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ABSTRACT

The cell membrane experiences deformation and poration due to electrical stress. In this Letter, we develop continuum simulations of the capacitance-transmembrane potential (TMP) characteristics of the cell membrane. The electromechanical properties of biological cells are gaining increasing visibility so that the utility of numerical models should not be underestimated as a means to check and vet experimental analysis. While several early experiments with solvent-containing bilayers have demonstrated a nonlinear electric field dependence of the capacitance of artificial bilayer membranes, it is noteworthy that the TMP dependence of the membrane capacitance and resistance is not commonly reported in the computational literature. We consider both nonuniform tension and compression of the membrane to determine the anisotropic variation of its thickness, which depends on TMP and Young modulus. The membrane capacitance per unit area of the order of 10^{-2} Fm^{-2} and the areal membrane resistance of the order of $10^{-2} \Omega \text{ m}^2$ can be explained by the core (cytoplasm)-shell (membrane) structure of the cell. Our results show that a quadratic dependence of membrane capacitance and conductance captures the impact of the strain state under electric field excitation. We, furthermore, discuss the different degrees of influence on membrane capacitance and resistance that different structural parameters (cell aspect ratio, membrane thickness, and surface area) have. The method presented here provides a path forward toward exploring different core-shell models of biological cells in order to optimize cell electrodeformation and electroporation.

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Biological cells are structurally complex composite materials that are characterized by unique physical properties, i.e., the dielectric membrane, a phospholipidic bilayer, surrounded by the conducting cytoplasm and extracellular medium. Over the last few decades, significant efforts have been dedicated to exploring their response to environmental cues, which involve multiple spatial or temporal scales.^{1–3} Due to their specific elasticity characteristics (2D fluid-like lipid bilayer) and ultra-small dimension (of around 5 nm size), cell membranes undergo mechanical deformation that severely limits their biological functions under extreme conditions involving electric field stimulation and mechanical force. Yet, there is no currently accepted adequate understanding of their electromechanical properties.

From an electrostatic standpoint, cell membranes are capacitors that can be charged by applying a field across the membrane.^{4,5} Since the transmembrane potential (TMP) was invoked by Schwan^{5,6} to explain a broad range of effects depending on the electric field intensity and duration of its application, this concept has wide applicability

to monitor the stress-distribution in the cell membrane at different strain states under electric field excitation. While the core (cytoplasm)-shell (membrane) structure is likely to be simplified to describe real biological cells, it is nonetheless a good starting point to deal with electromechanical modeling of the TMP dependent cell membrane capacitance. The great value of simple models for problems as perplexingly difficult to handle as the electromechanical properties of cells is self-evident, particularly for providing baseline checks to experimental methods that will be applied to the cell electrodeformation (ED) and electroporation (EP) problems.⁵

In this Letter, we develop a comprehensive understanding of the electromechanical behavior of the cell membranes as a function of their characteristic constituent materials (even if the compositional variation is somewhat limited in Nature) under a steady-state electric field. Here, using continuum multiphysics simulations, we explore the relationships between membrane capacitance C_m , membrane resistance R_m , and TMP V_m and reveal that the cell material parameters

play a substantial role in controlling these quantities. We consider that the membrane and cytoplasm are purely elastic materials with respective Young's moduli set to $Y_m = 19 \text{ MPa}$ and $Y_c = 1 \text{ kPa}$. We also assume that the ED of the cell does not degrade the mechanical properties of the membrane. We demonstrate (i) a quadratic dependence of C_m on V_m in the V_m range investigated from 1 mV to 0.1 V, i.e., far below from the membrane EP threshold V_{EP} (experimental estimates for V_{EP} fall in the range of 0.2–1.5 V^{9,10}), and show that $R_m(V_m)$ scales as V_m^{-2} ; (ii) the ellipticity dependence and associated 3D cell shapes (whether they are prolate or oblate depends on the polarization and magnitude of the uniform steady-state electric excitation); and (iii) that the relative change in membrane thickness $\delta d_m/d_{m0}$, where d_m is the effective membrane thickness under electric field excitation and d_{m0} is the unperturbed membrane thickness, decreases approximately quadratically with electric field strength. Its angular variation parametrically behaves as $d_m = d_{m0} - c \cos^2(\theta) + d \sin^2(\theta)$, where c and d are two constants and θ is a polar angle characterizing the spheroidal cell, showing a smooth tension to compression transition of the membrane.

