



Degree of truth and truth-functionality

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Summary

In this paper, the 'degree of truth' of a proposition is viewed as the result of a semantical matching procedure between a representation of the reality and a representation of what is said in the proposition. The considered matching procedures make use of possibility and/or necessity measures. Depending on the matching procedure which is chosen to define the degree of truth of a proposition, truth-functionality of the induced connectives may or may not be preserved. Thus, starting from a semantical interpretation of the degree of truth, a multivalent logic may be built. This approach strongly departs from studies of many-valued logics viewed as many-valued algebraic calculi without semantical rationale. Moreover, new relationships between multivalent logic and fuzzy set and possibility theory are investigated.

Introduction

Many-valued logics (perhaps, with the exception of trivalent logics) have been mainly studied from an algebraic point of view without usually taking care of their semantical interpretation. As emphasized by Rescher¹³, a many-valued logic without some semantical rationale for its truth-values, may just remain a many-valued calculus.

In the following, starting from a semantical interpretation, the 'degree of truth' of a proposition is viewed as the result of a matching procedure. The matching procedures which are considered make use of possibility measures (Zadeh¹⁷) and/or of necessity measures. Then, the definition of the degree of truth of a proposition naturally enables to determine the connective functions attached to the basic connectives. Depending on the chosen definition, truth-functionality may or may not be preserved: it is the topic of the second main part of this paper. Thus, starting from a semantical interpretation of the degree of truth, a multivalent logic may be built.

This paper shows new relationships between multivalent logics on the one hand and fuzzy set and possibility theory on the other hand. Some relationships between the two fields have been recently discussed at length by many authors, for instance Zadeh¹⁹, Skala¹⁵, Trillas¹⁶, Dubois and Prade^{4,5}, Baldwin¹.

Degree of truth

The truth may be defined as the conformity of what is said with actual fact. More precisely, the concept of truth may be viewed as a measure of conformity between a representation of the contents of a proposition and a representation of what is actually known of the reality.

A proposition concerns either concrete physical objects or less tangible abstractions and asserts some of their individual properties and/or of their interrelationships. A representation of the contents of a propo-

sition is any symbolic expression of the 'deep structure' and of the meaning of the proposition, which is convenient for some intended manipulations. PRUF (Zadeh¹⁸) is an example of representation language for the contents of propositions. How to extract the representation of a sentence automatically is a key-problem in Artificial Intelligence and is not the topic of this paper. What is actually known of the reality is supposed to be represented in a data base \mathcal{D} (i.e. the collection of existing items and of the known values of their attributes and of their interrelations). Thus, the truth of a proposition will be relative to our state of knowledge.

Given a representation of a proposition we need a matching procedure to evaluate its conformity with the current state of data base \mathcal{D} . Conformity may be partial and may be a matter of degree. Let us consider some examples to illustrate the problem.

1. Suppose we know that "John's height is 1.70 meter". Given this information (which is recorded in \mathcal{D}), the proposition P_1 : "John's height is 1.55 meter" is clearly false while proposition P_2 : "John's height is between 1.60 and 1.80 meter" is true. There is no conformity between 1.55 and 1.70, while the real interval $[1.60, 1.80]$ is in agreement with 1.70 in the sense that this latter value is included in the interval mentioned in P_2 (in other words, \mathcal{D} entails P_2). Note that conformity does not necessarily mean identity and that the roles played by the proposition and by \mathcal{D} (i.e. what is known) are not symmetrical.

2. Now, we are supposed to just know that "John's height is between 1.60 and 1.80 meter". It is what is recorded in \mathcal{D} . In other words, we are only aware of a possibility distribution (the characteristic function of the interval $[1.60, 1.80]$) concerning the value of John's height: every value in $[1.60, 1.80]$ is a possible value for John's height while every value outside this interval is not a possible value. Then, given \mathcal{D} , proposition P_1 : "John's height is 1.55 meter" is still false, Propositions P_3 : "John's height is 1.70 meter" and P_4 : "John's height is between 1.75 and 1.85 meter" are possibly true (in the sense that 1.70 or $[1.75, 1.85]$ are compatible with \mathcal{D}); however, P_3 or P_4 are also possibly false; then, P_3 or P_4 are, neither necessarily (or certainly) true nor necessarily false. Proposition P_4 "John's height is greater than 1.60 meter" is (both possibly and) necessarily true because $[1.60, 1.80]$ is included in $[1.60, \infty)$; thus P_5 is true with respect to \mathcal{D} .

