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# System-Optimal Dynamic Traffic Assignment with partial users control: an analysis of different strategies

Enrico Siri and Paola Goatin

**Abstract**—In the present work, we consider a System Optimum Dynamic Traffic Assignment optimization problem on road networks employing time-varying partial traffic flow control. Depending on the network performance, trajectory control between the relative origin and destination nodes is applied to a variable fraction (“compliant”) of the demand. Network dynamics is derived by applying a Godunov discretization of the well-known Lightwill-Williams-Richards model, where the fundamental flow-density diagram is of the triangular form. At each node, a multi-class priority-based solver handles flow routing according to an aggregate class-density weighted distribution matrix coupled with a priority vector associated to incoming links. The selfish response of the uncontrolled fraction of flows (“non-compliant”) is addressed by updating the class-specific distribution matrices according to changing traffic conditions and consistently with a multinomial Logit random choice model. The goal of the the partial control optimization problem is to globally improve the network congestion level by rerouting a variable fraction of flows over a set of pre-computed routes. The fraction of controlled users varies according to the trade-off between the rerouting effort and the network status improvement. Results on a synthetic network are then presented and discussed.

## I. INTRODUCTION

The problem of traffic assignment (TA), either in static (STA) or dynamic (DTA) settings, has been extensively studied over the last decades by researchers and practitioners with the aim of estimating the traffic patterns emerging from the interaction between the will of individuals grappling with a scarce resource. Scarcity takes the form of congestion. If we assume that users act selfishly, classical STA models provide an estimation of the resulting *user equilibrium* (UE) state [1], which generally leads to inefficient network utilization [2]. On the contrary, if we assume rather that users adopt strategies aiming at a global optimisation, the same model leads to a *system optimal* (SO) allocation prediction. Of course, it is difficult to reach an SO in practice, since users tend to pursue a personal advantage (i.e. minimising their travel time/cost) rather than a common goal. It is therefore interesting to study optimisation and control strategies designed to exogenously induce an efficient state for the network. To this end, a number of *Dynamic Traffic Assignment* (DTA) models have been proposed [3], [4], which, by integrating an explicit flow propagation model, enable the network dynamics to be represented beyond merely equilibrium states and therefore they are particularly suitable for developing SO-DTA strategies [5], [6], [7]. A viable approach is exploiting some incentive

mechanism in order to induce a fraction of drivers, referred to as *compliant*, to adopt globally efficient strategies, while assuming that *non-compliant* users maintain their original strategy [6] or adapt to the new condition, thus leading to a Stackelberg game [8], [9] assuming their reaction been predictable. These problems are usually tackled in a static framework for analytical convenience but at the cost of strongly simplifying assumptions. Yet, practical implementation especially when dealing with a fast evolution, for example in the case of temporary network disruption, often requires a dynamic setting [6], [10].

This paper aims to provide a methodology to tackle a SO-DTA problem with dynamic partial control on the flows incorporating the reaction of non-compliant users in a macroscopic and non-stationary framework. In recent years, macroscopic multi-population models have been developed to represent the interaction between different classes of vehicles such as cars and trucks [11], cars and motorbikes [12], [13] or drivers and autonomous vehicles [14]. Such approaches have then been generalised to road networks [15] with populations identified based on their origin-destination pair [16], a predefined set of paths [6] or class-specific utility functions [17].

In this paper, following [18], [19], we propose a multi-population traffic flow model on networks, where the flow of each population is determined based on a corresponding mass conservation equation on road segments (the standard Lighthill-Williams-Richards (LWR) model [20]). Each population is characterised by specific distribution matrices at junctions that can be predetermined by a controller for compliant users or adjust to traffic conditions according to a multinomial Logit random choice model [21] for non-compliant ones, thus resulting in possibly time-varying split ratios.

Our model allows assessing the effectiveness of system-optimal routing control strategies according to the estimated cost of the control action and under different assumptions regarding the individual behaviour of users. The rest of the paper is organized as follows. In Section II, we present the multi-population traffic flow model. In Section III, we formally define user classes as well as the associated routing strategies, while in Section IV we suggest an implementation on a test network and some numerical results are reported and discussed.

## II. A MULTI-POPULATION TRAFFIC FLOW MODEL ON NETWORKS

The road network is represented by an oriented graph consisting of a set of links  $\mathcal{I} := \{I_\ell\}_{\ell \in \mathcal{L}}$ , each one parameterized by segments  $I_\ell = \pi_\ell(]0, L_\ell[)$ . Let  $\mathcal{J} = \{J_k\}_{k \in K}$  be a set of nodes. We denote by  $Inc(J_k) := \{i \in \mathcal{L} : J_k \in I_i, \pi_i(L_i) = J_k\}$  the set of incoming links for node  $J_k$ , and by  $Out(J_k) := \{j \in \mathcal{L} : J_k \in I_j, \pi_j(0) = J_k\}$  the set of outgoing links. Lastly, let us denote by  $\mathcal{O}, \mathcal{D} \subset \mathcal{J}$  the set of origin and destination nodes respectively.

