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# Robust Passenger-Centric Road Traffic Assignment based on Alternative Paths

Jean-Claude Lebègue<sup>1</sup>, Daniel Delahaye<sup>2</sup>, Aurélie Peuaud<sup>3</sup> and Jacco Hoekstra<sup>4</sup>

**Abstract**—Accidents, bad weather, and temporary construction can disrupt road traffic. A driver’s original route can be disrupted and delayed by several minutes or even hours. How can traffic be robust to such disruptions? In this paper, we propose a static robust user traffic assignment based on alternative paths along routes. The bi-objective model of the problem minimizes the generalized cost and the robustness of the paths used to assign the passengers. The results show that 10% of robustness is sufficient to compensate for its cost and absorb some of the delays caused by network disruptions.

## I. INTRODUCTION

Moving people from one place to another has never been easier, thanks to multiple transportation options. The more integrated a transportation system is in a city, the more people move from the city center to the suburbs where housing prices are lower. A good quality of transport services helps to reduce the use of cars, which helps to reduce air pollution.

The quality of a transportation system depends on its robustness. Disruptions are sure to happen in our transportation networks. They are of different kinds, ranging from accidents and infrastructure breakdowns that take a few hours or days to recover from, to natural disasters (eruption, snow, flooding...) that can last several weeks. These disruptions make the networks partially unavailable for a long time. If there are no alternatives to the damaged infrastructure, people become isolated. It becomes difficult to rescue them; the options to run away from a risk area are limited... Because of the societal risks involved, robustness has become a challenge in the transportation industry.

The definition of robustness has been widely debated in the vulnerability analysis community. The word "vulnerability" is preferred to "robustness" in the literature. In this paper, the two terms are considered as opposites. For [Khaled et al.(2015)Khaled, Jin, Clarke, and Hoque], vulnerability is the remaining performance of a system during a failure. [D’este et al.(2003)D’este, , and Taylor] define vulnerability as an attribute that characterizes the performance of a transportation system in the case of random scarce events capable of reducing the capacity of an infrastructure element or group of elements. [Berdica(2002)] considers vulnerability as the susceptibility to failures that can result in a significant degradation of network services. [Besinovic(2020)] defines

robustness as the ability to mitigate the daily delays caused by failures. For [Husdal(2005)], vulnerability is the tendency of a transportation network to be ineffective during critical failures. [Taylor and D’Este(2007)] defines vulnerability in terms of accessibility. A network node is vulnerable if the loss (or significant degradation) of a small number of links significantly reduces the accessibility of the node, as measured by a standard accessibility index.

The problem tackled in this work is the following: given an empty road transportation network, how can we assign passengers to minimize the impact of potential disruptions? The question is twofold: firstly, it consists in identifying attributes to characterize criticality and then combining them into a model that describes the vulnerability.

Several vulnerability models have already been developed in the literature. These models may be split into two groups. The approach followed by the first group is based on transportation theory whereas the models of the other group are based on topological properties. Seldom models bridge both approaches over. In this paper, we are proposing a new robustness measure based on alternative paths along passenger trips. The model is passenger-centric and combines topology and transportation network approaches.

The paper is organized as follows: Section II presents an overview of robustness models in the literature. Then Section III presents the mathematical formulation of the robustness model, and Section IV presents the results of several robust assignment applications to the Sioux Falls road network. Finally, we summarize the work and conclude the paper in Section V.

## II. RELATED WORK

The models of robustness in transportation networks are numerous. The section is organized according to the two main approaches used in the literature, which are transportation and topological models.

