

## Appendix A. Additional information on methods

### Mathematical proofs

Consider, the same notations as in the main text. In Pavoine and Ricotta (2021) we introduced index  ${}^\alpha Deq_j^*$  as a parametric index of functional distinctiveness as follows:

$${}^\alpha Deq_j^* = \frac{\sum_{c=1}^{N-1} u_{c|j} (N^{\alpha-1} - c^{\alpha-1})}{N^{\alpha-1} - 1} \quad (\text{A.1})$$

where  $u_{c|j}$  amounts to how much more dissimilar  $c$  is to  $j$  than species  $c - 1$  is:  $u_{c|j} = d_{c,j} - d_{c-1,j}$ , with  $d_{0,j} = d_{jj} = 0$ . We considered  ${}^1 Deq_j^*$  equal to the limit of  ${}^\alpha Deq_j^*$  when  $\alpha$  tends to 1, which yields:  ${}^1 Deq_j^* = \sum_{c=1}^{N-1} u_{c|j} (1 - \log_N(c))$ .

### Proof that Eq. A.1 above and Eq. 1 of the main text are equivalent

We start with Eq. A.1, i.e.  ${}^\alpha Deq_j^* = \frac{\sum_{c=1}^{N-1} u_{c|j} (N^{\alpha-1} - c^{\alpha-1})}{N^{\alpha-1} - 1}$

where  $c$  is the  $c^{\text{th}}$  species most functionally similar to  $j$ ,  $d_{c,j}$  its dissimilarity with species  $j$  (posing  $d_{0,j} = d_{jj} = 0$ ) and  $u_{c|j} = d_{c,j} - d_{c-1,j}$  (Pavoine and Ricotta, 2021).

$$\begin{aligned} \sum_{c=1}^{N-1} u_{c|j} (N^{\alpha-1} - c^{\alpha-1}) &= \sum_{c=1}^{N-1} u_{c|j} \sum_{m=c}^{N-1} ((m+1)^{\alpha-1} - m^{\alpha-1}) \\ &= \sum_{m=1}^{N-1} ((m+1)^{\alpha-1} - m^{\alpha-1}) \sum_{c=1}^m u_{c|j} \\ &= \sum_{m=1}^{N-1} ((m+1)^{\alpha-1} - m^{\alpha-1}) d_{m,j} \end{aligned}$$

□

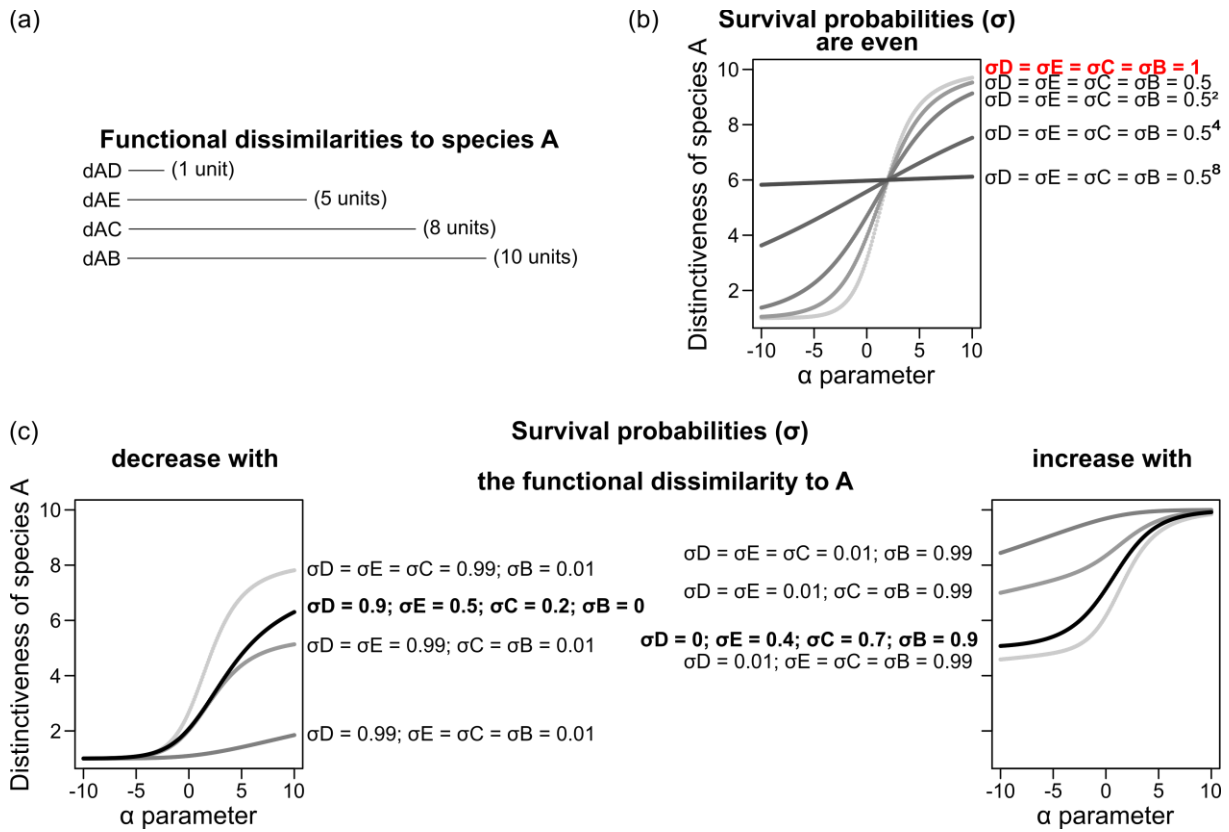
**Proof that  $\sum_{c=1}^{N-1} \pi_c = 1$  (Eq. 2 of the main text):**