For each link, a set of conservation equations are defined which, together with suitable transmission conditions at junctions, constitute the macroscopic framework to represent multi-classes flow propagation on the network. More in details, each class density  $\rho^c, c = 1, \dots, N_c$  is characterized by specific, possibly time-dependent, distribution coefficients at junctions, computed accordingly to the class specific routing strategy as outlined in Section III.

### A. Traffic dynamics on links

The traffic dynamics is modelled by a discretized LWR-type model [22], with flow functions of the form  $\rho^c v_\ell(r)$ , where  $r = \sum_{c=1}^{N_c} \rho_{c,h}^c \in [0, R_\ell]$  is the total traffic density, and the velocity  $v_\ell : [0, R_\ell] \rightarrow [0, V_\ell]$  is a non-increasing function such that  $v_\ell(0) = V_\ell$  and  $v_\ell(R_\ell) = 0$ . Furthermore, we assume that exist an unique point  $\hat{r}_\ell \in ]0, R_\ell[$  such that the flow function  $\rho \mapsto \rho v_\ell(\rho)$  is increasing for  $\rho \in [0, \hat{r}_\ell[$  and decreasing for  $\rho \in ]\hat{r}_\ell, R_\ell]$ .

Each link  $I_\ell$  is therefore uniformly divided in  $N_\ell$  cells of length  $\Delta x_\ell = L_\ell/N_\ell$ . Be  $\rho_{\ell,h}^{c,0}, h = 1, \dots, N_\ell, c = 1, \dots, N_c$  the initial density distribution then, at each time step  $t^\nu = \nu \Delta t, \nu \in \mathbb{N}$ , we update the class-specific traffic density using the conservative scheme

$$\rho_{\ell,h}^{c,\nu+1} = \rho_{\ell,h}^{c,\nu} - \frac{\Delta t}{\Delta x_\ell} \left( \frac{\rho_{\ell,h}^{c,\nu}}{r_{\ell,h}^\nu} F_{\ell,h}^\nu - \frac{\rho_{\ell,h-1}^{c,\nu}}{r_{\ell,h-1}^\nu} F_{\ell,h-1}^\nu \right), \quad (1)$$

for  $h = 2, \dots, N_\ell - 1$ , where  $F_{\ell,h}^\nu$  is the *Godunov* flow [23] corresponding to  $f_\ell(r) = r v_\ell(r)$  and defined by

$$F_{\ell,h}^\nu = F_\ell(r_{\ell,h}^\nu, r_{\ell,h+1}^\nu) := \min \{ D_\ell(r_{\ell,h}^\nu), S_\ell(r_{\ell,h+1}^\nu) \} \quad (2)$$

where  $D_\ell(r) = f_\ell(\min\{r, \hat{r}_\ell\})$  and  $S_\ell(r) = f_\ell(\max\{r, \hat{r}_\ell\})$  are respectively the (total) demand and supply functions.

Regarding the first and last cells of a link, the density is updated according to the incoming  $\bar{\gamma}_{\ell,1}^{c,\nu}$  and outgoing  $\bar{\gamma}_{\ell,N_\ell}^{c,\nu}$  flows, which in turns depend on the upstream and downstream node dynamics, see Section II-B, consequently we define

$$\rho_{\ell,1}^{c,\nu+1} = \rho_{\ell,1}^{c,\nu} - \frac{\Delta t}{\Delta x_\ell} \left( \frac{\rho_{\ell,1}^{c,\nu}}{r_{\ell,1}^\nu} F_{\ell,1}^\nu - \bar{\gamma}_{\ell,1}^{c,\nu} \right), \quad (3)$$

$$\rho_{\ell,N_\ell}^{c,\nu+1} = \rho_{\ell,N_\ell}^{c,\nu} - \frac{\Delta t}{\Delta x_\ell} \left( \bar{\gamma}_{\ell,N_\ell}^{c,\nu} - \frac{\rho_{\ell,N_\ell-1}^{c,\nu}}{r_{\ell,N_\ell-1}^\nu} F_{\ell,N_\ell-1}^\nu \right). \quad (4)$$

In order to guarantee the stability of the scheme (1)-(4), the standard Courant-Fredrichs-Lewy (CFL) conditions [24]

must be satisfied, therefore

$$\Delta t \leq \frac{\Delta x_\ell}{\max_\ell \|f'_\ell\|_{L^\infty}([0, R_\ell])}. \quad (5)$$

### B. Node dynamics

At each junction  $J_k$ , let the (possibly time dependent) distribution matrices relative to each class  $\rho^c, c = 1, \dots, N_c$  be defined as follows

$$A_k^{c,\nu} = \left\{ a_{k,ji}^{c,\nu} \right\}_{i,j}, \quad i \in Inc(J_k), j \in Out(J_k).$$

To ensure flow conservation the following must hold

$$a_{k,ji}^{c,\nu} \geq 0 \quad \text{for any } i, j \quad (6)$$

$$\sum_{j \in Out(J_k)} a_{k,ji}^{c,\nu} = 1 \quad \text{for all } i \in Inc(J_k). \quad (7)$$

Based on the class-specific matrices, it is now possible to define the total traffic distribution matrix as

$$A_k^\nu := \{a_{k,ji}^\nu\}, \quad a_{k,ji}^\nu := \sum_{c=1}^{N_c} a_{k,ji}^{c,\nu} \frac{\rho_{i,N_i}^{c,\nu}}{r_{i,N_i}^\nu} \quad (8)$$

where  $\rho_{i,N_i}^{c,\nu}/r_{i,N_i}^\nu$  is the class density ratio in the last cells of incoming link  $i \in Inc(J_k)$ .

