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# PREDICTIVE MODELLING APPROACH FOR A PIEZOELECTRIC BISTABLE ENERGY HARVESTER ARCHITECTURE

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## ABSTRACT

The goal of this paper is to present a new lumped model dedicated to Piezoelectric Bistable Energy Harvesters (or PBEHs). Such ambient vibration energy converters are interesting for their broadband frequency response. The PBEH architecture of interest in this paper uses post-buckled beams to create bistability, as well as two Amplified Piezoelectric Actuators (or APAs); one for energy harvesting and another to tune the buckling level. The proposed model provides information on how the beams' axial stiffness and tuning APA affect the performance and feasibility of the PBEH, which is lacking in literature. The lumped model is first formulated based on the Euler-Lagrange equations and then validated experimentally using a PBEH prototype. It is shown that the relative error between the predicted and measured characteristics and responses of the PBEH remains inferior to 10%. Moreover, the prototype presented produces around 4 mW at 4 m/s<sup>2</sup> with 22 % relative bandwidth.

## KEYWORDS

Piezoelectric energy harvester, bistability, dynamic lumped model, post-buckled beams

## INTRODUCTION

Ambient energy harvesters are being more and more considered as a viable power source for autonomous electronic devices, as well as more durable alternatives to chemical batteries. One commonly considered exploitation is the powering of the sensor nodes used in wireless sensor networks [1].

In particular, vibration energy harvesters have shown a very promising performance. These harvesters convert ambient vibration energy into electric energy with mechanical resonators coupled with electromagnetic induction, electrostatic mechanisms, or piezoelectric components, which is the focus of the present paper. A classic instance of this type of harvesters is the coupling of linear oscillators, such as cantilever beams, with piezoelectric patches [2]. In spite of the design simplicity it offers, this solution is not adapted to all ambient vibrations because of its narrow frequency band [3], and can have low electromechanical coupling levels due to the 3-1 modes used [4].

One alternative is the use of bistable oscillators which present a broadband frequency response. This non-linearity can be introduced in two ways; either by using magnetic mechanisms, or post-buckled structures, which is the approach adopted for this work. Incorporating a non-linearity in the vibrating structure does however imply higher modelling complexity.

Moreover, the use of Amplified Piezoelectric Actuators (or

APAs) is a solution to trigger 3-3 electromechanical coupling modes. It is nonetheless uncommon and thus less documented in literature.

Amongst the few works in which this solution is used, Liu et al's [5] contribution is the first that comes to mind. Although the bending stiffness of the beams is accounted for in this work, the effect of their axial stiffness is omitted. Moreover, no tuning APA is used in the structure. A second instance of this type of PBEH is Huguet et al's architecture [6], that includes an APA used to tune the buckling level for orbit jump purposes. Unfortunately, the lumped model used does not include the effect of the beams nor that of the tuning APA's.

The goal of this paper is to present a lumped model for this PBEH architecture, that uses post-buckled beams and APAs for energy harvesting and tuning purposes. The lumped model accounts for the bending and axial stiffnesses of the beams in addition to APAs' stiffnesses, and is formulated using Euler-Lagrange equations. Its accuracy is then tested by comparing the theoretical and numerical results it implies, to the experimental results found using a PBEH prototype.

## LUMPED MODEL FORMULATION

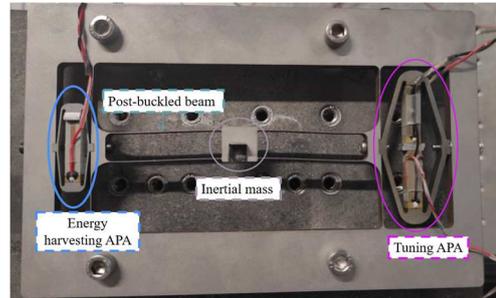


Figure 1: Image of a PBEH prototype with the architecture studied in this paper

Figure 1 shows a picture of the PBEH prototype with the studied architecture. The APA on the left is an energy harvesting APA linked to a resistance load. The APA on the right is solely used to vary the buckling level of the PBEH, and not at all for energy harvesting purposes.

Equivalent models of the PBEH architecture are represented in Figure 2. The variable  $x$  represents the displacement of the inertial mass  $M$ ,  $D$  represents the damping coefficient, and  $L(\bar{x})$  the mass frame distance for a given displacement of the mass.

Figure 2 (a) is a schematic of the initial equivalent model proposed by Huguet et al [6] where the tuning APA is considered infinitely rigid, and the post-buckled beams are assumed to behave like Perfect Revolute Joints (PRJs) with infinitely rigid bars. ( $K_2 \rightarrow \infty, K_f \rightarrow 0, K_t \rightarrow \infty$ ). In the lumped model proposed here and illustrated in the

schematic shown in Figure 2 (b), we move on from the previous assumption and consider that the stiffness of the tuning APA and the elasticity of the buckled beams affect the PBEH's dynamic behavior. The elasticity of each post-buckled beams is considered by adding springs for their bending and axial compression stiffnesses respectively named  $K_b$  and  $K_a$ .

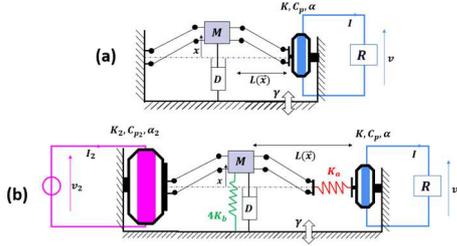


Figure 2: schematic representations of the modelling principles proposed by Huguet et al [6] with the Perfect Revolute Joint (or PRJ) assumption (a) and the one presented in this paper (b).

When considering the 4 identical beams and assuming symmetry around the inertial mass, the equivalent stiffnesses of the 4 beams are equal to  $4K_b$  and  $K_a$ , as shown in a previous study [7].

The spring  $K_a$  and both APAs are represented in the equivalent system represented in Figure 3

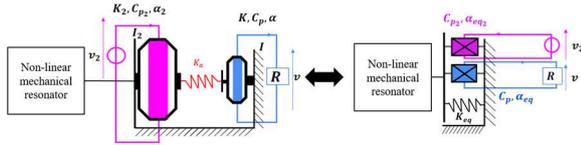


Figure 3: Schematic representation of the equivalent system

Static mechanics considerations dictate that if the elements of a system are in series, the forces each isolated element of the system is subjected to, as well as the force applied on the equivalent system, are equal. Therefore, the force  $F$  applied to the energy harvesting APA in series with  $K_a$  is equal to the one applied by the tuning APA and to the one applied to the equivalent system, as described by (1).  $\Delta L_1$ ,  $\Delta L_2$  and  $\Delta L$  respectively refer to the displacement variations of the energy harvesting APA in series with the spring  $K_a$ , the tuning APA and equivalent system.

$$\begin{cases} F = \frac{K K_a}{K_a + K} \Delta L_1 + \frac{\alpha K_a}{K_a + K} v = K_2 \Delta L_2 + \alpha_2 v_2 \\ = K_{eq} \Delta L + \alpha_{eq} v + \alpha_{eq2} v_2 \\ \Delta L = \Delta L_1 + \Delta L_2 \end{cases} \quad (1)$$

Resolving this equation system results in finding the characteristics of an equivalent system, given by (2) and illustrated in Figure 3, that will be considered for the rest of the study.

$$\begin{cases} K_{eq} = \frac{K_a K_2 K}{K_a K_2 + K} = \beta_c K \\ \alpha_{eq} = \frac{K_a K_2 \alpha}{K_a K_2 + K} = \beta_c \alpha \\ \alpha_{eq2} = \frac{K \alpha_2}{K_a K_2 + K} = (1 - \beta_c) \alpha_2 \end{cases} \quad (2)$$

With  $\beta_c$  a parameter which is added to facilitate comprehension, given by (3).

$$\beta_c = \frac{1}{1 + \frac{K}{K_2 K_a}} \quad (3)$$

The movements of the mass are considered to move along the  $\vec{x}$  direction only. Thusly, the generalized coordinates considered are the position of the mass  $x$  and two voltage related coordinate  $\lambda_1$  and  $\lambda_2$  as  $\dot{\lambda}_1 = v$  and  $\dot{\lambda}_2 = v_2$ . The corresponding Euler-Lagrange balance is described by equation system (4).

$$\begin{cases} M\ddot{x} - D\dot{x} = \frac{d}{dt} \left( \frac{\partial \mathbb{L}}{\partial \dot{x}} \right) - \frac{\partial \mathbb{L}}{\partial x} \\ I = \frac{d}{dt} \left( \frac{\partial \mathbb{L}}{\partial \dot{\lambda}} \right) - \frac{\partial \mathbb{L}}{\partial \lambda} \\ I_2 = \frac{d}{dt} \left( \frac{\partial \mathbb{L}}{\partial \dot{\lambda}_2} \right) - \frac{\partial \mathbb{L}}{\partial \lambda_2} \end{cases} \quad (4)$$

The Lagrangian function of the system is described by equation (5).

$$\mathbb{L} = T - S_{AK_f} - S_{K_{eq}} + W_c \quad (5)$$

$T$ ,  $S_{AK_f}$ ,  $S_{K_{eq}}$  and  $W_c$  represent respectively the kinetic energy of the system, the bending compression elastic energy stored in the beams, the axial compression elastic energy stored in the equivalent APA and the global potential electric energy of the APAs used in the structure.

$$T = \frac{1}{2} M \dot{x}^2 \quad (6)$$

$$S_{K_{eq}} = 2 \left( \frac{1}{2} K_{eq} \Delta L^2 \right) \quad (7)$$

$$S_{4K_b} = \frac{1}{2} (4K_b x^2) \quad (8)$$

$$W_c = \frac{1}{2} C_p v^2 + \alpha_{eq} v \Delta L + \frac{1}{2} C_{p2} v_2^2 + \alpha_{eq2} v_2 \Delta L \quad (9)$$

The expression of the length variation of the equivalent system  $\Delta L$  that appears in (1) stem from geometrical consideration relative to the movements of the inertial mass, and is given by (10).

$$\Delta L = 2(L - L(\vec{x})) = \left( 2L - 2\sqrt{L^2 + x_0^2 - x^2} \right) \quad (10)$$

The buckling level of the PBEH  $x_{01}$  is given by (11). It is a modified value of the buckling level  $x_0$  established when the PRJ assumption stands. In the case of the proposed model, the influence of the bending stiffness  $K_b$  and the voltage of the tuning APA  $v_2$  are non-negligible and consequently appear in (11).

$$x_{01} = \sqrt{\left( x_0^2 - \frac{2K_b L^2}{K_{eq}} - \frac{\alpha_{eq2} v_2 L}{K_{eq}} \right)} \quad (11)$$

This expression emphasizes the effect of the bending stiffness  $4K_b$  of the beams as it represents additional mechanical resistance, supplementary to the one intentionally added by the term containing the voltage of the tuning APA  $v_2$ .

In practice, the value of the buckling level is brought back to the desired value  $x_0$  set when the PRJ assumption stands. The same value of the buckling level can be established, provided that the force applied on the spring  $4K_b$  is higher. Consequently, the value of the bending stiffness  $K_b$  can be seen as an indicator of the force necessary for buckling a given PBEH, and is a function of the amount of strain put on the APAs or on the beams for a given value of  $x_{01}$ . In that manner, it can be considered a measure of feasibility. The equation system that stems from the Euler-Lagrange equations (4) when considering the displacements of the

mass and buckling level to be small with respect to  $L$ . ( $x_{01} \ll L$  et  $x \ll L$ ), is described by (12).

$$\begin{cases} M\gamma = M\ddot{x} - 2K\beta_c \frac{x_{01}^2}{L^2} x + \frac{2K\beta_c x^3}{L^2} + D\dot{x} \\ \quad + \frac{2\alpha\beta_c xv}{L} \\ I_1 = \frac{2\alpha\beta_c x\dot{x}}{L} - C_p \dot{v} \\ I_2 = \frac{2\alpha_2(1-\beta_c)x\dot{x}}{L} - C_{p2} \dot{v}_2 \end{cases} \quad (12)$$

The parameter  $\beta_c$ , which is linked to the axial stiffness  $K_a$  of the beams and the stiffness of the tuning APA  $K_2$ , is the only value that affects the key normalized parameters that define the PBEH's dynamic behavior.

It acts as a correction factor for the parameters affected, that accounts for the elastic energy stored in the beams and in the tuning APA. When  $\beta_c$  equals 1, the equations obtained are the ones described by Huguet et al [6] given by (13), seeing as the value of the tuning APA's voltage  $v_2$  is a constant.

$$\begin{cases} M\gamma = M\ddot{x} - 2K \frac{x_0^2}{L^2} x + \frac{2K}{L^2} x^3 + D\dot{x} + \frac{2\alpha}{L} xv \\ I = \frac{2\alpha}{L} x\dot{x} - C_p \dot{v} \end{cases} \quad (13)$$

When considering the normalized parameters of the PBEH, assuming the quality factor is a constant, The value of the correction factor  $\beta_c$  affects the electromechanical coupling coefficient  $k_m^2$ , the characteristic pulsation  $\omega_0$  described in (14) and (15).

$$k_m^2 = \frac{\alpha^2 \beta_c}{K C_p} = k_{mPRJ}^2 \beta_c \quad (14)$$

$$\omega_0 = \frac{x_{01}}{L} \sqrt{\frac{4K\beta_c}{M}} = \omega_{0PRJ} \sqrt{\beta_c} \quad (15)$$

A lower correction factor  $\beta_c$  implies a lower electromechanical coupling coefficient  $k_m^2$  and characteristic pulsation  $\omega_0$ , compared to the initial values  $k_{mPRJ}^2$  and  $\omega_{0PRJ}$  obtained when  $\beta_c$  equals 1.

This implies a bigger bandwidth but a lower maximum power output if the rest of the normalized parameters of the harvester are fixed (including the quality factor) as seen in Morel et al's work [8] and in a previous study [7].

In conclusion, the proposed lumped model shows that the stiffness  $K_b$  only affects feasibility. Moreover, low values of the stiffnesses  $K_a$  and  $K_2$ , compared to the value of the energy harvesting's stiffness  $K$ , can lead to drastic changes in the power output that becomes way lower than what is projected for  $K_a \rightarrow \infty$  and  $K_2 \rightarrow \infty$ .

## EXPERIMENTAL RESULTS

Two experimental test series were carried out to explore the predictive capability of the lumped model proposed, using the PBEH prototype represented in Figure 1.

The first tests are characterization tests that aim to compare the estimated parameters of the structure, based on the APA characteristics and beam geometry, to the ones obtained when subjecting the PBEH to an impedance analysis test using a Keysight E4990A impedance analyzer. For different buckling levels imposed by the tuning APA the PBEH is subjected to an impedance analyzer. The latter imposes a low voltage excitation of 5mV and gives us the resonant pulsation, the quality factor and mechanical coupling coefficient of the PBEH. Those values are used to estimate the value of the buckling level using (15).

According to theory,  $k_m^2$  should maintain a fixed value no matter what buckling level is imposed. The buckling level is estimated using the measured value of the resonant pulsation  $\omega_0$  and the equation described in (15).

According to Figure 4 we can conclude that the measured electromechanical coupling coefficient remains almost the same with a relative error inferior to 10%. This error is bigger for lower buckling levels, which suggests that it can stem from the precision of the impedance analyzer. Indeed, lower buckling levels imply lower values of the resonant pulsation and thus more noise.

The theoretical estimation of  $k_m^2$  is considered valid, which implies that the value of the correction factor  $\beta_c$  predicted in theory is correct. Consequently, the estimated values of the buckling level deduced from the values of the resonant pulsation using equation (15) is valid.

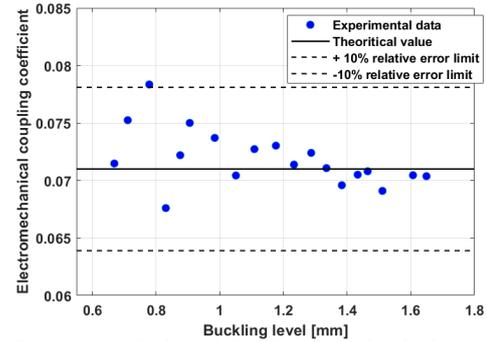


Figure 4: Evolution of the electromechanical coupling coefficient as a function of the estimated buckling level

Table 1 is a list of the parameters of the PBEH prototype. These parameters are all predicted by the lumped model for a fixed level of voltage within the tuning APA, with the exception of the quality factor of the structure which is identified using the impedance analyzer.

Table 1: The parameters of the PBEH prototype used for experimental validation

Parameter	Symbol	Value
Mass frame distance	$L$	35 mm
Inertial mass	$M$	6 g
Buckling level	$x_{01}$	0,7 mm
Electro-mechanical coupling coefficient	$k_m^2$	0.071
Quality factor	$Q_m$	160
Characteristic frequency	$f_0$	47

The second experimental test series consist of observing the displacement and responses of the PBEH prototype for a given sinusoidal ascending frequency sweep with a fixed acceleration amplitude. The results are then compared to the numerical estimations based on the lumped model.

The PBEH prototype is fixed on an electromagnetic shaker driven by a power amplifier. The shaker's acceleration is monitored by an accelerometer fixed on the shaker and linked to a dSpace controlling board, that drives the power amplifier. This permits the control of the acceleration of the shaker through a PI controller.

The displacement, velocity and acceleration of the inertial mass are monitored with a differential laser vibrometer, then recorded and sent to the dSpace controlling board. The

tuning APA is controlled by a power supply that imposes a fixed level of voltage; the buckling level of the beams is thus fixed for the duration of the experiments carried out. The energy harvesting APA is connected to a programmable resistance load, linked to the dSpace board that sets its value. The voltage across the energy harvesting APA is also monitored through a voltage follower connected to the dSpace board.

The tests are made for a fixed acceleration level of  $4 \text{ m/s}^2$  and for a resistance load of  $1 \text{ k}\Omega$ . The displacement, power response and phase of the PBEH based on lumped models are deduced from numerical simulation made using MATLAB.

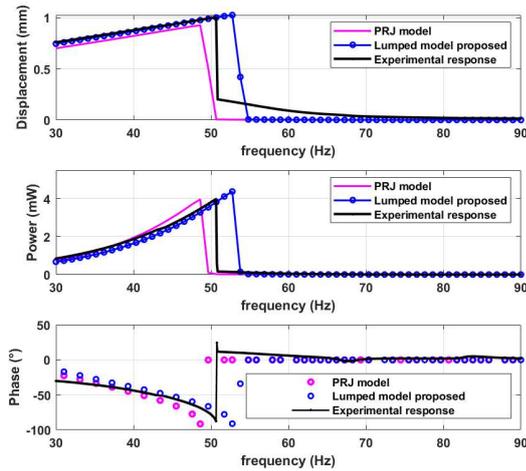


Figure 5: Measured and predicted displacement, power and phase evolution of the PBEH for an ascending sinusoidal frequency sweep with a fixed acceleration

Figure 5 shows that for  $4 \text{ m/s}^2$  the PBEH prototype produces around  $4 \text{ mW}$  with 22 % relative bandwidth. When using the lumped model proposed, the numerical response and experimental response match, with a relative error inferior to 6 %. The mismatch that occurs in the phase's evolution, most likely due to a lack of stability and robustness of the PBEH's inter-well motion.

Numerical results show that when considering the PRJ assumption by modifying the normalized values of the PBEH (rendering the correction factor equal to 1), the error increases up to 15.8%.

Consequently, the model assumptions can be considered valid. The relative error obtained with the enhanced lumped model is low enough for the model to be considered predictive for design purposes.

## CONCLUSION

This paper presents a dynamic lumped model suited for a specific type of PBEHs architectures that uses post-buckled beams to create bistability, and APAs to harvest the vibration energy and to tune the PBEH's buckling level. A few simplifying assumptions allow the introduction of the effect of the buckled beams' axial and bending stiffnesses, as well as the effect of the tuning APA. The lumped model proposed is established using Euler-Lagrange equations, and a system describing the mechanical and electrical aspect of the dynamic system's behavior are found. The

theoretical and numerical results that stem from the lumped model proposed are in good agreement with the experimental results obtained using a PBEH prototype with the architecture described. The proposed model facilitates the understanding of the effect of the post-buckled beams and tuning APA on the PBEH's performance and feasibility.

The predictive capacity of the proposed lumped model can thus facilitate the design optimization of similar PBEH architectures. Additionally, the inclusion of the tuning APA in the lumped model will facilitate the simulation of orbit jump scenarios, made to improve the PBEH's power response, using a rapid buckling level variation, such as the one describes in Huguet et al's work [6].

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