

# The influence of heterogeneity on the strength of volcanic rocks and the stability of lava domes

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1	The influence of heterogeneity on the strength of volcanic rocks and the stability
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27	Abstract
28	The collapse of lava domes, inherently heterogeneous structures, represents a significant volcanic hazard.
29	Numerical and analogue models designed to model dome instability and collapse have incorporated heterogeneity

30 in the form of discrete zones with homogeneous properties. Based on an assessment of dome rock heterogeneity, 31 we explore whether material property heterogeneity ("diffuse" heterogeneity) within these discrete zones can 32 promote dome instability. X-ray computed tomography shows that dome samples are characterised by high 33 microstructural heterogeneities; e.g., porosity varies from 0.07 to 0.20 over millimetric lengthscales. To explore how microstructural heterogeneity influences sample-scale strength, we performed numerical simulations using 34 35 Rock Failure and Process Analysis. The mean mechanical properties of the numerical samples were constant, and 36 we introduced heterogeneity by varying their distribution using a Weibull probability function. The models show 37 that increasing heterogeneity can reduce sample-scale strength by more than a factor of two. To explore the 38 influence of dome-scale heterogeneity, we numerically generated lava domes in Particle Flow Code. The domes 39 have the same bulk strength but are characterised by different degrees of heterogeneity by varying the distribution 40 of cohesion using a Weibull probability function. The models show that a greater degree of heterogeneity induces 41 higher dome-scale displacements and that, when there is also a discrete weakened zone, the addition of diffuse 42 heterogeneity leads to more widely distributed deformation. Therefore, alongside discrete zones defined by 43 different material properties, we find that the diffuse heterogeneity inherent to a dome is sufficient to compromise 44 dome stability and should be incorporated in future modelling endeavours.

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46 Keywords: lava dome; heterogeneity; uniaxial compressive strength; porosity; coefficient of variation

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### 48 **1 Introduction**

49 The collapse of a lava dome, which can result in rock falls, debris avalanches, laterally-directed 50 explosions, and pyroclastic density currents, represents a significant volcanic hazard (Sparks, 1997; Nakada et al., 51 1999; Voight, 2000; Voight et al., 2000; Voight and Elsworth, 2000; Calder et al., 2002; Saucedo et al., 2005; 52 Komorowski et al., 2013; Ogburn et al., 2015; Harnett et al., 2019a). The collapse of a lava dome can also rapidly 53 depressurise subsurface magma, triggering large and potentially devastating eruptions (Sparks et al., 2002). The 54 mechanisms of dome instability are numerous, and very often operate in concert. For example, dome stability can 55 be compromised by increases in magma extrusion rate (Calder et al., 2002), basal strike-slip movements (Lagmay 56 et al., 2000), the build-up of pressurised fluids (Sparks, 1997; Komorowski et al., 1997; Voight and Elsworth, 57 2000; Reid, 2004; Ball et al., 2018; Heap et al., 2021a; Mordensky et al., 2022), topographical, geometrical, and 58 structural factors (Walter et al., 2015; Darmawan et al., 2018; Harnett and Heap, 2021, Mériaux et al., 2022), 59 ground acceleration resulting from earthquakes (Wallace et al., 2021), hydrothermal alteration (Reid et al., 2001; Ball et al., 2013; Heap et al., 2021b; Darmawan et al., 2022; Harnett et al., 2022), and rain-induced pore-fluid
pressurisation (Matthews et al., 2002; Elsworth et al., 2004; Taron et al., 2007).

62 A characteristic common to all lava domes is that they are heterogeneous across a variety of scales. At 63 the microstructural scale (tens to hundreds of microns), microscopy and X-ray computed tomography have shown 64 that volcanic rocks, including dome rocks, are heterogeneous, typically containing a groundmass populated by 65 phenocrysts and pores in variable quantities and of variable sizes and shapes (Zandomeneghi et al., 2010; Shea et 66 al., 2010; Kushnir et al., 2016; Farguharson et al., 2016; Rhodes et al., 2018; Cashman, 2020). Moreover, the 67 observation of micro-breccia and micrometre-size pyroclast-filled fractures (tuffisites) in cryptodomes and lavas 68 (Heiken et al., 1988; Komorowski et al., 1997; Tuffen et al., 2003) are evidence of the presence of textural 69 discontinuities and the circulation of pressurised gas that may generate variations in mechanical properties and, 70 therefore, heterogeneity. At the sample scale (a few to tens of centimetres), laboratory studies have shown that the 71 physical and mechanical properties of rocks collected from the same dome can vary by several orders of magnitude 72 (Smith et al., 2011; Heap et al., 2014a; Farquharson et al., 2015; Harnett et al., 2019b; Bain et al., 2019; Heap et 73 al., 2020a; Kendrick et al., 2021; Heap and Violay, 2021). The large range in the physical and mechanical 74 properties typically observed for volcanic rocks of the same porosity is often considered to be the result of micro-75 and sample-scale heterogeneities (e.g., Heap and Violay, 2021). Numerical experiments using modelling software 76 Fast Lagrangian Analysis of Continua (FLAC; Itasca Consulting Group Ltd) (Villeneuve et al., 2012), UDEC and 77 3DEC (Distinct Element Modelling; Itasca Consulting Group Ltd) (Ghazvinian et al., 2014; Stavrou et al., 2019; 78 Xu et al., 2020), Rock Failure Process Analysis (RFPA; Mechsoft Technology) (Tang et al., 2000, 2007; Zhu, 79 2008), and Particle Flow Code (PFC; Itasca Consulting Group Ltd) (Peng et al., 2017; Liakas et al., 2017) have 80 also demonstrated that microstructural heterogeneity leads to a reduction in rock strength. Numerical experiments 81 have also shown that pore size and shape (Heap et al., 2014b; Griffiths et al., 2017; Heap et al., 2021c) and crystal 82 content (Heap et al., 2016) can influence mechanical behaviour of volcanic rocks.