To begin, it is worth noting that due to the very different length scales between the membrane thickness (set to 5 nm for the undeformed spherical cell) and cell size (in the undeformed state, its radius R is set to 5 μm), the membrane can be treated as a parallel plate capacitor with capacitance per unit area, $\tilde{C}_m = C_m/A_m = \epsilon_0 \epsilon_m/d_m$, where ϵ_m is the membrane permittivity and A_m is the surface area. Typical data obtained on biological cells indicate that \tilde{C}_m is of the order of 10^{-2} Fm^{-2} , which represents a very large capacitance per unit area because of the nanometric thickness of the membrane.^{6–8} The areal membrane resistance is of the order of $10^{-2} \Omega \text{ m}^2$.^{4,9–12} Assuming a uniform permittivity in the membrane ($\epsilon_0 \epsilon_m = 4.4 \times 10^{-11} \text{ Fm}^{-1}$, where ϵ_0 denotes the vacuum permittivity), the field across the membrane is $E_m = V_m/d_m$ and the force acting on this capacitor is $F_m = C_m V_m^2/2d_m$. This results in a pressure on the membrane of 10^4 Nm^{-2} at 100 mV. This implies that increasing the force results in a reduction of thickness and an increase in the area. In their physiological state, the membrane potential of quiescent cells is called the resting potential V_{rest} , i.e., reflecting the imbalance of electrical charge that exists between the interior of electrically excitable cells and their surroundings: V_{rest} is roughly -0.07 V and is constant everywhere on the cell membrane.^{6,7} When an external electric field E is applied to the cell, the TMP V_m superimposes to V_{rest} .

A schematic defining the key structural parameters and different views of an example numerical simulation are illustrated in Fig. 1. Since the field-induced change in TMP varies with the position over a cell membrane, we introduce a polar angle denoted by θ , which is the angle measured from the z axis with respect to the applied electric field direction.

One of the most reliable methods for studying the multiscale and multiphysics behavior of biological materials is the finite element based first-principles simulation.^{13,14} Our strategy is to first perform 3D finite element analysis on the geometry shown in Fig. 1 using the software Comsol Multiphysics.¹⁵ We describe the computational details in the supplementary material. When properly controlled, C_m gives a measure of the change in strain due to the change in structural parameters. From a computational standpoint, it is a nontrivial task to explore the membrane capacitance and conductance behavior due to the difficulty in dealing with the nonuniform variation of d_m . In the

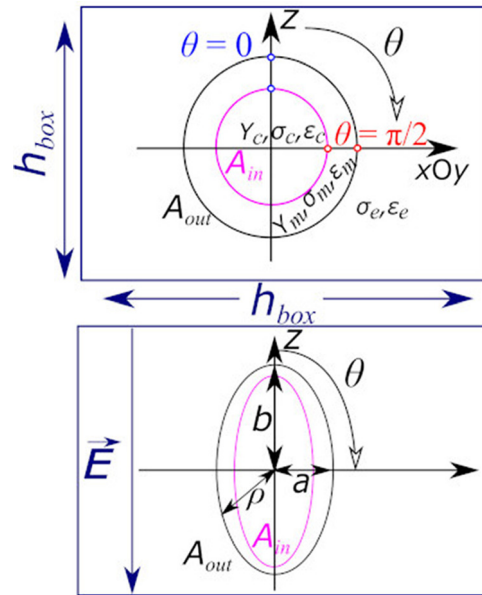


FIG. 1. Schematic diagrams illustrating the 3D projection of the cell geometry onto the xOy plane. (Top panel) The undeformed state has a spherical symmetry. The orientation angle, denoted by θ , refers to the angle measured from the z axis with respect to the applied electric field direction. The corresponding numerical values are $d_m = 5 \text{ nm}$, $R = 5 \mu\text{m}$, $\sigma_m = 5 \times 10^{-7} \text{ Sm}^{-1}$, $\epsilon_0 \epsilon_m = 4.4 \times 10^{-11} \text{ Fm}^{-1}$, $\sigma_c = 0.2 \text{ Sm}^{-1}$, $\epsilon_0 \epsilon_c = 7 \times 10^{-10} \text{ Fm}^{-1}$, $\sigma_e = 0.2 \text{ Sm}^{-1}$, $\epsilon_0 \epsilon_e = 7 \times 10^{-10} \text{ Fm}^{-1}$, $Y_m = 19 \text{ MPa}$, and $Y_c = 1 \text{ kPa}$, and the computational box h_{box} has size = 20 μm . (Bottom panel) For comparison, we show the deformed state under uniform steady-state electric excitation, where b and a are the two principal semi-axes along and perpendicular to the electric field direction.