By adopting any junction rules [25] exploiting the matrices  $A_k^\nu$ , it is then possible to compute the total traffic flows exiting an incoming link

$$\bar{\gamma}_{i,N_i}^\nu, \quad i \in Inc(J_k), \quad (9)$$

and redistribute them over the individual classes

$$\bar{\gamma}_{j,1}^{c,\nu} = \sum_{i \in Inc(J_k)} a_{k,ji}^{c,\nu} \bar{\gamma}_{i,N_i}^{c,\nu}, \quad j \in Out(J_k), \quad (10)$$

$$\bar{\gamma}_{i,N_i}^{c,\nu} = \frac{\rho_{i,N_i}^{c,\nu}}{r_{i,N_i}^\nu} \bar{\gamma}_{i,N_i}^\nu, \quad i \in Inc(J_k), \quad (11)$$

providing the boundary flows necessary in (3) and (4) respectively. In this paper, we make use of the *Priority based Riemann Solver* introduced in [26] maximizing the through flow and capable of handling an arbitrary configuration of incoming and outgoing links while accounting for priorities defined for the incoming ones.

### C. Network boundary conditions

In order to obtain a complete traffic flow model, it is necessary to establish boundary conditions at the origin and destination nodes.

Let  $Fin_o^\nu = \sum_{c=1}^{N_c} fin_o^{c,\nu}$  be the total amount of incoming flows willing to enter the network at origin node  $J_o$  at time  $t = t^\nu$  which depend on class-specific incoming flows  $fin_o^{c,\nu}$ .

In any given time the residual capacity of the first cells of a link  $\ell \in Out(J_o)$  might be insufficient to accommodate all the  $Fin_o^\nu$ . To guarantee flow conservation in case of a saturated network, it is sufficient to add a *buffer* associated to each origin node, modelled as a cell of infinite capacity. While solving the flow conservation issue, this approach may cause a consistent violation of the FIFO condition, commonly required in such models. This is because, once

into a cell, it is assumed that the vehicles are uniformly distributed regardless of the arrival order. One way to alleviate this problem is to replace the single buffer with a series of buffers with limited capacity whose number is dynamically controlled in order to accommodate all the necessary incoming flows. A buffer sequence is then treated like any other incoming link and the entering flow out of the upstream first buffer

$$\bar{\gamma}_o^\nu, \quad J_o \in \mathcal{O} \quad (12)$$

is managed by a solver and redistributed over the classes on each outgoing links

$$\bar{\gamma}_o^{c,\nu} = \frac{l_{o,1}^{c,\nu}}{l_{o,1}^\nu} \bar{\gamma}_o^\nu \quad \bar{\gamma}_{j,1}^{c,\nu} = a_{o,j1}^{c,\nu} \bar{\gamma}_o^{c,\nu}, \quad (13)$$

where  $l_{o,1}^c$  and  $l_{o,1}^{c,\nu}$  are total and  $c$  class number of vehicles in the first buffer, respectively. In [6], the algorithm applied to handle the load of the demand into the buffers, their allocation and the no-lag inter-buffer flow propagation is presented and discussed in details.

Regarding the outflow boundary conditions, it is sufficient to define for each destination node a *sink* as a cell of infinite capacity with, possibly, a limit on the amount of flow it can accommodate per unit of time. Therefore, let  $F_{out,d}^\nu$  be the boundary condition for destination node  $J_d$  at time instant  $t = t^\nu$ . Therefore, a solver can be applied to compute the total outflows leaving the network

$$\bar{\gamma}_{i,N_i}^\nu, \quad i \in Inc(J_d), \quad (14)$$

and redistribute it among the classes

$$\bar{\gamma}_{i,N_i}^{c,\nu} = \frac{\rho_{i,N_i}^{c,\nu}}{r_{i,N_i}^\nu} \bar{\gamma}_{i,N_i}^\nu, \quad \bar{\gamma}_d^{c,\nu} = \sum_{i \in Inc(J_d)} \alpha_{d,1i}^{c,\nu} \bar{\gamma}_{i,N_i}^{c,\nu}. \quad (15)$$

### III. ADMISSIBLE CLASS ROUTING STRATEGIES

Each class  $c = 1, \dots, N_c$  is represented by a tuple  $(J_{oc}, J_{dc}, \mathcal{A}^c)$  with  $J_{oc} \subseteq \mathcal{O}$ ,  $J_{dc} \subseteq \mathcal{D}$  and where the class-specific set of distribution matrices  $\mathcal{A}^c = \{A_k^{c,\nu}\}$  represent implicitly the routing strategy adopted by the class.