At the dome-scale (tens to hundreds of metres), field investigations and analogue and numerical modelling have shown that lava domes are structurally very complex, with mechanical properties that vary through space and time (Fink and Griffiths, 1998; Watts et al., 2002; Harnett et al., 2019b; Zorn et al., 2020; Tsepelev et al., 2021). Geophysical methods have also exposed the large-scale heterogeneity of lava domes. For example, muon tomography—a geophysical method that can provide the 3D density structure of a volcano—has shown that the density of a dome can be spatially very variable (Tanaka et al., 2007; Lesparre et al., 2012; Rosas-Carbajal et al., 2017), corresponding to spatial variations in compressive and tensile strength (Heap et al., 2021b). Electrical

90 resistivity and magnetic surveys have shown that portions of a lava dome can be affected by the presence of a 91 hydrothermal system, the location and/or extent of which can depend on internal structural features (Rosas-92 Carbajal et al., 2016; Byrdina et al., 2017; Finn et al., 2018; Peterson et al., 2021). Heterogeneity is commonly 93 reflected in numerical and analogue models of volcanoes and volcanic domes, typically designed to better 94 understand large-scale deformation (e.g., slow spreading, sliding on pre-existing faults or fractures, the formation 95 of large-scale fractures, partial collapse), as discrete zones defined by different material properties. For example, 96 analogue models have used different materials-silicon putty to represent weakened or altered zones and a sand-97 gypsum mixture to represent stronger and/or intact zones-to represent different discrete zones within the volcano 98 (Cecchi et al., 2004; van Wyk de Vries et al., 2000, Byrne et al., 2013). Numerical models typically separate the 99 volcanic edifice, flank, or dome into different discrete zones, often guided by geophysical data, and assign these 100 zones different rock physical and mechanical properties (Heap et al., 2021b; Harnett and Heap, 2021; Carr et al., 101 2022; Harnett et al., 2022; Mordensky et al., 2022). An example of this discrete heterogeneity is shown in Fig. 1a, 102 where a lava dome is split into two discrete zones characterised by different, but uniform, physical and mechanical 103 properties (a weak zone in yellow and a strong zone in white). In the context of a volcanic edifice, flank, or dome, 104 such discrete zones could represent, for example, volumes that are characterised by different rock types with 105 different properties, different lava units, different domes or spines within a dome complex, or zones variably 106 influenced by hydrothermal alteration. However, existing geophysical datasets suggest that, although volcanic 107 structures can, and should, be considered with discrete zones characterised by different physical and mechanical 108 properties, these properties also vary within these discrete zones (termed here as *diffuse heterogeneity*). An 109 example of *discrete and diffuse heterogeneity* is shown in Fig. 1b, where the lava dome is again split into two 110 discrete zones characterised by different physical and mechanical properties (i.e. discrete heterogeneity) but, in 111 this instance, the physical and mechanical properties within these zones are also heterogenous (i.e. diffuse 112 heterogeneity). However, the extent to which diffuse heterogeneity can influence dome stability remains 113 unexplored.

Our goal in this contribution is to better understand the role of heterogeneity on the stability of a lava dome. We first characterised the microstructural heterogeneity of dome rocks using X-ray micro computed tomography ( $\mu$ CT). We then performed numerical experiments, calibrated using laboratory stress-strain data for dome rock, to assess the influence of systematically varied microstructural heterogeneity on the strength of dome rocks at the sample-scale. Finally, we investigated the effect of heterogeneity at the dome scale. To do so, we simulated domes characterised by the same average bulk strength, but with different heterogeneities (i.e. diffuse heterogeneity), and domes characterised by both discrete and diffuse heterogeneities. These results reveal new insights on the influence of the internal structure and fabric of volcanic domes on their stability. As such they are aimed at providing one of the missing links to progress beyond the state of the art of modelling and the quantitative evaluation of dome volumes susceptible to instability and collapse and the modelling of resulting rockslide and debris avalanche emplacement as proposed by, for example, Peruzzetto et al. (2019). Understanding the mechanisms that can contribute to dome instability and collapse will help improve monitoring strategies and mitigate the hazards presented by lava domes worldwide.

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#### 128 2 Materials and Methods

129 2.1 Microstructural and compositional heterogeneity

130 To assess the microstructural heterogeneity of dome rocks, X-ray tomography images were acquired for 131 cylindrical samples (10 mm in diameter and 20 mm in length) prepared from three dome rocks from La Soufrière 132 de Guadeloupe, an active andesitic stratovolcano located on the French island of Guadeloupe in the Eastern 133 Caribbean (Moretti et al., 2020). Two of the three dome rocks were collected from a collapse scar to the northeast 134 of dome summit (samples H3 and H4A), and the third was collected from the dome summit near Cratère Sud 135 (sample H19). Samples from these blocks were recently analysed in terms of their compressive and tensile strength 136 (Heap et al., 2021b, 2022a, b) and thermal properties (Heap et al., 2022c). Fig. 2 shows backscattered scanning 137 electron microscope (SEM) images of each sample. All three dome rocks are porphyritic andesites characterised 138 by a microcrystalline groundmass with phenocrysts of dominantly plagioclase and pyroxene (orthopyroxene and 139 clinopyroxene) (Fig. 2). The connected porosity of each of the three cylindrical samples was calculated using their 140 bulk sample volumes and their skeletal (connected) volumes measured using a helium pycnometer. The connected 141 porosities were measured to be 0.18, 0.20, and 0.12 for samples H3, H4A, and H19, respectively.

142 X-ray tomography images were acquired for the three cylindrical samples at the High-Resolution X-ray 143 Computed Tomography Facility at the University of Texas at Austin (USA). X-rays were set to 90 kV and 12 W, 144 and the acquisition time was 0.07 s. A total of 4175 image slices, with a voxel side length of 5 μm, were taken of 145 each of the three cylindrical samples.

146 X-ray powder diffraction (XRPD) was used to provide the mineral content of all three samples (data from 147 Heap et al., 2021b, 2022c). These XRPD data reveal that all three samples are characterised by advanced argillic 148 alteration, the result of the efficient circulation of acidic sulfate-chloride-rich fluids (below 350 °C, and down to 149 150-200 °C; pH < 4) (Heap et al., 2021b). Sample H3 contains primary plagioclase (46.6 wt%), pyroxene (11.8