top panel of Fig. 2, we see the dependence of the relative change in membrane thickness $\delta d_m/d_{m0}$, which decreases approximately quadratically with electric field strength. Electrostriction theory shows that the cell distorts in the presence of electrical stimulation.¹⁶ For the case of the elastic membrane shown in Fig. 1, the strain along the field direction will be inhomogeneous, i.e., the membrane is under tensile strain along the x - y directions and under compressive strain in the z direction. This provides our rationale for using the expression $d_m = d_{m0} - c \cos^2(\theta) + d \sin^2(\theta)$, where c and d are constants, the second term is the electrostrictive contribution, and the third term accounts for volume conservation (see the supplementary material for a derivation of this expression). This analytic form contains the two main components of the electromechanical response. An example appears in the inset of the bottom panel of Fig. 2, which compares our calculations with this analytical expression. This approach provides a reliable way of interrogating the effect of different material parameters on the electromechanical properties of the cell. This is corroborated by experimental data showing that the thickness of the membrane does not change by more than a few percent throughout the EP process.^{6,7}

As shown in Fig. 3, C_m exhibits a nonlinear behavior with a strong deformation character at higher V_m . We see that $C_m = C_{m0}(1 + \alpha V_m^2)$, where C_{m0} denotes the zero voltage value of the capacitance, whereas R_m scales as $1/C_m$ (Fig. 4). We also indicate the calculated values of α , which are in the range of $5.3\text{--}8.2 \times 10^{-2} \text{ V}^{-2}$, i.e., completely consistent with experimental observations on artificial

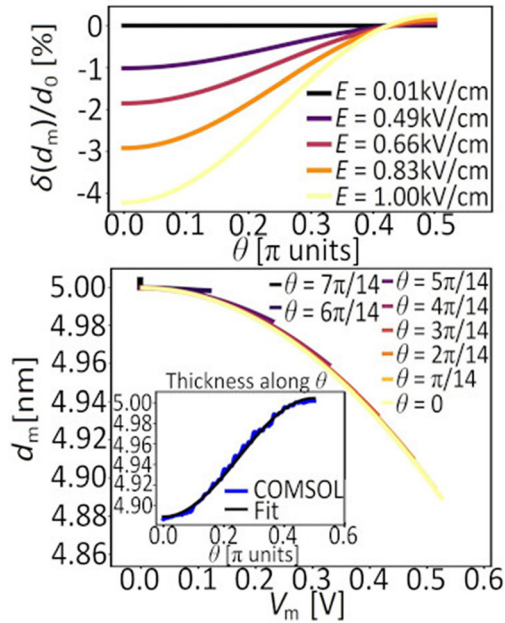


FIG. 2. (Top panel) Illustrating the nonuniform variation of the membrane thickness vs θ . $R = 5 \mu\text{m}$ and $Y_m = 19 \text{MPa}$. The different colors correspond to different electric field magnitudes ranging from 10^{-2} to 1kVcm^{-1} . (Bottom panel) Variation of d_m vs V_m for different values of θ . The inset shows the fit of the thickness data vs θ with the analytical expression $d_m = d_{m0}c\cos^2(\theta) + d\sin^2(\theta)$, where c and d are constants, the second term is the electrostrictive contribution, and the third term accounts for volume conservation. The electric field strength is set to 1kV/cm .

bilayer membranes.¹⁷ We attribute the increase in C_m to the simultaneous decrease in d_m and the increase in A_m . The applied electric field, and consequently TMP, can, thus, have a pronounced effect on controlling the mechanical state of the cell membrane.