Let us denote the possibility and the necessity (certainty) that proposition P is true in the context \mathcal{D} respectively by $\text{Pos}(P | \mathcal{D})$ and $\text{Nes}(P | \mathcal{D})$. $\text{Pos}(P | \mathcal{D})$ ($\text{Nes}(P | \mathcal{D})$ resp.) will be equal to 1 when it would be possible (necessary resp.) that P is true and to 0 when it would not be. With this convention,

$$\begin{aligned} \text{Pos}(P_1 | \mathcal{D}) &= \text{Nes}(P_1 | \mathcal{D}) = 0; \\ \text{Pos}(P_3 | \mathcal{D}) &= 1; \end{aligned}$$

$$\begin{aligned} \text{Nes}(P3 | \mathcal{D}) &= 0; \\ \text{Pos}(P5 | \mathcal{D}) &= \text{Nes}(P5 | \mathcal{D}) = 1. \end{aligned}$$

Assuming a uniform probability density, the probability of P4 true knowing \mathcal{D} would be $\text{Prob}(P4 | \mathcal{D}) = 0.25$, while $\text{Prob}(P5 | \mathcal{D}) = 1$.

3. Again, suppose we have the information "John's height is 1.70 meter" in \mathcal{D} , with the proposition P6: "John is tall". P6 is translated into a possibility distribution (whose domain is the universe of heights) which is nothing but the membership function μ_T of the fuzzy set labelled by 'tall' (Zadeh^{17 18}).

The possibility that P6 is true will be taken as equal to the possibility that the value of a variable restricted by the specification "tall" is 1.70 (i.e. the membership degree of 1.70 to the fuzzy set "tall"). Then, $\text{Pos}(P6 | \mathcal{D}) = \mu_T(1.70) \in [0, 1]$; the fuzziness of the proposition leads to a degree of possibility intermediary between 0 and 1. Associating 'not tall' with the membership function $1 - \mu_T$ and given that the necessity of an event corresponds to the impossibility of the opposite event, we have $\text{Nes}(P6 | \mathcal{D}) = \mu_T(1.70) = \text{Pos}(P6 | \mathcal{D})$ (provided that the reverse function $x \rightarrow 1 - x$ is also used to define the degree of impossibility from the degree of possibility). Then, $\mu_T(1.70)$ may be regarded as the degree of truth of the proposition P6.

4. Conversely, suppose we have only the information "John is tall" represented in \mathcal{D} and we consider the proposition P3: "John's height is 1.70 meter". The compatibility of P3 with \mathcal{D} is $\text{Pos}(P3 | \mathcal{D}) = \mu_T(1.70)$ because 1.70 is a possible value for a variable restricted by 'tall' with a degree of possibility equal to $\mu_T(1.70)$. However, other values than 1.70 are more or less possible for John's height; then $\text{Nes}(P3 | \mathcal{D}) = 0$ in this situation.

5. Let us consider the Aristotelian example: the proposition P7: "There will be a sea battle to-morrow". Clearly, the possibility distribution domain reduces here to two elements corresponding to the eventualities of a sea battle or of no sea battle. We have $\text{Pos}(P7 | \mathcal{D}) = 1$ and $\text{Nes}(P7 | \mathcal{D}) = 0$ where \mathcal{D} is our to-day state of knowledge. Future contingents appear to correspond to a situation similar to this of the example 2.

6. Let us consider the following example taken from Hodges⁸: "After the office party, a man admits to his wife (P8) I did kiss some of the girls (\mathcal{D}) in fact he kissed all nineteen of them".

What is the truth status of P8? Although 'some' is a quite fuzzy quantifier, it is not really possible that 'some' and 'all' denote the same quantity. Note that P8 does not mean "I only kissed some of the girls" which entails that there would exist one or several girls not kissed by the husband. Only, P8 may appear partial, incomplete with respect to \mathcal{D} ; but, if the truth status is based on a conformity with \mathcal{D} , certainly P8 is true (because the contents of \mathcal{D} entails P8). If the husband appears to be somewhat a liar, it is because he does not confess the whole fact. Thus, truth appears to be the conformity between what is said and what should be said with respect to \mathcal{D} but also with respect to other situational considerations. For instance if a man whose major belongings are a car and a house, says P9: "I own a car", depending on circumstances it may be regarded as completely true or only as partially true (if for instance P9 is the answer to the question "what are all your major belongings?").