### A. Transportation robustness models

Robustness models based on transportation theory are simulation oriented. Usually, the models are composed of the evaluation of a measure before and after removing transportation elements in the network. The most famous measure of this kind is the network robustness index (NRI) which is the difference in system assignment cost before and after removing a link [Mattsson and Jenelius(2015)], [Jafino et al.(2020)Jafino, Kwakkel, and Verbraeck]. The robustness is characterized by the increase in total cost done by link removal. [Voltes-Dorta et al.(2017)Voltes-Dorta,

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Rodriguez-Deniz, and Suau-Sanchez] use this approach to measure the vulnerability of the European air transportation network by the delay generated by the closure of the main airports. [Pien et al.(2015)Pien, Han, Shang, Majumdar, and Ochieng] define a vulnerability model for air transportation networks. The model quantifies the impact on the maximum network flow for a wide range of capacity reductions at a node. [Jenelius et al.(2006)Jenelius, Petersen, and Mattsson] develop two models for road vulnerability analysis. One global demand-weighted and one relative for importance and exposure. The main difference between both models is that importance is deterministic whereas exposure is stochastic. Importance measures the change in weighted total travel cost between all OD pairs due to disruption. Exposure quantifies the impact of disruptions experienced by all the users of the transportation network. [Erath et al.(2009)Erath, Birdsall, Axhausen, and Hajdin] suggest a model with three components: additional travel time, additional travel distance, and additional accident cost. [Luathep et al.(2011)Luathep, Sumalee, Ho, and Kurauchi] use the variation in topological accessibility caused by a breakdown as a measure of vulnerability. Finally, [Xu et al.(2017)Xu, Chen, and Yang] find critical elements by resolving a bi-level MILP problem where the master problem optimizes the traffic assignment and the slave minimizes the connectivity.

### B. Topological robustness models

Topological models take root in complex graph theory. They are generally only based on graph properties. The most famous models of this kind are called centrality measures [Héger and Bohác(2014)], [Riveros and Salas(2020)], [Bröhl and Lehnertz(2019)], [Zhang and Luo(2017)]. They came from social network analysis and quantifies to which extent the elements of a graph are central. The most commonly used in vulnerability analysis are degree, closeness, eigenvector, and finally betweenness centrality. The centrality degree quantifies to which extent a node is accessible. Closeness centrality measures how close an element is to all the other nodes. Eigenvector centrality can be seen as the influence of an element in the network. Betweenness quantifies how many times a node acts as a bridge along the shortest path between two other nodes.

Among these measures, betweenness centrality seems to be the reference. It is either used for comparison and validation of newly defined models [Pien et al.(2015)Pien, Han, Shang, Majumdar, and Ochieng] or used as a part of these models. [E. Yu and Zhao(2018)] design a link robustness model based on betweenness centrality, node degree, and clique size. Computing clique size is time-consuming and not applicable to large-scale networks. Three other models were used for comparison: the Jaccard coefficient, bridgeness index, and reachability index. The Jaccard coefficient is a statistical measure used to compare the neighbourhood of the links' nodes. The Bridgeness index estimates the extent to which a link acts as a bridge between cliques. Reachability measures the accessibility of the nodes in the network after a link disruption. [I. Kivimäki and Saerens(2016)] define two

betweenness centrality models based on randomized shortest paths. These models can converge to the betweenness centrality depending on the parameters.

However, robustness models are not limited to centrality indices. [Vodák et al.(2019)Vodák, Bíl, Svoboda, Křivánková, Kubeček, Rebok, and Hliněný] define a loss function that represents vulnerability. The function aims to find the minimal link cut set which maximizes the disintegration of the network in several connected components. However, the method seems to be applicable only to undirected networks. [Li et al.(2014)Li, Fu, Wang, Lu, Berezin, Stanley, and Havlin], [Q. Cai and Alam(2019)] use percolation theory to identify the bottleneck links by analyzing the network before and after reaching the critical threshold. These links seem to be responsible for network disintegration into connected components.