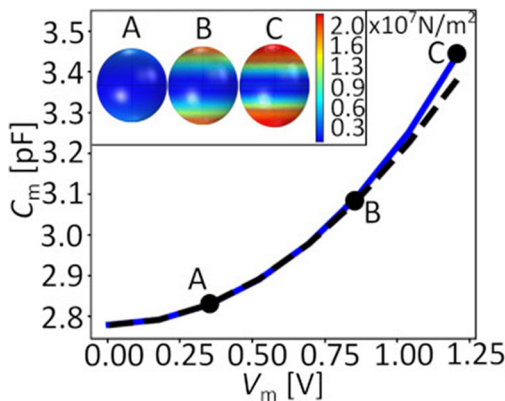


FIG. 3. Illustrating the variation of membrane capacitance as a function of V_m . C_m increases quadratically at low V_m value ($V_m < 1.2 \text{V}$) and then varies exponentially at larger field strengths. Inset: A, B, and C denote three deformed cell geometries for which the cell shape can be described by a prolate spheroid with the axis of rotational symmetry aligned with the electric field. The electric field strengths for A, B, and C are 0.7 , 1.7 , and 2.4kV/cm , respectively. The color code indicates the numerical values of the electromechanical stress acting on the membrane.

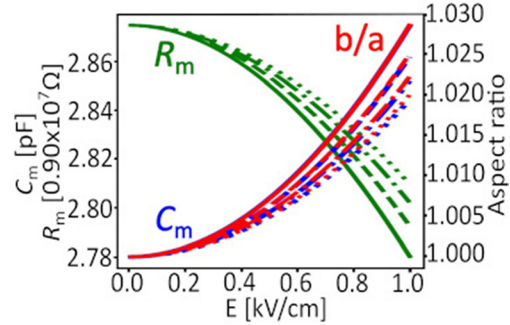


FIG. 4. Membrane capacitance C_m (blue lines), resistance R_m (green lines), and cell aspect ratio b/a (red lines) as a function of the exogenous electric field. $d_m = 5 \text{nm}$, $R = 5 \mu\text{m}$, and Y_m varies from 19MPa (solid line) to 22MPa (dashed line) to 25MPa (dash-dotted line) and 28MPa (dotted line).

Monitoring C_m and R_m of an initially spherical cell exposed to an electric field reflects a nonlinear behavior as a function of V_m , which reflects prolate morphologies of the cell up to a critical value of the electric field strength $E > E^*$ (Fig. 3) for which the stressed cell undergoes a significant change in shape.¹⁸ Using the procedure outlined above, we find that E^* scales as R^{-1} (not shown). For higher values of V_m , it is worth observing that the electric field-induced deformation produces complex axisymmetric cell shapes, which can be described using a variety of different parameterized forms, e.g., Cassini curves.¹⁸ The next thing to notice is that $R_m(V_m)$ does not scale as V_m^{-1} as suggested by Morshed and coworkers¹⁹ but as V_m^{-2} .

Next, we seek to unravel the role of the elastic modulus of the membrane Y_m in modulating the electromechanical behavior of the membrane. Few tests conducted on various types of eukaryotic cells show that they have a Young modulus ranging from 19MPa to 28MPa .¹ We note that the trend is consistent with earlier experimental observations showing that α is a parameter inversely proportional to the Young modulus.²⁰

Notice that we also investigate what happens if the material properties of the extracellular medium are varied, i.e., how the electrical parameters sense changes in their environment? In the [supplementary material](#), we provide another figure when the conductivity ratio between the cytoplasm and the extracellular medium varies by two orders of magnitude but keep all other structural and material parameters constant. We find that C_m and R_m are only weakly affected in the range of electric field magnitudes explored. Within the framework of our ED model, there can be hierarchies of geometrical parameters over electrical parameters because cell shape symmetry breaking (Fig. 1) is assumed to be mainly driven by the elastic response of the membrane to the electrostatic force (Maxwell stress tensor) excitation acting on it.

In summary, within the framework of a finite element simulation approach, our data and model provide direct evidence that the structural parameters of a single initially spherical cell, modeled as an elastic shell surrounding a core cell interior, have a significant and distinctive influence on membrane capacitance and resistance. The membrane capacitance and areal membrane resistance are found to be of the order of 10^{-2}Fm^{-2} and $10^{-2} \Omega \text{m}^2$, respectively, i.e., remarkably close to the estimates found in experimental analysis.^{2,3,9,17} Additionally, we find that a quadratic dependence of C_m and $1/R_m$ captures the impact of the strain state under steady-state electric excitation.