7. Again an example from Hodges⁸: "After another office party, another man boasts to his wife (P10) All girls kissed me

(\mathcal{D}) in fact there were no girls at the party".

Here, the matching procedure between P10 and \mathcal{D} fails not because the values of the asserted properties or relations are different from the actual ones but because of a more radical reason: some of the entities involved in the proposition do not exist in \mathcal{D} . Thus, P10 presupposes the presence of at least one girl (even perhaps two) at the party.

Basically, a proposition is true (with respect to our state of knowledge), if and only if we are certain that what is asserted in the proposition is in conformity with reality. However, the matching procedure between a proposition P and the reality (\mathcal{D}) may yield a result different from true or false in several situation as shown in the above examples:

i) When our perception of reality is imperfect (imprecise, fuzzy or even indeterminate: e.g. future contingents);

ii) When what is asserted is fuzzy, then quite naturally degrees of truth are introduced;

iii) When the proposition presupposes the existence of entities which do not exist.

Whatever the situation, it seems interesting to evaluate the possibility that the proposition P is true (i.e. $\neg P$ is false) and the possibility that $\neg P$ is true (which by complementation to 1 yields the necessity that P is true). By convention, in situation iii) corresponding to example 7, we shall take $\text{Pos}(P | \mathcal{D}) = \text{Pos}(\neg P | \mathcal{D}) = 0$. Possibility and necessity have been introduced in a very intuitive manner in the above examples. Their precise definitions are

$$\text{Pos}(P | \mathcal{D}) = \sup_{u \in U} \min(\mu_P(u), \mu_{\mathcal{D}}(u)) \quad (1)$$

$$\text{Nes}(P | \mathcal{D}) = \inf_{u \in U} \max(\mu_P(u), 1 - \mu_{\mathcal{D}}(u)) \quad (2)$$

where μ_P and $\mu_{\mathcal{D}}$ are respectively the possibility distributions attached to the proposition P and the data base \mathcal{D} on the universe of discourse U. It is easy to check that the above formulae give back the values corresponding to the above examples. A rationale for these formulae may be found in Prade¹¹ for the semantics (possibility and necessity) and for their use as comparison indices in Dubois and Prade⁶. They have been used in pattern matching procedures in Artificial Intelligence (see Cayrol, Farreny and Prade^{2, 3}, where approximate conformity is taken into account by introducing a fuzzy relation modeling approximate equality in (1) and (2)). Basically, $\text{Pos}(P | \mathcal{D})$ is an intersection index (is there something in common in P and in \mathcal{D} ?) and $\text{Nes}(P | \mathcal{D})$ an inclusion index (does \mathcal{D} entail P?). The following properties are easy to check and are in agreement with the semantics.

$$\text{Nes}(P | \mathcal{D}) = 1 - \text{Pos}(\neg P | \mathcal{D}) \quad (3)$$

$$0 \leq \text{Nes}(P | \mathcal{D}) \leq \text{Pos}(P | \mathcal{D}) \leq 1 \quad (4)$$

(4) holds whenever $\sup_{u \in U} \mu_{\mathcal{D}}(u) = 1$ (which means that there exists at least one value which is completely possible among the more or less possible values corresponding to reality).

$$\text{Pos}(P | \mathcal{D}) = \text{Pos}(\mathcal{D} | P) \quad (5)$$

but generally,

$$\text{Nes}(P | \mathcal{D}) \neq \text{Nes}(\mathcal{D} | P) \quad (6)$$

Let us consider now the following quantity

$$\partial(P|\mathcal{D}) = \frac{\text{Pos}(P|\mathcal{D}) + \text{Nes}(P|\mathcal{D})}{2} \quad (7)$$

A noticeable property of $\partial(P|\mathcal{D})$ is

$$\partial(\neg P|\mathcal{D}) = 1 - \partial(P|\mathcal{D}) \quad (8)$$

$\partial(P|\mathcal{D})$ may be viewed as a degree of truth of P. Indeed $\partial(P|\mathcal{D}) = 1$ (completely true) as soon as $\text{Nes}(P|\mathcal{D}) = 0$; $\partial(P|\mathcal{D}) = 0$ (completely false) as soon as $\text{Pos}(P|\mathcal{D}) = 0$. When $\mu_{\mathcal{D}}$ is the membership function of a singleton (i.e. the reality is precisely known): $\text{Pos}(P|\mathcal{D}) = \text{Nes}(P|\mathcal{D}) = \partial(P|\mathcal{D})$; then $\partial(P|\mathcal{D}) \in (0,1)$ only if P is fuzzy. When $\text{Pos}(P|\mathcal{D}) = 1$ and $\text{Nes}(P|\mathcal{D}) = 0$ (see examples 2 and 5), $\partial(P|\mathcal{D}) = \frac{1}{2}$ which is the truth-value traditionally attached to the "possible" (truth-indeterminacy) in some three-valued logics (Lukasiewicz⁹, see Rescher¹³). In spite of (8), $\partial(\cdot|\mathcal{D})$ is not a kind of probability measure as it will be shown in the next section.