### C. Mixed models

Robustness analysis is complex. The literature models cannot afford to explain all the aspects of robustness. That is why some papers use several measures in their analysis trying to capture each aspect by a single measure. [Wang et al.(2017)Wang, Koc, Derrible, Ahmad, Pino, and Kooij] analyze the robustness of the metro network via ten robustness metrics among which centrality indices, clustering coefficient, spectral indices... Another approach is to combine the different measures into a single formula to get a global view of robustness. [Van Mieghem et al.(2010)Van Mieghem, Doerr, Wang, Hernandez, Hutchison, Karaliopoulos, and Kooij] weight each robustness measure to capture all the aspects they want to analyze. However, this approach is tricky because it is sensitive to the weight chosen for each measure and the independence of the measures. In [E. Yu and Zhao(2018)], the authors define a new link centrality by combining its betweenness centrality and the number of cliques containing the link. This model combines two well-known topological metrics to capture a new aspect of vulnerability. In [Ma et al.(2022)Ma, Alam, Cai, and Delahaye] the approach is similar. The authors perform two topological analyses (betweenness centrality and percolation theory) to identify the set of critical links in airspace networks.

## III. MODEL

In this section, we are going to present the transportation network and a traffic assignment model from the literature, and the passenger-centric robustness model developed by the authors.

### A. Transportation network

A transportation network can be defined by a triplet  $\mathcal{T} = (\mathcal{G}, t, d)$  with :

- $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ : a strongly connected digraph where  $\mathcal{N}$  is the set of nodes and  $\mathcal{A}$  the set of links.
- $t$ : vector of the transportation network link travel cost.
- $d$ : vector of the user demand which is the number of people traveling from an origin  $p$  to a destination  $q$ .

TABLE I: Notations

Variable	Description
$\mathcal{T} = (\mathcal{G}, t, d)$	Transportation network
$\mathcal{G} = (\mathcal{N}, \mathcal{A})$	Strongly connected digraph of nodes and links
$\mathcal{N}, \mathcal{A}$	Set of nodes and links
$\mathcal{P}, \mathcal{Q}, \mathcal{C}$	Set of origins, destinations, and OD pairs
$\mathcal{R}_{pq}$	Set of simple routes between nodes p and q
$d = (d_{pq})$	User fixed demand
$t = (t_a)$	Link travel cost
$\tau_a, \beta_a, \gamma_a$	Link travel time parameters
$f = (f_a)$	Link flow
$c = (c_{pqr})$	Route travel cost
$h = (h_{pqr})$	Route flow
$\pi = (\pi_{pq})$	Shortest route cost
$s = (s_{nq})$	Number of alternative routes between nodes n and q
$w = (w_{pqr})$	Route robustness
$v = (v_{pqr})$	Route vulnerability
$\nu = (\nu_{pq})$	Maximum route robustness
$z = (z_{pqr})$	Multi-objective route cost
$\zeta = (\zeta_{pq})$	Shortest multi-objective route cost

We distinguish two types of transportation networks according to whether the transportation cost of a link depends or not on its flow. If it depends on the flow the transportation network is congested otherwise it is not which means  $t_a(f_a)$  is constant.

In our case, we consider the well-known Bureau of Public Roads (BPR) cost function to force congestion by making the cost too high when it is busy:

$$c_a(f_a) = \tau_a * (1 + \beta_a * (\frac{f_a}{k_a})^{\gamma_a}) \quad (1)$$

with  $\tau_a$ ,  $\beta_a$ , and  $\gamma_a$  being data of the link  $a$ . The constant  $\tau_a$  is known as the free flow travel time.

Often, the transportation demand ( $d_{pq}$ ) between an OD pair  $pq$  can be distributed among several paths  $r$  connecting  $p$  to  $q$ . Let's note  $h_{pqr}$  the flow along the path  $r$  leading from  $p$  to  $q$  then:

$$d_{pq} = \sum_{r \in \mathcal{R}_{pq}} h_{pqr} \quad (2)$$

The cost associated with the route  $r$  is equal to the sum of the costs of the links belonging to this route.

$$c_{pqr} = \sum_{a \in \mathcal{A}} \delta_{pqra} t_a \quad (3)$$

$\delta_{pqra}$  equals 1 if the link  $a$  belongs to path  $r$  and 0 otherwise.