150 and 5.6 wt% of orthopyroxene and clinopyroxene, respectively), and magnetite (0.8 wt%); secondary minerals 151 include kaolinite (17.4 wt%), cristobalite (10.6 wt%), pyrite (3.8 wt%), Na-alunite (2.8 wt%), and quartz (0.6 152 wt%). Sample H4A contains primary plagioclase (23.3 wt%) and pyroxene (11.8 and 4.9 wt% of orthopyroxene 153 and clinopyroxene, respectively); secondary minerals include kaolinite (43.4 wt%), cristobalite (11.8 wt%), pyrite 154 (2.3 wt%), Na-alunite (1.3 wt%), gypsum (0.7 wt%), and quartz (0.6 wt%). Sample H19 contains primary 155 plagioclase (22.0 wt%) and pyroxene (10.2 and 5.0 wt% of orthopyroxene and clinopyroxene, respectively), 156 secondary minerals include opal-A (33 wt%), Na-alunite (14.2 wt%), cristobalite (9.5 wt%), hematite (2.4 wt%), 157 kaolinite (2.0 wt%), and quartz (1.7 wt%). In all three samples, Na-alunite and cristobalite are observed as pore-158 filling precipitates and plagioclase phenocrysts are often altered and partially replaced by opal-A and/or kaolinite 159 (Fig. 2). The total wt% of secondary (alteration) minerals, a measure of their alteration, in samples H3, H4A, and 160 H19 is 35.2, 60, and 62.8 wt%, respectively. We highlight that, although these hydrothermally altered samples are 161 representative of the material forming the dome at La Soufrière de Guadeloupe, they may not represent freshly 162 extruded dome materials at other active volcanoes that can be characterised by initially low levels of alteration.

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164 2.2 Sample scale heterogeneity: Rock Failure and Process Analysis modelling

165 To explore the effect of heterogeneity on the sample scale, 2D numerical experiments were performed 166 using Rock Failure and Process Analysis (RFPA), a numerical code based on elastic damage mechanics (Tang, 167 1997, 2011). Numerical samples were prepared to the same dimensions as the uniaxial compressive strength tests 168 performed in Heap et al. (2021b), which were 20 mm in diameter and 40 mm in length. Before we performed the 169 numerical experiments, we ran some preliminary simulations to ascertain the appropriate mesh size for the model. 170 To do so, we performed simulations with the aim of using the model to capture stress-strain data from a laboratory 171 experiment performed on a sample of andesite from La Soufrière de Guadeloupe (sample H18; Heap et al., 2021b) 172 in which the numerical samples consisted of either 45,000 or 80,000 square elements, resulting in resolutions of 173 7.5 and 10 elements per mm, respectively. Both of these simulations accurately captured the mechanical behaviour 174 of the laboratory experiment and so we used numerical samples containing 45,000 square elements (resolution of 175 7.5 elements per mm) for our numerical experiments. In the model, each square element is assigned a set of 176 physical and mechanical input parameters. To obtain the mean values for these model input parameters, we 177 performed a calibration in which we modified the model input parameters until the stress-strain behaviour, 178 Young's modulus, and uniaxial compressive strength from the numerical experiment matched that of a laboratory 179 experiment performed on a sample of andesite from La Soufrière de Guadeloupe (sample H18; Heap et al., 2021b).

180 The stress-strain curves for the numerical and laboratory experiment are shown in Fig. 3a, and the mean input 181 parameters for the calibrated model are listed in Table 1. For the calibration model, the physical and mechanical 182 properties of each square element were not identical, and varied according to a Weibull probability density 183 function:

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$$x(u) = \frac{m}{u_0} \left(\frac{u}{u_0}\right)^{m-1} exp\left[-\left(\frac{u}{u_0}\right)^m\right].$$
 (1)

186

187 A Weibull probability density function contains a scale parameter and a shape parameter. Here, u is the scale 188 parameter of an individual element, and the scale parameter  $u_0$  is the average element parameter. x(u) is the 189 dependence of a given property on the distribution scale input, u, such that x(u) is replaced by the Young's 190 modulus,  $E_0(u)$ , or the uniaxial compressive strength,  $\sigma_c(u)$ , to determine the probability of a certain square 191 element having a certain property. The shape parameter is m. In material sciences, the shape parameter m is often 192 called the "homogeneity index". For the calibration model, we set m to be 5. As a result, although the mean uniaxial 193 compressive strength of an element in the numerical experiment shown in Fig. 3a was 147.3 MPa (Table 1), the 194 strength of an individual element varied according to Eq. (1), where m = 5. This microscale heterogeneity resulted 195 in a numerical sample with a macroscopic uniaxial compressive strength of 98.8 MPa (Fig. 3a). Therefore, 196 changing homogeneity index, m, and keeping all the other parameters in the model constant, allows us to explore 197 the influence of microscale heterogeneity on sample-scale uniaxial compressive strength, where high values of m198 correspond to very homogeneous samples, and low values of m correspond to very heterogeneous samples. An 199 extremely high homogeneity index yields a Gaussian distribution with high kurtosis (Xu et al., 2018). As the 200 homogeneity index decreases, the distribution becomes increasingly positively skewed (Fig. 3b). Indeed, although 201 the mean values remain constant, the median values of uniaxial compressive strength and Young's modulus 202 decrease as a function of decreasing homogeneity index. For example, at m = 1.1, the median uniaxial compressive 203 strength and Young's modulus are 105.7 MPa and 33.2 GPa, respectively. Owning to their success in simulating 204 the mechanical behaviour and failure of rock, the statistics of the mechanical properties of rock are often assumed 205 to follow Weibull distributions in numerical modelling (Tang, 1997; Zhu and Tang, 2004; Liu et al., 2004; Wong 206 et al., 2006; Tang, 2011). Weibull distributions have also been used in other aspects of volcanology. For example, 207 Weibull functions have been used to describe the distribution of dyke thickness (Krumbholz et al., 2014) and the 208 distribution of flaws surrounding magma chambers (Gray and Monaghan, 2004).

209 We performed numerical experiments in which deformed samples were characterised by different values 210 of m: 1.1 (most heterogeneous), 2, 3, 4, 5, 6, 7, and 8 (most homogeneous). The distribution of Young's modulus 211 (mean value of 46 GPa; Table 1) and uniaxial compressive strength (mean value of 147.3 MPa; Table 1) for each 212 of these numerical experiments are shown in Figs. 3b and 3c, respectively. For example, as shown in Fig. 3c, the 213 elements comprising the sample with the highest value of m are clustered around the mean value of 147.3 MPa, 214 whereas the elements comprising the sample with the lowest value of m can be much lower (close to 0 MPa) or 215 much higher (higher than 350 MPa) than this mean value. We note that the microstructural heterogeneity in our 216 numerical samples is provided by varying the physical and mechanical properties of square elements that have a 217 length of 133 µm and, as a result, the modelling assumes microstructural heterogeneities of the same size. In nature, 218 and in our dome samples (Fig. 2), volcanic rocks can contain heterogeneities on a number of scales, from the nano-219 or microscale to the scale of the sample (centimetric scale). The influence of larger heterogeneities, such as mm-220 scale pores (Heap et al., 2014b; Griffiths et al., 2017; Heap et al., 2021c) and crystals (Heap et al., 2016), on the 221 mechanical behaviour of volcanic rocks have been investigated in previous studies using a heterogeneous 222 groundmass with a constant homogeneity index (m = 3). The modelling presented here varies the homogeneity 223 index of the groundmass in numerical samples (m = 1.1, 2, 3, 4, 5, 6, 7, and 8) that do not contain larger pores and 224 crystals, or any other mm-scale heterogeneity.