A routing strategy  $\mathcal{A}^c$  is defined as an *admissible strategy* for the class  $c$  if it guarantee flow conservation while preventing flow dispersion on paths that do not lead to destination node  $J_{dc}$ . Thus, let us define for each class  $c$  and each node  $J_k$  the sets  $p_{k,s}^c = [J_k, \dots, J_{dc}]$  and  $\mathcal{P}_k^c = \{p_{k,s}^c\}$ , where  $s = 1, \dots, P_k^c$  while  $p_{k,s}^c$  is a path connecting node  $J_k$  to  $J_{dc}$  defined as a ordered sequence of nodes such that for any adjacent pair  $J_v, J_u \in p_{k,s}^c$ ,  $\exists \ell \in \mathcal{L} : \pi_\ell(0) = J_v$  and  $\pi_\ell(L_\ell) = J_u$ . Consequently  $\mathcal{P}_k^c$  is the set of all available paths connecting  $J_k$  to  $J_{dc}$ .

Therefore, a routing strategy  $\mathcal{A}^c$  is admissible if at each node  $J_k$  such that  $\mathcal{P}_k^c \neq \emptyset$  the following holds

$$a_{k,ji}^{c,\nu} \leq \sum_{s=1}^{P_k^c} \delta_{k,j_s} \quad (16)$$

together with (6) and (7), where

$$\delta_{k,j_s}^c = \begin{cases} 1 & \text{if } \pi_j(L_j) \in p_{k,s}^c \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

The above conditions binds the flow of a class  $c$  to be routed only along paths leading to its destination, assuming at least one such a path exists for each origin-destination pair. Therefore, since conditions (6)-(7) and (16)-(17) must holds also in origin nodes, it follows that as long as an admissible strategy is applied, flows of a class generated at the respective origin node  $J_{oc}$  can only move along paths connecting  $J_{oc}$  to  $J_{dc}$ .

The only remaining issue may come from the initial conditions if the initial densities of a class  $c$  are positive on a link from which  $J_{dc}$  cannot be reached. Without any loss of generality, let us define as *admissible initial conditions* any initial density pattern on links at time instant  $t^0 = 0$  such that

$$\rho_{\ell,h}^{c,0} = \begin{cases} 0 & \text{if } \mathcal{P}_k^c = \emptyset, J_k = \pi_\ell(L_\ell), \\ \bar{\rho}_{\ell,h}^c \geq 0 & \text{otherwise.} \end{cases} \quad (18)$$

Let us now distinguish between two class sets: *compliant* and *non-compliant* users where the former includes those classes of users whose routing strategies consist in following unique pre-determined paths while the latter user classes relying on a selfish, possibly time-varying, routing strategy.

#### A. Compliant users

For a compliant class, the associated strategy  $\mathcal{A}^c$  is characterized by a time-invariant set of distribution matrices satisfying (6)-(7) and (16)-(17) which prefigures a routing along a unique path  $\bar{p}^c \in \mathcal{P}_{oc}^c$ , thus for each  $J_k \in \bar{p}^c$

$$a_{k,ji}^{c,\nu} = \begin{cases} 1 & \text{if } \pi_j(L_j) \in \bar{p}^c, \\ 0 & \text{otherwise.} \end{cases} \quad (19)$$

We then assume that for each compliant class  $c$

$$\rho_{\ell,h}^{c,0} = 0, \quad h = 1, \dots, N_\ell, \ell \in \mathcal{L}. \quad (20)$$

The strategy defined in (19) is admissible and coupled with admissible initial conditions (20) binds compliant flows to travel only along  $\bar{p}^c$ .

#### B. Non-compliant users

A non-compliant class is characterized by a set  $\mathcal{A}^c$  of, possibly time-varying, distribution matrices that depend on some function of the network state (typically travel times). Within the scope of the present paper, non-compliant users adopt a stochastic wardropian behaviour in an attempt to minimize their perceived travel times based on a multinomial Logit distribution [21]. Let  $z_{k,s}^{c,\nu}$  be the choice probability associated with path  $p_{k,s}^c$  computed from

$$z_{k,s}^{c,\nu} = \frac{1}{1 + \sum_{y \neq s} e^{-\theta^c (d_{k,s}^\nu - d_{k,y}^\nu)}} \quad (21)$$

where  $d_{k,s}^\nu$  is the actual travel time between node  $J_k$  and destination node  $J_d^c$  and  $\theta^c$  can be interpreted as the  $c$ -class specific user sensitivity coefficient to marginal gains