Moreover, the flow on the link  $a$  is the sum of the flows of the paths passing through this link:

$$f_a = \sum_{r \in \mathcal{R}_{pq}} \delta_{pqra} h_{pqr} \quad (4)$$

### B. Deterministic user equilibrium

Many models (static, dynamic, deterministic, stochastic...) were defined to tackle traffic assignment problems. This paper pays attention only to the static deterministic user equilibrium.

The problem consists in determining the distribution of demand flows on the equilibrium network. In order to do so, two assumptions must be formulated:

- 1) each user has a global knowledge of the network (links, paths, costs ...)

- 2) all users have the same behavior

Moreover, each user is assumed to choose the minimum cost path. This strategy is not the most realistic but remains a good approximation. An equilibrium is reached when no user can reduce his transport cost by choosing an alternative path to the one he is already using. In this case, all paths of the same OD pair that are used have the same cost and those that are not used have higher costs. This balance is summarised by Wardrop's first principle [Wardrop(1952)] :

- **travel times on all used roads are equal to and smaller than those of unused roads.**

The user equilibrium is obtained by solving the following system:

$$(\mathcal{P}) \begin{cases} c_{pqr} = \pi_{pq} \text{ if } h_{pqr} > 0 & \forall p, q, r \\ c_{pqr} > \pi_{pq} \text{ if } h_{pqr} = 0 & \forall p, q, r \\ d_{pq} = \sum_{r \in \mathcal{R}_{pq}} h_{pqr} & \forall p, q \\ f_a = \sum_{r \in \mathcal{R}_{pq}} \delta_{pqra} h_{pqr} & \forall a \\ f_a \geq 0, h_{pqr} \geq 0, d_{pq} \geq 0 & \forall a, p, q, r \end{cases} \quad (5)$$

### C. Passenger-centric robustness model

The vulnerability to disturbances of a traffic flow on a path connecting an OD pair is related to the density of its flow and its alternative paths available. In particular, when density increases the costs get higher. Besides, when there are fewer shorter alternative paths available, the disruption of the flow has a great impact (more congestion and traffic re-routing, thus greater delays).

The model focuses on alternative paths between any transit path node and the final destination node. There is no interest in considering the robustness of the origin node because at this node the passenger can still choose between all other paths available. However, on the first transit node, the trip has already started thus he cannot go back anymore. Moreover, if we start from the origin node, the calculation of the robustness of a path would take into account the other paths that have been used for the assignment of the demand for the OD concerned. This poses a problem insofar as one wishes to have path robustness measures that are independent of each other.

More formally, if we note  $s_{nq}$  the number of alternative routes connecting nodes  $n$  and  $q$  whose cost is less than a threshold; let us consider the route  $r \in \mathcal{R}_{pq}$  then the robustness  $\rho_{pqr}$  of the route  $r$  is:

$$w_{pqr} = \sum_{n \in r \setminus (p, q)} s_{nq} \quad (6)$$

The variable  $s_{nq}$  makes the model passenger-centric. Disruptions may happen at each node of the route so a robust path should propose at least one alternative when it happens. All routes cannot be alternatives. The costs of these alternatives must be reasonable compared to the cost of the shortest path connecting node  $n$  to the destination node.

From now on, the model is purely topological. The transportation part which involves the flow assigned to the path is to be included.

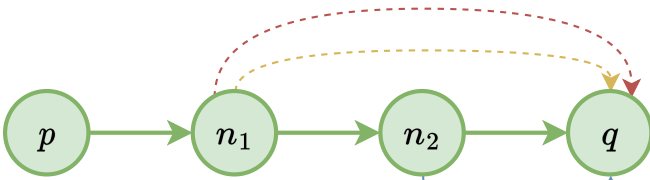


Fig. 1: An example of the computation of  $w_{pqr}$  of the green path  $r = (p, n_1, n_2, q)$ . Two alternative paths (dashed yellow and red paths) connect node  $n_1$  to node  $q$  and one (dashed blue path) connects node  $n_2$  to  $q$ , so  $w_{pqr} = 2 + 1 = 3$ .

