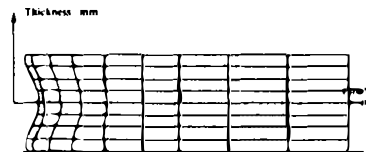


Figure 8

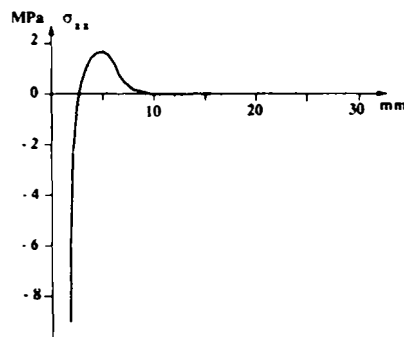


Figure 9

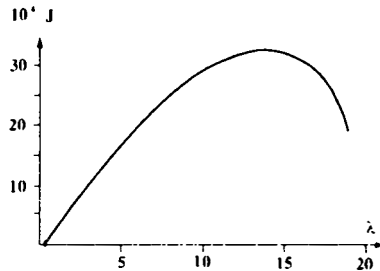


Figure 10

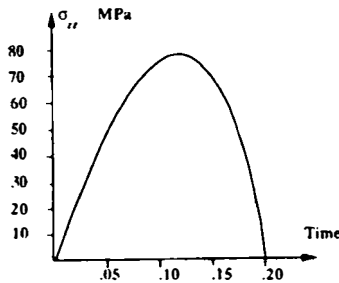


Figure 11

CONCLUSION

The mechanical behaviour study of any composite laminate is reduced to the modelling of two constituents: the elementary single layer and the interface. Damage Mechanics of Composite Materials allows us to model the elementary constituents behaviour whose main features are damage and plasticity. Thanks to this modelling it is easy to compare the behaviour of several carbon-epoxy laminates. This kind of approach can be extended to other type of materials, and has been successfully applied to 3D carbon-carbon composites.³⁸

From the different fracture modes that have been introduced to model the layer, a multicriterion approach, similar the one proposed by Hashin,³⁹ can be deduced for laminates. It consists of a brittle elastic criterion for fibres and a rupture criterion for the matrix. The latter is obtained through an instability condition caused by the damage behaviour of the matrix. This approach is global throughout the thickness of the laminate and thus is implicit. The general application of the single layer modelling is proposed to predict the inner fracture of a laminate by means of non-linear two-dimensional computation.²¹⁻²³

The interface modelling allows us to predict the initial stage of delamination due to the damage behaviour of connections between layers.^{27,28} This leads to a non-linear tridimensional evolution problem whose treatment is dealt with in this paper. Within this frame-work it is also possible to study the delamination propagation.

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