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#### 226 2.3 Dome scale heterogeneity: Particle Flow Code modelling

227 To explore the effect of heterogeneity on the dome scale, we used 2D Discrete Element Method (DEM) 228 models created in Particle Flow Code (PFC; Itasca Consulting Group Ltd), following the methods outlined by 229 Harnett et al. (2018) and Harnett and Heap (2021). The DEM allows us to model the post-failure detachment and 230 movement of blocks of rock, which cannot be simulated by the FEM-based software RFPA used for the sample-231 scale modelling. We suggest that DEM modelling is therefore more relevant for dome-scale simulations due to the 232 spectrum of collapse processes that can be expected at natural domes (e.g., Calder et al., 2002). Indeed, PFC 233 modelling has been previously, and successfully, used to model dome growth and collapse (Husain et al., 2014; 234 Harnett et al., 2018; Harnett and Heap, 2021; Harnett et al., 2022). These DEM models consider a particle-based 235 material in which circular particles interact at interparticle contacts. Contact behaviour is primarily governed by 236 stiffness and cohesion. At the boundary between the particles and walls (i.e. the ground surface), the contact 237 behaviour is cohesionless and governed by friction. The particle size in the model is not representative of individual 238 crystals, grains, or rock blocks, but rather represents discretisation of the medium for the purpose of computation. First, we numerically generated two fully solid domes (see Harnett and Heap, 2021) containing 9745 circular particles and 22710 contacts. These contacts are given a uniform strength of 1 MPa. We select here a much lower bulk strength for the dome-scale (1 MPa) than the sample-scale (tens to hundreds of MPa) following previous studies that have shown that a decrease in magnitude is required when upscaling laboratory parameters for use at the dome-scale (Walter et al., 2019; Villeneuve and Heap, 2021). The domes are stable, such that they will not deform unless perturbed in some way.

245 To explore the effect of heterogeneity on the stability of a dome, we used the Weibull probability density 246 function shown in Eq. 1 to provide a distribution of the cohesion parameter acting at each interparticle contact. 247 For one dome, we let m = 2, creating a heterogenous dome, where the lowest contact cohesion is close to 0 MPa 248 (representative of weak or altered rock) and the highest contact cohesion is 3 MPa (representative of strong or 249 fresh rock). For the other dome, we let m = 10, creating a relatively homogeneous dome, where the contact 250 cohesion values cluster more tightly around 1 MPa, with minimum cohesion values of ~0.5 MPa and maximum 251 cohesion values of <1.5 MPa. In both cases, the average strength remains equal (i.e., the mean bond strength is 1 252 MPa in both cases), allowing us to solely explore the influence of heterogeneity on dome strength. In each case, 253 we vary both the cohesive and tensile strength as a function of the Weibull distribution, where the compressive to 254 tensile strength ratio is constant at 10:1 (Heap et al., 2022a). The distributions of bond cohesion for the two domes 255 are shown in Fig. 4.

256

#### 257 3 Results

#### 258 3.1 Microstructural and compositional heterogeneity

259 We used X-ray micro computed tomography (µCT) to assess the microstructural and compositional 260 heterogeneity of three dome samples from La Soufrière de Guadeloupe. Using open-source software ImageJ, we 261 first plot grey level histograms for each sample using the entire  $\mu$ CT image volumes (Fig. 5). The grey level is a 262 value between 0 (lowest attenuation and lowest density) and 255 (highest attenuation and highest density). The 263 three dome rocks are characterised by three peaks: at values of grey level of  $\sim 20$ ,  $\sim 45-50$ , and  $\sim 90-95$  (Fig. 5). 264 Although we consider the peak at a grey level of  $\sim 20$  contains to correspond to the porosity within the sample, this 265 peak also includes voxels that contain both porosity and solid material (groundmass and/or crystals). The peaks at grey levels of ~45-50 and ~90-95 likely correspond to, respectively, the groundmass and higher-density crystals 266 267 (e.g., plagioclase crystals that are denser than the groundmass). The broad nature of these peaks, and the saddles between them, are likely the result of voxels containing mixtures of groundmass and crystals. The wide variationin grey level, and therefore density, within the samples speaks to their high microstructural heterogeneity.

270 Next, we stacked the µCT images in ImageJ and determined the coefficient of variation (COV) of the 271 grey level (the ratio of the standard deviation of the grey level to the mean grey level), the porosity, the average 272 equivalent pore diameter, the average pore aspect ratio, and the average pore circularity for each image in the stack 273 (a total of 4175 image slices). The COV of the grey level in µCT images of rocks has previously been used to 274 assess microstructural heterogeneity (e.g., Louis et al., 2007). In these analyses, and due to edge effects, we 275 removed 300 slices (i.e. 1.5 mm) from the top and bottom of each of the stacks. The equivalent pore diameter, d, 276 was determined using  $d = (3/2)d_F$ , where  $d_F$  is the average Feret diameter. The pore aspect ratio is the ratio of 277 the minor to major pore axis, where a perfect circle would have an aspect ratio of one. Finally, the pore circularity, 278 C, is given by  $C = 4\pi p^2$ , where p is the pore perimeter. A perfect circle will have a pore circularity of one. In all 279 the analyses of pore size and shape, we excluded all pores that were determined to have a diameter  $< 10 \mu m$ .