(or losses) of travel time. Thus, the associated non-compliant class-specific distribution coefficient at each node such that  $\mathcal{P}_k \neq \emptyset$  can be derived

$$\alpha_{k,ji}^{c,\nu} = \sum_{s=1}^{P_k^c} z_{k,s}^{c,\nu} \cdot \delta_{k,js}, \quad (22)$$

where  $\delta_{k,js}$  is the link-path incidence coefficient defined in (17). The strategy given in (22) is admissible. We then smooth out the distribution matrices update as follows

$$A_k^{c,\nu} = \omega^c \hat{A}_k^{c,\nu} + (1 - \omega^c) A_k^{c,\nu-1}, \quad (23)$$

where  $\hat{A}_k^{c,\nu}$  is the distribution matrix associated to class  $c$  at node  $J_k$  computed applying (22) and  $\omega^c \in [0, 1]$ . Clearly, if both  $\hat{A}_k^{c,\nu}$  and  $A_k^{c,\nu-1}$  are admissible, then necessarily  $A_k^{c,\nu}$  is admissible too.

#### IV. NUMERICAL RESULTS

The traffic flow model presented in this paper is implemented in Python 3.11 making use of the following main libraries: networkx [27], numpy [28], pandas [29] and scipy [30].

##### A. Optimization Problem

The control action is implemented by converting a possibly time varying fraction of non-compliant flows into compliant ones. We underline how the design of appropriate incentive mechanisms allying selfish behaviour to a global goal is beyond the scope of this work. The paths available to each non-compliant class are computed during the initialization phase and the associated path-related compliant classes are then initialized accordingly.

The optimal control vector is the result of a finite horizon optimization problem where the objective function can be subdivided into three main components: the total travel time of all vehicles travelling through the network

$$H^1(\bar{u}) = \sum_{\nu=1}^{N_\nu} \sum_{\ell \in \mathcal{L}} \sum_{h=1}^{N_\ell} r_{\ell,h}^\nu(\bar{u}) \Delta x_\ell \Delta t, \quad (24)$$

the total travel time of all vehicles waiting in the buffers

$$H^2(\bar{u}) = \sum_{\nu=1}^{N_\nu} \sum_{J_o \in \mathcal{O}} \sum_{b=1}^{B_o} l_{o,b}^\nu(\bar{u}) \Delta t, \quad (25)$$

and finally a cost associated to the control action

$$H^3(\bar{u}) = \Phi \sum_{\nu=1}^{N_\nu} \sum_{J_o \in \mathcal{O}} \sum_{c=1}^{N_c} \sum_{s=1}^{P_o^c} u_{o,s}^{c,\nu} \text{fin}_o^{c,\nu} \Delta t, \quad (26)$$

where  $N_\nu$  is the number of simulated instant,  $\bar{u} = \{u_{o,s}^{c,\mu}\}$  is the control vector and  $u_{o,s}^{c,\mu}$  represents the fraction of class  $c$  non-compliant users having  $J_o$  as origin node converted to the relative compliant ones associated to path  $p_o^c \in \mathcal{P}_o^c$  at time instant  $t = t^\nu$ . Then, in (26) coefficient  $\Phi$  represents a per-vehicle control cost which makes  $H^3(\bar{u})$  homogeneous with (24) and (25). As a reference, in the following we refer to the sum of  $H^1$  and  $H^2$  simply as total travel time ( $TTT$ ).

Therefore, the resulting finite horizon optimization problem is defined as follows

$$\begin{aligned} & \underset{\bar{u}}{\text{minimize}} && H(\bar{u}) = H^1(\bar{u}) + H^2(\bar{u}) + H^3(\bar{u}) \\ & \text{subject to} && u_{o,s}^{c,\mu} \geq 0, \\ & && \sum_{s \in \mathcal{P}_o^c} u_{o,s}^{c,\mu} \leq 1. \end{aligned} \quad (27)$$

It should be noted that the convexity of  $H(\bar{u})$  is not guaranteed. Therefore, we employed a *differential heuristic global optimization algorithm* [31].

##### B. Synthetic Network

We implemented the model on a Braess-like test network as illustrated in Fig. 1 where 1 and 8 are the origin and destination nodes respectively. All the links, with the exception of  $1 \rightarrow 2$  and  $7 \rightarrow 8$ , have the same functional characteristics namely  $\hat{r}_\ell = 82$  [veh/km],  $R_\ell = 100$  [veh/km] and  $L_\ell = 5$  [km], while  $R_{(1,2)} = R_{(7,8)} = 300$  [veh/km] so as to emphasise any potential congestion. Each link is discretized into 5 cells with  $\Delta x = 1$  [km] leading to  $\Delta t = 0.0125$  [h] ( $\approx 45$ [sec]). Two hours are simulated for a total of 160 time steps.

We then define a single non-compliant class whose users travelling from node 1 to node 8 adopt a routing strategy as described in Section III with  $\theta^c = 0.5$  and  $\omega^c = 0.2$ . The associated demand is 5000 [veh/h] up to  $\nu = 45$  time instants ( $\approx 33$ [min]) and 0 afterwards. This gives each user the time to leave the network, thereby emphasising the effectiveness of a routing strategy over another. The available routes connecting the origin to the destination are  $p_1 = [1, 2, 3, 5, 7, 8]$ ,  $p_2 = [1, 2, 4, 5, 7, 8]$  and  $p_3 = [1, 2, 4, 6, 7, 8]$ .

