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# 1 Constraint Automata on Infinite Data Trees: 2 From $\text{CTL}(\mathbb{Z})/\text{CTL}^*(\mathbb{Z})$ To Decision Procedures

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## 7 — Abstract —

8 We introduce the class of tree constraint automata with data values in  $\mathbb{Z}$  (equipped with the less  
9 than relation and equality predicates to constants), and we show that the nonemptiness problem is  
10 EXPTIME-complete. Using an automata-based approach, we establish that the satisfiability problem  
11 for  $\text{CTL}(\mathbb{Z})$  ( $\text{CTL}$  with constraints in  $\mathbb{Z}$ ) is EXPTIME-complete, and the satisfiability problem for  
12  $\text{CTL}^*(\mathbb{Z})$  is 2EXPTIME-complete (only decidability was known so far). By-product results with  
13 other concrete domains and other logics, are also briefly discussed.

14 **2012 ACM Subject Classification** Theory of computation  $\rightarrow$  Logic and verification

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## 17 **1** Introduction

18 In this paper, we study the satisfiability problem for the branching-time temporal logics  
19  $\text{CTL}(\mathbb{Z})$  and  $\text{CTL}^*(\mathbb{Z})$ , extending the classical temporal logics  $\text{CTL}$  and  $\text{CTL}^*$  in that atomic  
20 formulae express constraints about the relational structure  $(\mathbb{Z}, <, =, (=_{\mathfrak{d}})_{\mathfrak{d} \in \mathbb{Z}})$ . Formulae in  
21 these logics are interpreted over Kripke structures that are annotated with values in  $\mathbb{Z}$ . A  
22 typical  $\text{CTL}^*(\mathbb{Z})$  formula is the formula  $\text{AGF}(x < \text{X}x)$  stating that on all paths infinitely often  
23 the value of the variable  $x$  at the current position is strictly smaller than the value of  $x$  at the  
24 next position. Formalisms defined over relational structures, also known as *concrete domains*,  
25 are considered in many works, including works on temporal logics [40, 13, 54, 48, 19, 35],  
26 description logics [50, 51, 52, 53, 16, 45, 3], and automata [38, 61, 43, 71, 65, 57]. Combining  
27 reasoning in your favourite logic with reasoning in a relevant concrete domain reveals to  
28 be essential for numerous applications, for instance for reasoning about ontologies, see  
29 e.g. [52, 46], or data-aware systems, see e.g. [28, 34]. A brief survey can be found in [26].

30 Decidability results for concrete domains handled in [53, 38, 3] exclude the ubiquitous  
31 concrete domain  $(\mathbb{Z}, <, =, (=_{\mathfrak{d}})_{\mathfrak{d} \in \mathbb{Z}})$ . By contrast, decidability results for logics with concrete  
32 domain  $\mathbb{Z}$  require dedicated proof techniques, see e.g. [11, 23, 61, 46]. In particular, *fragments*  
33 of  $\text{CTL}^*(\mathbb{Z})$  are shown decidable in [11] using integral relational automata from [17], and  
34 the satisfiability problem for existential and universal  $\text{CTL}^*$  with gap-order constraints  
35 (more general than the ones in this paper) can be solved in PSPACE [12, Theorem 14].  
36 Another important breakthrough came with the decidability of  $\text{CTL}^*(\mathbb{Z})$  [15, Theorem 32]  
37 (see also [14]) by designing a reduction to a decidable second-order logic, whose formulae are  
38 made of Boolean combinations of formulae from MSO and from WMSO+U [10], where U is  
39 the unbounding second-order quantifier, see e.g. [8, 9]. This is all the more remarkable as  
40 the decidability result is part of a powerful general approach [15], but no sharp complexity  
41 upper bound can be inferred. More recently, the condition  $C_{\mathbb{Z}}$  [23] to approximate the set

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of satisfiable symbolic models of a given  $LTL(\mathbb{Z})$  formula is extended to the branching case in [46] leading to the  $EXPTIME$ -easiness of a major reasoning task for the description logic  $\mathcal{ALCF}^P(\mathbb{Z}_c)$ . However, no elementary complexity upper bounds for the satisfiability problem for  $CTL(\mathbb{Z})$  nor  $CTL^*(\mathbb{Z})$  were known since their decidability was established in [13, 15].

In this paper, we prove that the satisfiability problem for  $CTL(\mathbb{Z})$  is  $EXPTIME$ -complete, and the satisfiability problem for  $CTL^*(\mathbb{Z})$  is  $2EXPTIME$ -complete. We pursue the *automata-based approach* for solving decision problems for temporal logics, following seminal works for temporal logics, see e.g. [68, 69, 44]. This popular approach consists of reducing logical problems (satisfiability, model-checking) to automata-based decision problems while taking advantage of existing results and decision procedures from automata theory, see e.g. [67].

It is well-known that decision procedures for  $CTL^*$  are difficult to design, and the combination with the concrete domain  $\mathbb{Z}$  is definitely challenging. Moreover, we aim at proposing a general framework: we do not wish for every new logic with concrete domain to study again and again what is the proper way to define products of automata leading to optimal complexity. That is why our main goal in this work is to investigate a new class of *tree constraint automata*, understood as a target formalism in the pure tradition of the automata-based approach, and easy to reuse. The structures accepted by such tree constraint automata are *infinite* trees in which nodes are labelled by a letter from a finite alphabet and a tuple in  $\mathbb{Z}^\beta$  for some  $\beta \geq 1$  (this excludes the automata designed in [36, 37] dedicated to finite trees where no predicate  $<$  is involved). Decision problems for alternating automata over infinite alphabets are often undecidable, see e.g. [56, 47, 25, 41], and therefore we advocate the introduction of *nondeterministic* constraint automata without alternation. Our definition of tree constraint automata naturally extends the definition of constraint automata for words (see e.g. [17, 59, 61, 43, 57]) and as far as we know, the extension to infinite trees in the way done herein has not been considered earlier in the literature.

As a key result, we show that the nonemptiness problem for tree constraint automata over  $(\mathbb{Z}, <, =, (=_{\mathfrak{d}})_{\mathfrak{d} \in \mathbb{Z}})$  is  $EXPTIME$ -complete. In order to obtain the  $EXPTIME$  upper bound, we adapt results from [46, 45] (originally expressed in the context of interpretations for description logics) and we take advantage of several automata-based constructions for Rabin/Streest tree automata. As a corollary, we establish that the satisfiability problem for  $CTL(\mathbb{Z})$  is  $EXPTIME$ -complete (Theorem 14), which is one of the main results of the paper. As a by-product, it also allows us to conclude that the concept satisfiability problem w.r.t. general TBoxes for  $\mathcal{ALCF}^P(\mathbb{Z}_c)$  is in  $EXPTIME$ , a result known since [46].

Our main contribution is the characterisation of the complexity for  $CTL^*(\mathbb{Z})$  satisfiability, which is an open problem evoked in [15, Section 9] and [46, Section 5] (decidability was established ten years ago in [14]). In Section 6, we show that the satisfiability problem for  $CTL^*(\mathbb{Z})$  is in  $2EXPTIME$  by using Rabin tree constraint automata (introduced herein). We have to check that the essential steps for  $CTL^*$  can be lifted to  $CTL^*(\mathbb{Z})$  to get the optimal upper bound. In general, our contributions stem from the cross-fertilisation of automata-based techniques for temporal logics and reasoning about (infinite) structures made of  $\mathbb{Z}$ -constraints.

*A complete version with all the proofs can be found in [27].*

## 2 Temporal Logics with Numerical Domains

### 2.1 Concrete Domain $(\mathbb{Z}, <, =, (=_{\mathfrak{d}})_{\mathfrak{d} \in \mathbb{Z}})$ and Kripke Structures

In the sequel, we consider the concrete domain  $(\mathbb{Z}, <, =, (=_{\mathfrak{d}})_{\mathfrak{d} \in \mathbb{Z}})$  (also written  $\mathbb{Z}$ ), where  $=_{\mathfrak{d}}$  is a unary predicate stating the equality with the constant  $\mathfrak{d}$  and  $<$  and  $=$  are the usual

88 relations on  $\mathbb{Z}$ . Let  $\text{VAR} = \{x, y, \dots\}$  be a countably infinite set of variables. A *term*  $\mathbf{t}$   
 89 *over*  $\text{VAR}$  is an expression of the form  $X^i \mathbf{x}$ , where  $\mathbf{x} \in \text{VAR}$  and  $X^i$  is a (possibly empty)  
 90 sequence of  $i$  symbols ‘ $X$ ’. A term  $X^i \mathbf{x}$  should be understood as a variable (that needs to be  
 91 interpreted) but, later on, we will see that the prefix  $X^i$  will have a temporal interpretation.  
 92 We write  $T_{\text{VAR}}$  to denote the set of all terms over  $\text{VAR}$ . For all  $i \in \mathbb{N}$ , we write  $T_{\text{VAR}}^{\leq i}$  to  
 93 denote the subset of terms of the form  $X^j \mathbf{x}$ , where  $j \leq i$ . For instance,  $T_{\text{VAR}}^{\leq 0} = \text{VAR}$ . An  
 94 *atomic constraint*  $\theta$  *over*  $T_{\text{VAR}}$  is an expression of one of the forms below:

$$95 \quad \mathbf{t} < \mathbf{t}' \quad \mathbf{t} = \mathbf{t}' \quad =_{\mathfrak{d}}(\mathbf{t}) \text{ (also written } \mathbf{t} = \mathfrak{d}),$$

96 where  $\mathfrak{d} \in \mathbb{Z}$  and  $\mathbf{t}, \mathbf{t}' \in T_{\text{VAR}}$ . A *constraint*  $\Theta$  is defined as a Boolean combination of  
 97 atomic constraints. Constraints are interpreted on valuations  $\mathbf{v} : T_{\text{VAR}} \rightarrow \mathbb{Z}$  that assign  
 98 elements from  $\mathbb{Z}$  to the terms in  $T_{\text{VAR}}$ , so that  $\mathbf{v}$  *satisfies*  $\theta$ , written  $\mathbf{v} \models \theta$ , if and only if, the  
 99 interpretation of the terms in  $\theta$  makes  $\theta$  true in  $\mathbb{Z}$  in the usual way. The Boolean connectives  
 100 are interpreted as usual. A constraint  $\Theta$  is *satisfiable*  $\stackrel{\text{def}}{\iff}$  there is a valuation  $\mathbf{v} : T_{\text{VAR}} \rightarrow \mathbb{Z}$   
 101 such that  $\mathbf{v} \models \Theta$ . Similarly, a constraint  $\Theta_1$  *entails* a constraint  $\Theta_2$  (written  $\Theta_1 \models \Theta_2$ )  $\stackrel{\text{def}}{\iff}$   
 102 for all valuations  $\mathbf{v}$ , we have  $\mathbf{v} \models \Theta_1$  implies  $\mathbf{v} \models \Theta_2$ . The satisfiability problem restricted  
 103 to finite conjunctions of atomic constraints can be solved in PTIME (see e.g. [17, Lemma  
 104 5.5]) and entailment is in coNP. In the sequel, quite often, the valuations  $\mathbf{v}$  are of the form  
 105  $\{\mathbf{x}_1, \dots, \mathbf{x}_\beta\} \rightarrow \mathbb{Z}$  when we are only interested in the values for the variables in  $\{\mathbf{x}_1, \dots, \mathbf{x}_\beta\}$ .

106 **Kripke structures.** In order to define logics with the concrete domain  $\mathbb{Z}$ , the semantical  
 107 structures of such logics are enriched with valuations that interpret the variables by elements  
 108 in  $\mathbb{Z}$ . A  $\mathbb{Z}$ -*decorated Kripke structure* (or *Kripke structure* for short)  $\mathcal{K}$  is a triple  $(\mathcal{W}, \mathcal{R}, \mathbf{v})$ ,  
 109 where  $\mathcal{W}$  is a non-empty set of *worlds*,  $\mathcal{R} \subseteq \mathcal{W} \times \mathcal{W}$  is the accessibility relation and  
 110  $\mathbf{v} : \mathcal{W} \times \text{VAR} \rightarrow \mathbb{Z}$  is a valuation function. A Kripke structure  $\mathcal{K}$  is *total* whenever for all  
 111  $w \in \mathcal{W}$ , there is  $w' \in \mathcal{W}$  such that  $(w, w') \in \mathcal{R}$ . Given a Kripke structure  $\mathcal{K} = (\mathcal{W}, \mathcal{R}, \mathbf{v})$   
 112 and a world  $w \in \mathcal{W}$ , an *infinite path*  $\pi$  from  $w$  is an  $\omega$ -sequence  $w_0, w_1 \dots w_n, \dots$  such that  
 113  $w_0 = w$  and for all  $i \in \mathbb{N}$ , we have  $(w_i, w_{i+1}) \in \mathcal{R}$ . Finite paths are defined accordingly.

114 **Labelled trees.** Given  $D \geq 1$ , a *labelled tree of degree  $D$*  is a map  $\mathfrak{t} : \text{dom}(\mathfrak{t}) \rightarrow \Sigma$   
 115 where  $\Sigma$  is some (potentially infinite) alphabet and  $\text{dom}(\mathfrak{t})$  is an infinite subset of  $[0, D-1]^*$   
 116 such that  $\mathbf{n} \in \text{dom}(\mathfrak{t})$  and  $\mathbf{n} \cdot i \in \text{dom}(\mathfrak{t})$  for all  $0 \leq i < j$  whenever  $\mathbf{n} \cdot j \in \text{dom}(\mathfrak{t})$  for some  
 117  $\mathbf{n} \in [0, D-1]^*$  and  $j \in [0, D-1]$ . The elements of  $\text{dom}(\mathfrak{t})$  are called *nodes*. The empty  
 118 word  $\varepsilon$  is the *root node* of  $\mathfrak{t}$ . For every  $\mathbf{n} \in \text{dom}(\mathfrak{t})$ , the elements  $\mathbf{n} \cdot i$  ( $i \in [0, D-1]$ ) are  
 119 called the *children nodes of  $\mathbf{n}$* , and  $\mathbf{n}$  is called the *parent node of  $\mathbf{n} \cdot i$* . We say that the tree  
 120  $\mathfrak{t}$  is a *full  $D$ -ary tree* if every node  $\mathbf{n}$  has exactly  $D$  children  $\mathbf{n} \cdot 0, \dots, \mathbf{n} \cdot (D-1)$ . Given a  
 121 tree  $\mathfrak{t}$  and a node  $\mathbf{n}$  in  $\text{dom}(\mathfrak{t})$ , an infinite *path* in  $\mathfrak{t}$  starting from  $\mathbf{n}$  is an infinite sequence  
 122  $\mathbf{n} \cdot j_1 \cdot j_2 \cdot j_3 \dots$ , where  $j_i \in [0, D-1]$  and  $\mathbf{n} \cdot j_1 \dots j_i \in \text{dom}(\mathfrak{t})$  for all  $i \geq 1$ .

123 A *tree Kripke structure*  $\mathcal{K}$  is a Kripke structure  $(\mathcal{W}, \mathcal{R}, \mathbf{v})$  such that  $(\mathcal{W}, \mathcal{R})$  is a tree  
 124 (not necessarily a full  $D$ -ary tree). Tree Kripke structures  $(\mathcal{W}, \mathcal{R}, \mathbf{v})$  such that  $(\mathcal{W}, \mathcal{R})$   
 125 is isomorphic to the tree induced by  $[0, D-1]^*$  are represented by maps of the form  
 126  $\mathfrak{t} : [0, D-1]^* \rightarrow \mathbb{Z}^\beta$ . This assumes that we only care about the value of the variables  
 127  $\mathbf{x}_1, \dots, \mathbf{x}_\beta$  and  $\mathfrak{t}(\mathbf{n}) = (\mathfrak{d}_1, \dots, \mathfrak{d}_\beta)$  encodes that for all  $i \in [1, \beta]$ , we have  $\mathbf{v}(\mathbf{n}, \mathbf{x}_i) = \mathfrak{d}_i$ .

## 128 2.2 The Logic CTL\*( $\mathbb{Z}$ )

129 We introduce the logic CTL\*( $\mathbb{Z}$ ) extending the temporal logic CTL\* from [29] but with  
 130 constraints over  $\mathbb{Z}$ . *State formulae*  $\phi$  and *path formulae*  $\Phi$  of CTL\*( $\mathbb{Z}$ ) are defined below

$$131 \quad \phi := \neg\phi \mid \phi \wedge \phi \mid \mathbf{E}\Phi \quad \Phi := \phi \mid \mathbf{t} = \mathfrak{d} \mid \mathbf{t}_1 = \mathbf{t}_2 \mid \mathbf{t}_1 < \mathbf{t}_2 \mid \neg\Phi \mid \Phi \wedge \Phi \mid X\Phi \mid \Phi U\Phi,$$

132 where  $\mathbf{t}, \mathbf{t}_1, \mathbf{t}_2 \in T_{\text{VAR}}$ . The size of a formula is understood as its number of symbols with  
 133 integers encoded with a binary representation. We use also the universal path quantifier A

134 and the standard temporal connectives R and G ( $A\Phi \stackrel{\text{def}}{=} \neg E \neg \Phi$ ,  $\Phi_1 R \Phi_2 \stackrel{\text{def}}{=} \neg(\neg \Phi_1 U \neg \Phi_2)$ ,  
 135 and  $G\Phi \stackrel{\text{def}}{=} \perp R \Phi$  with  $\perp$  equal to  $E(x < x)$ ). No propositional variables occur in  $\text{CTL}^*(\mathbb{Z})$   
 136 formulae, but it is easy to simulate them with atomic formulae of the form  $E(x = 0)$ . We say  
 137 that a formula in  $\text{CTL}^*(\mathbb{Z})$  is in *simple form* if it is in negation normal form (using A, R and  
 138  $\vee$  as primitive) and all terms occurring in the formula are from  $T_{\text{VAR}}^{\leq 1}$ . State formulae are  
 139 interpreted on worlds from a Kripke structure, whereas path formulae are interpreted on  
 140 infinite paths. The two satisfaction relations are defined as follows (we omit the clauses for  
 141 Boolean connectives), where  $\mathcal{K} = (\mathcal{W}, \mathcal{R}, \mathbf{v})$  is a total Kripke structure, and  $w \in \mathcal{W}$ .

- 142 ■  $\mathcal{K}, w \models E\Phi \stackrel{\text{def}}{\iff}$  there is an infinite path  $\pi$  from  $w$  such that  $\mathcal{K}, \pi \models \Phi$ .
- 143 Let  $\pi = w_0, w_1, \dots$  be an infinite path of  $\mathcal{K}$ . Let us define  $\mathbf{v}(\pi, X^j \mathbf{x}) \stackrel{\text{def}}{=} \mathbf{v}(w_j, \mathbf{x})$ , for all terms  
 144 of the form  $X^j \mathbf{x}$ . For every  $n$ ,  $\pi[n, +\infty)$  is the suffix of  $\pi$  truncated by the  $n$  first worlds.
- 145 ■  $\mathcal{K}, \pi \models \mathbf{t} = \mathfrak{d} \stackrel{\text{def}}{\iff} \mathbf{v}(\pi, \mathbf{t}) = \mathfrak{d}$ ;  $\mathcal{K}, \pi \models \mathbf{t}_1 \sim \mathbf{t}_2 \stackrel{\text{def}}{\iff} \mathbf{v}(\pi, \mathbf{t}_1) \sim \mathbf{v}(\pi, \mathbf{t}_2)$  for all  $\sim \in \{<, =\}$ ,
- 146 ■  $\mathcal{K}, \pi \models \Phi U \Psi \stackrel{\text{def}}{\iff}$  there is  $j \geq 0$  such that  $\mathcal{K}, \pi[j, +\infty) \models \Psi$  and for all  $j' \in [0, j - 1]$ , we  
 147 have  $\mathcal{K}, \pi[j', +\infty) \models \Phi$ ;
- 148 ■  $\mathcal{K}, \pi \models X\Phi \stackrel{\text{def}}{\iff} \mathcal{K}, \pi[1, +\infty) \models \Phi$ .

149 Let us define two fragments of  $\text{CTL}^*(\mathbb{Z})$ . Formulae in the logic  $\text{CTL}(\mathbb{Z})$  are of the form

$$150 \quad \phi := E \Theta \mid A \Theta \mid \neg \phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid EX\phi \mid E\phi U\phi \mid E\phi R\phi \mid AX\phi \mid A\phi U\phi \mid A\phi R\phi,$$

151 where  $\Theta$  is a constraint.  $\text{LTL}(\mathbb{Z})$  formulae are defined from path formulae for  $\text{CTL}^*(\mathbb{Z})$   
 152 according to  $\Phi := \Theta \mid \Phi \wedge \Phi \mid \Phi \vee \Phi \mid X\Phi \mid \Phi U\Phi \mid \Phi R\Phi$ , where  $\Theta$  is a constraint. Negation  
 153 occurs only in constraints since the LTL logical connectives have their dual in  $\text{LTL}(\mathbb{Z})$ . In  
 154 contrast to  $\text{CTL}^*(\mathbb{Z})$  and  $\text{CTL}(\mathbb{Z})$ ,  $\text{LTL}(\mathbb{Z})$  formulae are evaluated over infinite paths of  
 155 valuations  $\mathbf{v} : \text{VAR} \rightarrow \mathbb{Z}$  (no branching involved).

156 The *satisfiability problem* for  $\text{CTL}^*(\mathbb{Z})$ , written  $\text{SAT}(\text{CTL}^*(\mathbb{Z}))$ , is defined as follows.

157 **Input:** A  $\text{CTL}^*(\mathbb{Z})$  state formula  $\phi$ .

158 **Question:** Is there a total Kripke structure  $\mathcal{K}$  and a world  $w$  such that  $\mathcal{K}, w \models \phi$ ?

159 The satisfiability problem  $\text{SAT}(\text{CTL}(\mathbb{Z}))$  for  $\text{CTL}(\mathbb{Z})$  is defined analogously; for  $\text{LTL}(\mathbb{Z})$ ,  
 160  $\text{SAT}(\text{LTL}(\mathbb{Z}))$  is the problem to decide whether there exists an infinite sequence of valuations  
 161  $\mathbf{v} : \text{VAR} \rightarrow \mathbb{Z}$  for a given  $\text{LTL}(\mathbb{Z})$  formula  $\Phi$ .

162 Decidability, and, more precisely, PSPACE-completeness of  $\text{SAT}(\text{LTL}(\mathbb{Z}))$  is shown in [24].  
 163 For some strict fragments of  $\text{CTL}^*(\mathbb{Z})$ , decidability is shown in [11, 12]. It is only recently  
 164 in [14, 13, 15], that decidability is established for the full logic using a translation into a  
 165 decidable second-order logic:

166 ► **Proposition 1** ([14, 15]).  $\text{SAT}(\text{CTL}^*(\mathbb{Z}))$  is decidable.

167 The proof in [14, 15] does not provide a complexity upper bound as the target decidable  
 168 2nd-order logic admits an automata-based decision procedure with open complexity [10, 8, 9].

169 Let us shortly explain why the satisfiability problem is challenging. First of all, observe  
 170 that  $\text{CTL}^*(\mathbb{Z})$  has atomic formulae in which integer values at the current and successor states  
 171 are compared. This prevents us from using a simple translation from  $\text{CTL}^*(\mathbb{Z})$  to  $\text{CTL}^*$   
 172 with new propositions. Models of  $\text{CTL}^*(\mathbb{Z})$  formulae can be viewed as an infinite network  
 173 of constraints on  $\mathbb{Z}$ ; even if a formula contains only a finite set of constants, a model may  
 174 contain an infinite set of values, as it is the case for, e.g., the formula  $EG(x < Xx)$ . Hence  
 175 a direct Boolean abstraction does not work. On the other hand,  $\text{CTL}^*(\mathbb{Z})$  has no freeze  
 176 quantifier and no data variable quantification, and hence no way to directly compare values  
 177 at unbounded distance (but this can only be done by propagating local constraints), unlike  
 178 e.g. the formalisms in [21, 63, 5, 1]. Hence, the lower bounds from [42] cannot apply either.

179 A problem related to satisfiability is the *model-checking problem*. Fragments of the  
 180 model-checking problem involving a temporal logic similar to  $\text{CTL}^*(\mathbb{Z})$  are investigated

181 in [17, 11, 12, 34] (see also [39, 20, 70, 2]). However, model-checking problems with  $\text{CTL}^*(\mathbb{Z})$ -  
 182 like languages are easily undecidable, see e.g. [17, Theorem 1] and [54, Theorem 4.1] (more  
 183 general constraints are used in [54] but undecidability proof uses only the constraints involved  
 184 herein). The difference between model-checking and satisfiability is subtle and underlines  
 185 that decidability/complexity of  $\text{CTL}(\mathbb{Z})/\text{CTL}^*(\mathbb{Z})$  satisfiability is not immediate.

186 In this paper, we prove the precise computational complexity of  $\text{SAT}(\text{CTL}^*(\mathbb{Z}))$  and  
 187  $\text{SAT}(\text{CTL}(\mathbb{Z}))$ . We follow the automata-based approach, that is, we translate formulae in  
 188 our logics into equivalent automata – tree constraint automata for  $\text{CTL}(\mathbb{Z})$ , and Rabin tree  
 189 constraint automata for  $\text{CTL}^*(\mathbb{Z})$  – so that we can reduce the satisfiability problem for the  
 190 logics to the nonemptiness problem for the corresponding automata.

### 191 3 Tree Constraint Automata

192 In this section, we introduce the class of tree constraint automata that accept sets of infinite  
 193 trees of the form  $\mathbb{t} : [0, D - 1]^* \rightarrow (\Sigma \times \mathbb{Z}^\beta)$  for some finite alphabet  $\Sigma$  and some  $\beta \geq 1$ .  
 194 The transition relation of such automata states constraints between the  $\beta$  integer values  
 195 at a node and the integer values at its children nodes. The acceptance condition is a  
 196 Büchi condition (applied to the infinite branches of the input tree), but this can be easily  
 197 extended to more general conditions (which we already consider by the end of this section).  
 198 Moreover, our definition is specific to the concrete domain  $\mathbb{Z}$  but it can be easily adapted to  
 199 other concrete domains. Formally, a *tree constraint automaton* (TCA, for short) is a tuple  
 200  $\mathbb{A} = (Q, \Sigma, D, \beta, Q_{\text{in}}, \delta, F)$ , where

- 201 ■  $Q$  is a finite set of locations;  $\Sigma$  is a finite alphabet,
- 202 ■  $D \geq 1$  is the (branching) degree of (the trees accepted by)  $\mathbb{A}$ ,
- 203 ■  $\beta \geq 1$  is the number of variables (a.k.a. registers),
- 204 ■  $Q_{\text{in}} \subseteq Q$  is the set of initial locations;  $F \subseteq Q$  encodes the Büchi acceptance condition,
- 205 ■  $\delta$  is a *finite* subset of  $Q \times \Sigma \times (\text{TreeCons}(\beta) \times Q)^D$ , the transition relation. Here,  
 206  $\text{TreeCons}(\beta)$  denotes the constraints (Boolean combinations of atomic constraints) built  
 207 over the terms  $\mathbf{x}_1, \dots, \mathbf{x}_\beta, \mathbf{x}'_1, \dots, \mathbf{x}'_\beta$ , where  $\mathbf{x}'_i$  denotes the term  $\mathbb{X}\mathbf{x}_i$ .  $\delta$  consists of  
 208 tuples  $(q, \mathbf{a}, (\Theta_0, q_0), \dots, (\Theta_{D-1}, q_{D-1}))$ , where  $q \in Q$  is called the source location,  $q_0, \dots,$   
 209  $q_{D-1} \in Q$ ,  $\mathbf{a} \in \Sigma$ , and  $\Theta_0, \dots, \Theta_{D-1}$  are constraints.

210 **Runs.** Let  $\mathbb{t} : [0, D - 1]^* \rightarrow (\Sigma \times \mathbb{Z}^\beta)$  be an infinite full  $D$ -ary tree over  $\Sigma \times \mathbb{Z}^\beta$ . A *run*  
 211 of  $\mathbb{A}$  on  $\mathbb{t}$  is a mapping  $\rho : [0, D - 1]^* \rightarrow \delta$  satisfying the following conditions:

- 212 ■  $\rho(\varepsilon) = (q_{\text{in}}, \dots)$  such that  $q_{\text{in}} \in Q_{\text{in}}$ ;
- 213 ■ for every  $\mathbf{n} \in [0, D - 1]^*$  with  $\rho(\mathbf{n}) = (q, \mathbf{a}, (\Theta_0, q_0), \dots, (\Theta_{D-1}, q_{D-1}))$ ,  $\mathbb{t}(\mathbf{n} \cdot i) = (\mathbf{a}_i, \mathbf{z}_i)$ ,  
 214 and  $\rho(\mathbf{n} \cdot i)$  starts by the location  $q_i$  for all  $0 \leq i < D$ , we have  $\mathbb{t}(\mathbf{n})$  of the form  
 215  $(\mathbf{a}, \mathbf{z})$  and  $\mathbb{Z} \models \Theta_i(\mathbf{z}, \mathbf{z}_i)$  for all  $0 \leq i < D$ . Here,  $\mathbb{Z} \models \Theta_i(\mathbf{z}, \mathbf{z}_i)$  is a shortcut for  
 216  $[\vec{\mathbf{x}} \leftarrow \mathbf{z}, \vec{\mathbf{x}}' \leftarrow \mathbf{z}_i] \models \Theta_i$  where  $[\vec{\mathbf{x}} \leftarrow \mathbf{z}, \vec{\mathbf{x}}' \leftarrow \mathbf{z}_i]$  is a valuation  $\mathbf{v}$  on the variables  
 217  $\{\mathbf{x}_j, \mathbf{x}'_j \mid j \in [1, \beta]\}$  with  $\mathbf{v}(\mathbf{x}_j) = \mathbf{z}(j)$  and  $\mathbf{v}(\mathbf{x}'_j) = \mathbf{z}_i(j)$  for all  $j \in [1, \beta]$ .

218 We show an example of a run  $\rho$  on  $\mathbb{t}$  in Figure 1. Suppose  $\rho$  is a run of  $\mathbb{A}$ . Given a path  
 219  $\pi = j_1 \cdot j_2 \cdot j_3 \dots$  in  $\rho$  starting from  $\varepsilon$ , we define  $\text{inf}(\rho, \pi)$  to be the set of locations that appear  
 220 infinitely often as the source locations of the transitions in  $\rho(\varepsilon)\rho(j_1)\rho(j_1 \cdot j_2)\rho(j_1 \cdot j_2 \cdot j_3) \dots$ .  
 221 A run  $\rho$  is *accepting* if for all paths  $\pi$  in  $\rho$  starting from  $\varepsilon$ , we have  $\text{inf}(\rho, \pi) \cap F \neq \emptyset$ . We  
 222 write  $L(\mathbb{A})$  to denote the set of trees  $\mathbb{t}$  that admit an accepting run.

223 **Nonemptiness problem.** As usual, the *nonemptiness problem for TCA* asks whether  
 224 a TCA  $\mathbb{A}$  satisfies  $L(\mathbb{A}) \neq \emptyset$ . To define the size of  $\mathbb{A}$  in a reasonably succinct encoding, we  
 225 need to consider the size of constraints from  $\text{TreeCons}(\beta)$ . Indeed, unlike (plain) Büchi tree  
 226 automata [68], the number of transitions in a tree constraint automaton is *a priori* unbounded

(TreeCons( $\beta$ ) is infinite) and the maximal size of a constraint occurring in transitions is unbounded too. In particular, this means that  $\text{card}(\delta)$  is a priori unbounded, even if  $Q$  and  $\Sigma$  are fixed. We write  $\text{MCS}(\mathbb{A})$  to denote the maximal size of a constraint occurring in  $\mathbb{A}$  (with binary encoding of the integers). The complexity of the nonemptiness problem should take into account these parameters. Note also that our automaton model differs from the Presburger Büchi tree automata from [62, 6] for which, in the runs, arithmetical expressions are related to constraints between numbers of children labelled by different locations. Herein, the arithmetical expressions state constraints between integer values.

Next, we introduce a variant of TCA by considering the Rabin acceptance condition (as opposed to the Büchi acceptance condition). A *Rabin tree constraint automaton* (Rabin TCA, for short) is a tuple  $\mathbb{A} = (Q, \Sigma, D, \beta, Q_{\text{in}}, \delta, \mathcal{F})$  defined as for TCA except that  $\mathcal{F}$  is a set of pairs of the form  $(L, U)$ , where  $L, U \subseteq Q$ . All the definitions about TCA apply except that a run  $\rho : [0, D - 1]^* \rightarrow \delta$  is *accepting* iff for all paths  $\pi$  in  $\rho$  starting from  $\varepsilon$ , there is some  $(L, U) \in \mathcal{F}$  such that  $\text{inf}(\rho, \pi) \cap L \neq \emptyset$  and  $\text{inf}(\rho, \pi) \cap U = \emptyset$ .

**Finite alphabet.** The set  $\Sigma$  in data trees  $\mathbb{t} : [0, D - 1]^* \rightarrow (\Sigma \times \mathbb{Z}^\beta)$  plays no specific role herein, especially that it could be encoded with simple constraints of the form  $\mathbf{x}^* = \mathfrak{d}$ , where  $\mathbf{x}^*$  is a distinguished variables. Its inclusion is more handy when the logical atomic formulae include constraints on variables *and* propositional variables, as done in [27, Section 5.2] dedicated to description logics (developments on description logics are very little in this paper, due to lack of space).

## 4 Complexity of the Nonemptiness Problem for TCA

This section is dedicated to prove the EXPTIME-completeness of the nonemptiness problem for TCA (Theorem 11) and Rabin TCA (Theorem 13) (we make a distinction between TCA and Rabin TCA because the complexity bounds differ slightly, see Lemma 10 and Lemma 12). Before we prove the EXPTIME upper bound, let us drop a few words on the lower bound. We show EXPTIME-hardness of the nonemptiness problem for TCA by reduction from the acceptance problem for alternating Turing machines running in polynomial space, see e.g. [18, Corollary 3.6]. Indeed, the polynomial-space tape using a finite alphabet  $\Sigma$  can be encoded by a polynomial amount of variables taking values in  $[1, \text{card}(\Sigma)]$ , details can be found in [27, Section 4.1]. EXPTIME-hardness for Rabin TCA follows, as every TCA with set  $F$  of accepting locations can be encoded as a Rabin TCA with a single Rabin pair  $(F, \emptyset)$ .

The proof of the EXPTIME upper bound is divided into two parts. In order to determine whether  $L(\mathbb{A})$  is nonempty for a given TCA  $\mathbb{A}$ , we first reduce the existence of some tree  $\mathbb{t} \in L(\mathbb{A})$  to the existence of some *regular symbolic tree* that is *satisfiable*, that is, it admits a concrete model (Sections 4.1 and 4.2). Second, we characterise the complexity of determining the existence of such satisfiable regular symbolic trees (Section 4.3). The result for Rabin TCA is presented in Section 4.4.

From now on, we assume a fixed TCA  $\mathbb{A} = (Q, \Sigma, D, \beta, Q_{\text{in}}, \delta, F)$  with the constants  $\mathfrak{d}_1, \dots, \mathfrak{d}_\alpha$  occurring in  $\mathbb{A}$  such that  $\mathfrak{d}_1 < \dots < \mathfrak{d}_\alpha$  (we assume there is at least one constant).

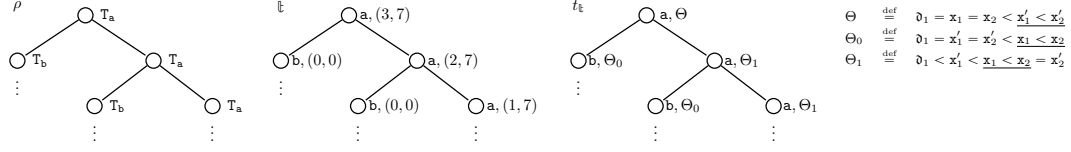
### 4.1 Symbolic Trees

A *type* over the variables  $\mathbf{z}_1, \dots, \mathbf{z}_n$  is an expression of the form

$$(\bigwedge_i \Theta_i^{\text{CST}}) \wedge (\bigwedge_{i < j} \mathbf{z}_i \sim_{i,j} \mathbf{z}_j), \text{ where}$$

■ for all  $i \in [1, n]$ ,  $\Theta_i^{\text{CST}}$  is equal to either  $\mathbf{z}_i < \mathfrak{d}_1$ , or  $\mathbf{z}_i > \mathfrak{d}_\alpha$  or  $\mathbf{z}_i = \mathfrak{d}$  for some  $\mathfrak{d} \in [\mathfrak{d}_1, \mathfrak{d}_\alpha]$ .

This definition goes a bit beyond the constraint language in  $\mathbb{Z}$  (because of expressions of



■ **Figure 1** A tree  $t$  (middle), a run  $\rho$  of some TCA on  $t$  (left), where,  $T_a = (q, \mathbf{a}, (\Theta_0, q), (\Theta_1, q))$  and  $T_b = (q, \mathbf{b}, (\Theta_0, q), (\Theta_1, q))$ , and the symbolic tree  $t_{\tilde{t}}$  (abstraction of  $t$ ) (right).

271 the form  $\mathbf{z}_i < \mathfrak{d}_1$  and  $\mathbf{z}_i > \mathfrak{d}_\alpha$ ), but this is harmless in the sequel. What really matters in  
 272 a type is the way the variables are compared to each other and to the constants.

273 ■  $\sim_{i,j} \in \{>, =, <\}$  for all  $i < j$ .

274 Checking the satisfiability of a type can be done in polynomial-time, based on a standard  
 275 cycle detection, see e.g. [17, Lemma 5.5]. The set of *satisfiable types* built over the variables  
 276  $\mathbf{x}_1, \dots, \mathbf{x}_\beta, \mathbf{x}'_1, \dots, \mathbf{x}'_\beta$  is written  $\text{SatTypes}(\beta)$  ( $n$  above is equal here to  $2\beta$ ). Observe that  
 277  $\text{card}(\text{SatTypes}(\beta)) \leq ((\mathfrak{d}_\alpha - \mathfrak{d}_1) + 3)^{2\beta} \times 3^{\beta^2}$ . The *restriction* of the type  $\Theta$  to some set  
 278 of variables  $X \subseteq \{\mathbf{x}_i, \mathbf{x}'_i \mid i \in [1, \beta]\}$  is made of all the conjuncts in which only variables in  
 279  $X$  occur. The type  $\Theta$  restricted to  $\{\mathbf{x}'_i \mid i \in [1, \beta]\}$  *agrees* with the type  $\Theta'$  restricted to  
 280  $\{\mathbf{x}_i \mid i \in [1, \beta]\}$  iff  $\Theta$  and  $\Theta'$  are logically equivalent modulo the renaming for which  $\mathbf{x}_i$  and  $\mathbf{x}'_i$   
 281 are substituted, for all  $i \in [1, \beta]$ . For instance, in Figure 1,  $\Theta$  restricted to  $\{\mathbf{x}'_1, \mathbf{x}'_2\}$  agrees  
 282 with  $\Theta_0$  restricted to  $\{\mathbf{x}_1, \mathbf{x}_2\}$ . The main properties we use about satisfiable types are stated  
 283 below.

284 (I) Let  $\mathbf{z}, \mathbf{z}' \in \mathbb{Z}^\beta$ . There is a unique  $\Theta \in \text{SatTypes}(\beta)$  such that  $\mathbb{Z} \models \Theta(\mathbf{z}, \mathbf{z}')$ .

285 (II) For every constraint  $\Theta$  built over the variables  $\mathbf{x}_1, \dots, \mathbf{x}_\beta, \mathbf{x}'_1, \dots, \mathbf{x}'_\beta$  and the constants  
 286  $\mathfrak{d}_1, \dots, \mathfrak{d}_\alpha$  there is a disjunction  $\Theta_1 \vee \dots \vee \Theta_\gamma$  logically equivalent to  $\Theta$  and each  $\Theta_i$   
 287 belongs to  $\text{SatTypes}(\beta)$  (empty disjunction stands for  $\perp$ ).

288 (III) For all  $\Theta \neq \Theta' \in \text{SatTypes}(\beta)$ , the constraint  $\Theta \wedge \Theta'$  is not satisfiable.

289 The proof is by an easy verification and this justifies the term ‘type’ used in this context.

290 **Abstraction with types.** A *symbolic tree*  $t$  is a map  $t : [0, D - 1]^* \rightarrow \Sigma \times \text{SatTypes}(\beta)$ .  
 291 Symbolic trees are intended to be abstractions of trees labelled with concrete values in  $\mathbb{Z}$ .  
 292 Given a tree  $\mathfrak{t} : [0, D - 1]^* \rightarrow \Sigma \times \mathbb{Z}^\beta$ , its *abstraction* is the symbolic tree  $t_{\tilde{t}} : [0, D - 1]^* \rightarrow$   
 293  $\Sigma \times \text{SatTypes}(\beta)$  such that for all  $\mathbf{n} \cdot i \in [0, D - 1]^*$  with  $\mathfrak{t}(\mathbf{n}) = (\mathbf{a}, \mathbf{z})$  and  $\mathfrak{t}(\mathbf{n} \cdot i) = (\mathbf{a}_i, \mathbf{z}_i)$ ,  
 294  $t_{\tilde{t}}(\mathbf{n} \cdot i) \stackrel{\text{def}}{=} (\mathbf{a}_i, \Theta_i)$  for the unique  $\Theta_i \in \text{SatTypes}(\beta)$  such that  $\mathbb{Z} \models \Theta_i(\mathbf{z}, \mathbf{z}_i)$ . Note that  
 295 the primed variables in  $\Theta_i$  refer to the  $\beta$  values at the node  $\mathbf{n} \cdot i$ , whereas the unprimed  
 296 ones refer to the  $\beta$  values at the parent node  $\mathbf{n}$ . At the root  $\varepsilon$  with  $\mathfrak{t}(\varepsilon) = (\mathbf{a}, \mathbf{z})$ , we have  
 297  $t_{\tilde{t}}(\varepsilon) \stackrel{\text{def}}{=} (\mathbf{a}, \Theta)$  for the unique  $\Theta \in \text{SatTypes}(\beta)$  such that  $\mathbb{Z} \models \Theta(\mathbf{0}, \mathbf{z})$ , where  $\mathbf{0} \in \mathbb{Z}^\beta$  is  
 298 arbitrary as there are actually no parent values at the root. A symbolic tree  $t$  is *satisfiable*  
 299  $\stackrel{\text{def}}{\iff}$  there is  $\mathfrak{t} : [0, D - 1]^* \rightarrow \Sigma \times \mathbb{Z}^\beta$  such that  $t_{\tilde{t}} = t$ . We say that  $\mathfrak{t}$  *witnesses the satisfaction*  
 300 *of*  $t$ , also written  $\mathfrak{t} \models t$ . A symbolic tree  $t$  is *regular* if its set of subtrees is finite.

301 **A-consistency.** In our quest to decide whether  $L(\mathbb{A}) \neq \emptyset$ , we are interested in symbolic  
 302 trees that satisfy certain properties that we subsume under the name *A-consistent*. A symbolic  
 303 tree  $t : [0, D - 1]^* \rightarrow \Sigma \times \text{SatTypes}(\beta)$  is *A-consistent* if the following conditions are satisfied:

304 ■  $t$  is *locally consistent*: for every node  $\mathbf{n}$ , the type  $\Theta$  labelling  $\mathbf{n}$  restricted to  $\mathbf{x}'_1, \dots, \mathbf{x}'_\beta$   
 305 agrees with all types  $\Theta_i$  labelling its children nodes  $\mathbf{n} \cdot i$  restricted to  $\mathbf{x}_1, \dots, \mathbf{x}_\beta$ , and

306 ■ there is an accepting run  $\rho$  of  $\mathbb{A}$  (but ignoring the conditions on data values) such that  
 307 for all  $\mathbf{n} \in [0, D - 1]^*$  with  $t(\mathbf{n}) = (\mathbf{a}, \Theta)$ ,  $t(\mathbf{n} \cdot i) = (\mathbf{a}_i, \Theta_i)$  for all  $i \in [0, D - 1]$ , and  
 308  $\rho(\mathbf{n}) = (q, \mathbf{a}, (\Theta'_0, q_0) \dots (\Theta'_{D-1}, q_{D-1}))$ , we have  $\Theta_i \models \Theta'_i$  for all  $i \in [0, D - 1]$ .



309 ▶ **Example 2.** In Figure 1, we show a tree  $\mathfrak{t}$  with concrete values in  $\mathbb{Z}^\beta$  for  $\beta = 2$  (middle) and  
 310 its abstraction  $t_{\mathfrak{t}}$  (right). We assume that  $\mathfrak{d}_1 = 0$  is the only constant; consequently,  $t_{\mathfrak{t}}$  uses  
 311 constraints in  $\text{SatTypes}(\beta)$  that are built with variables  $\mathbf{x}_1, \mathbf{x}_2$ , their primed variants  $\mathbf{x}'_1, \mathbf{x}'_2$ ,  
 312 and the constant  $\mathfrak{d}_1$ . We underline constraints to illustrate the property of local consistency.  
 313 It is not hard to prove that the set of all  $\mathbb{A}$ -consistent symbolic trees is  $\omega$ -regular, that is, it  
 314 can be accepted by a classical tree automaton without constraints. In the following, we use  
 315 the standard letter  $A$  to distinguish automata *without constraints* from TCA.

316 ▶ **Lemma 3.** *There exists a Büchi tree automaton (without constraints)  $A_{\text{cons}(\mathbb{A})}$  such that*  
 317  $L(A_{\text{cons}(\mathbb{A})})$  *is equal to the set of  $\mathbb{A}$ -consistent symbolic trees.*

318 The locations in  $A_{\text{cons}(\mathbb{A})}$  are from  $\text{SatTypes}(\beta) \times Q$  and the transition relation for  $A_{\text{cons}(\mathbb{A})}$   
 319 can be decided in polynomial-time in  $\text{card}(\delta) + \beta + \text{card}(\Sigma) + D + \text{MCS}(\mathbb{A})$ .

320 However, not every  $\mathbb{A}$ -consistent symbolic tree admits a concrete model. Thus the more  
 321 important property is to check whether  $L(A_{\text{cons}(\mathbb{A})})$  contains some *satisfiable* symbolic tree  
 322 (and we explain how to do this in the next two subsections). The result below is a variant  
 323 of many similar results relating symbolic models and concrete models in logics for concrete  
 324 domains, see e.g. [23, Corollary 4.1], [38, Lemma 3.4], [16, Theorem 25], [46, Theorem 11].

325 ▶ **Lemma 4.**  $L(\mathbb{A}) \neq \emptyset$  *iff there is a satisfiable symbolic tree in  $L(A_{\text{cons}(\mathbb{A})})$ .*

## 326 4.2 Satisfiability for Regular Locally Consistent Symbolic Trees

327 Below, we focus on deciding when  $L(A_{\text{cons}(\mathbb{A})})$  contains a *satisfiable* symbolic tree, while  
 328 evaluating the complexity to check its existence. Given a locally consistent symbolic tree  
 329  $t : [0, D - 1]^* \rightarrow \Sigma \times \text{SatTypes}(\beta)$ , we introduce an infinite labelled graph that contains  
 330 exactly the same types as  $t$  but expressed in a tree-like graph from which it is convenient  
 331 to characterize satisfiability in terms of paths, under the premise that  $t$  is regular. Similar  
 332 symbolic structures are introduced in [49, 23, 14, 46]. The graph is equal to the structure

$$333 G_t^c = (V_t, \overset{=}{\rightarrow}, \overset{<}{\rightarrow}, U_{<\mathfrak{d}_1}, (U_i)_{i \in [\mathfrak{d}_1, \mathfrak{d}_\alpha]}, U_{>\mathfrak{d}_\alpha}),$$

334 where  $V_t = [0, D - 1]^* \times (\{\mathbf{x}_1, \dots, \mathbf{x}_\beta\} \cup \{\mathfrak{d}_1, \mathfrak{d}_\alpha\})$ ,  $\overset{=}{\rightarrow}$  and  $\overset{<}{\rightarrow}$  are two binary relations over  $V_t$ ,  
 335 and  $\{U_{<\mathfrak{d}_1}, U_{\mathfrak{d}_1}, U_{\mathfrak{d}_1+1}, \dots, U_{\mathfrak{d}_\alpha}, U_{>\mathfrak{d}_\alpha}\}$  is a partition of  $V_t$ . Elements in  $\{\mathbf{x}_1, \dots, \mathbf{x}_\beta\} \cup \{\mathfrak{d}_1, \mathfrak{d}_\alpha\}$   
 336 are denoted by  $\mathbf{x}_d, \mathbf{x}_{d_1}, \mathbf{x}_{d_2}, \dots$  (variables or constants). Moreover,  $V_t^\beta \stackrel{\text{def}}{=} [0, D - 1]^* \times$   
 337  $\{\mathbf{x}_1, \dots, \mathbf{x}_\beta\}$ . The rationale behind the construction of  $G_t^c$  is to reflect the constraints between  
 338 parent and children nodes as well as the constraints regarding constants, in such a way that,  
 339 if  $\mathfrak{t}$  witnesses the satisfaction of  $t$ , then, e.g.,  $\mathfrak{t}(\mathbf{n})(\mathbf{x}_d) < \mathfrak{t}(\mathbf{n}')(\mathbf{x}_{d'})$  if  $(\mathbf{n}, \mathbf{x}_d) \overset{<}{\rightarrow} (\mathbf{n}', \mathbf{x}_{d'})$ , and  
 340  $\mathfrak{t}(\mathbf{n})(\mathbf{x}_d) = \mathfrak{d}_1$  if  $(\mathbf{n}, \mathbf{x}_d) \in U_{\mathfrak{d}_1}$ . Here are all conditions for building  $G_t^c$ .

341 **(VAR)** For all  $(\mathbf{n}, \mathbf{x}_i), (\mathbf{n}', \mathbf{x}_{i'}) \in V_t^\beta$ , for all  $\sim \in \{<, =\}$ ,  $(\mathbf{n}, \mathbf{x}_i) \overset{\sim}{\rightarrow} (\mathbf{n}', \mathbf{x}_{i'})$  iff either  $\mathbf{n}' = \mathbf{n} \cdot j$   
 342 and  $\mathbf{x}_i \sim \mathbf{x}_{i'}$  in  $\Theta$  with  $t(\mathbf{n}') = (\mathbf{a}, \Theta)$ , or  $\mathbf{n} = \mathbf{n}'$  and  $\mathbf{x}'_i \sim \mathbf{x}'_{i'}$  in  $\Theta$  with  $t(\mathbf{n}') = (\mathbf{a}, \Theta)$ , or  
 343  $\mathbf{n} = \mathbf{n}' \cdot j$  and  $\mathbf{x}'_i \sim \mathbf{x}_{i'}$  in  $\Theta$  with  $t(\mathbf{n}') = (\mathbf{a}, \Theta)$ .

344 **(P1)** For all  $\mathfrak{d} \in [\mathfrak{d}_1, \mathfrak{d}_\alpha]$  and  $(\mathbf{n}, \mathbf{x}_j) \in V_t^\beta$ ,  $(\mathbf{n}, \mathbf{x}_j) \in U_{\mathfrak{d}}$  iff  $\mathbf{x}'_j = \mathfrak{d}$  in  $\Theta$  with  $t(\mathbf{n}) = (\mathbf{a}, \Theta)$ .

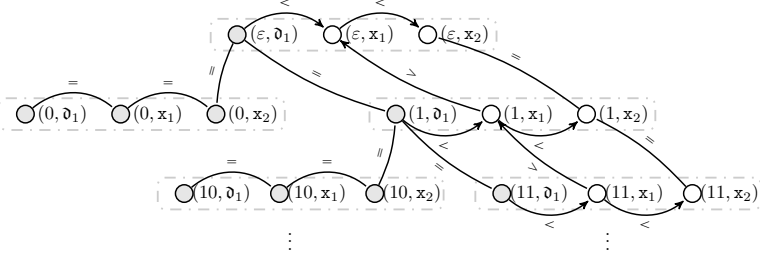
345 **(P2)** For all  $(\mathbf{n}, \mathbf{x}_j) \in V_t^\beta$ ,  $(\mathbf{n}, \mathbf{x}_j) \in U_{<\mathfrak{d}_1}$  iff  $\mathbf{x}'_j < \mathfrak{d}_1$  in  $\Theta$  with  $t(\mathbf{n}) = (\mathbf{a}, \Theta)$ .

346 **(P3)** For all  $(\mathbf{n}, \mathbf{x}_j) \in V_t^\beta$ ,  $(\mathbf{n}, \mathbf{x}_j) \in U_{>\mathfrak{d}_\alpha}$  iff  $\mathbf{x}'_j > \mathfrak{d}_\alpha$  in  $\Theta$  with  $t(\mathbf{n}) = (\mathbf{a}, \Theta)$ .

347 **(P4)** For all  $\mathbf{n} \in [0, D - 1]^*$ ,  $(\mathbf{n}, \mathfrak{d}_1) \in U_{\mathfrak{d}_1}$  and  $(\mathbf{n}, \mathfrak{d}_\alpha) \in U_{\mathfrak{d}_\alpha}$ .

348 **(CONS)** This is about elements of  $V_t$  labelled by constants and how the edge labels reflect the  
 349 relationships between the constants. Formally, for all  $((\mathbf{n}, \mathbf{x}_d), (\mathbf{n}', \mathbf{x}_{d'})) \in (V_t \times V_t) \setminus (V_t^\beta \times$   
 350  $V_t^\beta)$ , for all  $\mathfrak{d}^\dagger, \mathfrak{d}^{\dagger\dagger}$  in  $\{< \mathfrak{d}_1, \mathfrak{d}_1, \dots, \mathfrak{d}_\alpha, > \mathfrak{d}_\alpha\}$  s.t.  $(\mathbf{n}, \mathbf{x}_d) \in U_{\mathfrak{d}^\dagger}$  and  $(\mathbf{n}', \mathbf{x}_{d'}) \in U_{\mathfrak{d}^{\dagger\dagger}}$ , for  
 351 all  $\sim \in \{<, =\}$ ,  $(\mathbf{n}, \mathbf{x}_d) \overset{\sim}{\rightarrow} (\mathbf{n}', \mathbf{x}_{d'})$  iff either  $\mathfrak{d}^\dagger, \mathfrak{d}^{\dagger\dagger} \in [\mathfrak{d}_1, \mathfrak{d}_\alpha]$  and  $\mathfrak{d}^\dagger \sim \mathfrak{d}^{\dagger\dagger}$ , or  $\mathfrak{d}^\dagger = \{< \mathfrak{d}_1\}$ ,  
 352  $\mathfrak{d}^{\dagger\dagger} \neq \{< \mathfrak{d}_1\}$  and  $\sim$  is equal to  $<$  or  $\mathfrak{d}^\dagger \neq \{> \mathfrak{d}_\alpha\}$ ,  $\mathfrak{d}^{\dagger\dagger} = \{> \mathfrak{d}_\alpha\}$  and  $\sim$  is equal to  $<$ .

353 Below, we illustrate the definition of the graph  $G_{t_{\mathbb{k}}}^C$  for the symbolic tree  $t_{\mathbb{k}}$  in Figure 1.  
 354 The edges labelled with  $=$  or  $<$  reflect the constraints (we omit edges if they can be inferred  
 from the other edges).  
 For instance,  $(1, \mathbf{x}_1) \overset{<}{\sim} (\varepsilon, \mathbf{x}_1)$  corresponds to the  
 355 constraint  $\mathbf{x}'_1 < \mathbf{x}_1$ . Grey nodes are in  $U_{\mathfrak{d}_1}$ , all other  
 nodes are in  $U_{>\mathfrak{d}_1}$  (no nodes in  $U_{<\mathfrak{d}_1}$ ).



356 A map  $p : \mathbb{N} \rightarrow V_t$  is a *path map* in  $G_t^C \stackrel{\text{def}}{\iff}$  for all  $i \in \mathbb{N}$ , either  $p(i) \overset{=}{\rightarrow} p(i+1)$  or  
 357  $p(i) \overset{<}{\rightarrow} p(i+1)$  in  $G_t^C$ . Similarly,  $r : \mathbb{N} \rightarrow V_t$  is a *reverse path map* in  $G_t^C \stackrel{\text{def}}{\iff}$  for all  $i \in \mathbb{N}$ ,  
 358 either  $r(i) \overset{=}{\rightarrow} r(i+1)$  or  $r(i+1) \overset{<}{\rightarrow} r(i)$ . A path map  $p$  (resp. reverse path map  $r$ ) is  
 359 *strict*  $\stackrel{\text{def}}{\iff} \{i \in \mathbb{N} \mid p(i) \overset{<}{\rightarrow} p(i+1)\}$  (resp.  $\{i \in \mathbb{N} \mid r(i+1) \overset{<}{\rightarrow} r(i)\}$ ) is infinite. An *infinite*  
 360 *branch*  $\mathcal{B}$  is an element of  $[0, D-1]^\omega$ . We write  $\mathcal{B}[i, j]$  with  $i \leq j$  to denote the subsequence  
 361  $\mathcal{B}(i) \cdot \mathcal{B}(i+1) \cdots \mathcal{B}(j)$ . Given  $(\mathbf{n}, \mathbf{xd}) \in V_t$ , a path map  $p$  from  $(\mathbf{n}, \mathbf{xd})$  along  $\mathcal{B}$  is such that  
 362  $p(0) = (\mathbf{n}, \mathbf{xd})$  and for all  $i \geq 0$ ,  $p(i)$  is of the form  $(\mathbf{n} \cdot \mathcal{B}[0, i], \cdot)$ . A reverse path map  $r$  from  
 363  $(\mathbf{n}, \mathbf{xd})$  along  $\mathcal{B}$  admits a similar definition. We present the condition  $(\star^C)$  that is the central  
 364 property for characterising regular symbolic trees in  $L(A_{\text{cons}(\mathcal{A})})$  that are satisfiable, following  
 365 the remarkable result established in [46, Lemma 22] that non-satisfiability of a symbolic tree  
 366 can be witnessed along a *single branch*.

367  $(\star^C)$  There are *no* elements  $(\mathbf{n}, \mathbf{xd}), (\mathbf{n}, \mathbf{xd}')$  in  $G_t^C$  (same node  $\mathbf{n}$  from  $[0, D-1]^*$ ) and no  
 368 infinite branch  $\mathcal{B}$  such that  
 369 1. there exists a path map  $p$  from  $(\mathbf{n}, \mathbf{xd})$  along  $\mathcal{B}$ ,  
 370 2. there exists a reverse path map  $r$  from  $(\mathbf{n}, \mathbf{xd}')$  along  $\mathcal{B}$ ,  
 371 3.  $p$  or  $r$  is strict, and  
 372 4. for all  $i \in \mathbb{N}$ ,  $p(i) \overset{<}{\rightarrow} r(i)$ .

373 The following proposition states a key property: non-satisfaction of a regular locally consistent  
 374 symbolic tree can be witnessed along a *single branch* by violation of  $(\star^C)$ .

375 **► Proposition 5.** For every regular locally consistent symbolic tree  $t$ ,  $G_t^C$  satisfies  $(\star^C)$  iff  $t$  is  
 376 satisfiable.

377 A proof can be found in [27, Section 7].

378 **► Example 6.** Assume that every node along the rightmost branch in the symbolic tree  
 379  $t_{\mathbb{k}}$  in Figure 1 is labelled with  $(\mathbf{a}, \Theta_1)$ . Then  $t_{\mathbb{k}}$  is not satisfiable: in order to satisfy  $\Theta_1$ 's  
 380 conjunct  $\mathbf{x}'_1 < \mathbf{x}_1$ , the value of  $\mathbf{x}_1$  must inevitably become finally smaller than  $\mathfrak{d}_1$ , violating  
 381 the conjunct  $\mathfrak{d}_1 < \mathbf{x}_1$ . Consequently, the rightmost branch of  $G_{t_{\mathbb{k}}}^C$  presented above does not  
 382 satisfy  $(\star^C)$ : there exists a path map  $p$  from  $(\varepsilon, \mathfrak{d}_1)$  along  $1^\omega$ , there exists a strict reverse  
 383 path map  $r$  from  $(\varepsilon, \mathbf{x}_1)$  along  $1^\omega$ , and for all  $i \in \mathbb{N}$  we have  $p(i) \overset{<}{\rightarrow} r(i)$ .

384 **New constant nodes.** Proposition 5 above is a variant of [46, Lemma 22]. Before going  
 385 any further, let us in short explain the improvement of our developments compared to what is  
 386 done in [46, 45]. The *framified constraint graphs* defined in [46, Definition 14] correspond to  
 387 the above defined graph  $G_t^C$  without  $[0, D-1]^* \times \{\mathfrak{d}_1, \mathfrak{d}_\alpha\}$  and corresponding edges. However,  
 388 Example 6 illustrates the importance of taking into account these elements when deciding  
 389 satisfiability (without  $\mathfrak{d}_1$ , the graph would satisfy  $(\star^C)$ ). Actually, Example 6 invalidates  $(\star)$   
 390 as used in [46, 45] because the constants are missing to apply properly [15]. The problematic  
 391 part in [46, 45] is due to the proof of [45, Lemma 5.18] whose main argument takes advantage

of [15] but without the elements related to constant values (see also [24, Lemma 8]). With Proposition 5, we also propose a proof to characterise satisfiability of symbolic trees that is independent of [15]. Note also that the condition  $(\star)$  in [46, Section 3.3] generalises the condition  $C_Z$  from [23, Section 6] (see also the condition  $\mathcal{C}$  in [24, Definition 2] and a similar condition in [32, Section 2]). A condition similar to  $(\star)$  is also introduced recently in [7, Lemma 18] to decide a realizability problem based on  $LTL(\mathbb{Z}, <, =)$ .

We recall that there are *nonregular* locally consistent symbolic trees  $t$  such that  $G_t^c$  satisfies  $(\star^c)$  (see e.g. [23, 46]) but  $t$  is not satisfiable; indeed, satisfiability of symbolic trees is not an  $\omega$ -regular property. The next result states that  $(\star^c)$  is  $\omega$ -regular; hence, satisfiability of symbolic trees can be overapproximated advantageously.

► **Lemma 7.** *There is a Rabin tree automaton  $A_{\star^c}$  such that  $L(A_{\star^c}) = \{t \mid G_t^c \text{ satisfies } (\star^c)\}$ , the number of Rabin pairs is bounded above by  $8(\beta + 2)^2 + 3$ , the number of locations is exponential in  $\beta$ , the transition relation can be decided in polynomial-time in*

$$\max([\log(|\mathfrak{d}_1|)], [\log(|\mathfrak{d}_\alpha|)]) + \beta + \text{card}(\Sigma) + D.$$

**Proof sketch.** The proof of Lemma 7 is structured as follows (see [27, Section 4.3]).

(1) We construct a Büchi word automaton  $A_B$  accepting the complement of  $(\star^c)$  for  $D = 1$ .  
 (2)  $A_B$  is nondeterministic, but we can determinize it and get a deterministic Rabin word automaton  $A_{B \rightarrow R}$  such that  $L(A_B) = L(A_{B \rightarrow R})$  (using the determinisation construction from [60, Theorem 1.1]).  
 (3) By an easy construction, we obtain a deterministic Street word automaton  $A_S$  accepting the complement of  $L(A_{B \rightarrow R})$ ; it accepts words that satisfy  $(\star^c)$  for  $D = 1$ .  
 (4) By [60, Lemma 1.2], we construct a deterministic Rabin word automaton  $A_R$  s.t.  $L(A_S) = L(A_R)$ .  
 (5) Finally, we construct a Rabin tree automaton  $A_{\star^c}$ , the intuitive idea is to "let run the automaton  $A_R$ " along every branch of a run of  $A_{\star^c}$ , doable thanks to the determinism of  $A_R$ . Since  $(\star^c)$  states a property on every branch, we are done.

**Differences with [46].** Lemma 7 is similar to [46, Proposition 26] but there is an essential difference: the number of Rabin pairs in Lemma 7 is not a constant but a value depending on  $\beta$ , an outcome of our investigations. It is important to know the number of Rabin pairs in  $A_{\star^c}$  for our complexity analysis as checking nonemptiness of Rabin tree automata is *exponential* in the number of Rabin pairs [30, Theorem 4.1]. Our proof of Lemma 7 also proposes a slight novelty compared to the construction in [46]: we design  $A_{\star^c}$  without firstly constructing *a tree automaton for the complement language* (as done in [46]) and then using results from [55] (elimination of alternation in tree automata). Our new approach shall be rewarding: not only we can better understand how to express the condition  $(\star^c)$ , but also we control the size parameters of  $A_{\star^c}$  involved in our forthcoming complexity analysis. Furthermore, it may be useful to implement the decision procedure for solving the satisfiability problem for  $CTL(\mathbb{Z})$  (resp. for  $CTL^*(\mathbb{Z})$ ). Note also that the above analysis about the number of Rabin pairs is independent from the question discussed above about having the elements in  $[0, D - 1]^* \times \{\mathfrak{d}_1, \mathfrak{d}_\alpha\}$  within  $G_t^c$ .

Summarizing the developments so far, we can conclude this subsection as follows:

► **Lemma 8.**  $L(\mathbb{A}) \neq \emptyset$  iff  $L(A_{\text{cons}(\mathbb{A})}) \cap L(A_{\star^c}) \neq \emptyset$ .

For its proof, by way of example, if  $L(A_{\text{cons}(\mathbb{A})}) \cap L(A_{\star^c})$  is non-empty, then as  $L(A_{\text{cons}(\mathbb{A})}) \cap L(A_{\star^c})$  is regular, it contains a regular  $\mathbb{A}$ -consistent symbolic tree  $t$  (see e.g. [58] and [64, Section 6.3] for the existence of regular trees) and by Proposition 5,  $t$  is satisfiable. By Lemma 4, we get  $L(\mathbb{A}) \neq \emptyset$ . For the other direction, we use Lemma 3 as well as the property that for every satisfiable symbolic tree  $t$ ,  $G_t^c$  satisfies the condition  $(\star^c)$ .

### 4.3 ExpTime Upper Bound for TCAs

Lemma 8 justifies why deciding the nonemptiness of  $L(A_{\text{cons}(\mathbb{A})}) \cap L(A_{\star^c})$  is crucial. In the proof of Lemma 9 below (see [27, Section 4.4]), we propose a construction for the intersection of Rabin tree automata that only performs an exponential blow-up for the number of locations, which is fine for our purposes.

► **Lemma 9.** *There is a Rabin tree automaton  $A$  such that  $L(A) = L(A_{\text{cons}(\mathbb{A})}) \cap L(A_{\star^c})$  and the number of Rabin pairs is polynomial in  $\beta$ , the number of locations is in  $\mathcal{O}(\text{card}(\text{SatTypes}(\beta)) \times \text{card}(Q) \times 2^{P(\beta)})$  for some polynomial  $P(\cdot)$  and the transition relation can be decided in polynomial-time in  $\text{card}(\delta) + \beta + \text{card}(\Sigma) + D + \text{MCS}(\mathbb{A})$ .*

Nonemptiness of Rabin tree automata is polynomial in the cardinality of the transition relation and exponential in the number of Rabin pairs, see e.g. [30, Theorem 4.1]. More precisely, it is in time  $(m \times n)^{\mathcal{O}(n)}$ , where  $m$  is the number of locations and  $n$  is the number of Rabin pairs, see the statement [30, Theorem 4.1]. However, this is not exactly what we need herein, as the complexity expression above concerns *binary* trees, and it assumes that the transition relation  $\delta$  can be decided in constant time. If, as in our case,  $D \geq 1$  and deciding whether a tuple belongs to  $\delta$  requires  $\gamma$  time units, checking nonemptiness is actually in time  $(\text{card}(\delta) \times \gamma \times n)^{\mathcal{O}(n)}$  (by scrutiny of the proof of [30, Theorem 4.1], page 144). Here,  $\gamma$  may depend on parameters related to  $\mathbb{A}$  and in Lemma 10 below,  $\gamma$  takes the value  $\text{card}(\delta) + \beta + \text{card}(\Sigma) + D + \text{MCS}(\mathbb{A})$  (by Lemma 9). Hence the following result:

► **Lemma 10.** *The nonemptiness problem for TCA can be solved in time in  $\mathcal{O}(R_1(\text{card}(Q) \times \text{card}(\delta) \times \text{MCS}(\mathbb{A}) \times \text{card}(\Sigma) \times R_2(\beta))^{R_2(\beta) \times R_3(D)})$  for some polynomials  $R_1, R_2$  and  $R_3$ .*

Assuming that the size of the TCA  $\mathbb{A} = (Q, \Sigma, D, \beta, Q_{\text{in}}, \delta, F)$ , written  $\text{size}(\mathbb{A})$ , is polynomial in  $\text{card}(Q) + \text{card}(\delta) + D + \beta + \text{MCS}(\mathbb{A})$  (which makes sense for a reasonably succinct encoding), from the computation of the bound in Lemma 10, the nonemptiness of  $L(\mathbb{A})$  can be checked in time  $\mathcal{O}(R(\text{size}(\mathbb{A}))^{R'(\beta+D)})$  for some polynomials  $R$  and  $R'$ . The EXPTime upper bound of the nonemptiness problem for TCA is now a consequence of the above complexity expression.

► **Theorem 11.** *Nonemptiness problem for tree constraint automata is EXPTime-complete.*

### 4.4 Rabin Tree Constraint Automata

We can prove the EXPTime upper bound of the nonemptiness problem for Rabin TCA (Theorem 13) and follow the same lines of arguments as for TCA. Given a Rabin TCA  $\mathbb{A} = (Q, \Sigma, D, \beta, Q_{\text{in}}, \delta, \mathcal{F})$ , we define a Rabin tree automaton  $A'_{\text{cons}(\mathbb{A})}$  such that  $L(\mathbb{A}) \neq \emptyset$  iff there is  $t \in L(A'_{\text{cons}(\mathbb{A})})$  that is satisfiable (cf. Lemma 4 for TCA). Moreover, we take advantage of  $A_{\star^c}$  so that  $L(\mathbb{A}) \neq \emptyset$  iff  $L(A'_{\text{cons}(\mathbb{A})}) \cap L(A_{\star^c})$  is non-empty (cf. Lemma 8). It remains to determine the cost for testing nonemptiness of  $L(A'_{\text{cons}(\mathbb{A})}) \cap L(A_{\star^c})$ . Here is the counterpart of Lemma 9 (same kind of arguments).

► **Lemma 12.** *There is a Rabin tree automaton  $A$  s.t.  $L(A) = L(A'_{\text{cons}(\mathbb{A})}) \cap L(A_{\star^c})$ , the number of Rabin pairs is polynomial in  $\beta + \text{card}(\mathcal{F})$ , the number of locations is in  $\mathcal{O}(\text{card}(\text{SatTypes}(\beta)) \times \text{card}(Q) \times 2^{P(\beta + \text{card}(\mathcal{F}))})$  for some polynomial  $P(\cdot)$ , and the transition relation can be decided in polynomial-time in  $\text{card}(\delta) + \beta + \text{card}(\Sigma) + D + \text{MCS}(\mathbb{A})$ .*

As for Lemma 10, we conclude that the nonemptiness problem for Rabin TCA can be solved in time  $\mathcal{O}(R_1(\text{card}(Q) \times \text{card}(\delta) \times \text{MCS}(\mathbb{A}) \times \text{card}(\Sigma) \times R_2(\beta + \text{card}(\mathcal{F})))^{R_2(\beta + \text{card}(\mathcal{F})) \times R_3(D)})$  for polynomials  $R_1, R_2$  and  $R_3$ . The nonemptiness problem for Rabin TCA is also in EXPTime.

480 ▶ **Theorem 13.** *The nonemptiness problem for Rabin TCA is EXPTIME-complete.*

481 This result is mainly useful to characterize the complexity of  $\text{SAT}(\text{CTL}^*(\mathbb{Z}))$  in Section 6.

## 482 **5 Tree Constraint Automata for CTL( $\mathbb{Z}$ )**

483 Below, we harvest the first results from what is achieved in the previous section:  $\text{SAT}(\text{CTL}(\mathbb{Z}))$   
 484 is in EXPTIME. So, enriching the CTL models with numerical values interpreted in  $\mathbb{Z}$  does  
 485 not cause a complexity blow-up. We follow the automata-based approach and (after proving  
 486 a refined version of the tree model property for  $\text{CTL}(\mathbb{Z})$ ) the key step is to translate  $\text{CTL}(\mathbb{Z})$   
 487 formulae into equivalent TCA. Theorem 14 below is one of our main results.

488 ▶ **Theorem 14.** *The satisfiability problem for  $\text{CTL}(\mathbb{Z})$  is EXPTIME-complete.*

489 **Sketch.** EXPTIME-hardness is inherited from CTL. For EXPTIME-easiness, let  $\phi$  be a  $\text{CTL}(\mathbb{Z})$   
 490 formula. A first step is to preprocess the formula into a formula in *simple form* (see definition  
 491 in Section 2.2). Then, we can construct from a formula  $\phi$  in simple form a TCA  $A_\phi$  s.t.  $\phi$  is  
 492 satisfiable iff  $L(A_\phi) \neq \emptyset$  and  $A_\phi$  satisfies the following properties.

- 493 ■ The degree  $D$  and the number of variables  $\beta$  are bounded by  $\text{size}(\phi)$ .
- 494 ■ The number of locations is bounded by  $(D \times 2^{\text{size}(\phi)}) \times (\text{size}(\phi) + 1)$ .
- 495 ■ The number of transitions is in  $\mathcal{O}(2^{P(\text{size}(\phi))})$  for some polynomial  $P(\cdot)$ .
- 496 ■ The finite alphabet  $\Sigma$  in  $A_\phi$  is unary;  $\text{MCS}(A_\phi)$  is quadratic in  $\text{size}(\phi)$ .

497 By Lemma 10, the nonemptiness problem for TCA can be solved in time

$$498 \mathcal{O}(R_1(\text{card}(Q) \times \text{card}(\delta) \times \text{MCS}(A) \times \text{card}(\Sigma) \times R_2(\beta))^{R_2(\beta) \times R_3(D)}).$$

499 Since the transition relations of the automata  $A_{\text{cons}(A)}$  and  $A_{\star C}$  can be built in polynomial-time,  
 500 we get that nonemptiness of  $L(A_\phi)$  can be solved in exponential-time. ◀

501 Let  $\mathbb{N}$  be the concrete domain  $(\mathbb{N}, <, =, (=_{\partial})_{\partial \in \mathbb{N}})$  for which we can also show that nonemptiness  
 502 of TCA with constraints interpreted on  $\mathbb{N}$  has the same complexity as for TCA with constraints  
 503 interpreted on  $\mathbb{Z}$ . Let  $\text{CTL}(\mathbb{N})$  be the variant of  $\text{CTL}(\mathbb{Z})$  with constraints interpreted  
 504 on  $\mathbb{N}$ . As a corollary,  $\text{SAT}(\text{CTL}(\mathbb{N}))$  is EXPTIME-complete. With the concrete domain  
 505  $(\mathbb{Q}, <, =, (=_{\partial})_{\partial \in \mathbb{Q}})$ , all the trees in  $L(A_{\text{cons}(A)})$  are satisfiable (no need to intersect  $A_{\text{cons}(A)}$  with  
 506 a hypothetical  $A_{\star C}$ , see e.g. [49, 4, 23, 38]), and therefore  $\text{SAT}(\text{CTL}(\mathbb{Q}))$  is in EXPTIME too.  
 507 TCA can be also used to show that the concept satisfiability w.r.t. general TBoxes for the  
 508 description logic  $\mathcal{ALCF}^P(\mathbb{Z}_c)$  is in EXPTIME [46, 45], see more details in [27, Section 5.2].

## 509 **6 Complexity of the Satisfiability Problem for the Logic $\text{CTL}^*(\mathbb{Z})$**

510 We show that  $\text{SAT}(\text{CTL}^*(\mathbb{Z}))$  can be solved in  $2\text{EXPTIME}$ . We follow the automata-based  
 511 approach for  $\text{CTL}^*$ , see e.g. [31, 30], but adapted to Rabin TCA. The main challenge here  
 512 is to carefully check that essential steps for  $\text{CTL}^*$  can be lifted to  $\text{CTL}^*(\mathbb{Z})$ , but also that  
 513 computationally we are in a position to provide an optimal complexity upper bound.

514 Let us explain in short all steps necessary to obtain the result. We start by establishing a  
 515 special form for  $\text{CTL}^*(\mathbb{Z})$  formulae from which Rabin TCA will be defined, following ideas  
 516 from [31] for  $\text{CTL}^*$ . A  $\text{CTL}^*(\mathbb{Z})$  state formula  $\phi$  is in *special form* if it has the form below

$$517 \text{E} (x = 0) \wedge \left( \bigwedge_{i \in [1, D-1]} \text{AGE } \Phi_i \right) \wedge \left( \bigwedge_{j \in [1, D']} \text{A } \Phi'_j \right),$$

518 where the  $\Phi_i$ 's and the  $\Phi'_j$ 's are  $\text{LTL}(\mathbb{Z})$  formulae in simple form (see Section 2), for some  
 519  $D \geq 1, D' \geq 0$ . We can restrict ourselves to  $\text{CTL}^*(\mathbb{Z})$  state formulae in special form (see  
 520 the proof of [27, Proposition 6]).

521 ▶ **Proposition 15.** *For every  $\text{CTL}^*(\mathbb{Z})$  formula  $\phi$ , one can construct in polynomial time in*  
 522 *the size of  $\phi$  a  $\text{CTL}^*(\mathbb{Z})$  formula  $\phi'$  in special form s.t.  $\phi$  is satisfiable iff  $\phi'$  is satisfiable.*

523 So  $\phi'$  is also of polynomial size in the size of  $\phi$ . Let us state a tree model property of special  
 524 formulae, with a strict discipline on the witness paths. Proposition 16 below is a counterpart  
 525 of [31, Theorem 3.2] but for  $\text{CTL}^*(\mathbb{Z})$  instead of  $\text{CTL}^*$ , see also the variant [38, Lemma 3.3].

526 ▶ **Proposition 16.** *Let  $\phi$  be a  $\text{CTL}^*(\mathbb{Z})$  formula in special form built over  $\mathbf{x}_1, \dots, \mathbf{x}_\beta$ .  $\phi$  is*  
 527 *satisfiable iff there is a tree  $\mathbb{t} : [0, D - 1] \rightarrow \mathbb{Z}^\beta$  such that  $\mathbb{t}, \varepsilon \models \phi$  and for each  $i \in [1, D - 1]$ ,*  
 528  *$\mathbb{t}$  satisfies AGE  $\Phi_i$  via  $i$ , that is, if  $\mathbb{t}, \mathbf{n} \models \mathbf{E} \Phi_i$ , then  $\Phi_i$  is satisfied on the path  $\mathbf{n} \cdot i \cdot 0^\omega$ .*

529 Proposition 16 justifies our restriction to infinite trees and to TCA in the rest of this section.  
 530 Proposition 15 allows us to restrict our attention to constructing automata for formulae of  
 531 (only) the form AGE  $\Phi$  and A  $\Phi$ , where  $\Phi$  is a simple formula in  $\text{LTL}(\mathbb{Z})$ . The first step is to  
 532 translate simple formulae in  $\text{LTL}(\mathbb{Z})$  into equivalent *word* constraint automata (TCA with  
 533 degree  $D = 1$ ). Adapting the standard automata-based approach for  $\text{LTL}$  [69], we can show  
 534 the following proposition (see the proof of [27, Proposition 8]).

535 ▶ **Proposition 17.** *Let  $\Phi$  be an  $\text{LTL}(\mathbb{Z})$  formula in simple form. There is a constraint word*  
 536 *automaton  $\mathbb{A}_\Phi$  such that  $\{\mathbf{w} : \mathbb{N} \rightarrow \mathbb{Z}^\beta \mid \mathbf{w} \models \Phi\} = \text{L}(\mathbb{A}_\Phi)$ , and the following conditions hold.*  
 537 *(I) The number of locations in  $\mathbb{A}_\Phi$  is bounded by  $\text{size}(\Phi) \times 2^{2 \times \text{size}(\Phi)}$ .*  
 538 *(II) The cardinality of  $\delta$  in  $\mathbb{A}_\Phi$  is in  $\mathcal{O}(2^{P(\text{size}(\Phi))})$  for some polynomial  $P(\cdot)$ .*  
 539 *(III) The maximal size of a constraint in  $\mathbb{A}_\Phi$  is quadratic in  $\text{size}(\Phi)$ .*

540 We can now construct, for every  $i \in [0, D - 1]$ , a TCA  $\mathbb{A}_i$  such that  $\text{L}(\mathbb{A}_i) = \{\mathbb{t} : [0, D - 1]^* \rightarrow \mathbb{Z}^\beta \mid \mathbb{t} \models \text{AGE } \Phi_i \text{ and } \mathbb{t} \text{ satisfies AGE } \Phi_i \text{ via } i\}$ . The idea is to construct  $\mathbb{A}_i$  so  
 541 that it starts off the word constraint automaton  $\mathbb{A}_{\Phi_i}$  at each node  $\mathbf{n}$  of the tree and runs it  
 542 down the designated path  $\mathbf{n} \cdot i \cdot 0^\omega$  to check whether  $\Phi_i$  actually holds along this path. This  
 543 can be easily done for AGE  $\Phi_i$ ; however, for formulas of the form A  $\Phi'_j$ , for this construction  
 544 to be correct, the underlying constraint word automaton  $\mathbb{A}'_j$  must be *deterministic*, that  
 545 is, for all locations  $s$ , letters  $\mathbf{a}$  and pairs of valuations  $(\mathbf{z}, \mathbf{z}') \in \mathbb{Z}^{2\beta}$ , there exists in  $\mathbb{A}'_j$  at  
 546 most a single transition  $(s, \mathbf{a}, \Theta, s')$  such that  $\mathbb{Z} \models \Theta(\mathbf{z}, \mathbf{z}')$ . A well-known construction to  
 547 transform nondeterministic Büchi automata to equivalent deterministic Rabin automata  
 548 is due to Safra [60, Theorem 1.1]. An important step towards the optimal complexity for  
 549  $\text{CTL}^*(\mathbb{Z})$  is to show that it is possible to lift this construction to word constraint automata,  
 550 which is a result of its own interest. A special attention is given to the cardinality of the  
 551 transition relation and to the size of the constraints in transitions, as these two parameters  
 552 are, a priori, unbounded in constraint automata but essential to perform a forthcoming  
 553 complexity analysis.  
 554

555 ▶ **Theorem 18.** *Let  $\mathbb{A} = (Q, \Sigma, \beta, Q_{in}, \delta, F)$  be a Büchi word constraint automaton involving*  
 556 *the constants  $\mathfrak{d}_1, \dots, \mathfrak{d}_\alpha$ . There is a deterministic Rabin word constraint automaton  $\mathbb{A}' =$*   
 557  *$(Q', \Sigma, \beta, Q'_{in}, \delta', \mathcal{F})$  such that  $\text{L}(\mathbb{A}) = \text{L}(\mathbb{A}')$  verifying the following quantitative properties.*  
 558 *(I)  $\text{card}(Q')$  is exponential in  $\text{card}(Q)$  and the number of Rabin pairs in  $\mathbb{A}'$  is bounded by*  
 559  *$2 \cdot \text{card}(Q)$  (same bounds as in [60, Theorem 1.1]).*  
 560 *(II) The constraints in the transitions are from  $\text{SatTypes}(\beta)$ , are of size cubic in  $\beta +$   
 561  *$\max(\lceil \log(\lceil \mathfrak{d}_1 \rceil) \rceil, \lceil \log(\lceil \mathfrak{d}_\alpha \rceil) \rceil)$  and  $\text{card}(\delta') \leq \text{card}(Q')^2 \times \text{card}(\Sigma) \times ((\mathfrak{d}_\alpha - \mathfrak{d}_1) + 3)^{2\beta} \times 3^{\beta^2}$ .**

562 This and Proposition 17 lead us to the result below on  $\text{LTL}(\mathbb{Z})$  formulae in simple form.

563 ▶ **Corollary 19.** *Let  $\Phi$  be an  $\text{LTL}(\mathbb{Z})$  formula in simple form built over the variables  $\mathbf{x}_1, \dots, \mathbf{x}_\beta$*   
 564 *and the constants  $\mathfrak{d}_1, \dots, \mathfrak{d}_\alpha$ . There exists a deterministic Rabin word constraint automaton*  
 565  *$\mathbb{A}_\Phi$  such that  $\{\mathbf{w} : \mathbb{N} \rightarrow \mathbb{Z}^\beta \mid \mathbf{w} \models \Phi\} = \text{L}(\mathbb{A}_\Phi)$ , and the following conditions hold.*

- 566 (I) The number of locations in  $\mathbb{A}_\Phi$  is bounded by  $2^{2^{P^\dagger(\text{size}(\Phi))}}$  for some polynomial  $P^\dagger(\cdot)$ .  
 567 (II) The number of Rabin pairs is bounded by  $2 \times \text{size}(\Phi) \times 2^{2 \times \text{size}(\Phi)}$ .  
 568 (III) The cardinality of  $\delta$  in  $\mathbb{A}_\Phi$  is bounded by  $\text{card}(\text{SatTypes}(\beta)) \times 2^{2^{P^\dagger(\text{size}(\Phi))+1}}$ .  
 569 (IV)  $\text{MCS}(\mathbb{A}_\Phi)$  is cubic in  $\beta + \max(\lceil \log(|\mathfrak{D}_1|) \rceil, \lceil \log(|\mathfrak{D}_\alpha|) \rceil)$ , i.e. polynomial in  $\text{size}(\Phi)$ .

570 This enables us to use the idea illustrated above for formulas of the form AGE  $\Phi_i$  also for  
 571 formulas of the form A  $\Phi'_j$ , and define Rabin TCA  $\mathbb{A}'_j$  such that  $L(\mathbb{A}'_j) = \{\mathfrak{t} : [0, D-1]^* \rightarrow$   
 572  $\mathbb{Z}^\beta \mid \mathfrak{t} \text{ satisfies A } \Phi'_j\}$ . We are now ready to perform the final step towards the main result of  
 573 this section. Let us recapitulate what we have so far.

- 574 ■ One can define a TCA  $\mathbb{A}_0$  with two locations such that  $L(\mathbb{A}_0)$  is the set of trees  $\mathfrak{t} :$   
 575  $[0, D-1]^* \rightarrow \mathbb{Z}^\beta$  such that  $\mathfrak{t}(\varepsilon)(x_1) = 0$ , to handle  $E(x_1 = 0)$  in formulae in special form.
- 576 ■ For all  $1 \leq i < D$ , there are (Büchi) TCA  $\mathbb{A}_i$  such that  $L(\mathbb{A}_i)$  is the set of trees  
 577  $\mathfrak{t} : [0, D-1]^* \rightarrow \mathbb{Z}^\beta$  such that  $\mathfrak{t}, \varepsilon \models \text{AGE } \Phi_i$  and  $\mathfrak{t}$  satisfies AGE  $\Phi_i$  via  $i$ . Recall that  
 578 TCA can be seen as Rabin TCA with a single Rabin pair.
- 579 ■ For all  $1 \leq j \leq D'$ , there are Rabin TCA  $\mathbb{A}'_j$  such that  $L(\mathbb{A}'_j)$  is the set of trees  $\mathfrak{t}$  such  
 580 that  $\mathfrak{t}$  satisfies A  $\Phi_j$ , with an exponential number of Rabin pairs in  $\text{size}(\Phi)$ .

581 To define a Rabin TCA  $\mathbb{A}$  such that  $L(\mathbb{A}) = L(\mathbb{A}_0) \bigcap_{i \in [1, D-1]} L(\mathbb{A}_i) \bigcap_{j \in [1, D'] } L(\mathbb{A}'_j)$ , and  
 582 then use the complexity bounds previously established, we need the result below (see the full  
 583 proof in [27, Section 6.5]).

584 ► **Lemma 20.** *Let  $(\mathbb{A}_k)_{1 \leq k \leq n}$  be a family of Rabin TCA such that  $\mathbb{A}_k = (Q_k, \Sigma, D, \beta, Q_{k, \text{in}}, \delta_k, \mathcal{F}_k)$ ,  
 585  $\text{card}(\mathcal{F}_k) = N_k$  and  $N = \prod_k N_k$ . There is a Rabin TCA  $\mathbb{A}$  such that  $L(\mathbb{A}) = \bigcap_k L(\mathbb{A}_k)$  and*

- 586 ■ *the number of Rabin pairs is equal to  $N$ ;  $\text{MCS}(\mathbb{A}) \leq n + \text{MCS}(\mathbb{A}_1) + \dots + \text{MCS}(\mathbb{A}_n)$ ,*
- 587 ■ *the number of locations (resp. transitions) is less than  $(\prod_k \text{card}(Q_k))(2n)^N$  (resp.  $\prod_k \text{card}(\delta_k)$ ).*

588 Putting all results together, the nonemptiness of  $L(\mathbb{A})$  can be checked in double-exponential  
 589 time in  $\text{size}(\phi)$ , leading to Theorem 21 below, which is the main result of the paper. It  
 590 answers open questions from [11, 15, 16, 46].

591 ► **Theorem 21.** *SAT(CTL\*( $\mathbb{Z}$ )) is 2EXPTIME-complete.*

592 2EXPTIME-hardness is from SAT(CTL\*) [66, Theorem 5.2]. As a corollary, SAT(CTL\*( $\mathbb{N}$ ))  
 593 is also 2EXPTIME-complete. Furthermore, assuming that  $<_{\text{pre}}$  is the prefix relation on  $\{0, 1\}^*$ ,  
 594 we can use the reduction from [22, Section 4.2] to conclude SAT(CTL\*( $\{0, 1\}^*, <_{\text{pre}}$ )) is  
 595 2EXPTIME-complete too. Furthermore, as observed earlier, when the concrete domain is  
 596  $(\mathbb{Q}, <, =, (=_{\mathfrak{d}})_{\mathfrak{d} \in \mathbb{Q}})$ , all the trees in  $L(\mathbb{A}_{\text{cons}(\mathbb{A})})$  are satisfiable, and therefore SAT(CTL\*( $\mathbb{Q}$ )) is  
 597 also in 2EXPTIME, which is already known from [38, Theorem 4.3].

## 598 7 Concluding Remarks

599 We developed an automata-based approach to solve SAT(CTL( $\mathbb{Z}$ )) and SAT(CTL\*( $\mathbb{Z}$ )), by  
 600 introducing tree constraint automata that accept infinite data trees with data domain  $\mathbb{Z}$ . The  
 601 nonemptiness problem for tree constraint automata with Büchi acceptance conditions (resp.  
 602 with Rabin pairs) is EXPTIME-complete, see Theorem 11 (resp. Theorem 13). The difficult  
 603 part consists in proving the EXPTIME-easiness for which we show how to substantially adapt  
 604 the material in [45, Section 5.2] that guided us to design the correctness proof of ( $\star^c$ ). The  
 605 work [46] was indeed a great inspiration but we adjusted a few statements from there (see  
 606 also [27]). We recall that ( $\star$ ) in [46] is not fully correct (see Section 4.2) as we need to  
 607 add constants (leading to the variant condition ( $\star^c$ )). Moreover, our construction of the  
 608 automaton in Lemma 7 does depend on the number of variables unlike [46, Proposition 26].

609 This is crucial for complexity, as it is related to the number of Rabin pairs. We also use [30]  
 610 more precisely than [46, p.621] as we handle non-binary trees. In short, we introduced  
 611 TCA for which we characterise complexity of the non-emptiness problem (providing a few  
 612 improvements to [46]). We left aside the question of the expressiveness of TCA, which is  
 613 interesting but out of the scope of this paper.

614 This lead us to show that  $\text{SAT}(\text{CTL}(\mathbb{Z}))$  is  $\text{EXPTIME}$ -complete (Theorem 14), and  
 615  $\text{SAT}(\text{CTL}^*(\mathbb{Z}))$  is  $2\text{EXPTIME}$ -complete (Theorem 21). The only decidability proof for  
 616  $\text{SAT}(\text{CTL}^*(\mathbb{Z}))$  done so far, see [15, Theorem 32], is by reduction to a decidable second-order  
 617 logic. Our complexity characterisation for  $\text{SAT}(\text{CTL}^*(\mathbb{Z}))$  provides an answer to several  
 618 open problems related to  $\text{CTL}^*(\mathbb{Z})$  fragments, see e.g. [11, 38, 15, 16, 46]. We believe that  
 619 our results on TCA can help to establish complexity results for other logics (see also Section 6  
 620 about a domain for strings and [33, Section 4] to handle more concrete domains).

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