280 Using the voxel side length of 5  $\mu$ m as the thickness of each image, we can plot these microstructural 281 parameters as a function of sample length. We plot the COV and porosity as a function of sample length for each 282 of the three dome samples in Figs. 6 and 7, respectively (plots for the average equivalent pore diameter, the average 283 pore aspect ratio, and the average pore circularity are provided in the Supplementary Information). We also provide 284 the image slice with the highest and lowest value for each sample in Figs. 6 and 7. Fig. 6 shows that the COV 285 varies from 0.27–0.31 to 0.48–0.52, and that the variation in COV is approximately the same for each of the three 286 samples. The image slices with the lowest values of COV are those without high-density pyrite and hematite and 287 with few plagioclase phenocrysts (Fig. 6). Conversely, the image slices with the highest values of COV are those 288 with large amounts of high-density pyrite and hematite and large plagioclase phenocrysts (Fig. 6). Fig. 7 shows 289 that the porosity of samples H3, H4A, and H19 varies from 0.07–0.20, 0.04–0.10, and 0.05–0.11, respectively. 290 The image slices with the lowest and highest porosities are those with a small and large number of macropores, 291 respectively (Fig. 7). The average porosity calculated from the stacked CT images are 0.11, 0.07, and 0.08 for H3, 292 H4A, and H19, respectively (Fig. 7). These calculated porosities are lower, and much lower in the case of H4A, 293 than the connected porosities measured in the laboratory (0.18, 0.20, and 0.12 for H3, H4A, and H19, respectively). 294 The lower porosities calculated using the CT images, compared to the laboratory-measured values, is the result of 295 the presence of  $< 10 \,\mu$ m-diameter micropores that are, presumably, abundant in sample H4A. 296 We find that the average equivalent pore diameters of samples H3, H4A, and H19 are 82, 63, and 99  $\mu$ m,

290 We find that the average equivalent pore diameters of samples H3, H4A, and H19 are 82, 63, and 99 μm,
 297 respectively. The average pore aspect ratio of each sample is 0.52. And, finally, the average pore circularities of

samples H3, H4A, and H19 are 0.73, 0.74, and 0.68, respectively. We find that the equivalent pore diameter, the pore aspect ratio, and the pore circularity do not vary considerably along the sample lengths (see data in Supplementary Information). Although the proportion and size of the macropores varies from slice to slice (see the image slices in Fig. 7), the large number of smaller pores results in a similar average equivalent pore diameter for each of the slices along the length of the samples.

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#### 304 3.2 Impact of heterogeneity on the sample scale

305 To explore the effect of microstructural and compositional heterogeneity on the sample scale, we 306 performed calibrated 2D numerical experiments using RFPA. The samples in these numerical experiments were 307 all characterised by the same average Young's modulus and strength, but we varied their homogeneity by changing 308 the range of these two parameters using a Weibull probability density function (Fig. 2; Eq. 1). The results of the 309 numerical experiments are shown in Fig. 8 (see also Table 2). We first note that all the numerical samples failed 310 in a brittle manner, whereby sample failure was promoted by the formation of a throughgoing macroscopic fracture 311 (see inset on Fig. 8b). The data of Fig. 8 show that the uniaxial compressive strength of the numerical samples 312 decreases from ~110 MPa at a homogeneity index of 8 (the most homogeneous sample) to ~45 MPa at a 313 homogeneity index of 1.1 (the most heterogeneous sample). In other words, increasing sample heterogeneity can 314 reduce sample-scale strength by more than a factor of two. Fig. 8 also shows that the reduction in strength as the 315 homogeneity index is decreased is non-linear: strength is decreased by only a couple of MPa when the homogeneity 316 index is decreased from 8 to 7, but decreases by almost 20 MPa when the homogeneity index is decreased from 2 317 to 1.1 (see also Table 2). We also highlight that the post-peak portion of the numerical experiments performed at 318 m = 7 and 8 are different to those performed at lower values of m (Fig. 8). The stress drop associated with 319 macroscopic sample failure at m < 7 occurs almost immediately after the peak stress is reached whereas, at m = 7320 and 8, the sample reaches the peak and there is an appreciable strain softening period prior to the stress drop that 321 accompanies macroscopic failure (Fig. 8).

322

#### 323 3.3 Impact of heterogeneity on the dome-scale

To explore the effect of heterogeneity at the dome scale, we performed 2D simulations using PFC. The model starting condition in each stage is a quasi-stable dome, which has already settled under gravitational forces and will not deform unless intrinsic or extrinsic factors are changed. The initial modelled dome has a width of 440 m and a maximum height of 130 m. The new models presented here show domes characterised by the same average bond cohesion; this is analogous to the uniaxial compressive strength of the macroscale material. To explore the influence of heterogeneity, we prepared a reasonably homogenous dome (for which m = 10; Fig. 4; Eq. 1) and a very heterogeneous dome (for which m = 2; Fig. 4; Eq. 1). The distribution of bond cohesion in these models is shown in Fig. 9. The domes analysed in this study are unbuttressed domes that were emplaced on a flat surface (Fig. 9).

333 The influence of dome heterogeneity on an otherwise-stable dome is presented in Fig. 10. The images in 334 Fig. 10 show the displacement in the dome when the contact cohesion between particles was changed from a 335 uniform value of 1 MPa (the quasi-stable dome; perfectly homogeneous in terms of particle contact cohesion) to 336 values assigned by the Weibull probability function (mean bond strength in both scenarios is 1 MPa; but contact 337 cohesion varies from  $\sim 0.5$  MPa to < 1.5 MPa for the relatively homogenous dome (Fig. 10a) and from close to 0 338 MPa to 3 MPa for the heterogeneous dome (Fig. 10b)). The average displacement is 0.0036 m for the relatively 339 homogenous dome (Fig. 10a) and 0.0198 m for the heterogeneous dome (Fig. 10b). The dome with higher 340 heterogeneity shows higher displacements across the dome, and particularly at its centre (Fig. 10b), and shows that 341 heterogeneity alone is enough to destabilise the dome, even when the bulk strength remains equal. By contrast, the 342 relatively homogeneous dome remains stable, with only minor displacements toward the left flank of the dome 343 (Fig. 10a).

