

Viscous theory for 3 layers Rayleigh Taylor instability

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Objective :

Model for Liquid Metal Battery (LMB)

Issue :

Need for a reliable energy storage system for a stationary application

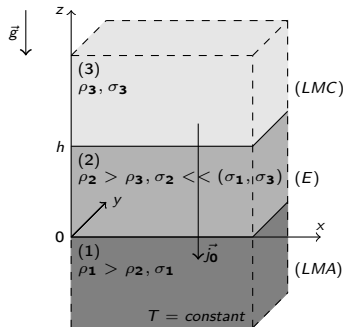


Figure : Sketch of the LMB model

Composition :

A molten salt (the electrolyte E) between 2 Liquid Metals (the anode LMA and the cathode LMC)

Studied case :

Stability analysis to prevent short circuits

Inviscid theory without magnetic field by Mikaelian, Phys. Rev. A 1982

Experimental data by Adkins et al., Phys. Rev. Fluids 2 2016



Viscid theory without magnetic field

Inviscid theory with a magnetic field for 2 systems close to LMB by Sneyd, J. Fluid Mech 1985

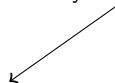
(3) \equiv gas \rightarrow electric arc furnace

(3) \equiv solid \rightarrow aluminium reduction cell



(3) \equiv liquid \rightarrow LMB

Inviscid theory with a magnetic field



Viscid theory with a magnetic field

\Rightarrow A model for LMB

Studied case :

Viscid theory without magnetic field

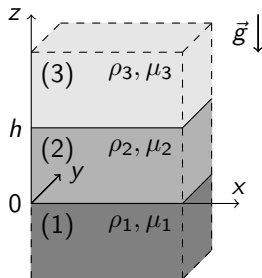


Figure : Sketch of the system

Assumptions :

- Incompressible fluids
- Infinite depth for extreme layers
- Isothermal conditions

Equations of the problem

Volume equations :

$$\vec{\nabla} \cdot \vec{u} = 0 \quad (1)$$

$$\rho \left(\vec{u}_{,t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} \right) = -\vec{\nabla} p + \vec{\nabla} \bar{\tau} + \rho \vec{g} \quad (2)$$

With

$$(\bar{\tau})_{jk} = \mu (u_{k,x_j} + u_{j,x_k}) \quad (3)$$

Jump conditions :

$$\vec{u}_i \cdot \vec{n}_i = z_{ii+1,t} = \vec{u}_{i+1} \cdot \vec{n}_i \text{ at } z = z_{ii+1} \quad (4)$$

$$\left(p_{i+1} - p_i - \gamma_{ii+1} \left(\frac{z_{ii+1,x,x}}{(1 + z_{ii+1,x}^2)^{\frac{3}{2}}} + \frac{z_{ii+1,y,y}}{(1 + z_{ii+1,y}^2)^{\frac{3}{2}}} \right) \right) \vec{n}_i = (\bar{\tau}_{i+1} - \bar{\tau}_i) \cdot \vec{n}_i \text{ at } z = z_{ii+1} \quad (5)$$

With $i = \{1; 2\}$ and $\vec{n}_i = {}^t(-z_{ii+1,x}, -z_{ii+1,y}, 1)$

Assumptions :

$$\vec{u}_i^* = \vec{0}$$

$$z_{12}^* = 0$$

$$z_{23}^* = h$$

Equations:

$$\vec{\nabla} p_i^* = -\rho_i g \vec{e}_z$$

$$p_2^* - p_1^* = 0 \text{ at } z = 0$$

$$p_3^* - p_2^* = 0 \text{ at } z = h$$

Solution:

$$p_1^* = -\rho_1 g z + p_c$$

$$p_2^* = -\rho_2 g z + p_c$$

$$p_3^* = -\rho_3 g z + (\rho_3 - \rho_2) g h + p_c$$

With $p_c = p_1^*(z = 0)$, a constant

Linearized volume equations :

$$u_{ix,x} + u_{iy,y} + u_{iz,z} = 0 \quad (6)$$

$$\rho u_{x,t} = -p'_{,x} + \mu \nabla^2 u_x \quad (7)$$

$$\rho u_{y,t} = -p'_{,y} + \mu \nabla^2 u_y \quad (8)$$

$$\rho u_{z,t} = -p'_{,z} + \mu \nabla^2 u_z \quad (9)$$

Linearized jump conditions :

$$u_{iz} = z_{ii+1,t} = u_{i+1z} \text{ at } z = z_{ii+1} \quad (10)$$

$$\begin{aligned} & (p'_{i+1} - p'_i - (\rho_{i+1} - \rho_i)gz_{ii+1} - \gamma_{ii+1} (z_{ii+1,x,x} + z_{ii+1,y,y})) \vec{e}_z \\ & = (\bar{\tau}_{i+1} - \bar{\tau}_i) \cdot \vec{e}_z \text{ at } z = (i-1)h \end{aligned} \quad (11)$$

With $i = \{1; 2\}$

Analysing the perturbation into normal modes \Rightarrow Solutions whose dependance on x , y and t is given by $e^{ik_x x + ik_y y + \omega t}$ are seeked

General case :