$$\begin{aligned}
\pi_c &= \frac{(c+1)^{\alpha-1} - (c)^{\alpha-1}}{N^{\alpha-1} - 1} \\
\sum_{c=1}^{N-1} \pi_c &= \sum_{c=1}^{N-1} \frac{(c+1)^{\alpha-1} - (c)^{\alpha-1}}{N^{\alpha-1} - 1} \\
&= \frac{\sum_{c=1}^{N-1} (c+1)^{\alpha-1} - \sum_{c=1}^{N-1} (c)^{\alpha-1}}{N^{\alpha-1} - 1} = \frac{\sum_{c=2}^N (c)^{\alpha-1} - \sum_{c=1}^{N-1} (c)^{\alpha-1}}{N^{\alpha-1} - 1} \\
&= \frac{(N)^{\alpha-1} + \sum_{c=2}^{N-1} (c)^{\alpha-1} - \sum_{c=2}^{N-1} (c)^{\alpha-1} - (1)^{\alpha-1}}{N^{\alpha-1} - 1} = 1
\end{aligned}$$

□

### Illustration of the behavior of function ${}^{\alpha}f$

In Fig. A.1., we use the theoretical example of Fig. 1 in the main text but modify the distribution of extinction probabilities (and thus that of survival probabilities) to illustrate the influence extinction probabilities have on the measurement of the functional distinctiveness of a species. As in Fig. 1, we focus on the measurement of the functional distinctiveness of species A. The functional dissimilarity to species A increases from species D, through E, C, to B in that order (Fig. A.1a). When the extinction probabilities of species D, E, C and B are null, their survival probabilities are maximum (=1). In that case (bold, red distribution in Fig. A.1b)  ${}^{\alpha}f$  is equal to  ${}^{\alpha}Deq_j^*$ . Varying the values of parameter  $\alpha$  from  $-\infty$  to  $\infty$ , the distinctiveness of species A then varies from its functional dissimilarity with species D (=1) to its functional dissimilarity with species B (=10). Assume that the survival probabilities of species D, E, C and B are even, then the influence of parameter  $\alpha$  on the functional distinctiveness of species A decreases with the extinction probability of the species. In the extreme scenario where their extinction probability would tend to 1 (= their survival probability would tend to 0) then the functional distinctiveness of species A, as measured by  ${}^{\alpha}f$ , would tend to its unweighted average functional dissimilarity with other species ( $= (d_{AD} + d_{AE} + d_{AC} + d_{AB}) / 4 = (1 + 5 + 8 + 10) / 4 = 6$ ) (Fig. A.1b). Compared to  ${}^{\alpha}Deq_j^*$ , the functional distinctiveness of species A is decreased in  ${}^{\alpha}f$  as soon as the survival probabilities of the species functionally close to A stay high while those of species functionally distant to A become low. In contrast, compared to  ${}^{\alpha}Deq_j^*$ , the functional distinctiveness of species A is increased in  ${}^{\alpha}f$  as soon as the survival probabilities of the species functionally close to A become low while those of species functionally distant to A stay high (Fig. A.1c).



**Fig. A.1.** Illustration of the behavior of function  ${}^{\alpha}f$  on a theoretical case study. Species here are the same as that of Fig. 1 in the main text but are reordered from the most functionally similar (species D) to the most functionally dissimilar (species B) to species A. As in the main text, the functional dissimilarity from a species  $i$  to species A is noted  $d_{iA}$  and the survival probability of a species  $i$  is noted  $\sigma_i$ . The link between the survival ( $\sigma_i$ ) and the extinction probability  $\varepsilon_i$  of the species is:  $\sigma_i = 1 - \varepsilon_i$ . Panel a) recalls the functional dissimilarities between species A and the other species. Panels b) and c) provide the values of the functional distinctiveness of species A according to index  ${}^{\alpha}f$ , values for its parameter  $\alpha$ , and different specified scenarios of species survival probabilities. Each curve of index  ${}^{\alpha}f$  as a function of its parameter  $\alpha$  in panels b) and c) corresponds to the scenario of species survival probabilities indicated next to it. The red, bold distribution of survival probability in panel b) corresponds to the case where  ${}^{\alpha}f = {}^{\alpha}Deq_j^*$ . Black curves in c) correspond to scenarios where the survival probability of a species ( $\sigma_i$ ) varies linearly with its functional dissimilarity to A ( $d_{iA}$ ):  $\sigma_i = 1 - (d_{iA}/10)$  (left side) and c)  $\sigma_i = (d_{iA}-1)/10$  (right side).