These trends, we believe, are real despite the limitations. First, our analysis has focused on nonporated membranes. In Ref. 21, Weaver and coworkers estimated the contribution of transient aqueous pores to capacitance and found that the time average property of a large population of transient aqueous pores has similar features with a quadratic dependence of capacitance. Second, an important constraint of the continuum model comes from the fact that the MC and membrane resistance features are independent of the molecular details of the membrane in which they occur but are dominated by the geometry and can be explained with simple mechanical models. It is also worth referring to a recent study of Liang and coworkers.²² In this study, the authors reported on a 3D analysis of the positive and negative optically induced dielectrophoresis (ODEP) forces on cells stimulated by non-uniform ac bias potential. The MC and areal membrane conductance per unit area of four types of cells were derived by characterizing their ODEP crossover frequencies using microvision techniques and were found to be of the order of 10^{-2} Fm^{-2} and $10^2 \text{ } \Omega \text{ m}^2$, respectively, i.e., remarkably close to the estimates found in this work. Third, we made the assumption about the scalar nature of the membrane permittivity. It can be argued that this assumption is not very physical (i.e., the cell membrane is heterogeneous, and the electric-field induced stretching of the membrane exhibits a peculiar anisotropy), but, nevertheless, our results allow us to shed some light on the mechanical strain involved in the ED.⁴ Note also that this model is flexible to incorporate a variety of other biological attributes such as internal organelles and the nucleus membrane. An obvious next step is to consider viscoelastic (or even poroelastic²²) cell materials; however, this is a much more difficult problem because the mechanism by which the membrane is deformed is crucially affected by the detailed temporal sequence of electric field excitation. Similarly, the contribution of electric double layers on both sides of the membrane to C_m needs to be better understood.²³ Ultimately, it is experiments that will shed light on the precise nature of the electric characteristics of the cell membrane, which is an important regulator of cell functionality.

See the [supplementary material](#) for the computational method for obtaining the capacitance and resistance-transmembrane potential characteristics of the cell membrane in greater detail. It also contains an additional figure showing C_m and R_m as a function of the exogenous electric field when the conductivity ratio between the cytoplasm and the extracellular medium is varied.

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DATA AVAILABILITY

The data that support the findings of this study are available within the article and its [supplementary material](#).