344

#### 345 4 Discussion

#### 346 4.1 Microstructural heterogeneity of volcanic rocks

347 Our SEM images (Fig. 2) and  $\mu$ CT data (Figs. 5, 6, and 7) have shown that dome rocks can be 348 characterised by a high degree of microstructural heterogeneity. As a comparison, the COV values calculated for 349 the four sandstones analysed in Louis et al. (2007), using images acquired at a very similar resolution to those 350 presented here (a voxel side length of 5.7 µm, compared to 5 µm for the images acquired for this study), were 351 0.186, 0.255, 0.303, and 0.313, respectively. The maximum COV values determined for our three dome rocks 352 were 0.48, 0.52, and 0.51 for H3, H4A, and H19, respectively (Fig. 6). The domes rock from La Soufrière de 353 Guadeloupe were also characterised by three broad X-ray attenuation coefficient peaks associated with 354 macropores, plagioclase phenocrysts, and high-density Fe-Ti oxides (Fig. 5), as also observed for dome rock from 355 Volcán de Colima (Mexico; Heap et al., 2020b). By contrast, X-ray attenuation coefficient peaks in porous 356 sandstones are typically strongly bimodal, with narrow peaks associated with macropores and grains (Louis et al., 357 2007). Two X-ray attenuation coefficient peaks are also observed in porous limestones (Ji et al., 2015). We note that the range of COV values for the more altered dome rocks (H4A and H19; characterised by an alteration of 60 and 62.8 wt.%, respectively) is larger than for the least altered dome rock (H3; characterised by an alteration of 35.2 wt.%) (Fig. 6), suggesting that hydrothermal alteration may increase the heterogeneity of dome rock (although more data are now required to test this hypothesis).

362 The variation in porosity along the length of a 10 mm-long sample of dome rock is particularly noteworthy 363 (Fig. 7). We find that the porosity of samples H3, H4A, and H19 varies from 0.07–0.20, 0.04–0.10, and 0.05–0.11, 364 respectively (Fig. 7). For comparison, the porosity measured using µCT data along the length of sandstone samples 365 was found to be  $\sim 0.04 \pm 0.002$  (Long et al., 2009),  $\sim 0.3 \pm 0.03$  (Appoloni et al., 2007), and the porosity of a suite 366 of low-porosity (< 0.1) sandstones was found to vary by up to about  $\pm 0.025$  (Zhao et al., 2019). The porosities of 367 three porous limestones were found to be 0.077–0.103 (average 0.1), 0.170–0.236 (average 0.2), and 0.237–0.288 368 (average 0.26) (Baud et al., 2017). Because porosity plays a first-order role in dictating physical properties such 369 as sample strength and Young's modulus, the wide range of porosity found within a 10 mm-long sample questions 370 the use of the average sample porosity when interpreting these data, and could help explain the large scatter in 371 strength and Young's modulus when plotted as a function of sample porosity (Heap and Violay, 2021). In other 372 words, samples with the same average sample porosity likely have different strengths simply due to whether the 373 porosity is homogeneously or heterogeneously distributed throughout the sample. Finally, we note that the 374 difference in porosity between that calculated from the stacked CT images and that measured in the laboratory 375 (Fig. 7) implies that the dome samples studied herein contain significant microporosity (pores that are  $< 10 \,\mu m$  in 376 diameter). This is particularly true for sample H4A (Fig. 7).

377 To conclude, and as discussed in Heap and Violay (2021), volcanic rocks are typically much more 378 microstructurally heterogeneous than, for example, granites or sedimentary rocks. For example, the porosity of 379 volcanic rocks can vary significantly (from almost 0 to almost 1), they can contain a wide distribution of pore sizes 380 that can be characterised by different geometries (Shea et al., 2010; Zandomeneghi et al., 2010), they can contain 381 both pores and microcracks (Kushnir et al., 2016), they can contain a variety of different types of crystals in 382 different abundances that can be characterised by different crystal sizes, shapes, and size distributions (Cashman, 383 2020), their groundmass can be variably crystallised (Clarke et al., 2007; Zorn et al., 2018), and they can be 384 variably influenced by hydrothermal alteration (Heap et al., 2021b).

385

386 4.2 Impact of heterogeneity on the strength of volcanic rocks and dome stability

387 Our calibrated 2D numerical experiments have shown that heterogeneity alone can reduce the uniaxial 388 compressive strength of dome rock by more than a factor of two (Fig. 8). Previous modelling studies also found 389 that microstructural heterogeneity leads to a reduction in rock strength (Tang et al., 2007, 2000; Zhu, 2008; 390 Villeneuve et al., 2012; Ghazvinian et al., 2014; Peng et al., 2017; Liakas et al., 2017; Xu et al., 2020), and 391 modelling designed to investigate the mechanical behaviour of volcanic rocks showed that pore size and shape 392 (Heap et al., 2014b; Griffiths et al., 2017) and crystal content (Heap et al., 2016) can also influence rock strength. 393 In the RFPA model, the numerical samples characterised by a high heterogeneity contain weaker elements (Fig. 394 3), which fail at lower stresses. The failure of a weak element results in a stress redistribution that can damage or 395 fail neighbouring elements, even those with higher strengths. As a result, heterogeneity can provide weak elements, 396 or weak zones, that can act as the locus for the formation of larger fractures that can result in macroscopic sample 397 failure. In terms of a volcanic rock, microstructural heterogeneities such as pores and fractured crystals or, in the 398 case of altered volcanic rocks, crystals that have suffered dissolution or partial or complete replacement with weak 399 minerals such as clays, could act as the loci for damage that eventually results in macroscopic sample failure.

400 Our 2D dome simulations have shown that diffuse heterogeneity alone can promote lava dome instability 401 (Fig. 10). Lava domes are structurally complex and comprise rocks with different strengths and, as such, are 402 inherently heterogeneous. Our dome simulations (Fig. 10) highlight the need for not only assessing dome 403 heterogeneity, but also monitoring processes that act to increase the heterogeneity of a dome, including processes 404 that increase heterogeneity without changing the bulk strength of the dome. For example, the circulation of 405 hydrothermal fluids may act to remove precipitates and/or sublimates within fractures in one part of the dome, 406 resulting in weakening, which are then precipitated within fractures elsewhere in the dome, resulting in 407 strengthening (although this will depend on the nature of the precipitated minerals). Although the bulk strength of 408 the dome may not be greatly affected in this scenario, the increase in heterogeneity can weaken the entire structure, 409 resulting in instability and potentially collapse.

