The coupling phase in cavity magnonics

Alan Gardin, Jeremy Bourhill, V. Vlaminck, Christian Person, Christophe Fumeaux, Vincent Castel, Giuseppe Tettamanzi

The coupling phase in cavity magnonics

Alan Gardin[1,2], Jeremy Bourhill[2,3], Vincent Vlaminck[2], Christian Person[2], Christophe Fumeaux[1], Vincent Castel[2] & Giuseppe Tettamanzi[1]

Abstract

- Cavity magnonics relevant for quantum transducers, quantum memories (using dark mode physics), non-reciprocal devices [1]: such applications justify studying cavity magnonics in multimode systems.
- Microwave photon/magnon coupling due to Zeeman coupling involves a *coupling phase* factor which is usually rightfully neglected.
  $\Rightarrow$ unless the system is "loop-coupled" (see next section for definition and examples), coupling phases are unobservables.
- (Experimentally) Simplest system in which the coupling phases manifest: two magnon modes coupling simultaneously to two cavity modes.
  The various coupling phases accumulate in a single quantity θ: *the physical phase*. Physical phase found to influence the presence of dark modes, and the strength of the cavity-mediated coupling.

Relevance of the coupling phase

- Origin of the coupling phase φ: Zeeman coupling between macrospin $\mathbf{S} = S_x\hat{\mathbf{x}} + S_y\hat{\mathbf{y}} + S_z\hat{\mathbf{z}}$ and the cavity mode's RF magnetic field $\mathbf{h} = h_x\hat{\mathbf{x}} + h_y\hat{\mathbf{y}}$ ($\mathbf{h} \perp H_0\hat{\mathbf{z}}$ static applied magnetic field)
- Interaction Hamiltonian using the rotating wave approximation [2]:

$$H_I = \int_{V_m} \mathrm{d}^3 r\, \mathbf{S}\cdot\mu_0\mathbf{h} \tag{1}$$

$$= \hbar g\left(e^{i\varphi} c m^\dagger + e^{-i\varphi} c^\dagger m\right) \tag{2}$$

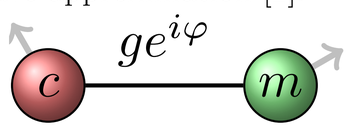

Figure 1: Representation of the magnon/photon coupling. Coupling phase φ measures the difference in direction of the (local) yellow arrows.

with coupling strength $g/2\pi \propto \eta\sqrt{\omega_c}$ and

$$\eta = \sqrt{\frac{\left(\int_{V_m} \mathrm{d}^3 r h_x\right)^2 + \left(\int_{V_m} \mathrm{d}^3 r h_y\right)^2}{V_m \int_{V_c} \mathrm{d}^3 r |\mathbf{h}|^2}},\quad \varphi = \arg\left\{\int_{V_m} \mathrm{d}^3 r h_x + i\int_{V_m} \mathrm{d}^3 r h_y\right\} \tag{3}$$

- A unitary transformation can gauge the phase away by rotating one of the modes $\Rightarrow$ if φ can be eliminated, it cannot have physical consequences
- Coupling phase can be "rotated away" when the system is not "loop-coupled", i.e. we have n modes but $k < n$ couplings between modes: we have n degrees of freedom but only $k < n$ constraints

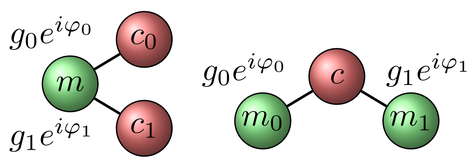

Figure 2: Non loop-coupled systems ($k < n$)

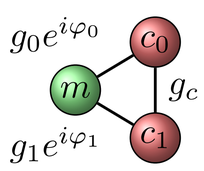

Figure 3: Loop-coupled systems ($k \geqslant n$)

An example with a physical phase

System definition

- System under study: two magnon modes (annihilation operators m_0 and m_1, angular frequencies $\omega_m - \delta_m$ and $\omega_m + \delta_m$) coupling to two cavity modes (c_0 and c_1 , angular frequencies $\omega_c - \delta_c$ and $\omega_c + \delta_c$), with coupling strength $g_0/2\pi$ to c_0 and $g_1/2\pi$ to c_1.
- Loop-coupled system: the various coupling phases combine to form a *physical phase* θ characterising the physics:

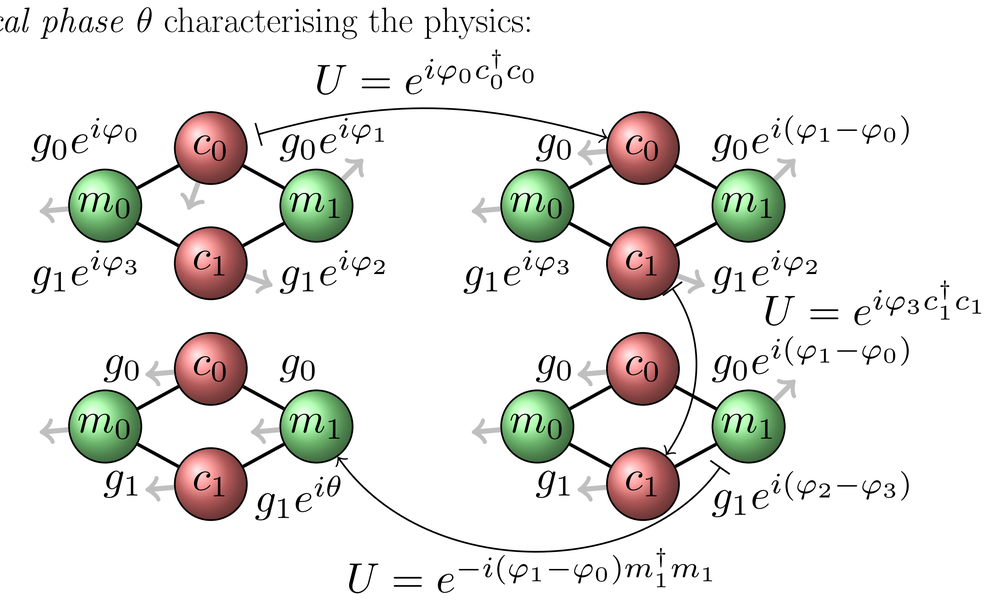

Figure 4: Reduction of the coupling phases to a single physical phase θ through unitary transformations.

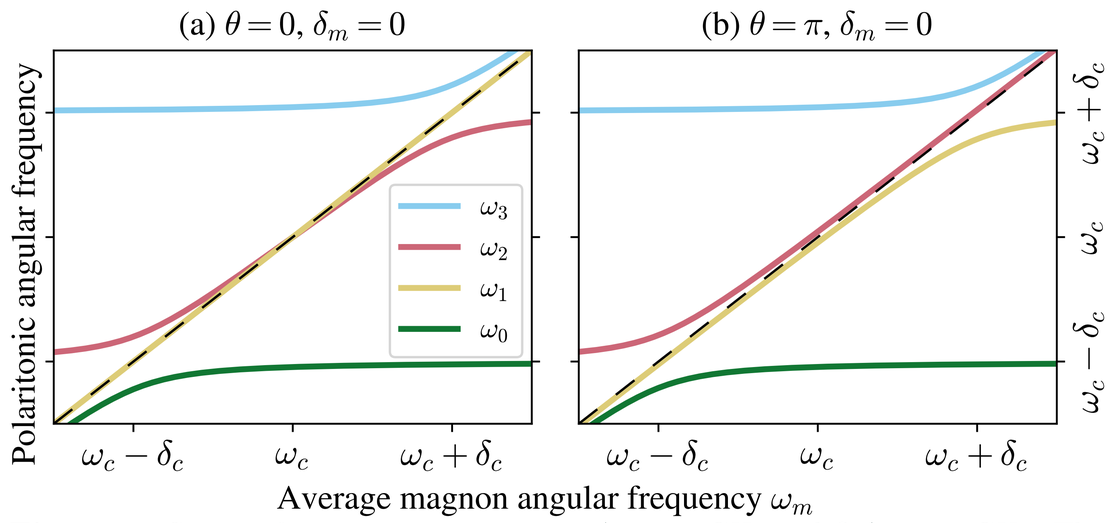

Figure 5: Spectra for $g_0 = g_1 = 0.03\omega_c$, $\omega_c/2\pi = 5$ GHz and $\delta_c/2\pi = 1$ GHz. The black dashed line is ω_m. Polaritons ω_1, ω_2 cross for $\theta = 0$ and anti-cross for $\theta = \pi$.