REFERENCES

- W. Hoiles, V. Krishnamurthy, and B. Cornell, *Dynamics of Artificially Engineered Membranes and Biosensors* (Cambridge University Press, 2018).
- L. D. Mosgaard, K. A. Zecchi, and T. Heimburg, "Mechano-capacitive properties of polarized membranes," *Soft Matter* **11**, 7899–2976 (2015).
- R. Benz and U. Zimmermann, "Relaxation studies on cell membranes and lipid bilayers in the high electric field range," *Bioelectrochem. Bioenergy* **7**, 723–739 (1980).
- C. Brosseau, E. Sabri, and S. Lasquelles, "A portrait of Nature's ultimate stretchable resistor-capacitor network: Cell membrane" (in press).
- J. Gimsa, M. Stubbe, and U. Gimsa, "A short tutorial contribution to impedance and AC-electrokinetic characterization and manipulation of cells and media: Are electric methods more versatile than acoustic and laser methods?," *J. Electr. Bioimpedance* **5**, 74–91 (2019).
- H. P. Schwan, *Adv. Biol. Med. Phys.* **5**, 147 (1957); H. P. Schwan, Historical review, state of the art, open problems, in *Interactions Between Electromagnetic Fields and Cells*, edited by A. Chiabrera, C. Nicolini, and H. P. Schwan (Plenum Press, New York, 1985), pp.1–18; H. P. Schwan, *Electroporation and Electrofusion in Cell Biology* (Plenum Press, 1989).
- K. R. Foster and H. P. Schwan, *Crit. Rev. Biomed. Eng.* **17**, 25 (1989); H. Pauly and H. P. Schwan, *Z. Naturforsch. B* **14**, 125 (1959); K. R. Foster and H. P. Schwan, in *Handbook of Biological Effects of Electromagnetic Fields*, edited by C. Polk and E. Postow (CRC Press, New York, 1996), pp. 25–102.
- T. J. Lewis, "Interfaces are the dominant feature of dielectrics at the nanometric level," *IEEE Trans Dielectr. Electr. Insul.* **11**, 739–753 (2004).
- T. Kotnik and D. Miklavčič, "Theoretical evaluation of voltage induction on internal membranes of biological cells exposed to electric fields," *Biophys. J.* **71**, 868–877 (1996); T. Kotnik, G. Pucihar, and D. Miklavčič, "The cell in the electric field," in *Clinical Aspects of Electroporation*, edited by S. T. Kee *et al.* (Springer Verlag, 2011).
- R. W. Glaser and J. Gimsa, "Gradual changes of membrane properties at high transmembrane electric potential simulate a breakdown threshold," in *Electricity and Magnetism in Biology and Medicine*, edited by M. Blank (San Francisco Press, 1993), 135–137.
- B. Alberts, J. H. Wilson, and T. Hunt, *Molecular Biology of the Cell*, 5th ed. (Garland Science, New York, 2008).
- T. R. Gowrishankar and J. C. Weaver, "An approach to electrical modeling of single and multiple cells," *Proc. Natl. Acad. Sci. U. S. A.* **100**, 3203–3208 (2003).
- T. Murovec and C. Brosseau, "Spectral fingerprint of electrostatic forces between biological cells," *Phys. Rev. E* **92**, 2015, 042717; D. Shamoon, J. Dermol-Cerne, L. Rems, M. Rebersek, T. Kotnik, S. Lasquelles, C. Brosseau, and D. Miklavčič, "Assessing the electro-deformation and electro-permeabilization of biological cells using a three dimensional finite element model," *Appl. Phys. Lett.* **114**, 2019, 063701; D. Shamoon, S. Lasquelles, and C. Brosseau, "A multiphysics analysis of the strain energy in multicellular environments," *Appl. Phys. Lett.* **115**, 043701 (2019).
- T. Murovec, D. C. Sweeney, E. Latouche, R. V. Davalos, and C. Brosseau, "Modeling of transmembrane potential in realistic multicellular structures before electroporation," *Biophys. J.* **111**, 2286–2295 (2016).
- COMSOL Multiphysics version 5.2.
- T. Heimburg, "The capacitance and electromechanical coupling of lipid membranes close to transitions: The effect of electrostriction," *Biophys. J.* **103**, 918 (2012).
- Experiences with solvent-containing bilayers have demonstrated that the capacitance of artificial bilayer membranes has a quadratic TMP dependence where the coefficient α is typically about 10 V^{-2} . S. M. White and T. E. Thompson, "Capacitance, area, and thickness variations in thin lipid films," *Biochim. Biophys. Acta* **323**, 7–22 (1973); D. Rosen and A. M. Sutton, "The effects of a direct potential bias on the electrical properties of bimolecular lipid membranes," *Biochim. Biophys. Acta* **163**, 226–233 (1968); O. Alvarez and R. Latorre, "Voltage-dependent capacitance in lipid bilayers made from monolayers," *Biophys. J.* **21**, 1–17 (1978).
- A. Di Biasio, L. Ambrosone, and C. Cametti, "Numerical simulation of dielectric spectra of aqueous suspensions of non-spheroidal differently shaped biological cells," *J. Phys. D: Appl. Phys.* **42**, 025401 (2009); M. Essone Mezeme and C. Brosseau, "Simulation of a toy model of cylindrical cells submitted to non-ionizing electromagnetic field: Effect of membrane cell disruption," *J. Appl. Phys.* **107**, 014701 (2010).
- B. I. Morshed, M. Shams, and T. Mussivand, "Deriving an electric circuit equivalent model of cell membrane pores in electroporation," *Biophys. Rev. Lett.* **8**, 21–12 (2013).

- ²⁰R. Benz and K. Janko, "Voltage-induced capacitance relaxation of lipid bilayer membranes: Effects on membrane composition," *Biochim. Biophys. Acta* **455**, 721–738 (1976).
- ²¹J. C. Weaver, K. T. Powell, R. A. Mintzer, H. Ling, and S. R. Loan, "The electrical capacitance of bilayer membranes: The contribution of transient aqueous pores," *Bioelectrochem. Bioenerg.* **12**, 393–404 (1984).
- ²²W. Liang, Y. Zhao, L. Liu, Y. Wang, W. J. Li, and G.-B. Lee, "Determination of cell membrane capacitance and resistance via optically induced electrokinetics," *Biophys. J.* **113**, 1531–1539 (2017).
- ²³E. Gongadze, A. Velikonja, S. Perutkova, P. Kramar, A. Maček-Lebar, V. Kralj-Iglič, and A. Iglič, "Ions and water molecules in an electrolyte solution in contact with charged and dipolar surfaces," *Electrochim. Acta* **126**, 42–60 (2014).