Dome-scale displacement as a result of increased diffuse heterogeneity shows downward and outward movement of previously stable dome material (Fig. 10). This type of displacement was also found when modelling the stability of a dome containing a discrete zone characterised by a lower, but uniform, strength (Harnett et al., 2022), although displacement was focussed toward the flank of the dome where the weakened zone was located. We suggest, therefore, that small magnitude, but widespread, deformation of a lava dome may be the result of diffuse heterogeneity or may indicate a progressive increase in diffuse heterogeneity linked, perhaps, to a progressive increase in hydrothermal alteration. 417 Based on the results of our study, we suggest that existing numerical models that simulate discrete zones 418 with different properties within a volcano need to also consider diffuse heterogeneity that might exist within these 419 zones. Material heterogeneity in analogue experiments could also be considered by using, for example, sand-sized 420 particles with different sizes and/or material properties. To demonstrate how discrete and diffuse heterogeneity 421 could influence the results of numerical models of dome stability, we provide the dome-scale displacement and 422 strain for (1) a homogenous dome with heterogeneous material properties (m = 2; Eq. 1) (Fig. 11a), (2) a dome 423 with an internal, off-centre weak zone in which both discrete zones have uniform material properties (Fig. 11b; 424 model results from Harnett et al., 2022), and (3) a dome with an internal, off-centre weak zone in which both 425 discrete zones have heterogeneous material properties (m = 2; Eq. 1) (Fig. 11c).

426 For the homogeneous dome with heterogeneous material properties, deformation and fracturing can be 427 observed on the flanks of the dome (Fig. 11a). Heterogeneous material properties induce an average displacement 428 of 0.18 m, while the maximum displacement for any one particle is 0.87 m. As discussed in Harnett et al. (2022), 429 a weak zone within the dome results in significant deformation and very large, dome-splitting fractures within and 430 above the off-centre weakened zone when both discrete zones have uniform material properties (Fig. 11b). In this 431 scenario, the introduction of a weakened discrete zone induces an average displacement of 0.29 m and a maximum 432 displacement of 3.22 m. When uniform material properties are considered in the modelling, deformation is very 433 much confined to the area adjacent to the weakened zone, and little to no deformation is observed on the opposite 434 flank (Fig. 11b). When the model from Harnett et al. (2022) (Fig. 11b) is re-run but, this time, incorporating 435 heterogeneous material properties in both zones (m = 2; Eq. 1), the deformation and fracturing within and above 436 the off-centre weakened zone is reduced, but there is more deformation and fracturing on the opposite flank of the 437 dome (Fig. 11c). The combined model showing both discrete and diffuse heterogeneity exhibits an average 438 displacement of 0.36 m and a maximum displacement of 1.42 m. Interestingly, the strain resulting from the diffuse 439 heterogeneity model (Fig. 11a) is not simply superimposed on the strain resulting from the discrete weakened zone 440 (Fig. 11b): when diffuse heterogeneity is considered, a weakened zone on one side of the dome results in more 441 fracturing on the opposite side of the dome (Fig. 11c). In other words, the introduction of diffuse heterogeneity in 442 a dome that also contains a discrete weakened zone has resulted in damage that is more distributed throughout the 443 dome, affecting a larger area, and less pronounced localisation in the form of dome-splitting fractures. This is 444 because, when diffuse heterogeneity is considered, there are weak, failure-prone contacts throughout the dome that 445 are not present when the material properties of the discrete zones are considered to be uniform. We speculate that, 446 although a reduction in dome-splitting fractures may in the short-term reduce the probability of partial flank collapses, distributed dome deformation may, in the long-term, mobilise larger volumes and promote larger-scalecollapses.

449 These results highlight that deformation and fracturing within a dome may not necessarily indicate that 450 there is a proximal weak zone, potentially complicating monitoring strategies at active lava domes. Further 451 complications arise from the fact that this deformation pattern can be enhanced by the morpho-structural 452 relationship between the dome and its surroundings (i.e. a buttressing effect provided by the paleo-collapse 453 horseshoe-shape structure within which the dome was built or the inclination of the substratum of the dome; e.g., 454 Heap and Harnett, 2021). Another important issue is the likely development of a collapse sequence with successive 455 retrogressive slide blocks retreating into the dome. Our new modelling (Fig. 11) also suggests that future dome 456 stability modelling should consider diffuse material property heterogeneity within discrete zones comprising the 457 modelled dome. An estimation of the different rock masses potentially involved in different collapse scenarios 458 (e.g., Peruzzetto et al., 2019) could also be refined to integrate the zonation of this dome heterogeneity. Ongoing 459 research is now underway to develop strategies and methods to improve the capacity and resolution of static and 460 functional imagery of volcanic domes (e.g., muon tomography and electrical conductivity), to find upscaling 461 relations between numerical models and the geophysical characterisation scale, and to couple the results with 462 thermo-hydro-mechanical simulations of fluid circulation within a lava dome.

463

#### 464 Concluding remarks

465 The mechanisms considered responsible for dome instability and collapse are numerous and often operate 466 in concert. Although the mechanisms may vary, understanding their impact on stability typically involves 467 incorporating heterogeneity into the models, such as including material with a different rheology in analogue 468 models (van Wyk de Vries et al., 2000; Cecchi et al., 2004) or by lowering the strength of material within a specific 469 zone in numerical models (Harnett et al., 2022). The heterogeneities within these dome stability models are 470 modelled as discrete zones with different, but uniform, material properties. These discrete zones could represent, 471 for example, zones weakened by hydrothermal alteration, zones characterised by different rock types with 472 disparate physical and mechanical properties, or zones characterised by a higher fracture density or pressurised 473 fluids. Here, we consider the influence of diffuse heterogeneity on dome stability, motivated by the high 474 heterogeneity of volcanic rock samples. In other words, although zones of discrete heterogeneities exist, there is 475 also an underlying inherent heterogeneity in the material properties of a dome. Our numerical models show that 476 this inherent diffuse heterogeneity can result in dome instability, highlighting that (1) domes are inherently 477 unstable, (2) dome instability can occur in the absence of obvious discrete weakened and/or fractured zones hence 478 potentially producing volumes of unstable collapse-ready rock masses larger than otherwise expected, (3) when 479 diffuse heterogeneity is considered, the impact of an off-centre weakened zone within the dome is more distributed 480 throughout the dome (when compared to modelling that assumes uniform material properties for the discrete zones 481 comprising the dome), and (4) future numerical modelling should consider heterogeneity within the discrete zones 482 comprising the modelled dome. These results underscore the importance of understanding the range of volumes 483 of material that can be involved in collapse scenarios. Furthermore, an increase in the heterogeneity in lava domes 484 will likely reduce cohesion and, combined with other factors, may enhance the mobility of a collapse debris-485 avalanche, thereby increasing its runout and hazard potential. Understanding the mechanisms that can contribute 486 to dome instability and collapse will help improve monitoring strategies and mitigate the hazards presented by 487 lava domes worldwide.