Numerical solutions for the growth rate

Simplified case :

$$\begin{aligned} &\Rightarrow \frac{\omega^4}{(gk)^2} (\rho_1 \rho_2 \cosh(kh) + \rho_2^2 \sinh(kh)) \\ &\quad + \rho_2 \rho_3 \cosh(kh) + \rho_1 \rho_3 \sinh(kh) \\ &\quad + \frac{\omega^2}{gk} (\rho_2 (\rho_1 - \rho_3) (\cosh(kh) + \sinh(kh))) \\ &\quad + (\rho_1 - \rho_2) (\rho_2 - \rho_3) \sinh(kh) = 0 \end{aligned} \tag{12}$$

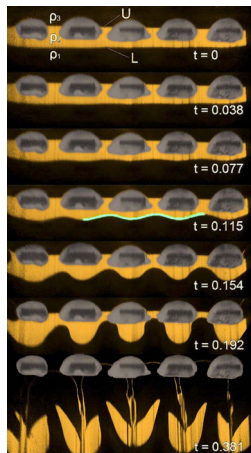
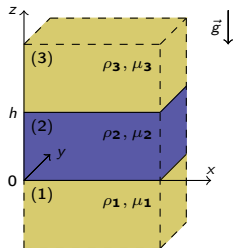
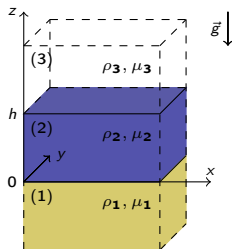


Figure : Set of images with respect to time of the Adkins et al. work



" $\rho_2 > \rho_1 = \rho_3$ case " :

$$\begin{aligned}\rho_1 &= \rho_3 = 773 \text{ kg.m}^{-3} \\ \rho_2 &= 1398 \text{ kg.m}^{-3} \\ \gamma_{12} &= \gamma_{23} = 3,5 \text{ mN.m}^{-1}\end{aligned}$$



" $\rho_2 > \rho_1 > \rho_3$ case " :

$$\begin{aligned}\rho_1 &= 773 \text{ kg.m}^{-3} \\ \rho_2 &= 1398 \text{ kg.m}^{-3} \\ \rho_3 &= 1,3 \text{ kg.m}^{-3} \\ \gamma_{12} &= 3,5 \text{ mN.m}^{-1} \\ \gamma_{23} &= 72 \text{ mN.m}^{-1}\end{aligned}$$

Figure : The 2 studied cases

Comparison with theories

Viscid theory more accurate mainly when λ is small:

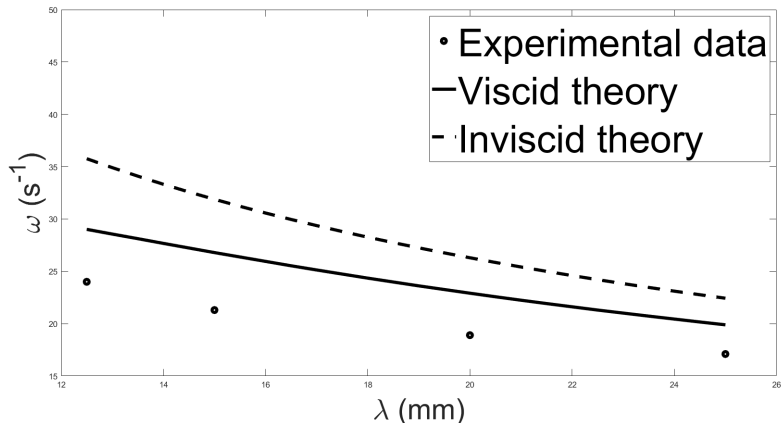


Figure : Comparison between the theories and the experimental data for the second case and $h = 2mm$

Comparison with theories

Viscid theory all the more accurate when h is small:

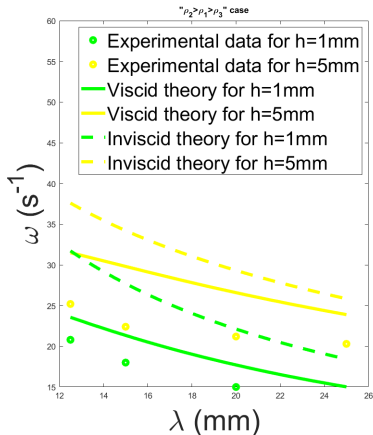
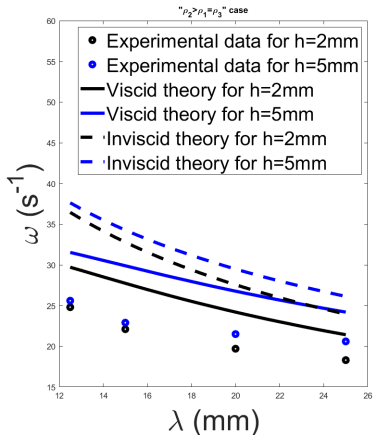


Figure : Same comparison for both cases and for different values of h

Conclusion :

- The results improve the theoretical predictions.

Perspectives :

- Cross-validate the theoretical results with OpenFoam.
- Viscid model for LMB.

- [1] Mikaelian, Rayleigh-Taylor instabilities in stratified fluids, Physical Review A (1982)
- [2] Adkins et al., Interface coupling and growth rate measurements in multilayer Rayleigh-Taylor instabilities, Physical Review Fluids 2 (2017)
- [3] Chandrasekhar, Hydrodynamic and hydromagnetic stability (1961)