# Multiscale Modelling of Nonlinear Eddy Current Problems Using the h-conforming Formulation

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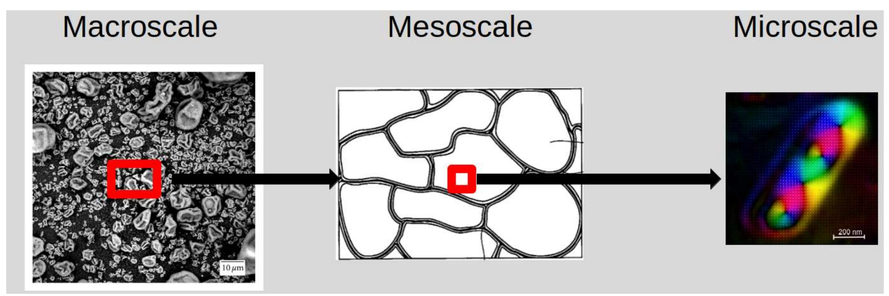
## Abstract

This paper deals with an ***h***-conforming multiscale formulation for nonlinear eddy current problems in idealized soft magnetic composites using the heterogeneous multiscale method. The classical ***h***-φ formulation is recalled before presenting the different components of the ***h***-φ multiscale formulation including the governing weak forms, the upscaling of quantities of interest and details some on the numerical implementation. Validation results are then presented before a conclusion.

## 1. Introduction

- Electrical devices with soft magnetic composites (SMC) can operate at higher frequencies with reduced eddy current losses.
- SMC exhibit isotropic nonlinear/hysteretic behaviour.
- Classical numerical methods such as FEM are too costly and multiscale methods are more convenient to characterize SMC. In this research work, we used the heterogeneous multiscale method (HMM).




## 2. Classical formulation

- Maxwell's equations and constitutive law:

$$\boldsymbol{curl}\, \boldsymbol{h}^\varepsilon = \boldsymbol{j}^\varepsilon \ , \ \boldsymbol{curl}\, \boldsymbol{e}^\varepsilon = -\partial_t \boldsymbol{b}^\varepsilon,$$

$$\boldsymbol{b}^\varepsilon = \boldsymbol{\mathcal{B}}(\boldsymbol{h}^\varepsilon) \ , \ \boldsymbol{e}^\varepsilon = \rho^\varepsilon \boldsymbol{j}^\varepsilon = \rho^\varepsilon\, \boldsymbol{curl}\, \boldsymbol{h}^\varepsilon.$$

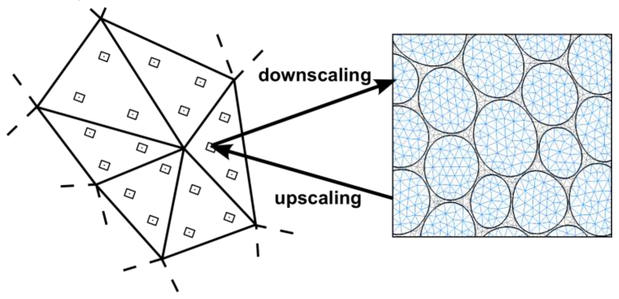
- Weak form: $\left(\frac{\partial}{\partial t}\boldsymbol{\mathcal{B}}\left(\boldsymbol{h}^\varepsilon\right), \boldsymbol{v}\right)_\Omega + \left(\rho^\varepsilon\, \mathbf{curl}\, \boldsymbol{h}^\varepsilon, \mathbf{curl}\, \boldsymbol{v}\right)_{\Omega_c} + \sum_{i\in\mathcal{C}} \mathcal{I}_i V_i = 0.$

- Discretization of the magnetic field:

$$\boldsymbol{h}^\varepsilon(\boldsymbol{x}) = \sum_{e\in\Omega_c} h_e \boldsymbol{S}_e(\boldsymbol{x}) + \sum_{n\in\Omega_c^C} \phi_n \boldsymbol{grad}\, S_n(\boldsymbol{x}) + \sum_{i\in\mathcal{C}} \mathcal{I}_i \boldsymbol{c}_i(\boldsymbol{x})$$

## 3. Multiscale formulation

**Scale separation** :
Length of the microstructure << length of device / characteristic length of external loadings.




- At each iteration: 1 macroproblem and many microproblems (1 per Gauss point of the Macro mesh). Couple these problems (**upscaling-downscaling**).

**Strong form** :

- **Macro** : $\boldsymbol{curl}_x\, \boldsymbol{h}_M = \boldsymbol{j}_M \ , \ \boldsymbol{curl}_x\, \boldsymbol{e}_M = -\partial_t \boldsymbol{b}_M,$

$$\boldsymbol{b}_M = \boldsymbol{\mathcal{B}}_M(\boldsymbol{h}_M) \ , \ \boldsymbol{e}_M = \rho \boldsymbol{j}_M = \rho\, \boldsymbol{curl}_x\, \boldsymbol{h}_M.$$

- **Meso** : $\boldsymbol{curl}_y\, \boldsymbol{h}_c = \boldsymbol{j}_c \ , \ \boldsymbol{curl}_y\, (\boldsymbol{e}_c - \kappa \partial_t \boldsymbol{\mathcal{B}}_M \times \boldsymbol{y}) = -\partial_t \boldsymbol{b}_m,$

$$\boldsymbol{b}_m = \boldsymbol{\mathcal{B}}(\boldsymbol{h}_M + \boldsymbol{h}_c) \ , \ \boldsymbol{e}_c = \rho \boldsymbol{j}_c = \rho\, \boldsymbol{curl}_y\, \boldsymbol{h}_c.$$

**Weak form** :

- **Macro** : $\left(\frac{\partial}{\partial t}\boldsymbol{\mathcal{B}}_M\left(\boldsymbol{h}_M\right), \boldsymbol{v}_M\right)_\Omega + \left(\rho\, \boldsymbol{curl}_x\, \boldsymbol{h}_M, \boldsymbol{curl}_x\, \boldsymbol{v}_M\right)_{\Omega_c} + \sum_{i\in\mathcal{C}} \mathcal{I}_i V_i = 0.$
- **Meso 1** : $\left(\frac{\partial}{\partial t}\boldsymbol{\mathcal{B}}\left(\boldsymbol{h}_M + \boldsymbol{h}_c\right), \boldsymbol{v}_c\right)_{\Omega_m} + \left(\rho\, \boldsymbol{curl}_y\, \boldsymbol{h}_c, \boldsymbol{curl}_y\, \boldsymbol{v}_c\right)_{\Omega_{mc}} = \left(\kappa\sigma \partial_t \boldsymbol{\mathcal{B}}_M \times \boldsymbol{y}, \boldsymbol{curl}_y\, \boldsymbol{v}_c\right)_{\Omega_{mc}}$
- **Meso 2** : $\left(\boldsymbol{\mathcal{B}}\left(\boldsymbol{h}_M + \boldsymbol{grad}_y\, \phi_c\right), \boldsymbol{grad}_y\, \phi_c'\right)_{\Omega_m} = 0$

**Upscaling** :

Upscaling of $\boldsymbol{\mathcal{B}}_M$ using Meso 1: $\boldsymbol{\mathcal{B}}_M = \frac{1}{|\Omega_m|} \iiint_{\Omega_m} \boldsymbol{\mathcal{B}}\left(\boldsymbol{h}_M + \boldsymbol{h}_c\right)$

Upscaling of $\frac{\partial \boldsymbol{\mathcal{B}}_M}{\partial \boldsymbol{h}_M}$ using problem Meso 2 and the finite difference method.

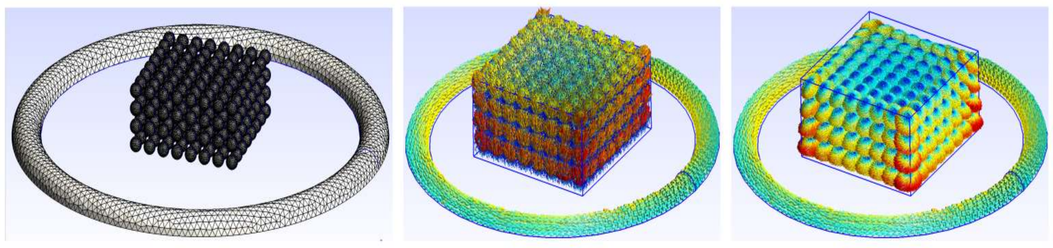
**Implementation** :

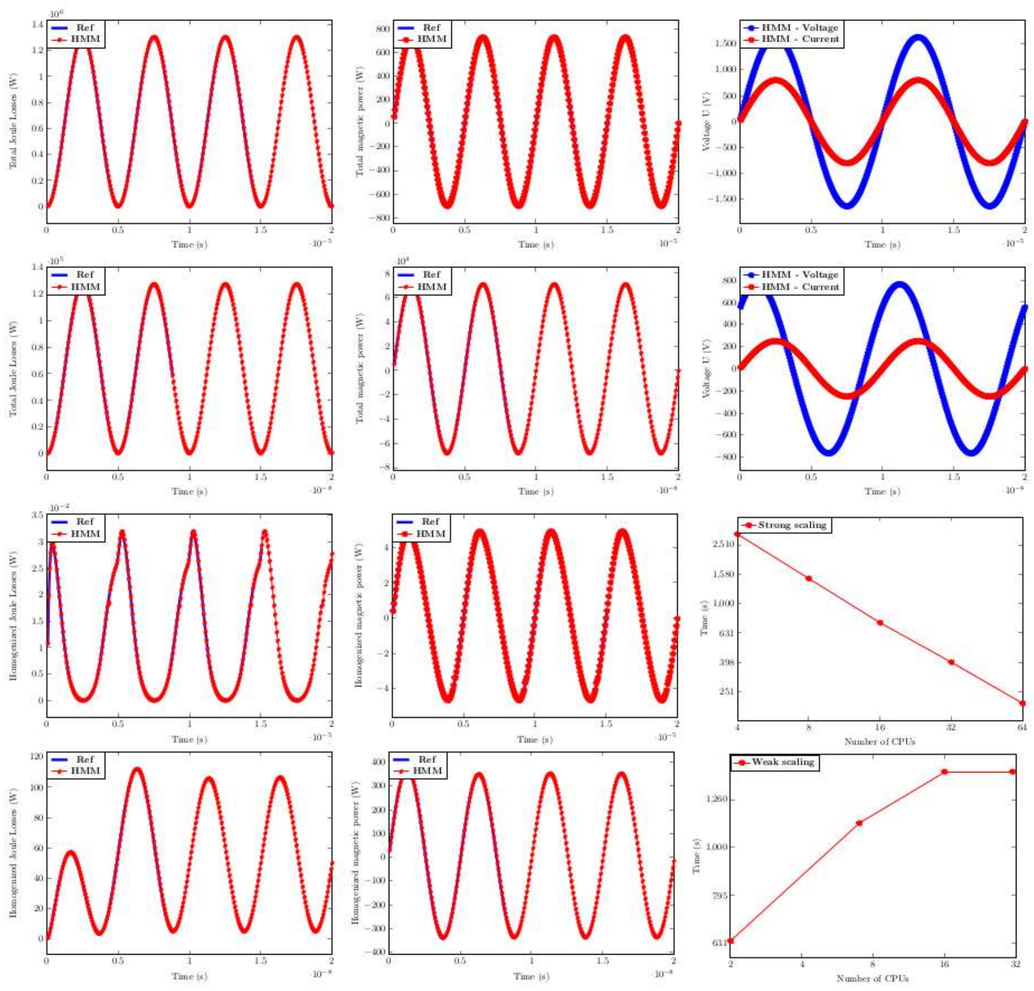
- Temporal discretization using the Backward Euler method
- Nonlinear problem solved using a relaxed Newton-Raphson scheme

## 4. Numerical Results

**Geometry** : 8 x 8 x 8 conductive spheres (radius r = 40μm) surrounded by a massive inductor.

**Material laws** : conductivity σ = $10^7$Sm, nonlinear Fröhlich-Kennely law.






## Conclusions:

- HMM yields accurate local fields and global quantities (Joule losses, voltage, etc.) for nonlinear eddy current problems with idealized SMC.
- $\boldsymbol{\mathcal{B}}_M$ is upscaled using fields of the magnetodynamic problem whereas $\frac{\partial \boldsymbol{\mathcal{B}}_M}{\partial \boldsymbol{h}_M}$ is obtained using fields of the magnetostatic problem.
- Adaptive relaxation should be used for the convergence of the nonlinear meso-problems and of macro-problem (if there is too much saturation).
- The HMM algorithm scales very well on clusters.

## Perspectives :

- Use of the HMM method to model closed magnetic circuits such as the transformers.
- Extension of the formulation to account for macroscale currents (e.g., problems with macroscale displacement currents).
- Use of the model order reduction (MOR) for further reduction of the computational cost.