488

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#### 502 Statements and Declarations

503 The authors declare no financial or non-financial interests that are directly or indirectly related to this work.

504

#### 505 Supplementary Information

506 Our manuscript is accompanied by two Supplementary Information files. Supplementary Information file 1 507 contains additional figures showing equivalent pore diameter, pore aspect ratio, and pore circularity as a function 508 of sample length. Supplementary Information file 2 is an Excel spreadsheet containing the data for Figures 5, 6, 7, 509 and 8.

510

#### 511 Author contributions

512 M. Heap and C. Harnett conceived and developed the idea for this study. Numerical experiments using PFC were 513 performed by C. Harnett. Numerical experiments using RFPA were performed by Z. Heng and T. Xu. Image 514 analysis was performed by T. Nazarbayov. M. Heap and C. Harnett wrote the first draft of the manuscript, with 515 contributions from all authors. All authors read and approved the final manuscript. 516

- 517 Figures
- 518



519

Fig 1. Schematic diagrams explaining the different types of lava dome heterogeneity discussed in this contribution.
(a) Discrete heterogeneity: the lava dome is split into discrete zones (yellow and white) that are characterised by
different, but uniform, physical and mechanical properties. (b) Discrete and diffuse heterogeneity: the lava dome
is split into two discrete zones (dark yellow and grey) characterised by different physical and mechanical properties
(discrete heterogeneity). In this case, the physical and mechanical properties within these zones are also
heterogenous (diffuse heterogeneity).



528 Fig 2. Backscattered scanning electron microscope images of the three dome rocks (H3, H4A, and H19) from La

- 529 Soufrière de Guadeloupe (Eastern Caribbean) analysed in this study. Black void space; greyscale groundmass
- and crystals.
- 531



**Fig. 3.** (a) Uniaxial stress-strain curves from a laboratory experiment performed on an andesite sample (H18; data from Heap et al., 2021b) (black curve) and from a numerical experiment using Rock Failure and Process Analysis (RFPA). The parameters for the model (listed in Table 1) were modified until the modelled curve closely matched the experimental data. Panels (b) and (c) show the distribution of Young's modulus and uniaxial compressive in a numerical sample with a given homogeneity index, m (1.1, 2, 3, 4, 5, 6, 7, and 8).





539 Fig. 4. The distribution of bond cohesion in the Particle Flow Code (PFC) domes characterised by different 540 homogeneity indices, m (the mean strength is 1 MPa in both cases). m = 2 (heterogeneous dome); m = 10541 (homogeneous dome).



Fig. 5. Grey level histograms for three dome rocks from La Soufrière de Guadeloupe. Frequency is the relative frequency. (a) Sample H3. (b) Sample H4A. (c) Sample H19. (d) Synopsis plot containing all the data (note that the scale on x-axis is not the same as for the other panels). Insets show photographs of the 10 mm-diameter samples.



Fig. 6. The variation in coefficient of variation (COV) along the length of the 20 mm-long samples. (a) Sample
H3. (b) Sample H4A. (c) Sample H19. The average COV and the location of the image slice with the highest and
lowest COV are indicated on the plots. Below the plots are the image slices corresponding to the highest and lowest
COV for each sample.





**Fig. 7.** The variation in porosity along the length of the 20 mm-long samples. (a) Sample H3. (b) Sample H4A. (c) Sample H19. The average porosity (from the  $\mu$ CT volume and the laboratory) and the location of the image slice with the highest and lowest porosity are indicated on the plots. Below the plots are the image slices corresponding to the highest and lowest porosity for each sample.





Fig. 8. Results of the 2D numerical experiments performed using Rock Failure and Process Analysis (RFPA). (a) Uniaxial stress-strain curves for numerical samples prepared with the same input parameters, but with different homogeneity indices (Table 1). The homogeneity index, *m*, varies from 8 (the most homogeneous sample) to 1.1 (the most heterogeneous sample). (b) Uniaxial compressive strength as a function of the homogeneity index (for the same numerical experiments shown in panel (a) (data available in Table 2). Inset shows an image of the postfailure numerical sample.



569 Fig. 9. Distribution of bond cohesion in (a) a homogeneous dome (m = 10) and (b) a heterogeneous dome (m = 2).

570 Dome width ~440 m.

571



Fig. 10. Results of the 2D dome simulations (dome width ~440 m) performed using Particle Flow Code (PFC). (a)
A homogeneous dome with a homogeneity index of 10. (b) A heterogeneous dome with a homogeneity index of
The colour indicates the displacement, where blue indicates low displacement and yellow indicates high
displacement.



579

580 Fig. 11. Results of the 2D dome simulations (dome width ~440 m) performed using Particle Flow Code (PFC). 581 Left plots show displacement from 0-1.5 m (yellow indicates higher displacement) and right plots show 582 normalised strain, where darker areas indicate areas of high strain accumulation (dark brown indicates higher 583 strain). (a) Dome with diffuse heterogeneity, as in Fig. 10. This dome is calibrated to unaltered rock as described 584 by Harnett et al. (2022), with a heterogeneity index of m = 2, about a mean contact strength of 84.7 MPa. (b) 585 Dome with a discrete zone of weaker material (both discrete zones have uniform material properties), identical to 586 that presented by Harnett et al. (2022). (c) The combined effect of a discrete zone of weaker material and 587 heterogeneous material properties in all zones. The dome rock is heterogeneous with m = 2 about a mean strength 588 of 84.7 MPa and the weak rock is heterogeneous with m = 2 about a mean strength of 29 MPa. Thumbnail 589 sketches of each scenario are provided in the middle of the modelled results.

## 591 Tables

## 

Property	Value
Homogeneity index	1.1, 2, 3, 4, 5, 6, 7, 8
Mean uniaxial compressive strength (MPa)	147.3
Mean Young's modulus (GPa)	46.0
Poisson's ratio	0.25
Ratio of compressive to tensile strength	10
Friction angle (°)	30

- **Table 1.** Model input parameters for the Rock Failure and Process Analysis (RFPA) modelling.

Homogeneity index, m	Uniaxial compressive strength (MPa)
8 (most homogeneous)	108.9
7	106.7
6	104.3
5	98.4
4	90.8
3	80.8
2	62.3
1.1 (most heterogeneous)	45.5

**Table 2.** Results of the 2D numerical experiments performed using Rock Failure and Process Analysis (RFPA).

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