Truth-functionality

Truth-functionality of propositional connectives is a very desirable property for a many-valued logic. Let $d(P|\mathcal{D})$ denote the degree of truth of P with respect to \mathcal{D} , yielded by some matching procedure; $d(P|\mathcal{D})$ is assumed to belong to $[0,1]$; $\partial(P|\mathcal{D})$ defined by formula (7) is an example of such a degree of truth. Truth-functionality means for the three basic connectives, negation (\neg), conjunction (\wedge), disjunction (\vee):

$$\exists f, d(\neg P|\mathcal{D}) = f[d(P|\mathcal{D})] \quad (9)$$

$$\exists g, d(P \wedge Q|\mathcal{D}) = g[d(P|\mathcal{D}), d(Q|\mathcal{D})] \quad (10)$$

$$\exists h, d(P \vee Q|\mathcal{D}) = h[d(P|\mathcal{D}), d(Q|\mathcal{D})] \quad (11)$$

where f is a mapping from $[0,1]$ to $[0,1]$, g and h are mappings from $[0,1]^2$ to $[0,1]$. Moreover, f, g and h must fulfil some natural requirements such as: coincidence with usual connective functions on $\{0,1\}$, symmetry and associativity for g and h, non-decreasingness for g and h, involution and decreasingness for f.

If we choose the possibility measure $\text{Pos}(P|\mathcal{D})$ or the necessity measure $\text{Nes}(P|\mathcal{D})$ (introduced in the precedent section) for $d(P|\mathcal{D})$, (10) and (11) are satisfied with $g = \min$ and $h = \max$ because of the following results (Prade^{11,12})

$$\text{Pos}(P + Q|\mathcal{D}) = \max(\text{Pos}(P|\mathcal{D}), \text{Pos}(Q|\mathcal{D})) \quad (12)$$

$$\text{Pos}(P \times Q|\mathcal{D}) = \min(\text{Pos}(P|\mathcal{D}), \text{Pos}(Q|\mathcal{D})) \quad (13)$$

$$\text{Nes}(P + Q|\mathcal{D}) = \max(\text{Nes}(P|\mathcal{D}), \text{Nes}(Q|\mathcal{D})) \quad (14)$$

$$\text{Nes}(P \times Q|\mathcal{D}) = \min(\text{Nes}(P|\mathcal{D}), \text{Nes}(Q|\mathcal{D})) \quad (15)$$

where

$$\forall u \in U, \forall v \in V \quad \mu_{P+Q}(u,v) = \max(\mu_P(u), \mu_Q(v)) \quad (16)$$

$$\forall u \in U, \forall v \in V \quad \mu_{P \times Q}(u,v) = \min(\mu_P(u), \mu_Q(v)) \quad (17)$$

In the above equations we are using the same notation to denote a proposition and the fuzzy set whose membership function is the possibility distribution (Zadeh^{17,18}) attached to the representation of the proposition.

Thus, $P \times Q$ is a Cartesian product of fuzzy sets (corresponding to a conjunction) on the Cartesian product of universes $U \times V$ and $P + Q$ is the dual operation (corresponding to the disjunction) (12) and (15) hold provided that $\sup \mu_{\mathcal{D}} = 1$.

However, $\text{Pos}(\neg P|\mathcal{D})$ ($\text{Nes}(\neg P|\mathcal{D})$ resp.) is not functionally related to $\text{Pos}(P|\mathcal{D})$ ($\text{Nes}(P|\mathcal{D})$ resp.). Thus, using $\text{Pos}(\cdot|\mathcal{D})$ or $\text{Nes}(\cdot|\mathcal{D})$ does not lead to a truth-functional calculus. Truth-functionality can be

recovered if we are using $\partial(\cdot|\mathcal{D})$ for instance, provided some restrictions. Let us prove this result. First, because of (8), (9) is satisfied with $f(x) = 1-x$. Indeed, because of (12-15), (10) and (11) hold respectively with $g = \min$ and $h = \max$, if