For each path, a vulnerability measure is defined. This measure takes into account the flow of passengers on the path and its robustness. Usually, traffic assignment problems are minimization problems. So, we define a model that fits this kind of optimization. The vulnerability of a path  $r$  during the path selection process of a path connecting an OD pair  $pq$  is defined as follows:

$$v_{pqr} = (h_{pqr} + 1)(v_{pq} - w_{pqr}) \quad (7)$$

By integrating the route vulnerability in the cost function, we define the new generalized bi-objective route cost:

$$z_{pqr} = \alpha v_{pqr} + (1 - \alpha)c_{pqr} \quad (8)$$

where  $\alpha$  is a parameter to be set according to the importance one wishes to give to robustness.

Finally, the robust version ( $\mathcal{P}_r$ ) of the problem ( $\mathcal{P}$ ) can be formulated the following way:

$$(\mathcal{P}_r) \begin{cases} z_{pqr} = \zeta_{pq} & \text{if } h_{pqr} > 0 & \forall p, q, r \\ z_{pqr} > \zeta_{pq} & \text{if } h_{pqr} = 0 & \forall p, q, r \\ d_{pq} = \sum_{r \in \mathcal{R}_{pq}} h_{pqr} & & \forall p, q \\ f_a = \sum_{r \in \mathcal{R}_{pq}} \delta_{pqra} h_{pqr} & & \forall a \\ f_a \geq 0, h_{pqr} \geq 0, d_{pq} \geq 0 & & \forall a, p, q, r \end{cases} \quad (9)$$

#### IV. RESULTS

The open database [for Research Core Team(2022)] gathered several road transportation network data: network, demand, and sometimes the node coordinates. The size of these networks varies from 4 to 33837 nodes.

We decided to test our model on the Sioux Falls road network. This network is composed of 24 nodes and 76 links. All the nodes are origins and destinations of at least one OD pair. Figure 2 plots the demand to assign between the OD pairs. We can remark there is no loop demand, the diagonal values of the matrix are null. Moreover, nodes 10, 16, and 22 have the highest number of departing and arriving passengers.

Figure 3 plots the results of two different assignments: the least-cost assignment and the robust assignment. This later is based on the alternative paths model developed in Section III. The links of the robust assignment are more loaded, their widths are thicker. Therefore, their costs are higher because

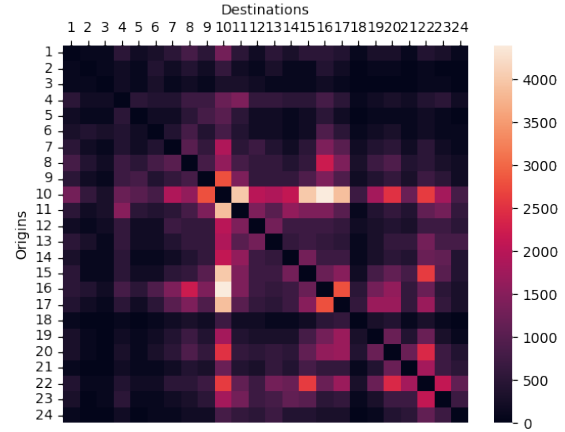


Fig. 2: OD matrix of the Sioux Falls road network

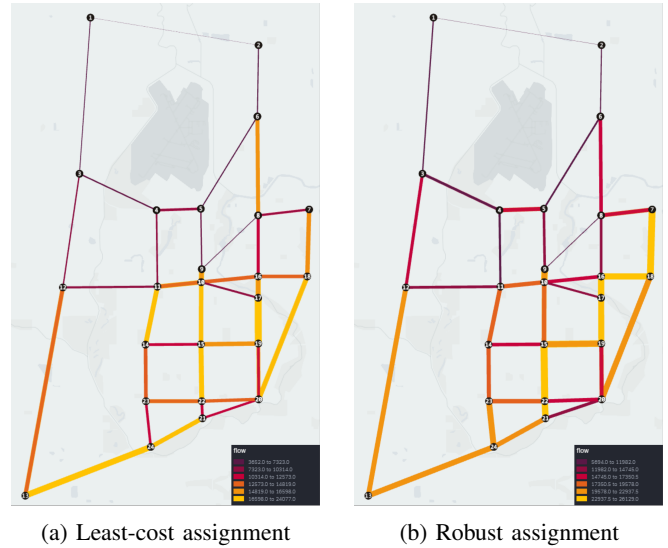


Fig. 3: Flow comparison between least-cost and robust assignment on the Sioux Falls road network

