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## Fractional dynamics of a Chikungunya transmission model

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### ABSTRACT

In this paper, we introduce a Caputo-sense fractional derivative to an existing model for the Chikungunya transmission dynamics, replacing integer derivative. After reviewing previous work on the integer order model, we suggest a change whereby the integer derivative is replaced by the Caputo derivative operator, and then we obtain results about the asymptotic stability of equilibrium points in this new fractional model. We prove the existence and uniqueness of the solution as well as the global stability of the fractional system. We then construct a numerical scheme for the fractional model, and use parameter values from the Chikungunya epidemic in Chad to perform numerical simulations. This allows us to validate our theoretical results and compare the dynamics behaviour of both models. We find from numerical simulations, that for the fractional-order parameter  $\eta$  in the range  $(0.85; 1)$ , daily detected cases are closer to those the model predict. Thus in the case of Chikungunya epidemic in Chad, the model with fractional derivative produces better results than which with integer derivative.

### Introduction

Chikungunya, or “man who bends up” in the Makonde language (Tanzania), is an arboviral disease that appears epidemically in many countries around the world. It was spread by the bite of a female mosquito that looked like an aedes (*Aedes albopictus* and *Aedes aegypti*). After the initial outbreak in Tanzania in 1952, the chikungunya virus (CHV) was described by researchers Marion Robinson and Lumsden [1,2]. Since this date, several Chikungunya epidemic episodes were observed in Europe (Italy and France), Asia (Bangkok, Thailand, Vellore, Calcutta and Maharashtra, Yemen Pakistan), South America (Brazil) and Africa (Comoros islands, Tanzania, Kenya, Democratic Republic of Congo, Chad), and in Reunion Islands in the Indian ocean [3–8]. In all these epidemic cases, the main control measures are vector control (use of insecticide, larvicide, sterile insect technique, etc.) and individual protection, since vaccination against Chikungunya, although in the development phase, is not yet available [9,10].

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To understand the transmission dynamics of Chikungunya and also evaluate the existing control measures, several mathematical models were proposed and studied (see [9–16] and the references therein). All of the works mentioned above are extensions of the SEIR-SEI type model, in which the authors include some existing control measures to combat disease transmission in human populations. Indeed, Agosto *et al.* in [11] developed a model structured by age. Mathematical analysis of the model indicated that the DFE is GAS whenever  $R_0 < 1$ . However, if the disease-induced death rate is taking account, they found that the backward bifurcation phenomenon occurs, when the DFE co-exists with two others points (one is unstable and other is stable) when  $R_0 < 1$ . They also found that the inclusion of age structure does not altered qualitative behaviour of the model. Dumont *et al.* in [9] proposed a model which take into account only mosquito aquatic stage, vector control and individual protections. They performed local asymptotic stability of the Chikungunya-free equilibrium whenever the basic reproduction number ( $R_0$ ) is less than one, while necessary condition to the GAS of Chikungunya-free equilibrium is obtained. They constructed numerical scheme based on the non-standard method, and estimated numerically  $R_0$ . They finally obtained that the value of  $R_0$  depend is place-dependent. In [13], Yakob and Clements formulated a transmission model of the Chikungunya virus between humans and mosquitoes. Using real data, they fitted the model and obtain that the type reproduction number is equal to 4.1. Through sensitivity analysis, they found that epidemics of Chikungunya occurred in Réunion Island were strongly influenced by latent period in humans and pre-patent period. The extension of these model were considered by other authors who included also control measures and applied optimal control [14,17]. In [17], Moulay and co-workers formulated a model including three control measures (vector control, individual protection and treatment) to fight the spread of Chikungunya in Réunion island. They use optimal control tools to analyse the optimal control model. Numerically, they obtained that there exist cost effective control efforts. The above models did not consider the density-dependent death rates of the populations involved (mosquitoes and humans). Recently, Abboubakar *et al.* in [18] proposed a SEIR-SEI-ELP model that includes density-dependent death rates, and four control measures. They performed an optimal control and efficiency analysis to figure out which combination of controls works best and permit a decrease in the total numbers of infected humans as well as the total numbers of mosquitoes in the adult and aquatic phases.

All the above cited models are principally formulated using classical derivatives (ordinary differential equations). Since more than two last decades, several works have shown that fractional derivatives are better than classical derivative when we want to model some Biological phenomenon like infectious diseases (see [19–30]). Compared to classical derivatives, FDs present a great advantage which is the memory effect [31,32]. One of disadvantage is that models with FDs do not have exact analytic solutions. Thus, numerous numerical methods are developed to approximate solutions [33,34].