Indirect magnon/magnon coupling at $\omega_m = \omega_c$

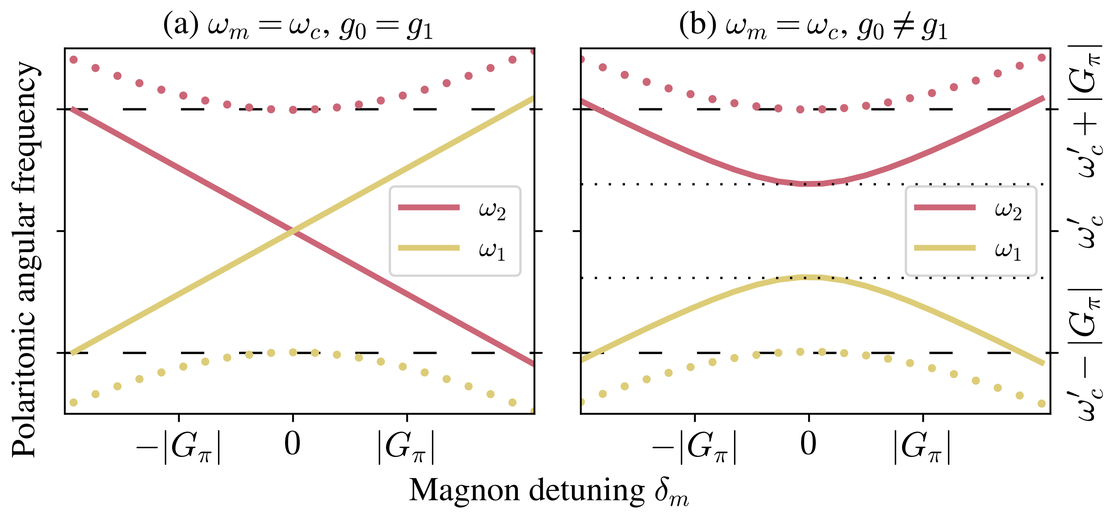

Figure 6: Spectra for $\theta = 0$ in solid lines and $\theta = \pi$ in dots. Other parameters same as in Fig 5. G_θ is given by equation (4) below.

with θ-dependent indirect magnon/magnon coupling strength $G_\theta/2\pi$:

$$G_\theta = \frac{\delta_c}{\delta_c^2 - \delta_m^2}\left(g_0^2 - e^{i\theta} g_1^2\right). \tag{4}$$

Dark mode physics

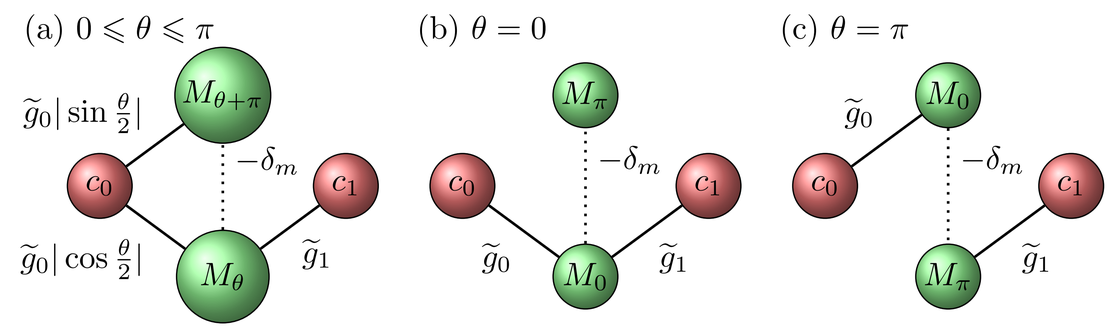

Figure 7: Illustration of the different couplings in the system.

where

$$M_\theta = \frac{m_0 + e^{-i\theta} m_1}{\sqrt{2}},\quad \widetilde{g}_i = \sqrt{2} g_i. \tag{5}$$

- Dark mode: eigenmode that does not couple to the readout mechanism
- Cavity magnonics readout: cavity photons
- For $\delta_m = 0$ and $\theta = 0$, M_π does not couple: dark mode

Outlook

- Coupling phase can have physical consequences in loop-coupled multimode systems
- Resulting discrete gauge-invariant phase θ akin to a synthetic gauge field, which can break time-reversal symmetry for specific values [3]
- Synthetic gauge field+dissipation engineering $\Rightarrow$ non-reciprocal behaviour [4]
  However, the physical phase is fixed by the cavity geometry...

References

[1] Babak Zare Rameshti, Silvia Viola Kusminskiy, James A. Haigh, Koji Usami, Dany Lachance-Quirion, Yasunobu Nakamura, Can-Ming Hu, Hong X. Tang, Gerrit E.W. Bauer, and Yaroslav M. Blanter. Cavity magnonics, September 2021.

[2] Alan Gardin, Jeremy Bourhill, Vincent Vlaminck, Christian Person, Christophe Fumeaux, Vincent Castel, and Giuseppe C. Tettamanzi. Manifestation of the coupling phase in microwave cavity magnonics. *Physical Review Applied*, 19(5):054069, may 2023.

[3] Jens Koch, Andrew A. Houck, Karyn Le Hur, and S. M. Girvin. Time-reversal-symmetry breaking in circuit-QED-based photon lattices. *Physical Review A*, 82(4):043811, oct 2010.

[4] Aashish Clerk. Introduction to quantum non-reciprocal interactions: from non-hermitian hamiltonians to quantum master equations and quantum feedforward schemes. *SciPost Physics Lecture Notes*, mar 2022.

[1]The University of Adelaide, Adelaide, SA 5005, Australia.
[2]IMT Atlantique & Lab-STICC (UMR 6285) CNRS, Technopole Brest-Iroise, Brest, France.
[3]University of Western Australia, Western Australia 6009, Australia.
[†]Corresponding author: `alan.gardin@adelaide.edu.au`.

Link to paper: