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► **To cite this version:**

Ilyès Berhouni, Didier Bailly, Ilias Petropoulos. On the Definition of Exergy in the Field of Aerodynamics. *AIAA Journal*, 2023, 61 (10), pp.4356-4366. 10.2514/1.J062833 . hal-04199417

**HAL Id: hal-04199417**

**<https://hal.science/hal-04199417v1>**

Submitted on 8 Sep 2023

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# On the definition of exergy in the field of aerodynamics

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The exergy concept originates from the field of static thermodynamics and expresses the maximum theoretically recoverable mechanical work from a system while it evolves towards its dead thermodynamic state. It accounts for both mechanical and thermal mechanisms, and allows to separate reversible and irreversible losses in the system's transformations. The physical insight provided by this concept motivated the development of an exergy-based performance evaluation method in the field of aerodynamics. The resulting formulation has the advantage of being independent of the feasibility of a drag/thrust breakdown (ambiguous for highly-integrated engine concepts) and includes thermal effects in the performance metrics. It however relies on an adapted definition of exergy, in particular involving a dead state in motion. This adapted definition is not trivial and raises theoretical concerns due to fundamental thermodynamic properties of exergy not being always satisfied. This paper aims at proposing a corrected version of this definition which ensures that the fundamental properties of exergy are respected. First, the exergy concept is presented alongside the concerns raised by its original adaptation, which to the best of the authors' knowledge has been used in all exergy-based flowfield analyses in the field of applied aerodynamics. Then, an unsteady exergy balance is derived in the geocentric reference frame (in which there are no ambiguities in the definition of exergy) and then transformed to a reference frame in translation. The corrected adaptation of the exergy definition for aerodynamics applications is extracted from this transformation and the impact on the exergy balance is analysed.

## I. Nomenclature

$$\dot{\mathcal{A}}_{tot} = \dot{\mathcal{A}}_{\phi} + \dot{\mathcal{A}}_{\nabla T} + \dot{\mathcal{A}}_w, \text{ Rate of total anergy generation, J.s}^{-1}$$

$$\dot{\mathcal{A}}_w = \text{Rate of anergy generation by shockwaves, J.s}^{-1}$$

$$\dot{\mathcal{A}}_{\nabla T} = \text{Rate of anergy generation by thermal mixing, J.s}^{-1}$$

$$\dot{\mathcal{A}}_{\phi} = \text{Rate of anergy generation by viscous dissipation, J.s}^{-1}$$

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$E$	=	Mass-specific total energy, $\text{J.kg}^{-1}$
$\dot{E}_u$	=	Streamwise kinetic energy outflow, $\text{J.s}^{-1}$
$\dot{E}_{vw}$	=	Transverse kinetic energy outflow, $\text{J.s}^{-1}$
$\dot{E}_{p\tau}$	=	Boundary surface forces' rate of work, $\text{J.s}^{-1}$
$e$	=	Mass-specific internal energy, $\text{J.kg}^{-1}$
$H$	=	Mass-specific total enthalpy, $\text{J.kg}^{-1}$
$h$	=	Mass-specific static enthalpy, $\text{J.kg}^{-1}$
$\bar{\bar{I}}$	=	Identity tensor
$k$	=	Thermal conductivity, $\text{W.m}^{-1}.\text{K}^{-1}$
$M$	=	Mach number
$\mathbf{n}$	=	Unit normal vector
$p$	=	Static pressure, $\text{kg.m.s}^{-2}$
$\mathbf{q}$	=	Heat flux by conduction, $\text{J.s}^{-1}$
$S_b$	=	Body surface
$S_o$	=	Outer boundary of the control volume
$S_w$	=	Shockwave surface
$s$	=	Mass-specific entropy, $\text{J.K}^{-1}.\text{kg}^{-1}$
$T$	=	Static temperature, K
$\mathcal{V}$	=	Control volume
$\mathbf{V}$	=	Fluid velocity vector, $\text{m.s}^{-1}$
$\mathbf{W}$	=	Shockwave velocity vector, $\text{m.s}^{-1}$
$\dot{\chi}_m$	=	Rate of mechanical exergy outflow, $\text{J.s}^{-1}$
$\dot{\chi}_q$	=	Rate of thermal exergy supplied by conduction, $\text{J.s}^{-1}$
$\dot{\chi}_{tfb}$	=	Rate of throughflow exergy, $\text{J.s}^{-1}$
$\dot{\chi}_{th}$	=	Rate of thermocompressible exergy outflow, $\text{J.s}^{-1}$
$\chi$	=	Mass-specific exergy, $\text{J.kg}^{-1}$
$\chi_f$	=	Mass-specific flow exergy, $\text{J.kg}^{-1}$
$\delta(\ )$	=	Perturbation of a quantity relative to the freestream, $= (\ ) - (\ )_\infty$
$\mu$	=	Fluid dynamic viscosity, $\text{kg.m.s}^{-1}$
$\rho$	=	Density, $\text{kg.m}^{-3}$
$\bar{\bar{\tau}}$	=	Viscous stress tensor, N
$\phi$	=	Dissipation rate per unit volume, $\text{J.s}^{-1}.\text{m}^{-3}$

- $( )_\infty$  = Quantity  $( )$  at freestream conditions
- $( \tilde{ } )$  = Quantity  $( )$  expressed in the geocentric frame  $\tilde{R}$
- $\nabla$  = Nabla operator
- $:=$  = Equal by definition
- $\llbracket \ ]$  = Discontinuous jump of a quantity

## II. Introduction

The precise investigation of aircraft performance through relevant metrics has been a driving factor of research since the birth of the aviation sector. While the efficiency of various aircraft configurations was initially quantified experimentally, the development of numerical tools has progressively allowed to perform accurate simulations for precise performance evaluations. This improved precision, together with the lower costs associated to numerical computations compared to experiments, led to the incorporation of Computational Fluid Dynamics (CFD) into design processes. Consequently, post-processing tools were developed in parallel to enhance the accuracy of physical and numerical analyses of the results obtained from numerical simulations or experiments. The possibility to link any performance degradation to physical aspects of the flow field via appropriate efficiency metrics quickly became a major stake of such post-processing tools.

The most mature approach developed and adopted in the case of aircraft consists in analysing aerodynamic performance with force-based methods. Such a method was developed with a near-field/far-field balance by van der Vooren and Destarac [1, 2], which allows to perform a phenomenological breakdown of drag components. By quantifying the components of the generated drag, as well as providing a clear link to the physical phenomena in the flow, their formulation gives indications on the design modifications which could be carried out in order to achieve a better aircraft performance. Since then, a large amount of work was dedicated to the refinement and extension of this drag decomposition method [3], such as an extension to rotating frames of reference [4, 5] or to unsteady flow analyses [6–8]. These force-based methods allow to consider the mechanical aspects of the performance analysis, yet are less adapted to configurations involving significant thermal exchanges (e.g. aerothermal effects in heat exchanger flows) or for which a clear thrust/drag separation is not possible (e.g. boundary-layer-ingesting configurations).

Another analysis, based on a mechanical energy balance and referred to as power balance, was more recently developed by Drela [9]. This approach gives a clear physical breakdown of the flow of mechanical energy, and is not dependent on the feasibility of a thrust/drag separation. As in far-field drag decomposition methods, sources of loss can be identified by a separation between reversible and irreversible losses (the latter being linked to viscous and shock phenomena). The power balance physical interpretation was since further studied and used to evaluate the performance benefits from the Boundary Layer Ingestion (BLI) concept [10–13]. While this method allows to study

concepts involving strong engine-airframe interactions, it still does not account for thermal effects present in the flow field, which are often small for adiabatic cases, yet non-negligible for a wide range of aerodynamics applications.

These considerations led to the development of another approach at ONERA based on the concept of exergy, which corresponds to the maximum mechanical work that can theoretically be extracted from the system while it evolves towards its dead state. The exergy concept is inherited from thermodynamics [14, 15], where it is commonly used in order to perform static analyses. Arntz et al. [16, 17] proposed an adaptation of the exergy definition for aerodynamics applications, and developed an exergy balance adapted to such configurations. This allows to perform a separation between reversible and irreversible losses of exergy, with a clear link to the physical phenomena involved. This method has been applied to conventional and disruptive aircraft configurations [17–20], as well as improved and studied from a numerical point of view [21]. In recent years, further work was performed to use this exergy balance for various numerical and experimental applications [22–24], extend it to rotating frames of reference [25, 26] and for the analysis of unsteady flows [27]. These developments are all based on the exergy definition of Arntz et al., yet the latter raises questions as it does not guarantee that fundamental properties of exergy are always satisfied (such as the guaranteed positivity of exergy [15]).

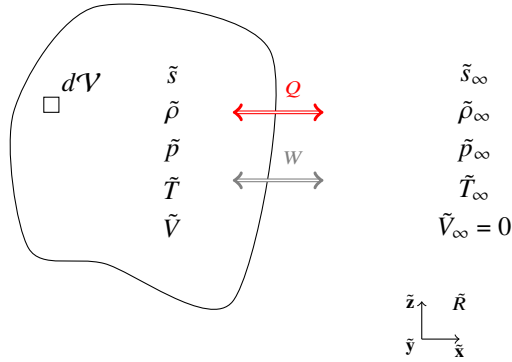
The main objective of this paper is to present a rigorous clarification on the adaptation of the exergy definition for flowfield analyses in the context of aerodynamics studies. The exergy concept as defined in thermodynamics, alongside its adaptation by Arntz et al. and the concerns it raises are detailed in Sec. III. Sec. IV then presents the derivation of an unsteady exergy balance for an aircraft configuration in the geocentric reference frame, for which there is no ambiguity in the exergy definition, as the reference state used in the exergy definition possesses no kinetic energy. In Sec. V, this balance is transformed in a frame of reference attached to an aircraft-type configuration, from which the correct exergy definition is extracted and the impact on the exergy balance (derivation and final form) is evaluated. Finally, the theoretical evolution of the exergy balance components in the schematic case of a converging nozzle is detailed in Sec. VI.

### **III. On the definition of exergy in aerodynamics applications**

#### **A. Definition of an exergy balance**

Let us consider a volume of fluid surrounded by a thermodynamic reservoir with which it exchanges mechanical work (noted  $W$ ), heat (noted  $Q$ ) and mass, as illustrated in Fig. 1. An inertial reference frame  $\tilde{R}$  is defined so that the reservoir is considered to be at rest with no velocity. The dead state of the system under study (here the fluid) is defined as the thermodynamic state in which the fluid is at a thermodynamic equilibrium with its environment, i.e. there is no potential for work recovery as no thermodynamic exchanges take place between the system and its environment. In this particular case, the dead state of the fluid corresponds to the thermodynamic state of the reservoir surrounding it, since

the latter is supposed to be large enough (in comparison to the system studied) for its thermodynamic state variables to be considered constant.



**Fig. 1** Volume of fluid exchanging work and heat with a thermodynamic reservoir at rest.

The goal is to quantify the maximum mechanical work that could be extracted from the mechanical and thermal thermodynamic processes that the fluid undergoes while it evolves towards its dead state. This is equivalent to quantifying the exergy of the fluid [15], which corresponds to the case where the mechanical work available is completely recovered and the thermal exchanges are converted into useful work by the use of a Carnot machine. When considering a perfect gas and neglecting the kinetic and gravitational potential energy, specific exergy is defined as:

$$\tilde{\chi} = (\tilde{e} - \tilde{e}_\infty) + \tilde{p}_\infty \left( \frac{1}{\tilde{\rho}} - \frac{1}{\tilde{\rho}_\infty} \right) - \tilde{T}_\infty (\tilde{s} - \tilde{s}_\infty) \quad (1)$$

This quantity is always positive [15]. It means that physically, there is always a potential for work recovery as long as the fluid is not completely at a thermodynamic equilibrium with its environment. If the fluid is considered to be in motion with respect to its dead state, the kinetic energy cannot be neglected, but must be included in the above exergy definition to get:

$$\tilde{\chi} = (\tilde{e} - \tilde{e}_\infty) + \tilde{p}_\infty \left( \frac{1}{\tilde{\rho}} - \frac{1}{\tilde{\rho}_\infty} \right) - \tilde{T}_\infty (\tilde{s} - \tilde{s}_\infty) + \frac{1}{2} \tilde{V}^2 \quad (2)$$

where  $\tilde{V}$  is defined as the fluid velocity relative to its velocity in its dead state. As kinetic energy is also positive, the specific exergy is mathematically always positive. This is physically correct as it represents the work that can be extracted from the thermodynamic transformations of the fluid between its considered state and its dead state. It cannot be negative mathematically or physically, as it would not be possible to extract a negative quantity of useful work. An exergy balance can be directly established from this definition as:

$$\frac{d}{dt} \int_{\mathcal{V}} \tilde{\rho} \tilde{\chi} dV = \underbrace{\dot{W}}_{\text{Rate of mechanical work extracted}} + \underbrace{\left( 1 - \frac{\tilde{T}_\infty}{\tilde{T}} \right) \dot{Q}}_{\text{Rate of mechanical work extracted from thermal exchanges using a Carnot Machine}} - \underbrace{\dot{D}^*}_{\text{Rate of exergy destruction in the volume}} \quad (3)$$

where  $\dot{W}$  can be further decomposed by introducing the boundary pressure mechanical work  $\dot{W}_p$  as  $\dot{W} = \dot{W}' + \dot{W}_p$ . Thus,  $\dot{W}'$  represents the part of mechanical work exchanged which is not linked to boundary pressure work. When considering a steady-flow open system, another quantity referred to as *flow exergy* [15] can be introduced. It is defined by including the transfer power (corresponding to the pressure forces driving the flow) to the exergy definition of Eq. (2). This gives:

$$\tilde{\chi}_f = \delta\tilde{e} + \tilde{p}_\infty\delta\left(\frac{1}{\tilde{\rho}}\right) - \tilde{T}_\infty\delta\tilde{s} + \frac{1}{2}\tilde{\mathbf{V}}^2 + \underbrace{\frac{\delta\tilde{p}}{\tilde{\rho}}}_{\text{Transfer power}} \quad (4)$$

$$= \delta\tilde{h} - \tilde{T}_\infty\delta\tilde{s} + \frac{1}{2}\tilde{\mathbf{V}}^2 \quad (5)$$

where  $\delta(\cdot) = (\cdot) - (\cdot)_\infty$  is the perturbation of the quantity  $(\cdot)$  relative to the dead state. This quantity represents the exergy of a steady stream of matter and can be positive or negative. The exergy balance can then be reexpressed for a volume without the presence of a discontinuity in the flow field as:

$$\int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\chi}}{\partial \tilde{t}} d\mathcal{V} + \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{\chi}_f \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS = \dot{W}' + \left(1 - \frac{\tilde{T}_\infty}{\tilde{T}}\right) \dot{Q} - \dot{D}^* \quad (6)$$

In this form of the exergy balance, the boundary pressure work is included in the exergy flux through the volume boundaries in order to let the flow exergy appear. The equation established above shows that, in the case of a steady-flow hypothesis, the specific flow exergy is conserved. In non-steady flows, the conserved quantity is rather the specific exergy. The link between specific exergy and specific flow exergy is thus similar to the link between total energy and total enthalpy.

## B. Previous work on exergy analysis in the field of aerodynamics

The use of an exergy balance for flow field analyses in aerodynamics applications was first introduced by Arntz et al. [16, 17]. The equations for a steady flow were derived in a reference frame  $R$  attached to a body in translation and the formulation was applied to several configurations with different levels of geometrical and physical complexity [18, 19]. These derivations were based on the conservation equations, the entropy equation and a definition of exergy where the atmosphere is considered as the fluid's dead state (which is not at rest for a translating frame of reference). Due to the kinetic energy of the atmosphere being non-zero in the reference frame in translation (attached to an object moving with velocity  $-V_\infty$ ), the specific exergy was defined by Arntz et al. as:

$$\chi_f^A = (e - e_\infty) + \left(\frac{p}{\rho} - \frac{p_\infty}{\rho_\infty}\right) - T_\infty(s - s_\infty) + \frac{1}{2}\mathbf{V}^2 - \frac{1}{2}\mathbf{V}_\infty^2 \quad (7)$$

Note that this definition of exergy rather corresponds to a flow exergy, the use of which is still valid in those authors' derivation since it was made for a steady flow (cf. Eq (6)). Since then, multiple works have been based on this definition

of exergy in order to use the same exergy-based formulation for experimental and numerical applications [21–24] as well as unsteady flow analyses [27]. Other works have considered a reference state at rest, but have formulated an unsteady balance of flow exergy rather than exergy (cf. Sec. III.A) [25, 28]. Defining exergy through Eq. (7) leads to several unclear considerations:

- As explained in Sec. III.A, an energy balance leads to the conservation of total energy and not total enthalpy, for which a conservation equation can be written only under a steady flow assumption. Similarly, in the exergy balance, the quantity transported is exergy and not flow exergy. Hence, for unsteady (and steady to be fully rigorous) applications, the exergy definition used as a starting point when deriving the exergy balance must be Eq. (2).
- Exergy is always positive by definition (cf. Eq. (2)), where  $\tilde{\mathbf{V}}$  is considered to be the velocity *relative to the dead state*. For external aerodynamics applications, the dead state is commonly taken as the atmosphere’s thermodynamic state. Considering a fixed frame of reference with no fluid velocity in the atmosphere, this means that the flow under study has no kinetic energy in its dead state. This case is similar to the case detailed in Fig. 1, which means that Eq. (2) defines exergy.

Now let us consider the same dead state with a reference frame  $R$  moving at a velocity  $-V_\infty$ . In this case, Arntz et al. implicitly adapted the flow exergy definition by subtracting the kinetic energy of the flow in its dead state to the flow exergy (cf. Eq. (7)). Doing the same for the specific exergy definition would lead to:

$$\chi^A = (e - e_\infty) + p_\infty \left( \frac{1}{\rho} - \frac{1}{\rho_\infty} \right) - T_\infty (s - s_\infty) + \frac{1}{2} \mathbf{V}^2 - \frac{1}{2} \mathbf{V}_\infty^2 \quad (8)$$

This definition, rather than the one of flow exergy (Eq. (7)), was the one used as a starting point for the exergy balance extension to steady flows in rotating reference frames investigated at ONERA [26]. This however is not trivial and leads to the loss of a fundamental property of exergy, as the specific exergy is not guaranteed to remain positive regardless of the reference state considered. As a result, the exactness of the above definition raises questions and requires careful investigation.

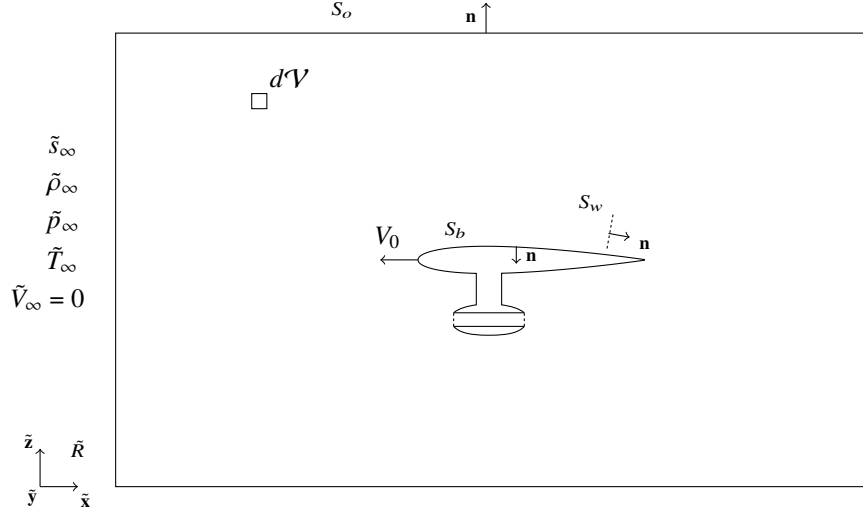
The following sections aim at clarifying these issues and at evaluating the potential impact of an error in the exergy definition used for the derivation of an exergy balance adapted to aerodynamics applications. To this purpose, the derivation of an unsteady exergy balance is first presented in a reference frame for which the atmosphere, considered as the dead state of the gas under study, has no kinetic energy. In this case, it is clear that the exergy of the fluid is defined by Eq. (2) (where velocity is defined in the fixed frame of reference) excluding any ambiguity in the final balance. The final balance equation is then transformed to a reference frame in translation, which corresponds to the classical approach in applied aerodynamics studies. From this transformation, a new definition of exergy expressed in a translating frame of reference (thus different from Eq. (8)) is then introduced.



## IV. Derivation of an unsteady exergy balance in the geocentric reference frame

### A. System definition

The analysis is performed in a continuous control volume  $\mathcal{V}$  limited by an outer boundary  $S_o$  and the aircraft surface  $S_b$ , as shown in Fig. 2. This volume is closed and its limits can be permeable, with a vector  $\mathbf{n}$  normal to the surface and pointing outwards of the volume. A shockwave discontinuity surrounded by a surface  $S_w$  with a normal vector  $\mathbf{n}$  is also considered in the calculations. This last normal vector is pointing outwards of the control volume, i.e. towards the interior of the shockwave volume.



**Fig. 2** 2D cross section of a 3D control volume surrounding the aircraft.

The system is thermodynamically open as it exchanges mass, work and heat with the surrounding fluid across its boundaries. The atmosphere is considered as a thermal and mechanical reservoir which also corresponds to the dead state in the exergy definition. In this case, the analysis is carried out in a fixed reference frame  $\tilde{R}$ , assumed inertial. The control volume is therefore moving with the aircraft at a velocity  $\mathbf{V}_0 = V_0 \tilde{\mathbf{x}}$ . It is supposed that the shockwave discontinuity is attached to the aircraft and moves with a velocity  $\tilde{\mathbf{W}}$  in  $\tilde{R}$ , which is equal to  $\mathbf{V}_0$  in the case of a steady shockwave. As a consequence of this choice of reference frame, the atmosphere is considered to be at rest with  $\tilde{\mathbf{V}}_\infty = \mathbf{0}$ , and the flow is unsteady in  $\tilde{R}$ .

The divergence theorem and the definition of the material derivative in this case are written as:

$$\int_{\mathcal{V}} \nabla \cdot (\cdot) d\mathcal{V} = \int_{S_b} (\cdot) \cdot \mathbf{n} dS + \int_{S_o} (\cdot) \cdot \mathbf{n} dS - \int_{S_w} \llbracket \cdot \rrbracket \cdot \mathbf{n} dS \quad (9)$$

$$\frac{d}{dt} \int_{\mathcal{V}} (\cdot) d\mathcal{V} = \int_{\mathcal{V}} \frac{\partial (\cdot)}{\partial t} d\mathcal{V} + \int_{\partial \mathcal{V}} (\cdot) \mathbf{V} \cdot \mathbf{n} dS - \int_{S_w} \llbracket \cdot \rrbracket \mathbf{W} \cdot \mathbf{n} dS \quad (10)$$

where  $(\cdot)$  is a continuous tensor and  $\llbracket \cdot \rrbracket$  is the jump across the discontinuous shockwave surface  $S_w$ .

## B. Conservation equations

The Navier-Stokes conservation equations for the mass, momentum and energy of the fluid expressed in the reference frame  $\tilde{R}$  are:

$$\left\{ \begin{array}{l} \int_{\mathcal{V}} \frac{\partial \tilde{\rho}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS + \int_{S_w} \llbracket \tilde{\rho} \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS \\ \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\mathbf{V}}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} (\tilde{\rho} \tilde{\mathbf{V}} \otimes \tilde{\mathbf{V}} + (\tilde{p} - \tilde{p}_{\infty}) \tilde{\tilde{I}} - \tilde{\tilde{\tau}}) \cdot \tilde{\mathbf{n}} dS \\ \quad + \int_{S_w} \llbracket \tilde{\rho} \tilde{\mathbf{V}} \otimes \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS \\ \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{E}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} (\tilde{\rho} \tilde{E} \tilde{\mathbf{V}} + (\tilde{p} \tilde{\tilde{I}} - \tilde{\tilde{\tau}}) \cdot \tilde{\mathbf{V}} + \tilde{\mathbf{q}}) \cdot \tilde{\mathbf{n}} dS \\ \quad + \int_{S_w} \llbracket \tilde{\rho} \tilde{E} \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS \end{array} \right. \quad (11)$$

$$\left\{ \begin{array}{l} \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\mathbf{V}}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} (\tilde{\rho} \tilde{\mathbf{V}} \otimes \tilde{\mathbf{V}} + (\tilde{p} - \tilde{p}_{\infty}) \tilde{\tilde{I}} - \tilde{\tilde{\tau}}) \cdot \tilde{\mathbf{n}} dS \\ \quad + \int_{S_w} \llbracket \tilde{\rho} \tilde{\mathbf{V}} \otimes \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS \end{array} \right. \quad (12)$$

$$\left\{ \begin{array}{l} \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{E}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} (\tilde{\rho} \tilde{E} \tilde{\mathbf{V}} + (\tilde{p} \tilde{\tilde{I}} - \tilde{\tilde{\tau}}) \cdot \tilde{\mathbf{V}} + \tilde{\mathbf{q}}) \cdot \tilde{\mathbf{n}} dS \\ \quad + \int_{S_w} \llbracket \tilde{\rho} \tilde{E} \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS \end{array} \right. \quad (13)$$

With the corresponding compatibility equations for the jump across the shockwave:

$$\left\{ \begin{array}{l} \llbracket \tilde{\rho} (\tilde{\mathbf{V}} - \tilde{\mathbf{W}}) \rrbracket_w \cdot \mathbf{n} = 0 \end{array} \right. \quad (14)$$

$$\left\{ \begin{array}{l} \llbracket \tilde{\rho} \tilde{\mathbf{V}} \otimes (\tilde{\mathbf{V}} - \tilde{\mathbf{W}}) + \tilde{p} \tilde{\tilde{I}} - \tilde{\tilde{\tau}} \rrbracket_w \cdot \mathbf{n} = 0 \end{array} \right. \quad (15)$$

$$\left\{ \begin{array}{l} \llbracket \tilde{\rho} \tilde{E} (\tilde{\mathbf{V}} - \tilde{\mathbf{W}}) + (\tilde{p} \tilde{\tilde{I}} - \tilde{\tilde{\tau}}) \cdot \tilde{\mathbf{V}} + \tilde{\mathbf{q}} \rrbracket_w \cdot \mathbf{n} = 0 \end{array} \right. \quad (16)$$

Due to Eq. (11), the energy equation can be rewritten as:

$$\int_{\mathcal{V}} \frac{\partial \tilde{\rho} \delta \tilde{E}}{\partial \tilde{t}} d\mathcal{V} = - \int_{\partial \mathcal{V}} (\tilde{\rho} \delta \tilde{E} \tilde{\mathbf{V}} + (\tilde{p} \tilde{\tilde{I}} - \tilde{\tilde{\tau}}) \cdot \tilde{\mathbf{V}} + \tilde{\mathbf{q}}) \cdot \tilde{\mathbf{n}} dS + \int_{S_w} \llbracket \tilde{\rho} \delta \tilde{E} \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS \quad (17)$$

where, under the assumptions made previously,  $\delta \tilde{E} = \delta \tilde{e} + \frac{1}{2} \tilde{\mathbf{V}}^2$ .

## C. Entropy equation

In addition to the conservation laws, the equation for the entropy evolution inside the control volume is needed to quantify the energy production in the exergy balance. This equation is derived from the Gibbs equation applied to a fluid element during a time variation  $d\tilde{t}$ :

$$\tilde{T} \frac{d\tilde{s}}{d\tilde{t}} = \frac{d\tilde{e}}{d\tilde{t}} - \frac{\tilde{p}}{\tilde{\rho}^2} \frac{d\tilde{\rho}}{d\tilde{t}} \quad (18)$$

The local forms of the mass, momentum and energy equations are injected in this expression, and the equation is integrated over the control volume  $\mathcal{V}$  to obtain:

$$\int_{\mathcal{V}} \left( \frac{\partial \tilde{\rho} \tilde{s}}{\partial \tilde{t}} + \tilde{\mathbf{V}} \cdot (\tilde{\rho} \tilde{s} \tilde{\mathbf{V}}) \right) d\mathcal{V} = \int_{\mathcal{V}} -\frac{1}{\tilde{T}} \tilde{\mathbf{V}} \cdot \tilde{\mathbf{q}} d\mathcal{V} + \int_{\mathcal{V}} \frac{1}{\tilde{T}} (\tilde{\tilde{\tau}} \cdot \tilde{\mathbf{V}}) \cdot \tilde{\mathbf{V}} d\mathcal{V} \quad (19)$$

The next step is to introduce Fourier's law  $\tilde{\mathbf{q}} = -k \tilde{\nabla} \tilde{T}$  where  $k$  is the thermal conductivity and  $\tilde{\Phi} = (\tilde{\tilde{\tau}} \cdot \tilde{\mathbf{V}}) \cdot \tilde{\mathbf{V}}$  as the

dissipation rate per unit volume. Then, using the divergence theorem, Eq. (11) and multiplying Eq. (19) with  $\tilde{T}_\infty$  leads to:

$$\begin{aligned} \tilde{T}_\infty \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \delta \tilde{s}}{\partial \tilde{t}} d\mathcal{V} = & -\tilde{T}_\infty \int_{\partial \mathcal{V}} \tilde{\rho} \delta \tilde{s} \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS - \int_{\partial \mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}} \tilde{\mathbf{q}} \cdot \tilde{\mathbf{n}} dS \\ & + \int_{\mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}^2} k (\tilde{\nabla} \tilde{T})^2 d\mathcal{V} + \int_{\mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}} \tilde{\Phi} d\mathcal{V} + \tilde{T}_\infty \int_{S_w} \left[ \frac{1}{\tilde{T}} \tilde{\mathbf{q}} + \tilde{\rho} \delta \tilde{s} \tilde{\mathbf{V}} \right] \cdot \tilde{\mathbf{n}} dS \quad (20) \end{aligned}$$

#### D. Exergy balance

The goal of this section is to derive an exergy balance based on the most straightforward definition of exergy for a reference state at rest. In  $\tilde{R}$ , the fluid is at rest in its dead state, meaning that specific exergy is defined as:

$$\tilde{\chi} = \delta \tilde{e} + \tilde{p}_\infty \delta \left( \frac{1}{\tilde{\rho}} \right) - \tilde{T}_\infty \delta \tilde{s} + \frac{1}{2} \tilde{\mathbf{V}}^2 \quad (21)$$

without any ambiguity regarding the kinetic energy at its reference state.

The unsteady variation of specific exergy in the control volume can be expressed using the above definition as:

$$\begin{aligned} \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\chi}}{\partial \tilde{t}} d\mathcal{V} + \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{\chi} \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS - \int_{S_w} \left[ \tilde{\rho} \tilde{\chi} \tilde{\mathbf{W}} \right] \cdot \tilde{\mathbf{n}} dS = & \int_{\mathcal{V}} \frac{\partial}{\partial \tilde{t}} \left( \tilde{\rho} \delta \tilde{e} - \tilde{\rho} \tilde{T}_\infty \delta \tilde{s} + \frac{1}{2} \tilde{\rho} \tilde{\mathbf{V}}^2 \right) d\mathcal{V} \\ & + \int_{\partial \mathcal{V}} \left( \tilde{\rho} \delta \tilde{e} - \tilde{\rho} \tilde{T}_\infty \delta \tilde{s} + \frac{1}{2} \tilde{\rho} \tilde{\mathbf{V}}^2 \right) \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS + \int_{\mathcal{V}} \frac{\partial}{\partial \tilde{t}} \left( \tilde{\rho} \tilde{p}_\infty \delta \left( \frac{1}{\tilde{\rho}} \right) \right) d\mathcal{V} + \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{p}_\infty \delta \left( \frac{1}{\tilde{\rho}} \right) \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS \\ & - \int_{S_w} \left[ \tilde{\rho} \left( \delta \tilde{e} - \tilde{T}_\infty \delta \tilde{s} + \frac{1}{2} \tilde{\mathbf{V}}^2 \right) \tilde{\mathbf{W}} \right] \cdot \tilde{\mathbf{n}} dS - \int_{S_w} \left[ \tilde{\rho} \tilde{p}_\infty \delta \left( \frac{1}{\tilde{\rho}} \right) \tilde{\mathbf{W}} \right] \cdot \tilde{\mathbf{n}} dS \quad (22) \end{aligned}$$

The following terms are grouped as:

$$\int_{\mathcal{V}} \frac{\partial}{\partial \tilde{t}} \left( \tilde{\rho} \tilde{p}_\infty \delta \left( \frac{1}{\tilde{\rho}} \right) \right) d\mathcal{V} + \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{p}_\infty \delta \left( \frac{1}{\tilde{\rho}} \right) \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS - \int_{S_w} \left[ \tilde{\rho} \tilde{p}_\infty \delta \left( \frac{1}{\tilde{\rho}} \right) \tilde{\mathbf{W}} \right] \cdot \tilde{\mathbf{n}} dS = \int_{\partial \mathcal{V}} \tilde{p}_\infty \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS \quad (23)$$

where several terms are simplified due to Eq. (11) and  $\tilde{p}_\infty$  being constant. Eqs. (23), (17) and (20) are then injected into Eq. (22):

$$\begin{aligned} \int_{\mathcal{V}} \frac{\partial \tilde{\rho} \tilde{\chi}}{\partial \tilde{t}} d\mathcal{V} = & - \int_{\partial \mathcal{V}} \tilde{\rho} \tilde{\chi} \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS + \int_{S_w} \left[ \tilde{\rho} \tilde{\chi} \tilde{\mathbf{W}} \right] \cdot \tilde{\mathbf{n}} dS \\ & - \int_{\partial \mathcal{V}} \left( (\tilde{p} \tilde{\mathbf{I}} - \tilde{\tau}) \cdot \tilde{\mathbf{V}} \right) \cdot \tilde{\mathbf{n}} dS + \int_{\partial \mathcal{V}} \tilde{p}_\infty \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS - \int_{\partial \mathcal{V}} \left( 1 - \frac{\tilde{T}_\infty}{\tilde{T}} \right) \tilde{\mathbf{q}} \cdot \tilde{\mathbf{n}} dS \\ & - \int_{\mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}} \tilde{\Phi} d\mathcal{V} - \int_{\mathcal{V}} \frac{\tilde{T}_\infty}{\tilde{T}^2} k (\tilde{\nabla} \tilde{T})^2 d\mathcal{V} - \tilde{T}_\infty \int_{S_w} \left[ \frac{1}{\tilde{T}} \tilde{\mathbf{q}} + \tilde{\rho} \delta \tilde{s} (\tilde{\mathbf{V}} - \tilde{\mathbf{W}}) \right] \cdot \tilde{\mathbf{n}} dS \quad (24) \end{aligned}$$

This equation is an exergy balance expressed in a fixed reference frame in which the atmosphere possesses no kinetic energy. Its terms can be interpreted as follows:

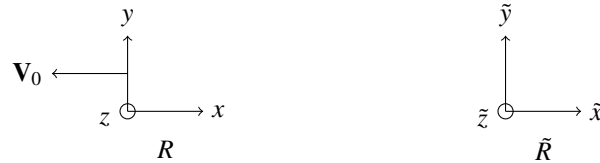
- $\int_{\mathcal{V}} \frac{\partial \bar{\rho} \bar{\chi}}{\partial t} d\mathcal{V}$  is the time rate of specific exergy change inside the control volume.
- $\int_{\partial \mathcal{V}} \bar{\rho} \bar{\chi} \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS - \int_{S_w} \llbracket \bar{\rho} \bar{\chi} \tilde{\mathbf{W}} \rrbracket \cdot \tilde{\mathbf{n}} dS$  is the exergy flux through the boundaries of the control volume. It can be an overall exergy inflow or outflow.
- $\int_{\partial \mathcal{V}} ((\bar{p} \bar{I} - \bar{\tau}) \cdot \tilde{\mathbf{V}}) \cdot \tilde{\mathbf{n}} dS$  is the rate of work done by the pressure and viscous forces on the boundaries of the control volume.
- $\int_{\partial \mathcal{V}} \bar{p}_\infty \tilde{\mathbf{V}} \cdot \tilde{\mathbf{n}} dS$  is the rate of isobaric work done at the boundary of the control volume. In the case where the work is performed in order to compress the gas in its transformation towards its dead state, this term represents a usable rate of work done by the atmospheric pressure. In the case where the gas is expanding against the atmospheric pressure, this term is an additional work that has to be provided for the transformation towards the dead state to take place.
- $\int_{\partial \mathcal{V}} \left(1 - \frac{\bar{T}_\infty}{\bar{T}}\right) \tilde{\mathbf{q}} \cdot \tilde{\mathbf{n}} dS$  is the thermal exergy provided or retrieved by thermal conduction through  $\partial \mathcal{V}$ .
- $\int_{\mathcal{V}} \frac{\bar{T}_\infty}{\bar{T}} \tilde{\Phi} d\mathcal{V}$  is the rate of anergy generated (or equivalently the rate of exergy destroyed irreversibly) due to viscous effects inside the control volume.
- $\int_{\mathcal{V}} \frac{\bar{T}_\infty}{\bar{T}^2} k (\tilde{\nabla} T)^2 d\mathcal{V}$  is the rate of anergy generated by thermal mixing inside the control volume.
- $\bar{T}_\infty \int_{S_w} \llbracket \frac{1}{\bar{T}} \tilde{\mathbf{q}} + \bar{\rho} \delta \tilde{s} (\tilde{\mathbf{V}} - \tilde{\mathbf{W}}) \rrbracket \cdot \tilde{\mathbf{n}} dS$  is the rate of anergy generated by the shockwave(s) inside the control volume.

In physical terms, this balance equation expresses that the change of exergy with respect to time inside the control volume is due to any inflow/outflow of exergy through the external surfaces, the rate of work done by external surface forces and irreversible exergy losses inside the control volume.

## V. Transformation of the exergy balance for a reference frame in translation

### A. System definition

Let us consider a frame of reference  $R$  in translation at a velocity  $\mathbf{V}_0$  with respect to the fixed reference frame  $\tilde{R}$  defined in Fig. 2 (as shown in Fig. 3):



**Fig. 3** Moving frame  $R$  at a velocity  $\mathbf{V}_0$  with respect to the fixed frame  $\tilde{R}$ .

This reference frame is commonly used in aerodynamics studies to simplify analyses. In this frame the aircraft is motionless, and the flow is steady. However, the fluid is *not* at rest in its dead state, but in motion with a velocity  $\mathbf{V}_\infty = -\mathbf{V}_0$ .

At  $t = 0$ , the two frames are considered to be coincident with a common origin. Under these conditions, the variables

in the translating frame of reference  $R$  can be expressed as functions of the ones linked to  $\tilde{R}$ :

$$\begin{cases} t = \tilde{t} \\ x = \tilde{x} - V_0 \tilde{t} \\ y = \tilde{y} \\ z = \tilde{z} \end{cases} \quad (25)$$

The time derivative must be transformed using:

$$\begin{aligned} \frac{\partial K}{\partial \tilde{t}} &= \frac{\partial K}{\partial t} \frac{\partial t}{\partial \tilde{t}} + \frac{\partial K}{\partial x} \frac{\partial x}{\partial \tilde{t}} + \frac{\partial K}{\partial y} \frac{\partial y}{\partial \tilde{t}} + \frac{\partial K}{\partial z} \frac{\partial z}{\partial \tilde{t}} \\ &= \frac{\partial K}{\partial t} - \mathbf{V}_0 \cdot \nabla K \\ &= \frac{\partial K}{\partial t} - \nabla \cdot (K \mathbf{V}_0) + K \nabla \cdot \mathbf{V}_0 \end{aligned} \quad (26)$$

where  $K$  is any continuous scalar quantity and the third equality is obtained due to  $\mathbf{V}_0$  being constant in space and time.

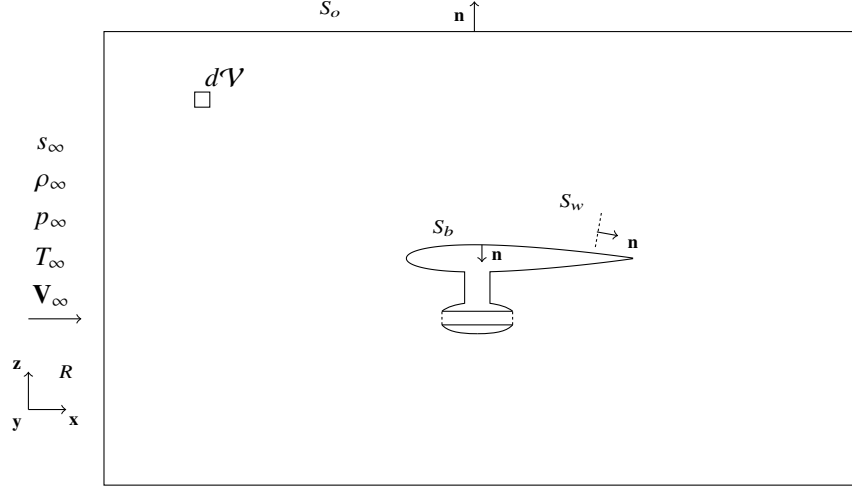
When considering an integral expression, this gives:

$$\begin{aligned} \int_{\mathcal{V}} \frac{\partial K}{\partial \tilde{t}} d\mathcal{V} &= \int_{\mathcal{V}} \frac{\partial K}{\partial t} d\mathcal{V} - \int_{\mathcal{V}} \nabla \cdot (K \mathbf{V}_0) d\mathcal{V} \\ &= \int_{\mathcal{V}} \frac{\partial K}{\partial t} d\mathcal{V} - \int_{\partial \mathcal{V}} K \mathbf{V}_0 \cdot \mathbf{n} dS + \int_{S_w} \llbracket K \mathbf{V}_0 \rrbracket \cdot \mathbf{n} dS \end{aligned} \quad (27)$$

The system presented in Fig. 2 is then considered with respect to  $R$ , as shown in Fig. 4. The system is thermodynamically open as it exchanges mass, work and heat with the surrounding fluid across its boundaries. The atmosphere is considered as a thermal and mechanical reservoir which also corresponds to the dead state in the exergy definition. In this case the analysis is carried out in  $R$ , assumed inertial. The control volume is therefore fixed with the fluid flowing in and out of it.  $R$  is chosen so that the aircraft velocity is null, whereas the atmosphere is considered to have a velocity  $\mathbf{V}_\infty = -\mathbf{V}_0$ . In general, the fluid velocity expressed in the reference frame in translation is  $\mathbf{V} = \tilde{\mathbf{V}} - \mathbf{V}_0 = \tilde{\mathbf{V}} + \mathbf{V}_\infty$ , and similarly the shock velocity is expressed as  $\mathbf{W} = \tilde{\mathbf{W}} + \mathbf{V}_\infty$ . In the case where the shock is steady in  $R$ , this gives  $\mathbf{W} = 0$ .

The gradient operator is not affected when moving from  $\tilde{R}$  to  $R$ , nor is any quantity that is not directly dependent on  $\tilde{\mathbf{V}}$  (i.e.  $\tilde{\nabla} = \nabla$  and  $\tilde{b} = b$  where  $b$  is any scalar quantity not dependent on the fluid velocity). As a consequence, the conservative variables are not affected by this change of reference frame, with the exception of the total energy which is expressed in  $R$  as:

$$E = e + \frac{1}{2} \mathbf{V}^2 = \tilde{E} + \mathbf{V} \cdot \mathbf{V}_\infty - \frac{1}{2} \mathbf{V}_\infty^2 \quad (28)$$



**Fig. 4** 2D cross section of a 3D control volume surrounding the aircraft as seen in  $R$ .

The above equation highlights that the value of the total energy changes between  $\tilde{R}$  and  $R$  (i.e.  $\tilde{E} \neq E$ ). An additional component depending on  $V_\infty$  appears in the definition of  $E$ , which is linked to the kinetic energy difference that is perceived due to the translation of  $R$ . The forms of the conservation equation obtained separately for  $\tilde{E}$  in  $\tilde{R}$  and  $E$  in  $R$  are however identical, as the terms depending of  $V_\infty$  are cancelled due to the mass and momentum conservation equation being expressed in  $R$ .

## B. Exergy balance

### 1. Exergy definition

As discussed in Sec. III, exergy is a scalar quantity that represents an amount of recoverable mechanical work from a system as it evolves towards its dead state through thermodynamic processes. Thus, it is not dependent on the reference frame it is expressed in (i.e.  $\chi = \tilde{\chi}$ ). The exergy definition must however be written with the fluid velocity expressed in  $R$ :

$$\chi = \tilde{\chi} = \delta e + p_\infty \delta \left( \frac{1}{\rho} \right) - T_\infty \delta s + \frac{1}{2} (\mathbf{V} - \mathbf{V}_\infty)^2 \quad (29)$$

$$= \chi^A - \mathbf{V} \cdot \mathbf{V}_\infty + \mathbf{V}_\infty^2 \quad (30)$$

In contrast to the definition of total energy in  $R$ , which is expressed as a function of  $\mathbf{V}$  (cf. Eq. (28)), the above expression is not further modified. This is because, as mentioned previously, the specific exergy quantifies the maximum amount of work that could be recovered as the system evolves towards its dead state. Thus, the consideration of a relevant dead state for the calculations is essential, and the dead state must correspond to the atmosphere thermodynamic state in aerodynamics applications.

To further detail this explanation, let us consider a fluid system which would only be composed of the atmosphere at rest. In this case, Eq. (29) guarantees that the exergy of the system equals zero independently of the choice of reference frame between  $\tilde{R}$  and  $R$ . In the case where any additional thermodynamic perturbation is considered (such as a compression or an expansion), the exergy would still be independent on the choice of reference frame, and its thermodynamic properties (in particular the guaranteed positivity of exergy) would be respected.

Another option would have been to adapt the exergy definition to the reference frame considered, in which case the kinetic energy component would be expressed in  $R$  as  $\frac{1}{2}V^2$  (similarly to what is done for the total energy conservation). In that case, an identical form of the exergy balance could have been derived for this new definition of exergy, as the terms depending of  $V_\infty$  would have been cancelled using the conservation equations. However, adapting the definition of specific exergy to each reference frame (similarly to what is performed for the total energy) would lead to a modification of the system's exergy depending on the chosen frame of reference. Considering the case of a system only composed of the atmosphere as above, the exergy would still be equal to zero in  $\tilde{R}$  but would be equal to  $\frac{1}{2}V_\infty^2$  in  $R$ , even though this would not correspond to a physical acceleration of the fluid. Thus, a fictive non-zero exergy resulting from the change of reference frame would have been mistakenly interpreted as a potential for mechanical work recovery. A different option would have been to adopt the definition  $\chi^A$ . This remediates the reference frame inconsistency, since the exergy computed in each reference frame correctly equals zero as the atmosphere kinetic energy is subtracted. Nevertheless, it leads to the loss of the guarantee that specific exergy is always positive when considering more complex systems.

As a consequence, Eq. (29) constitutes the only definition of the specific exergy that both guarantees the thermodynamic properties of  $\tilde{\chi}$  (and in particular its positivity) and does not lead to an inconsistency when a change of reference frame is performed. This reasoning is essential, as it shows that the exergy definition introduced by Arntz et al. [16, 17] and used in following aerodynamics studies is not consistent in that aspect.

## 2. Transformed exergy balance

The time derivative in Eq. (24) is transformed in the reference frame  $R$  in translation, and the fluid velocity expressed in  $R$  is injected to obtain:

$$\begin{aligned} \int_{\mathcal{V}} \frac{\partial \rho \chi}{\partial t} d\mathcal{V} + \int_{\partial \mathcal{V}} \rho \chi \mathbf{V} \cdot \mathbf{n} dS - \int_{S_w} \llbracket \rho \chi \mathbf{W} \rrbracket \cdot \mathbf{n} dS = & - \int_{\partial \mathcal{V}} ((p\bar{\bar{I}} - \bar{\bar{\tau}}) \cdot (\mathbf{V} - \mathbf{V}_\infty)) \cdot \mathbf{n} dS \\ & + \int_{\partial \mathcal{V}} p_\infty (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS - \int_{\partial \mathcal{V}} \left(1 - \frac{T_\infty}{T}\right) \mathbf{q} \cdot \mathbf{n} dS - \int_{\mathcal{V}} \frac{T_\infty}{T} \Phi d\mathcal{V} \\ & - \int_{\mathcal{V}} \frac{T_\infty}{T^2} k (\nabla T)^2 d\mathcal{V} - T_\infty \int_{S_w} \llbracket \frac{1}{T} \mathbf{q} + \rho \delta s (\mathbf{V} - \mathbf{W}) \rrbracket \cdot \mathbf{n} dS \quad (31) \end{aligned}$$

Note that since  $V_\infty$  is constant:

$$\bar{\bar{\tau}} = \mu \left( -\frac{2}{3} \nabla \cdot (\mathbf{V} - \mathbf{V}_\infty) + \nabla(\mathbf{V} - \mathbf{V}_\infty) + (\nabla(\mathbf{V} - \mathbf{V}_\infty))^T \right) = \bar{\bar{\tau}} \quad (32)$$

and thus:

$$\tilde{\Phi} = (\bar{\bar{\tau}} \cdot \nabla) \cdot (\mathbf{V} - \mathbf{V}_\infty) = \Phi \quad (33)$$

Eq. (31) is an exergy balance that is obtained by projecting Eq. (24) in  $R$ . It can also be derived by starting from the conservation equations expressed in  $R$  and by deriving an exergy balance as was done for  $\tilde{R}$  in Sec. IV. The exergy balance in the form of Eq. (31) is rather compact, but it can be further detailed in order to allow fine near-field/far-field analyses. For this purpose, the balance must be further decomposed in order to separate contributions on  $S_b$  and  $S_o$  as well as allow a clear identification of physical mechanisms in the flow associated to the components obtained.

### C. Further decomposition of the exergy balance

#### 1. Phenomenological decomposition

Whereas Eq. (31) allows a clear physical interpretation by itself, the decomposition is carried on in order to further refine the exergy balance interpretation. The exergy flux is decomposed using the exergy definition in  $R$  as:

$$\chi = \underbrace{\delta e + p_\infty \delta \left( \frac{1}{\rho} \right) - T_\infty \delta s}_{\chi_s} + \underbrace{\frac{1}{2} (\mathbf{V} - \mathbf{V}_\infty)^2}_{KE} \quad (34)$$

where  $\chi_s$  is referred to as the static exergy and  $KE$  is the kinetic energy.

Static exergy is the exergy that would have been available if the gas had no kinetic energy and no flowstream was considered. This corresponds to the useful work that could be retrieved from a gas compressed by (or expanding against) a piston in a closed container by recovering the mechanical work (equal to exergy) and transforming the thermal transfers into useful work (by a Carnot machine).

Kinetic energy can also be rewritten as  $\frac{1}{2} (\mathbf{V} - \mathbf{V}_\infty)^2 = \frac{1}{2} \tilde{\mathbf{V}}^2$ . This highlights that the actual kinetic energy, as seen in the fixed reference frame  $\tilde{R}$ , is perceived as a perturbation kinetic energy when seen from the reference frame in translation  $R$ . This observation is useful for the following comparisons between this derivation and previous studies performed on the exergy balance.

The kinetic energy is directly expressed using  $\tilde{\mathbf{V}} = \delta \mathbf{V} = (u, v, w)^T$ , giving:

$$\frac{1}{2} \tilde{\mathbf{V}}^2 = \frac{1}{2} (u^2 + v^2 + w^2) \quad (35)$$



The exergy flux appearing in Eq. (31) can thus be decomposed to get:

$$\begin{aligned}
\int_{\mathcal{V}} \frac{\partial \rho \left( \chi_s + \frac{1}{2}(u^2 + v^2 + w^2) \right)}{\partial t} d\mathcal{V} = & - \int_{\partial \mathcal{V}} \rho \chi_s \mathbf{V} \cdot \mathbf{n} dS + \int_{S_w} \llbracket \rho \chi_s \mathbf{W} \rrbracket \cdot \mathbf{n} dS \\
& - \int_{\partial \mathcal{V}} \frac{1}{2} \rho (u^2 + v^2 + w^2) \mathbf{V} \cdot \mathbf{n} dS + \int_{S_w} \llbracket \frac{1}{2} \rho (u^2 + v^2 + w^2) \mathbf{W} \rrbracket \cdot \mathbf{n} dS \\
& - \int_{\partial \mathcal{V}} \left( (p - p_\infty) \bar{\bar{I}} - \bar{\bar{\tau}} \right) \cdot (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS - \int_{\partial \mathcal{V}} \left( 1 - \frac{T_\infty}{T} \right) \mathbf{q} \cdot \mathbf{n} dS \\
& - \int_{\mathcal{V}} \frac{T_\infty}{T} \Phi d\mathcal{V} - \int_{\mathcal{V}} \frac{T_\infty}{T^2} k (\nabla T)^2 d\mathcal{V} - T_\infty \int_{S_w} \llbracket \frac{1}{T} \mathbf{q} + \rho \delta s (\mathbf{V} - \mathbf{W}) \rrbracket \cdot \mathbf{n} dS \quad (36)
\end{aligned}$$

Several notations are introduced from this equation:

- $\dot{X}_v(t) := \int_{\mathcal{V}} \frac{\partial \rho \chi_s}{\partial t} d\mathcal{V} + \int_{\mathcal{V}} \frac{\partial \frac{1}{2} \rho (u^2 + v^2 + w^2)}{\partial t} d\mathcal{V}$  is the unsteady rate of change of exergy inside the control volume. It is composed of two terms respectively representing a static and a kinetic contribution.
- $\dot{X}_{th} := - \int_{\partial \mathcal{V}} \rho \chi_s \mathbf{V} \cdot \mathbf{n} dS + \int_{S_w} \llbracket \rho \chi_s \mathbf{W} \rrbracket \cdot \mathbf{n} dS$  is the static exergy flux. This term is usually referred to as thermal or thermocompressible exergy flux.
- $\dot{X}_m := - \int_{\partial \mathcal{V}} \rho \frac{1}{2} (u^2 + v^2 + w^2) \mathbf{V} \cdot \mathbf{n} dS + \int_{S_w} \llbracket \frac{1}{2} \rho (u^2 + v^2 + w^2) \mathbf{W} \rrbracket \cdot \mathbf{n} dS - \int_{\partial \mathcal{V}} ((p - p_\infty) \bar{\bar{I}} - \bar{\bar{\tau}}) \cdot (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS$  is the mechanical exergy flux, composed of the flow of kinetic perturbation exergy and the transfer power linked to velocity perturbations on the volume boundaries. It is purely dependent on the velocity as perceived in the fixed reference frame  $\tilde{R}$ , which is considered to be a perturbation velocity when analysing the configuration in  $R$ .
- $\dot{X}_q := - \int_{\partial \mathcal{V}} \left( 1 - \frac{T_\infty}{T} \right) \mathbf{q} \cdot \mathbf{n} dS$  is the exergy transferred by thermal conduction through non-adiabatic boundaries.
- $\dot{A}_\phi := \int_{\mathcal{V}} \frac{T_\infty}{T} \Phi d\mathcal{V}$  is the viscous energy, corresponding to the rate of irreversible exergy destruction due to viscous effects.
- $\dot{A}_{\nabla T} := \int_{\mathcal{V}} \frac{T_\infty}{T^2} k (\nabla T)^2 d\mathcal{V}$  is the thermal energy, corresponding to the rate of irreversible exergy destruction due to thermal mixing.
- $\dot{A}_w := T_\infty \int_{S_w} \llbracket \frac{1}{T} \mathbf{q} + \rho \delta s (\mathbf{V} - \mathbf{W}) \rrbracket \cdot \mathbf{n} dS$  is the wave energy, corresponding to the rate of irreversible exergy destruction due to the presence of shockwaves.

Using these notations, the exergy balance is rewritten in a compact form as:

$$\underbrace{\dot{X}_v(t)}_{\text{Unsteady exergy variation in the control volume}} = \underbrace{\dot{X}_m + \dot{X}_{th} + \dot{X}_q}_{\text{Exergy flux through the control volume limits}} - \underbrace{(\dot{A}_\phi + \dot{A}_{\nabla T} + \dot{A}_w)}_{\text{Exergy destruction (i.e. energy generation) inside the control volume}} \quad (37)$$

The above equation corresponds to a classical balance of a physical quantity in which any unsteady variation comes from its fluxes at the domain borders and its production or destruction within the volume. It is a more detailed equivalent of Eq. (6) in  $R$ .

## 2. Separation of the contributions on $S_b$ and $S_o$

The above analysis can be refined by separating the contributions on  $S_b$  (referred to as near field) and  $S_o$  (referred to as far field) for the exergy flux terms. For example, the mechanical exergy is decomposed as  $\dot{\chi}_m = \dot{\chi}_{mb} - \dot{\chi}_{mo}$  where:

$$\dot{\chi}_{mb} = - \int_{S_b} \left( \rho \frac{1}{2} (u^2 + v^2 + w^2) \mathbf{V} + ((p - p_\infty) \bar{\bar{\mathbf{I}}} - \bar{\bar{\boldsymbol{\tau}}}) \cdot (\mathbf{V} - \mathbf{V}_\infty) \right) \cdot \mathbf{n} \, dS \quad (38)$$

$$\dot{\chi}_{mo} = \int_{S_o} \left( \rho \frac{1}{2} (u^2 + v^2 + w^2) \mathbf{V} + ((p - p_\infty) \bar{\bar{\mathbf{I}}} - \bar{\bar{\boldsymbol{\tau}}}) \cdot (\mathbf{V} - \mathbf{V}_\infty) \right) \cdot \mathbf{n} \, dS - \int_{S_w} \left[ \frac{1}{2} \rho (u^2 + v^2 + w^2) \mathbf{W} \right] \cdot \mathbf{n} \, dS \quad (39)$$

The sign convention is chosen so that any near-field contribution is considered as an exergy inflow while any far-field contribution is considered as an exergy outflow. The exergy balance then becomes:

$$\dot{\chi}_{mb} + \dot{\chi}_{thb} + \dot{\chi}_{qb} = \dot{\chi}_v(t) + \dot{\chi}_{mo} + \dot{\chi}_{tho} + \dot{\chi}_{qo} + \dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T} + \dot{\mathcal{A}}_w \quad (40)$$

where the farfield terms are arbitrarily placed on the same side as the unsteady exergy variation. For the development of a phenomenological drag breakdown adapted to unsteady flows, this choice was made by Toubin et al. [6–8] to account for delay effects (any drag generated by the body studied  $S_b$  is perceived on  $S_o$  with a time delay). However, in the case of exergy-based analyses, placing the unsteady variation on the side of near-field integrals (i.e. on  $S_b$ ) would also make a physical sense since the evolution of exergy in the volume is also considered through anergy generation. Further studies would thus be required to provide additional physical insight on this choice.

## 3. Decomposition of the near-field mechanical exergy

While the exergy balance derived above is complete, the analysis can be further refined. In particular, a desirable feature for the investigation of external aerodynamic flows in previous studies is the clear appearance of the contribution of the aerodynamic force in the balance, which allows to link the exergy balance to force-based analyses. This is achieved by first decomposing the mechanical contribution to the near-field exergy inflow (defined as  $\dot{\chi}_b = \dot{\chi}_{mb} + \dot{\chi}_{thb}$ ) as:

$$\begin{aligned} \dot{\chi}_b = & - \int_{S_b} \left( \rho \left( \chi_\xi + \frac{1}{2} V^2 \right) \mathbf{V} + ((p - p_\infty) \bar{\bar{\mathbf{I}}} - \bar{\bar{\boldsymbol{\tau}}}) \cdot \mathbf{V} \right) \cdot \mathbf{n} \, dS \\ & + \int_{S_b} (\rho \mathbf{V}_\infty \cdot \mathbf{V}) \mathbf{V} \cdot \mathbf{n} \, dS - \int_{S_b} \frac{1}{2} V_\infty^2 \rho \mathbf{V} \cdot \mathbf{n} \, dS + \mathbf{V}_\infty \cdot \int_{S_b} ((p - p_\infty) \bar{\bar{\mathbf{I}}} - \bar{\bar{\boldsymbol{\tau}}}) \cdot \mathbf{n} \, dS \end{aligned} \quad (41)$$

This decomposition consists in separating the contributions depending on  $\mathbf{V}_\infty$ , i.e. related to the change of reference frame, and the ones depending on  $V$ . The second and third terms of the above equation are obtained by expanding the kinetic contribution to specific exergy as  $\chi = \chi_\xi + V^2/2 - \mathbf{V} \cdot \mathbf{V}_\infty + V_\infty^2/2$ . They represent fluxes of the terms responsible

for the kinetic energy difference (associated to a reference frame transformation) through permeable parts of  $S_b$ . The second term of Eq. (41) represents a kinetic energy difference associated to a variation of momentum, while the third one is representative of a kinetic energy difference associated to a non-zero massflow variation along  $S_b$  (e.g. unbalanced massflow rate across the propulsion system permeable boundaries). The last term of Eq. (41) is finally representative of the variation of work rate of surface forces on  $S_b$  associated to the reference frame transformation. From this decomposition and by separating the contributions on permeable and non-permeable surfaces of  $S_b$  (respectively noted  $S_{bp}$  and  $S_{bnp}$ ),  $\dot{\mathcal{X}}_b$  can be expressed as  $\dot{\mathcal{X}}_b = \dot{\mathcal{X}}_{tfb} - \dot{\mathcal{X}}_{trb}$ , with:

$$\dot{\mathcal{X}}_{tfb} := - \int_{S_{bp}} \left( \rho \left( \chi_s + \frac{1}{2} V^2 \right) \mathbf{V} + ((p - p_\infty) \bar{\bar{I}} - \bar{\bar{\tau}}) \cdot \mathbf{V} \right) \cdot \mathbf{n} dS \quad (42)$$

$$\dot{\mathcal{X}}_{trb} := -\mathbf{V}_\infty \cdot \int_{S_{bp}} (\rho \mathbf{V} \otimes \mathbf{V} + (p - p_\infty) \bar{\bar{I}} - \bar{\bar{\tau}}) \cdot \mathbf{n} dS - \mathbf{V}_\infty \cdot \int_{S_{bnp}} ((p - p_\infty) \bar{\bar{I}} - \bar{\bar{\tau}}) \cdot \mathbf{n} dS + \int_{S_{bp}} \frac{1}{2} V_\infty^2 \rho \mathbf{V} \cdot \mathbf{n} dS \quad (43)$$

$\dot{\mathcal{X}}_{tfb}$  is referred to as the throughflow exergy, and represents the flux of a flow exergy defined for a reference state at rest in  $R$  (instead of  $\tilde{R}$ ). As this is not the real flow exergy flux (which is defined with respect to a reference state at rest in  $\tilde{R}$ , cf. Sec. V.B.1), an exergy difference  $\dot{\mathcal{X}}_{trb}$  composed of three different terms which are linked to the momentum and mass conservation in  $R$  appears in the decomposition. This difference appears for the sole reason that the analysis is performed in a reference frame in translation, in which the atmosphere is not at rest. The first two terms of Eq. (43) represent the difference of the rate of work of surface forces on walls due to the reference frame transformation from  $\tilde{R}$  to  $R$ . Note that the first term is representative of the interior of the propulsion system (limited by permeable surfaces). In the specific case where  $R$  translates at the same velocity as the body, the sum of these first two terms corresponds to the rate of work of aerodynamic forces applied to the body as perceived in  $\tilde{R}$  (no aerodynamic work is perceived in  $R$  as the body is motionless). The rate of work of aerodynamic forces perceived in  $\tilde{R}$  is different from zero in cases where thrust and drag do not balance each other, leading to the need for a non-aerodynamic force to work for the body to be at equilibrium (as also mentioned by Drela [9], Arntz [17], Sanders and Laskaridis [13]). Note that the third term in Eq. (43) is unchanged with respect to Eq. (41). Thus, in the case where  $R$  translates at the same velocity than the body, the exergy difference associated to the reference frame transformation can be written as:

$$\dot{\mathcal{X}}_{trb} = -\mathbf{V}_\infty \cdot \mathbf{F}_{aero} - \frac{1}{2} V_\infty^2 \dot{m}_b \quad (44)$$

where  $\mathbf{F}_{aero}$  is the overall aerodynamic force acting on the body and  $\dot{m}_b = - \int_{S_{bp}} \rho \mathbf{V} \cdot \mathbf{n} dS$  is the mass flow variation on  $S_{bp}$ .

$\dot{\mathcal{X}}_{trb}$  can vanish in different cases, the first being the trivial case where  $\mathbf{V}_\infty = \mathbf{0}$ , i.e.  $\tilde{R} = R$  and thus no exergy difference arises from the reference frame transformation. Otherwise, it can vanish if  $\dot{m}_b = 0$  and  $\mathbf{F}_{aero} = \mathbf{0}$ . The

condition  $\mathbf{F}_{aero} = \mathbf{0}$  corresponds to the case where the body is at equilibrium due to aerodynamic forces, i.e. thrust balances drag. In this case, there is no overall work exchange between the body and the fluid in  $\tilde{R}$  or in  $R$ , as aerodynamic forces are not modified between the two reference frames. It should however be noted that thrust and drag applied to the body separately lead to non-zero rates of work (modified when moving from  $\tilde{R}$  to  $R$ ), but the two rates of work compensate each other in both reference frames. It is important to note that  $\mathbf{F}_{aero} = \mathbf{0}$  does not lead to  $\dot{\mathcal{X}}_{trb} = 0$  if  $\dot{m}_b \neq 0$  (or vice versa), thus both effects should be considered in the evaluation of the exergy difference associated to the reference state not being at rest in  $R$ .

The cases where  $\dot{\mathcal{X}}_{trb} = 0$  and  $\dot{\mathcal{X}}_{trb} \neq 0$  should also be discussed in terms of near-field exergy inflow variation. If  $\dot{\mathcal{X}}_{trb} = 0$ , we obtain  $\dot{\mathcal{X}}_b = \dot{\mathcal{X}}_{tfb}$ , i.e. the near-field flow exergy flux is equal between choosing a reference state which is at rest in  $R$  or one which is at rest in  $\tilde{R}$  for the definition of exergy (even though both the exergy and flow exergy themselves are not the same, cf. Sec. V.B.1). If  $\dot{\mathcal{X}}_{trb} \neq 0$  and  $\dot{m}_b = 0$ , the body is not at an aerodynamic force equilibrium. This force is balanced by a non-aerodynamic force (e.g. its weight [9]), the work rate of which is equal in magnitude to the work rate of aerodynamic forces on the body. When the reference frame is moved from  $\tilde{R}$  to  $R$ , the near-field aerodynamic rate of work is no longer perceived (as the body appears motionless) but can be estimated using the exergy difference  $\dot{\mathcal{X}}_{trb}$ .  $\dot{\mathcal{X}}_{tfb}$  is then representative of the near-field exergy inflow that would be perceived if the body was at an aerodynamic equilibrium. Although the decomposition into  $\dot{\mathcal{X}}_{tfb}$  and  $\dot{\mathcal{X}}_{trb}$  is practical because it allows to isolate the aerodynamic force contribution, it should be noted that the real near-field exergy inflow corresponds to  $\dot{\mathcal{X}}_b$  rather than  $\dot{\mathcal{X}}_{tfb}$ . Finally, if  $\dot{\mathcal{X}}_{trb} \neq 0$ ,  $\mathbf{F}_{aero} \neq 0$  and  $\dot{m}_b \neq 0$ , an additional fictive (and thus unavailable) kinetic energy associated to a massflow variation across  $S_b$  is considered in  $\dot{\mathcal{X}}_{trb}$ . This exergy difference consideration is similar to the one discussed in Sec. V.B.1.

Overall, this reasoning highlights a fundamental difference between the thermodynamic variables in the dead state and  $\mathbf{V}_\infty$  for aerodynamics applications. Whereas  $p_\infty$ ,  $\rho_\infty$ ,  $T_\infty$  and  $s_\infty$  have a direct physical link with the thermodynamic dead state of the gas,  $V_\infty$  will always be equal to zero in the reference frame in which exergy is correctly defined (i.e.  $\tilde{R}$ ). The non-zero reference state velocity  $\mathbf{V}_\infty$  is thus representative of the change of reference frame that is performed to simplify aerodynamics analyses (e.g.  $R$  is usually chosen in order to carry out a steady analysis of the flow). As such,  $\mathbf{V}_\infty$  does not correspond to a physically different thermodynamic dead state.

The unsteady exergy balance is finally written as:

$$\dot{\mathcal{X}}_{tfb} + \dot{\mathcal{X}}_{qb} = \dot{\mathcal{X}}_v(t) + \dot{\mathcal{X}}_{trb} + \dot{\mathcal{X}}_{mo} + \dot{\mathcal{X}}_{tho} + \dot{\mathcal{X}}_{qo} + \dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T} + \dot{\mathcal{A}}_w \quad (45)$$

The unsteady exergy variation can also be distributed between the mechanical and thermocompressible exergy outflow to get:

$$\dot{\mathcal{X}}_{tfb} + \dot{\mathcal{X}}_{qb} = \dot{\mathcal{X}}_{trb} + \dot{\mathcal{X}}_{mo}(t) + \dot{\mathcal{X}}_{tho}(t) + \dot{\mathcal{X}}_{qo} + \dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T} + \dot{\mathcal{A}}_w \quad (46)$$

where  $\dot{\mathcal{X}}_{mo}(t) := \int_{\mathcal{V}} \frac{\partial \frac{1}{2} \rho (u^2 + v^2 + w^2)}{\partial t} d\mathcal{V} + \dot{\mathcal{X}}_{mo}$  and  $\dot{\mathcal{X}}_{tho}(t) := \int_{\mathcal{V}} \frac{\partial \rho \chi_s}{\partial t} d\mathcal{V} + \dot{\mathcal{X}}_{tho}$ .

#### 4. Link with the exergy balance derived by Arntz et al.

Eqs. (45) and (46) are not directly equivalent to the exergy balance derived by Arntz et al. [16, 17]. This equivalence could however be recovered by adding and subtracting  $\frac{1}{2} V_\infty^2$  to the near-field mechanical exergy of Eq. (41) and performing a different separation of the contributions as:

$$\dot{\mathcal{X}}_b = - \underbrace{\int_{S_b} \left( \rho \left( \chi_s + \frac{1}{2} (V^2 - V_\infty^2) \right) \mathbf{V} + ((p - p_\infty) \bar{\bar{\mathbf{I}}} - \bar{\bar{\boldsymbol{\tau}}}) \cdot \mathbf{V} \right) \cdot \mathbf{n} dS}_{\dot{\mathcal{X}}_{prop}} + \underbrace{\mathbf{V}_\infty \cdot \int_{S_b} (\rho (\mathbf{V} - \mathbf{V}_\infty) \otimes \mathbf{V} + (p - p_\infty) \bar{\bar{\mathbf{I}}} - \bar{\bar{\boldsymbol{\tau}}}) \cdot \mathbf{n} dS}_{-W\dot{\Gamma}} \quad (47)$$

where  $\dot{\mathcal{X}}_{prop}$  is referred to as the propulsive exergy by Arntz et al. [16, 17] and  $W\dot{\Gamma}$  is another definition of the exergy difference. This separation is consistent with the flow exergy definition  $\chi_f^A$  introduced by Arntz et al. (cf. Eq. (7)) in which the reference state kinetic energy is subtracted from the flow exergy expressed in  $R$ , a choice that does not guarantee the positivity of exergy (as discussed in Section V.B.1). Consequently,  $\dot{\mathcal{X}}_{prop}$  corresponds to  $\dot{\mathcal{X}}_{fb}$  (i.e. the flow exergy flux defined for a dead state that is at rest in  $R$  instead of  $\tilde{R}$ ) from which the reference state kinetic energy is subtracted.  $W\dot{\Gamma}$  then corresponds to the associated exergy difference linked to the reference frame translation, which was introduced as part of the far-field mechanical exergy outflow by Arntz et al.

In the case of a steady flow analysis in external aerodynamics, a hypothesis was made by Arntz et al. on the fact that the fuel mass flow rate is negligible with respect to the air mass flow rate, giving  $\dot{m}_b \approx 0$ . For studies in which this hypothesis is respected (as those performed by Arntz et al.), this gives  $\dot{\mathcal{X}}_{fb} = \dot{\mathcal{X}}_{prop}$  and  $\dot{\mathcal{X}}_{trb} = W\dot{\Gamma}$ . Thus, a direct equivalence can be retrieved between the results obtained by Arntz et al. and the exergy balance derived in this paper from the corrected exergy definition. Additionally, in this specific case, the expression of  $\dot{\mathcal{X}}_{trb}$  is identical to that of the potential energy rate introduced by Drela in the power balance method [9]. Note however that the equivalence between  $\dot{\mathcal{X}}_{trb}$  and  $W\dot{\Gamma}$  is not respected for cases where  $\dot{m}_b \neq 0$ .

Finally, both Eqs. (45) and (46) allow to recover the exergy balance of Arntz et al. [16, 17] when considering a steady-state flow. The derivation is however more direct with the corrected exergy definition of Eq. (29), as the different contributions can be directly decomposed without the need to add and subtract any term to retrieve the  $\dot{\mathcal{X}}_{trb}$  expression. Additionally, the  $\dot{\mathcal{X}}_{trb}$  contribution appears naturally with its near-field expression (as part of the mechanical exergy provided/consumed by the configuration under study), while  $W\dot{\Gamma}$  appeared with its far-field expression in previous works (after specific operations to extract this term from a total enthalpy flux term). The corrected definition of exergy also

provides a clearer and more general physical interpretation of the term  $\dot{X}_{trb}$ , which is different from the one suggested in previous works.

It should finally be noted that similarly to the work of Drela and Arntz et al., the mechanical far-field exergy outflow can be decomposed as:

$$\dot{X}_{mo} = \underbrace{\int_{S_o} \rho \frac{1}{2} u^2 \mathbf{V} \cdot \mathbf{n} dS}_{\dot{E}_u} + \underbrace{\int_{S_o} \rho \frac{1}{2} (v^2 + w^2) \mathbf{V} \cdot \mathbf{n} dS}_{\dot{E}_{vw}} + \underbrace{\int_{S_o} ((p - p_\infty) \bar{\bar{I}} - \bar{\bar{\tau}}) \cdot (\mathbf{V} - \mathbf{V}_\infty) \cdot \mathbf{n} dS}_{\dot{E}_{p\tau}} \quad (48)$$

where  $\dot{E}_u$  and  $\dot{E}_{vw}$  are respectively the axial and transversal perturbation kinetic energy, while  $\dot{E}_{p\tau}$  is the associated surface forces rate of work. Note that, in contrast to previous works, it has been chosen to define the last term as an overall surface force rate of work, including the viscous contribution.

### 5. Fully-transformed exergy balance in the reference frame $R$

The exergy balance of Eq. (46) is practical in the frame of near-field/far-field analyses, with a clear separation of contributions on  $S_b$  and  $S_o$ . In particular, a desirable feature is that the contribution from the aerodynamic force acting on  $S_b$  in the balance is clearly identified through the term  $\dot{X}_{trb}$  when  $\mathbf{V}_\infty$  corresponds to the body's translation velocity. Nonetheless, the exergy balance letting  $\dot{X}_{trb}$  appear is not fully transformed in the reference frame attached to the body, as the entrainment velocity is still explicitly present in the equation through  $\mathbf{V}_\infty$ . In this section, the shock velocity  $\mathbf{W}$  is assumed to be null for simplicity (it should however be noted that the following manipulations can still be applied without this hypothesis for integrals on  $S_w$ ). An additional step towards the complete exergy balance transformation consists in decomposing the mechanical component of the far-field exergy outflow as:

$$\dot{X}_o = \dot{X}_{tho} + \dot{X}_{mo} = \int_{S_o} \left( \rho \left( \chi_s + \frac{1}{2} (\mathbf{V} - \mathbf{V}_\infty)^2 \right) \mathbf{V} + \mathbf{V} \cdot ((p - p_\infty) \bar{\bar{I}} - \bar{\bar{\tau}}) \right) \cdot \mathbf{n} dS - \mathbf{V}_\infty \cdot \int_{S_o} ((p - p_\infty) \bar{\bar{I}} - \bar{\bar{\tau}}) \cdot \mathbf{n} dS \quad (49)$$

where the second integral represents the variation of the rate of work of surface forces on  $S_o$  associated to the reference frame transformation. This term can be grouped with its near-field equivalent appearing in  $\dot{X}_{trb}$  (cf. Eq. (43)), and the momentum conservation equation expressed in  $R$  then gives:

$$- \mathbf{V}_\infty \cdot \int_{\partial V} ((p - p_\infty) \bar{\bar{I}} - \bar{\bar{\tau}}) \cdot \mathbf{n} dS = \int_V \frac{\partial \rho \mathbf{V}_\infty \cdot \mathbf{V}}{\partial t} dV + \int_{\partial V} \rho (\mathbf{V}_\infty \cdot \mathbf{V}) \mathbf{V} \cdot \mathbf{n} dS \quad (50)$$

The above equation shows that the variation of the rate of work of surface forces at the limits of the domain due to the reference frame transformation is equivalent to a variation of kinetic energy in the domain (itself due to a variation of momentum). This kinetic energy variation is composed of a local unsteady variation as well as a flux variation through

the permeable limits of the domain. Using the above relation allows to rewrite the following right-hand-side terms of Eq. (45) as:

$$\begin{aligned} \dot{\chi}_v(t) + \dot{\chi}_{trb} + \dot{\chi}_o = & \int_{\mathcal{V}} \frac{\partial \rho \left( \chi_s + \frac{1}{2}(\mathbf{V} - \mathbf{V}_\infty)^2 + \mathbf{V}_\infty \cdot \mathbf{V} \right)}{\partial t} d\mathcal{V} + \int_{S_b} \frac{1}{2} V_\infty^2 \rho \mathbf{V} \cdot \mathbf{n} dS \\ & + \int_{S_o} \left( \rho \left( \chi_s + \frac{1}{2}(\mathbf{V} - \mathbf{V}_\infty)^2 + \mathbf{V}_\infty \cdot \mathbf{V} \right) \mathbf{V} + \mathbf{V} \cdot ((p - p_\infty)\bar{\bar{I}} - \bar{\bar{\tau}}) \right) \cdot \mathbf{n} dS \end{aligned} \quad (51)$$

This expression is further modified by developing the term  $\frac{1}{2}(\mathbf{V} - \mathbf{V}_\infty)^2$ :

$$\begin{aligned} \dot{\chi}_v(t) + \dot{\chi}_{trb} + \dot{\chi}_o = & \int_{\mathcal{V}} \frac{\partial \rho \left( \chi_s + \frac{1}{2} \mathbf{V}^2 \right)}{\partial t} d\mathcal{V} + \int_{S_o} \left( \rho \left( \chi_s + \frac{1}{2} \mathbf{V}^2 \right) \mathbf{V} + \mathbf{V} \cdot ((p - p_\infty)\bar{\bar{I}} - \bar{\bar{\tau}}) \right) \cdot \mathbf{n} dS \\ & + \frac{1}{2} V_\infty^2 \underbrace{\left( \int_{\mathcal{V}} \frac{\partial \rho}{\partial t} d\mathcal{V} + \int_{\partial \mathcal{V}} \rho \mathbf{V} \cdot \mathbf{n} dS \right)}_0 \end{aligned} \quad (52)$$

where the last term is cancelled by using the mass conservation equation expressed in  $R$ . Two terms are introduced from this manipulation:

- $\dot{\chi}_v^R(t) := \int_{\mathcal{V}} \frac{\partial \rho (\chi_s + \frac{1}{2} V^2)}{\partial t} d\mathcal{V}$  is the unsteady rate of exergy variation defined with respect to a reference state at rest in  $R$ .
- $\dot{\chi}_{mo}^R := \int_{S_o} \left( \rho \frac{1}{2} V^2 \mathbf{V} + \mathbf{V} \cdot ((p - p_\infty)\bar{\bar{I}} - \bar{\bar{\tau}}) \right) \cdot \mathbf{n} dS$  is the dynamic exergy flux defined for a reference state at rest in  $R$ .

The transformed exergy balance in  $R$  is then written:

$$\dot{\chi}_{tfb} + \dot{\chi}_{qob} = \dot{\chi}_v^R(t) + \dot{\chi}_{tho} + \dot{\chi}_{mo}^R + \dot{\chi}_{qo} + \dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T} + \dot{\mathcal{A}}_w \quad (53)$$

The contribution of the aerodynamic force does not explicitly appear in this form of the exergy balance, as there is no work from near-field surface forces on  $S_b$  in  $R$  (due to the body being motionless). Instead, the variation of the rate of work of surface forces on the limits of the domain is implicitly contained in the kinetic energy computed at the permeable limits of the domain (cf. Eq. (50)). In the case of an airfoil with adiabatic walls (i.e. a body with no permeable surface nor heat conduction), the left-hand side of the above equation would be null. Indeed, there would be no exchange of exergy between the body and the fluid since the former would appear motionless in  $R$ . The exergy-related contribution from drag acting on the body would then be implicitly contained in the term  $\dot{\chi}_{mo}^R$  (through the far-field kinetic energy flux), and cannot be separately quantified with Eq. (53). This quantification would require to retrieve the

term  $\dot{\mathcal{X}}_{trb}$  via its far-field contribution by explicitly adding and subtracting its components (as was done in previous studies based on the original definition of exergy).

Overall, the use of the transformed exergy balance in  $R$  is not desirable in near-field/far-field analyses. Indeed Eq. (50) highlights that, as a result of the transformation, near-field and far-field contributions due to surface forces acting on the limits of the domain are mixed in a kinetic energy variation at permeable limits of the domain. As a result, it is not possible to clearly identify which kinetic variation is due to the variation of the rate of work of surface forces in the near- or the far-field. Nonetheless, Eq. (53) possesses the advantage of being independent of  $\mathbf{V}_\infty$  (this is a direct consequence of defining exergy with respect to a reference state at rest in  $R$ ). This advantage is very useful for the analysis of cases where the translation velocity of the body is not known (e.g. internal turbomachinery elements). In such cases, the interpretation of  $\dot{\mathcal{X}}_{trb}$  is limited to an exergy difference associated to a hypothesis made on the body's translation velocity. It is then useful to work with Eq. (53) (which is independent of this hypothesis and actually corresponds to Eq. (45) with  $\mathbf{V}_\infty = \mathbf{0}$ ). This aspect is further detailed in Sec. VI with the theoretical analysis of a schematic case of a converging nozzle.

## VI. Evolution of the exergy balance components in the schematic case of a converging nozzle

As detailed in Sec. V.C.3, the interpretation of  $\dot{\mathcal{X}}_{trb}$  in the case of internal flows cannot always be directly linked to the total streamwise aerodynamic force applied on the aircraft. This situation occurs when the external aerodynamic state, and therefore the aircraft velocity, is unknown. To illustrate the interpretation of the exergy balance components and the transfers that occur between them, the schematic case of a steady flow analysis inside an axisymmetrical converging nozzle is considered (as illustrated in Fig. 5).

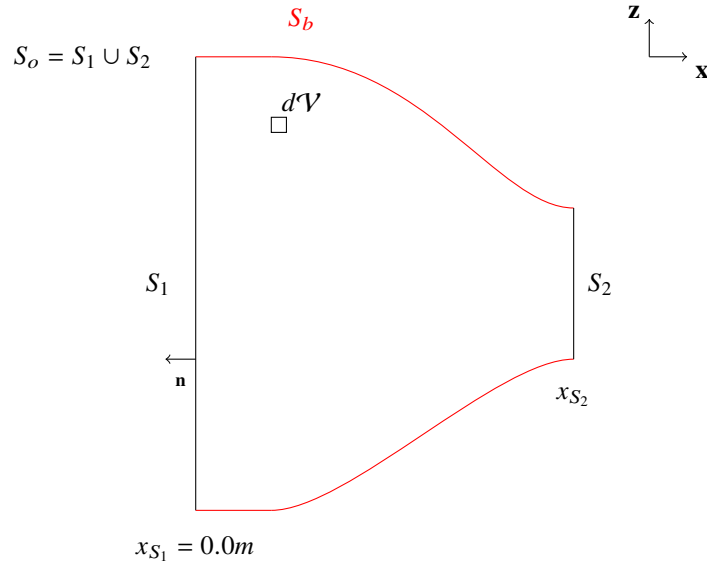


Fig. 5 2D cross section of a 3D control volume representative of a converging nozzle.



The surface  $S_o$  is composed of the inlet and the outlet surfaces (respectively noted  $S_1$  and  $S_2$ ), and the wall surface is noted  $S_b$ . The walls are considered adiabatic ( $\dot{\chi}_{qb} = 0$ ) and non-permeable ( $\dot{\chi}_{fb} = 0$ ). It is also considered that no shockwave discontinuity is present in this configuration ( $\dot{\mathcal{A}}_w = 0$ ), and that thermal transfers by conduction through  $S_o$  are negligible ( $\dot{\chi}_{qo} = 0$ ). In this case, Eq. (45) is simplified to:

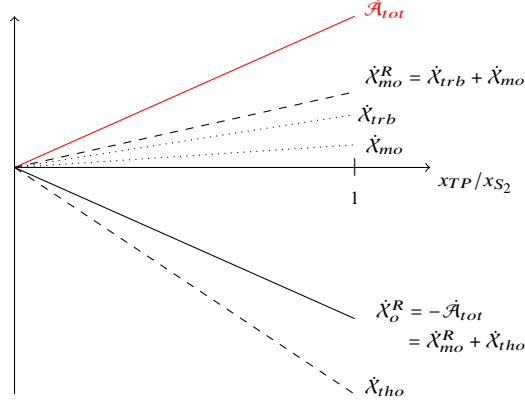
$$0 = \dot{\chi}_{trb} + \dot{\chi}_{mo} + \dot{\chi}_{tho} + \dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T} \quad (54)$$

As discussed in Sec. V.C.5, the transformed exergy balance in  $R$  is also useful for the investigation of this case since  $\mathbf{V}_\infty$  is not known. Under the hypotheses described above, it is written as:

$$0 = \underbrace{\dot{\chi}_{mo}^R + \dot{\chi}_{tho}}_{\dot{\chi}_o^R} + \underbrace{\dot{\mathcal{A}}_\phi + \dot{\mathcal{A}}_{\nabla T}}_{\dot{\mathcal{A}}_{tot}} \quad (55)$$

with  $\dot{\chi}_{mo}^R = \dot{\chi}_{trb} + \dot{\chi}_{mo}$ . Note that for  $\mathbf{V}_\infty = \mathbf{0}$ ,  $\dot{\chi}_{trb} = 0$  and  $\dot{\chi}_{mo} = \dot{\chi}_{mo}^R$  (since  $\tilde{R} = R$ ), i.e. there is no exergy difference linked to the translation of the reference frame. The sum of  $\dot{\chi}_{mo}^R$  and  $\dot{\chi}_{tho}$  then represents the flux of flow exergy defined with respect to a reference state at rest in  $R$  through  $S_o$ , whereas the sum of  $\dot{\mathcal{A}}_\phi$  and  $\dot{\mathcal{A}}_{\nabla T}$  is the total anergy generated in the control volume.

A common analysis consists in performing a series of exergy balance computations for different fixed  $x$ -coordinate positions (noted  $x_{TP}$ ) of a transverse plane limiting the control volume downstream [17]. For each control volume, the different terms are evaluated and the trend obtained is plotted as a function of  $x_{TP}$ . The goal of the analysis below is to illustrate the expected trends of such a study in the case of a converging nozzle (presented in Fig. 5), in order to highlight the transfers occurring between the different terms as the limiting plane is moved downstream. To keep this illustration as simple as possible, the unrealistic hypothesis is made that the evolutions of all the terms are linear (as shown in Fig. 6).



**Fig. 6 Schematic illustration of transfers between exergy balance components in the theoretical case of a converging nozzle**

The first value of  $x_{TP}$  is assumed to be close enough to the nozzle inlet so that every component of the exergy balance is null. As the limiting plane position is moved further downstream, an increase of anergy generation  $\dot{\mathcal{A}}_{tot}$  is expected in the nozzle due to irreversible losses (viscous dissipation and thermal mixing). Equivalently, the flux of flow exergy  $\dot{\mathcal{X}}_o^R = -\dot{\mathcal{A}}_{tot}$  would decrease by the same amount between the inlet and the outlet section. This first step represents an overall interpretation in which any exergy decrease in the flow is due to anergy generation in the volume.

The flux of flow exergy defined for a reference state at rest in  $R$  can then be decomposed as  $\dot{\mathcal{X}}_o^R = \dot{\mathcal{X}}_{mo}^R + \dot{\mathcal{X}}_{tho}$  to refine this interpretation. Between the inlet and outlet sections, the thermocompressible exergy outflow is expected to decrease. Consequently,  $\dot{\mathcal{X}}_{mo}^R$  is expected to increase. One mechanism involved in this process is the conversion of thermocompressible into mechanical exergy as the flow is accelerated (i.e. there is a change in the nature of exergy), the other being the destruction of part of it through irreversible processes (i.e. anergy generation). In an ideal case, for which losses are neglected, the decrease of  $\dot{\mathcal{X}}_{tho}$  and the increase of  $\dot{\mathcal{X}}_{mo}^R$  would have been equal (i.e.  $\dot{\mathcal{X}}_{tho} = -\dot{\mathcal{X}}_{mo}^R$ ).

Finally, in the case where the external aerodynamic state is known,  $\dot{\mathcal{X}}_{mo}^R$  can be decomposed between  $\dot{\mathcal{X}}_{mo}$  and  $\dot{\mathcal{X}}_{trb}$ . This allows to quantify the part of the mechanical exergy flux defined for a reference state at rest in  $R$  that is not available for conversion into useful work downstream of the exit section (i.e.  $\dot{\mathcal{X}}_{trb}$ ), which is linked to the reference frame moving at a velocity  $-\mathbf{V}_\infty$  (making the configuration appear motionless and the flow analysis steady).  $\dot{\mathcal{X}}_{mo}$  then represents the part of  $\dot{\mathcal{X}}_{mo}^R$  that is actually convertible into mechanical work downstream (which is wasted in case the exhaust directly leads to the atmosphere). In the present case, the total streamwise force inside the nozzle is assumed to be directed opposite to the  $\mathbf{x}$  axis (i.e.  $\mathbf{V}_\infty = V_\infty \mathbf{x} > 0$ ). This corresponds to an increase of  $\dot{\mathcal{X}}_{trb}$  between the inlet and the outlet sections (in Fig. 6, it is arbitrarily assumed that  $\dot{\mathcal{X}}_{trb} > \dot{\mathcal{X}}_{mo}$ , this being dependent on the value of  $\mathbf{V}_\infty$ ). In physical terms, this means that over the mechanical exergy outflow computed with respect to a reference state at rest in  $R$ , an amount  $\dot{\mathcal{X}}_{trb}$  would have already been converted to useful work as this part would allow the configuration to translate at a velocity  $\mathbf{V}_0 = -\mathbf{V}_\infty$  in  $\tilde{R}$  (which is why it appears motionless in  $R$ ). As such, in the case where no device

capable of recovering the mechanical exergy outflow exists downstream, it is only the part equal to  $\dot{\chi}_{mo}$  that is indeed wasted out of the total mechanical exergy flux  $\dot{\chi}_{mo}^R$ . If such a device is present downstream, then  $\dot{\chi}_{mo}$  is the part of  $\dot{\chi}_{mo}^R$  that can be converted into mechanical work.

## VII. Conclusion and perspectives

This paper presents a clarification of the notion of exergy and its difference with the notion of flow exergy, with a particular focus on aerodynamics applications. The theoretical derivation of an unsteady exergy balance is first carried out in the geocentric reference frame, in which exergy is defined without an ambiguity. At a second step, the definition of exergy is transformed in the case of a translating frame of reference, resulting in a corrected definition with respect to previous exergy-based flowfield analyses in aerodynamics (in particular respecting the positivity of exergy). The corrected definition also highlights that the freestream fluid velocity is directly linked to the choice of a translating reference frame different from the one in which exergy is defined (in which the atmosphere is always at rest), whereas the rest of the dead state variables are representative of the thermodynamic state of the atmosphere. The impact of this modification is then assessed on the exergy balance expressed in a reference frame in translation, showing that the derivation, decomposition and interpretation of the different terms is more direct compared to previous works. In particular, the term  $\dot{\chi}_{irb}$  is introduced as a part of the near-field mechanical exergy, and is interpreted in physical terms as an exergy difference linked to the atmosphere not being at rest in the translating reference frame. The relation between the newly derived exergy definition and previous works is also investigated in detail, showing that the same form of exergy balance is obtained in the case of steady flow with no near-field mass flow variation. Differences should however appear in studies where the previously mentioned conditions are not respected. Finally, the specific case of internal flow is discussed, with different levels of decomposition suggested depending on the availability of external aerodynamic state data (and in particular the value of the global configuration velocity). Future works should focus on cases for which the correction of the exergy definition has a clear impact on the balance. This includes both cases with a non-negligible mass flow rate variation in the near field and unsteady flows. The new interpretation of  $\dot{\chi}_{irb}$  provided by the correction of the exergy definition, as well as the different levels of decomposition of the exergy balance, will also allow to investigate confined flows for which knowledge of the external reference state aerodynamic data is limited.

## Acknowledgments

The work presented in this paper was funded by the French Directorate General for Civil Aviation (DGAC) through the SUBLIME convention.

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