Firstly, the stochastic equilibrium state is computed when no control is applied and the relative distribution matrices are shown in Fig. 1. The users are equally distributed over the three available paths, as they are perfectly equivalent. We consider then a capacity loss on the fifth cell of link  $4 \rightarrow 5$  occurring after 15 time-steps ( $\approx 11$ [min]). As shown in Fig. 2, after a transitory phase of about 30 time-instants ( $\approx 22.5$ [min]) almost all users avoid using path  $p_2$  ( $TTT = 1556.4$ ).

It is now interesting to evaluate the variations of  $TTT$ , shown in Fig. 3, when one of the two coefficients characterising user routing strategy ( $\theta$  or  $\omega$ ) varies, while the other is

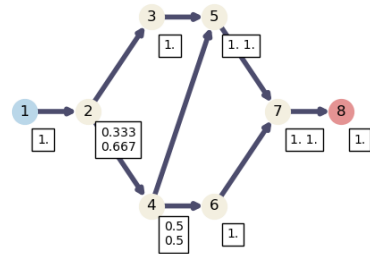


Fig. 1. Braess-like synthetic network. Next to each node are the distribution matrices at the equilibrium.

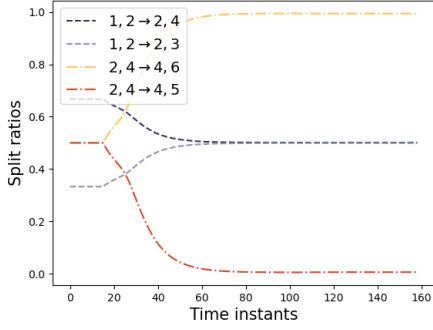


Fig. 2. Evolution of non-compliant split ratios at nodes 2 and 4.

kept fixed. As it might be expected, in both cases the  $TTT$  decreases when the responsiveness ( $\omega$ ) or the sensitivity to variations in travel time of the users ( $\theta$ ) increases. However, by far the most significant variation occurs when the coefficients are very close to zero. In the case of  $\omega = 0$  or  $\theta = 0$  users do not adapt their routing strategy to changing conditions and the three routes still have equal probability of being chosen, regardless of the amount of congestion.

Let us now consider the opportunity to control a fraction of users. Thus, 3 compliant classes are defined, each associated with one of the previously defined paths. The control action is updated every 15 time steps ( $\approx 11[\text{min}]$ ) until the traffic demand stops, for a total of 12 control variables. It is reasonable to assume that the greatest improvement to the network is achieved when all vehicles are controlled. However, such a scenario is far from being applicable in a realistic setting. Here, the parameter  $\Phi$  represents a hypothetical cost associated with the control of a single vehicle. Then, it is interesting to evaluate the effectiveness of the control action as  $\Phi$  varies. Controlling only a fraction of the users presents the challenge to account for the reaction of the non-compliant ones. Three scenarios are therefore considered: in *scenario 1* users continue to use the strategy adopted at equilibrium even after the disruption while this information is exploited in the optimization; in *scenario 2* users apply a reactive strategy while the optimization is still based on the information at the equilibrium; in *scenario 3* the optimization is based on the

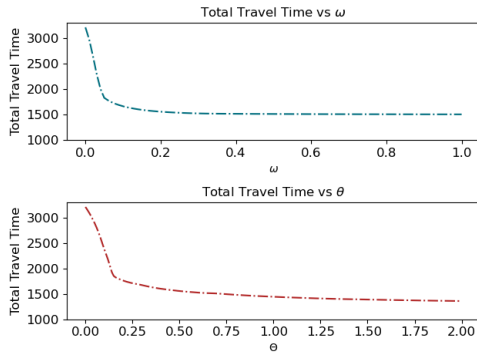


Fig. 3. Total Travel Time vs  $\omega$  and  $\Theta$

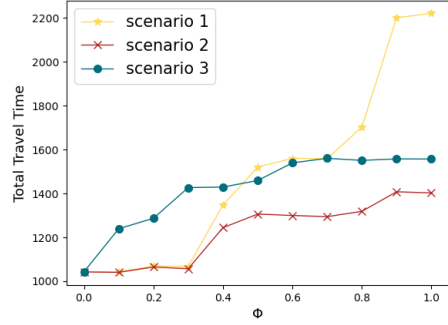


Fig. 4. Total Travel Time vs  $\Phi$ .