they increase with the flow (see Equation (1)). This behavior was expected. The robust assignment considers the cost and the alternatives along the path so the resulting selected path is usually longer than the least-cost path.

As we can see in Figure 4, more paths are used to assign passengers on the robust model than on the least-cost model. Besides, the demand being the same for both assignments, we can deduce there are fewer people on these routes. In case of a disruption, there would be fewer people to re-route and it is already known that these alternative costs are close to the disrupted path. Nevertheless, using multiple routes increases the odds to face a disruption.

We were interested in analyzing the global cost of the traffic for different disruption cases. In Figure 5, we ran several simulations with different values of parameters. A simulation is a three-step process. It starts with an assignment for a given cost and vulnerability parameter. Then, some links of the

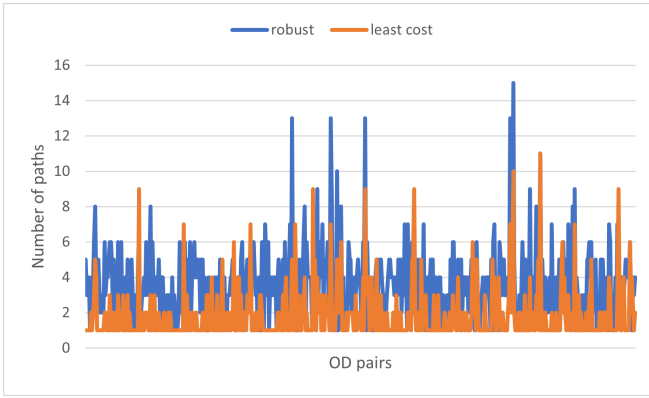
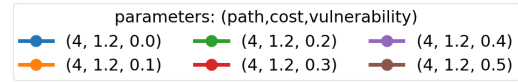


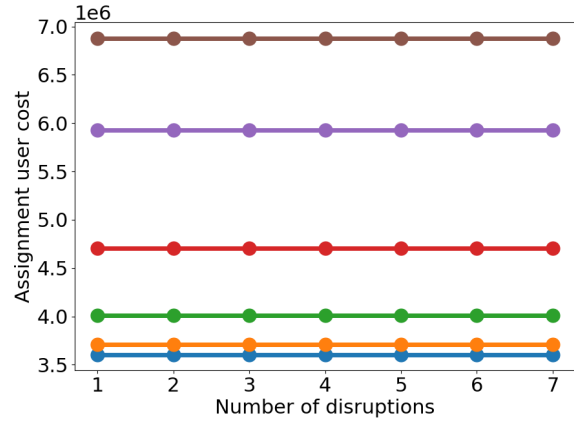
Fig. 4: Number of paths used for least-cost (orange) and robust (blue) assignments depending on the OD pairs

network are disrupted. And finally, the stranded passengers are reassigned from the node where the disruption happens. The cost parameter is used to compute the alternative paths so that their costs do not deviate too far away from the least-cost path. This parameter is related to  $s_{nq}$  in Equation (6). The vulnerability parameter is exactly the parameter  $\alpha$  in Equation (8). We set the cost parameter to 1.2 so that the cost of the alternatives is at a maximum 20% higher than the least-cost path and made the vulnerability parameter vary from 0 to 0.5. The vulnerability parameter set to 0 means that the robustness is not considered therefore it is equivalent to a least-cost assignment.