$$\frac{1}{2} [\min(p,q) + \min(p',q')] = \min\left(\frac{p+p'}{2}, \frac{q+q'}{2}\right) \quad (18)$$

$$\frac{1}{2} [\max(p,q) + \max(p',q')] = \max\left(\frac{p+p'}{2}, \frac{q+q'}{2}\right) \quad (19)$$

where

$$\text{Pos}(P|\mathcal{D}) = p; \quad \text{Pos}(Q|\mathcal{D}) = q;$$

$$\text{Nes}(P|\mathcal{D}) = p'; \quad \text{Nes}(Q|\mathcal{D}) = q'.$$

It is clear that (18) and (19) hold provided that if $p \leq q$, then $p' \leq q'$ and if $p \geq q$, then $p' \geq q'$ (20).

1) ($p' = 0$ or $p = 1$) and ($q' = 0$ or $q = 1$). It means that for each proposition, its negation or itself is completely possible (i.e. $\text{Nes}(P|\mathcal{D}) = 0$ which entails $\text{Pos}(\neg P|\mathcal{D}) = 1$ because of (3), or $\text{Pos}(P|\mathcal{D}) = 1$). It is the case if P is non-fuzzy (i.e. the excluded-middle law holds), but, this condition is sufficient, not necessary. Examples of this situation are examples 1, 2, 4 and 5 of the preceding section.

2) $p' = p$ and $q' = q$. It is the situation of example 3 where P is fuzzy and our knowledge \mathcal{D} is precise ($\mu_{\mathcal{D}}$ is the membership function of a singleton). Note that when mixing situations 1) and 2) (e.g. $p' = 0$; $p = 1$; $q = q' = \frac{1}{2}$), truth functionality is lost.

N.B.1 Gaines⁷ pointed out that the fuzzification (in the sense of the extension principle) of propositional calculus yields the multivalent logic whose connective functions are $f(x) = 1-x$, $g = \min$ and $h = \max$ provided that only normalized fuzzy sets of $\{0,1\}$ are considered and using a formula similar to (7) to aggregate the membership degree $\mu(0)$ attached to 0 (i.e. false) and this $\mu(1)$ attached to 1 (i.e. true) of each fuzzy set of $\{0,1\}$ (viewed as a 'fuzzy' truth-value). However, this result was not discussed in the framework of possibility/necessity theory or in relation with a semantical interpretation of the degree of truth.

N.B.2. In Prade¹⁰, new justifications of the interpretation of $1/2$ as 'possible' in some trivalent logics (Lukasiewicz⁹) are given in the framework of Zadeh's possibility theory.

Viewing the degree of truth of a proposition as the conformity of what is said with fact leads to use some measure(s) of similarity. It seems that possibility and necessity measures offer complementary points of view to evaluate this conformity (it is clear that one of them would not be sufficient). A priori, arithmetic mean is not the unique way to aggregate them in order to get a single number interpretable as a degree of truth. More generally, we have to look for an aggregation function m such that

$$m(1-p', 1-p) = f(m(p,p')) \quad (21)$$

$$m[\min(p,q), \min(p',q')] = g[m(p,p'), m(q,q')] \quad (22)$$

$$m[\max(p,q), \max(p',q')] = h[m(p,p'), m(q,q')] \quad (23)$$

where f,g,h are connective functions also to determine, if we want to preserve truth-functionality. The complete discussion of this system of equations is a topic for further research. Nevertheless, assuming m symmetrical and $f(x) = 1-x$, equation (21) is the characteristic equation of the symmetrical sums in the sense of Silver¹⁴.

We may also think of using some probabilistic

measure in order to evaluate the conformity (see Dubois and Prade⁶), as suggested in the example 2 of the preceding section. Thus, the degree of truth of a proposition would correspond to a probability of conformity of the contents of the proposition with respect to facts. But, as in Reichenbach's probabilistic logic (see Rescher¹³), truth-functionality would be lost.

Concluding remarks

This paper stresses the problem of the semantical interpretation of the notion of degree of truth in multivalent logic. The interest of a possibility/necessity-based approach is emphasized.

Apart from discussion of equations (21-23), topics for further research may deal with slightly different expressions (Prade¹¹) of possibility and necessity measures (namely, where other triangular norms than min and other triangular conorms than max are used in formulae (1) and (2) respectively) or with the use of the notion of compatibility (Zadeh¹⁹) to evaluate the conformity with fact in a fuzzy way.

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