actual adaptive strategy employed by the users. In Fig. 4 the variation of  $TTT$  against  $\Phi$  in the three scenarios is shown while in Fig. 5 the total number of controlled vehicles for each value of  $\Phi$  is reported as a reference. Firstly, it should be noted how the effectiveness of the control is the same across all the scenarios ( $TTT \approx 1040$ ) when  $\Phi = 0$ , i.e. when almost all vehicles are controlled and the assumptions about non-compliant drivers are irrelevant. Regarding scenario 1, as the cost of control increases fewer vehicles are rerouted (for  $\Phi > 0.3$ ) and as non-compliant drivers apply a non-adaptive strategy despite the disruption, network conditions deteriorate rapidly, as expected. The comparison between scenario 2 and scenario 3, on the other hand, is more interesting. In Fig. 4 it can be seen that scenario 2 produces a consistently lower  $TTT$  compared to scenario 3. Apparently, employing incomplete or inaccurate information results in better network efficiency. This is because in scenario 2 the responsiveness of users is neglected and the same massive number of vehicles, like in scenario 1, is controlled as shown in Fig. 5. However, this results in a significantly higher total cost of control, i.e.  $H^3$ , leading to a much higher overall cost, i.e.  $H$ , as shown in Fig. 6. Conversely, providing the right assumptions enables a balanced rerouting in terms of network benefit and control effort, leading to a significantly lower overall cost for scenario 3.

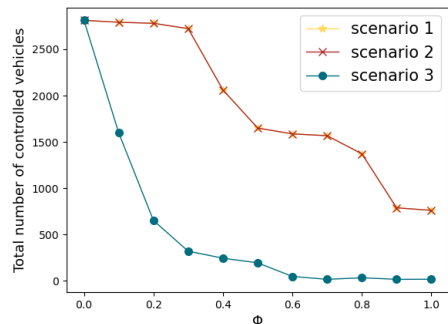


Fig. 5. Number of controlled vehicles vs  $\Phi$ .

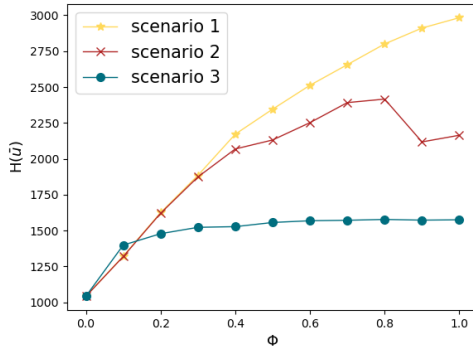


Fig. 6.  $H(\bar{u})$  vs  $\Phi$ .

## V. CONCLUSIONS

In the present paper, we proposed a dynamic macroscopic multi-population traffic model for road networks. The model distinguishes between non-compliant users, following a predetermined path, and compliant users, applying a routing strategy to minimize their own travel cost. The study evaluates how network performance depends on the behavior of non-compliant drivers and proposes a finite horizon optimization problem to mitigate performance degradation after a disruptive event. The proposed solution involves converting dynamically a portion of non-compliant flows into compliant ones. The results indicate that even with limited information on non-compliant user reactions, the control strategy is effective, though it may require a disproportionate effort. Future work aims to test the model on more complex networks and consider additional scenarios.

## REFERENCES

- [1] J. G. Wardrop, "Some theoretical aspects of road traffic research," *Proceedings of the Institution of Civil Engineers*, vol. 1, no. 3, pp. 325–362, 1952.
- [2] J. Zhang, S. Pourazarm, C. G. Cassandras, and I. C. Paschalidis, "The price of anarchy in transportation networks: Data-driven evaluation and reduction strategies," *Proceedings of the IEEE*, vol. 106, no. 4, pp. 538–553, 2018.
- [3] D. K. Merchant and G. L. Nemhauser, "A model and an algorithm for the dynamic traffic assignment problems," *Transportation science*, vol. 12, no. 3, pp. 183–199, 1978.
- [4] D. K. Merchant and G. L. Nemhauser, "Optimality conditions for a dynamic traffic assignment model," *Transportation Science*, vol. 12, no. 3, pp. 200–207, 1978.
- [5] Z. S. Qian, W. Shen, and H. Zhang, "System-optimal dynamic traffic assignment with and without queue spillback: Its path-based formulation and solution via approximate path marginal cost," *Transportation research part B: methodological*, vol. 46, no. 7, pp. 874–893, 2012.
- [6] S. Samaranyake, J. Reilly, W. Krichene, M. L. Delle Monache, P. Goatin, and A. Bayen, "Discrete-time system optimal dynamic traffic assignment (SO-DTA) with partial control for horizontal queuing networks," *Transportation Science*, 2018.
- [7] M. Carey and D. Watling, "Dynamic traffic assignment approximating the kinematic wave model: System optimum, marginal costs, externalities and tolls," *Transportation Research Part B: Methodological*, vol. 46, no. 5, pp. 634–648, 2012.
- [8] C. Swamy, "The effectiveness of stackelberg strategies and tolls for network congestion games," *ACM Transactions on Algorithms (TALG)*, vol. 8, no. 4, pp. 1–19, 2012.
- [9] W. Krichene, J. D. Reilly, S. Amin, and A. M. Bayen, "Stackelberg routing on parallel networks with horizontal queues," *IEEE Transactions on Automatic Control*, vol. 59, no. 3, pp. 714–727, 2014.