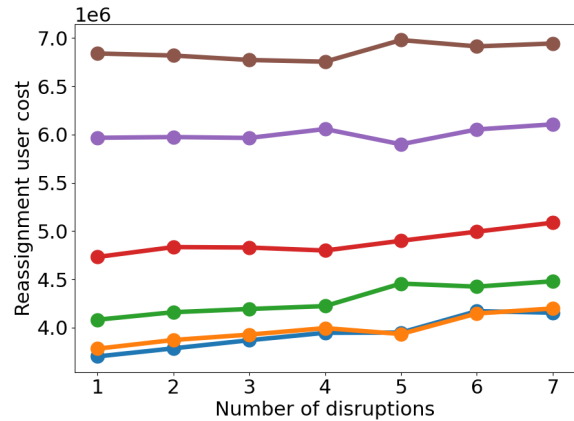
Figure 5b is the overall network cost after the robust assignment process. First, we can notice that the more robustness is considered the higher the cost. The second thing to remark is that the assignment is deterministic. For a given set of parameters, no matter the number of disruptions to be done, the cost is the same. Figure 5c represents the global reassignment cost after performing disruptions on the network. This cost is generally slightly increasing. However, we can observe a decreasing trend: from 1 to 4 on the brown curve; from 2 to 4 on the red curve; at 5 for the orange and purple curves. Last, but not least, the orange curve goes below the blue curve for 5 and 6 disruptions. This means the reassignment is less expensive when robustness is considered. The reassignment extra cost has been plotted in Figure 5d. The extra cost is the difference between the reassignment cost after a disruption and the assignment cost. Like the reassignment, the extra cost is generally increasing with the number of disruptions with the same decreasing trends as the reassignment. The curves are sorted in the opposite direction of the curves in the two previous graphs: the least robust (blue curve) is at the top and the most robust (brown curve) is at the bottom. The more robust the assignment, the lower the cost of reassignment. It happens sometimes that the extra cost is negative (brown and violet curve). The paths used to reassign the passengers are less expensive than the paths they were initially assigned. In this case, the travel time is shorter when disruptions happen on the network.



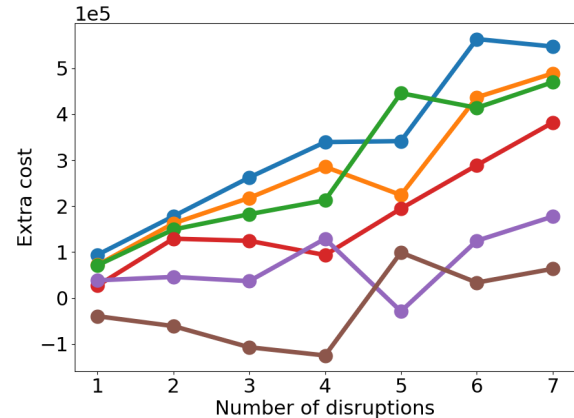
(a) Parameters used for the different curves: the path parameter is the number of paths computed at each iteration, the cost parameter is used to compute  $s_{nq}$  in Equation (6), and the vulnerability parameter is  $\alpha$  in Equation (8)



(b) Assignment cost



(c) Reassignment cost



(d) Extra cost

Fig. 5: Overall transportation cost after the traffic assignment and reassignment steps of the Sioux Falls road network disruption simulation for different parameter values

In this section, we have shown that robustness has a price. To take a robust path, one may be ready to travel longer. Here is the dilemma to be confronted with when a passenger looks for a robust trip. However, the results have indicated it is possible to benefit from robustness by slightly considering it (just 10%).

## V. CONCLUSION

We present in this paper a robust user assignment model based on alternative paths. The model is path-based and considers alternatives at each step of a passenger's trip to his destination. The results showed considering robustness has a price but great benefits can be achieved with only 10% of robustness, which seems promising.

The model presented is static. The next step is to extend it to a dynamic version which is more realistic. We are also interested in plugging this model into a dynamic traffic assignment tool to analyze disruption case scenarios.

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