- [10] S. Samaranyake, J. Reilly, W. Krichene, J. B. Lespiau, M. L. D. Monache, P. Goatin, and A. Bayen, "Discrete-time system optimal dynamic traffic assignment (SO-DTA) with partial control for horizontal queuing networks," in *2015 American Control Conference (ACC)*, pp. 663–670, July 2015.
- [11] S. Benzioni-Gavage and R. M. Colombo, "An  $n$ -populations model for traffic flow," *European Journal of Applied Mathematics*, vol. 14, no. 05, pp. 587–612, 2003.
- [12] R. Nair, H. S. Mahmassani, and E. Miller-Hooks, "A porous flow approach to modeling heterogeneous traffic in disordered systems," *Transportation Research Part B: Methodological*, vol. 45, no. 9, pp. 1331–1345, 2011.
- [13] S. Gashaw, P. Goatin, and J. Härrri, "Modeling and analysis of mixed flow of cars and powered two wheelers," *Transportation Research Part C: Emerging Technologies*, vol. 89, pp. 148–167, 2018.
- [14] M. W. Levin and S. D. Boyles, "A multiclass cell transmission model for shared human and autonomous vehicle roads," *Transportation Research Part C: Emerging Technologies*, vol. 62, pp. 103 – 116, 2016.
- [15] J. Van Lint, S. Hoogendoorn, and M. Schreuder, "Fastlane: New multiclass first-order traffic flow model," *Transportation Research Record: Journal of the Transportation Research Board*, no. 2088, pp. 177–187, 2008.
- [16] M. Garavello and B. Piccoli, "Source-destination flow on a road network," *Commun. Math. Sci.*, vol. 3, no. 3, pp. 261–283, 2005.
- [17] N. Zheng, G. R erat, and N. Geroliminis, "Time-dependent area-based pricing for multimodal systems with heterogeneous users in an agent-based environment," *Transportation Research Part C: Emerging Technologies*, vol. 62, pp. 133–148, 2016.
- [18] A. Festa, P. Goatin, and F. Vicini, "Navigation system based routing strategies in traffic flows on networks." working paper or preprint, Mar. 2023.
- [19] A. Festa and P. Goatin, "Modeling the impact of on-line navigation devices in traffic flows," in *2019 IEEE 58th Conference on Decision and Control (CDC)*, pp. 323–328, 2019.
- [20] M. J. Lighthill and G. B. Whitham, "On kinematic waves II. a theory of traffic flow on long crowded roads," *Proc. R. Soc. Lond. A*, vol. 229, no. 1178, pp. 317–345, 1955.
- [21] R. B. Dial, "A probabilistic multipath traffic assignment model which obviates path enumeration," *Transportation research*, vol. 5, no. 2, pp. 83–111, 1971.
- [22] C. F. Daganzo, "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory," *Transportation Res. Part B*, vol. 28, no. 4, pp. 269–287, 1994.
- [23] S. K. Godunov, "A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics," *Mat. Sb. (N.S.)*, vol. 47 (89), pp. 271–306, 1959.
- [24] R. Courant, K. Friedrichs, and H. Lewy, " ber die partiellen Differenzgleichungen der mathematischen Physik," *Math. Ann.*, vol. 100, no. 1, pp. 32–74, 1928.
- [25] M. Garavello, K. Han, and B. Piccoli, *Models for vehicular traffic on networks*, vol. 9. American Institute of Mathematical Sciences (AIMS), Springfield, MO, 2016.
- [26] M. L. Delle Monache, P. Goatin, and B. Piccoli, "Priority-based Riemann solver for traffic flow on networks," *Commun. Math. Sci.*, vol. 16, no. 1, pp. 185–211, 2018.
- [27] A. Hagberg, P. Swart, and D. S. Chult, "Exploring network structure, dynamics, and function using networkx," tech. rep., Los Alamos National Lab.(LANL), Los Alamos, NM (United States), 2008.
- [28] C. R. Harris, K. J. Millman, S. J. van der Walt, R. Gommers, P. Virtanen, D. Cournapeau, E. Wieser, J. Taylor, S. Berg, N. J. Smith, R. Kern, M. Picus, S. Hoyer, M. H. van Kerkwijk, M. Brett, A. Haldane, J. F. del R o, M. Wiebe, P. Peterson, P. G erard-Marchant, K. Sheppard, T. Reddy, W. Weckesser, H. Abbasi, C. Gohlke, and T. E. Oliphant, "Array programming with NumPy," *Nature*, vol. 585, pp. 357–362, Sept. 2020.
- [29] T. pandas development team, "pandas-dev/pandas: Pandas," Feb. 2020.
- [30] T. S. development team, "SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python," *Nature Methods*, vol. 17, pp. 261–272, 2020.
- [31] R. Storn and K. Price, "Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces," *Journal of global optimization*, vol. 11, no. 4, p. 341, 1997.