To our knowledge, there are few works on mathematical modelling of Chikungunya transmission dynamics using fractional derivatives. We found, for the moment, the works of Helikumi and coworkers [35,36]. In [35], Helikumi *et al.* proposed and analysed a fractional Chikungunya model which include asymptotic individuals. Their main objective was investigated the memory effect on the disease dynamics. Then, in [36], they modified the previous model by including one compartment for aquatic stages of mosquitoes, as well as control measures. Model parameters were considered time-dependent. They calibrated the model using real data and thus performed sensitivity analysis. The fractional optimal control model was then formulated and analysed. They concluded through numerical simulations that memory effects affect the dynamics of Chikungunya. Contrary to [18], the Chikungunya fractional models cited above [35,36] does not distinguish all aquatic stages (eggs, larvae and pupae). In a control point of view, this can represent a problem since the eggs, larvae, and pupae do not respond in the same manner to larvicides or other chemical products whose principal role is to reduce these mosquito populations in their aquatic stage.

The goal of this work is to extend the model proposed by Abboubakar *et al.* in [18], which describes the transmission dynamics of the recent outbreak of Chikungunya in Chad, by changing the integer-derivative to a fractional derivative in the sense of Caputo. And thus, to compare both the qualitative and quantitative dynamics of both models, We begin by recalling the theoretical results of the integer model and then formulate the model with a fractional derivative. We compute the offspring reproduction number  $N$  and the basic reproduction number  $R_0$ , both of which are proportional to the fractional order  $\eta$ . Using the Banach contraction theorem and the Ulam–Hyers stability criterion, we show the solutions existence and uniqueness of the fractional model, as well as the uniform stability of the fractional model in the Lyapunov sense. The numerical method is built using the Generalized Fractional Euler Method (GFEM). We finally perform several simulations to validate our theoretical results as well as compare the two dynamics qualitatively and quantitatively.

The following outline constitutes the paper’s structure: In Section g, we will review a few of the fundamental definitions and ideas of fractional calculus. Model formulation is discussed in Section g. The fractional model for Chikungunya is analysed in Section g. The Existence and uniqueness of solution as well as the global stability of the fractional model is also proved in Section g. Numerical scheme as well as numerical results are performed in Section g. A conclusion accomplishes the work.

**Preliminaries on fractional calculus**

Fractional calculus extends ordinary calculus by examining integrals and derivatives of any real or complex order. It has been applied to real-world problems. In this section, we provide few notions and useful results needed to analyse fractional models.

**Definition 1** (Riemann–Liouville fractional integral [26]). Assume that  $\alpha > 0$  be a real number and  $f : [a, b] \rightarrow \mathbb{R}$  an integrable function. The Riemann–Liouville fractional integral of  $f$  of order  $\alpha$  is defined by :

$$I_a^\alpha f(t) := \frac{1}{\Gamma(\alpha)} \int_a^t (t - \tau)^{\alpha-1} f(\tau) d\tau, \tag{1}$$

where  $\Gamma$  denotes the Gamma function defined as follows:  $\Gamma(t) := \int_0^\infty \tau^{t-1} \exp(-\tau) d\tau$  for  $t > 0$ .  $\Gamma$  has the following properties :  $\Gamma(t + 1) = t\Gamma(t)$ , for all  $t > 0$ , and  $\Gamma(n + 1) = n!$  for  $n$  a positive integer.

**Definition 2** (Caputo derivative [26]). Let  $f : [a, b] \rightarrow \mathbb{R}$  of class  $C^n$ . The Caputo fractional derivative of  $f$  of order  $\alpha$  is defined by :

$${}^C D_t^\alpha f(t) = \frac{1}{\Gamma(n - \alpha)} \int_{t_0}^t (t - \tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau, \tag{2}$$

with  $\alpha \notin \mathbb{N}, n = [\alpha] + 1$ .

**Definition 3** (Generalized Caputo derivative [37]). The new modified Caputo-type fractional derivative of order  $\alpha$  is given as follows

$$({}^C D_a^{\alpha,\rho} f)(\xi)(t) = \frac{\rho^{\alpha-m+1}}{\Gamma(m - \alpha)} \int_a^\xi (\xi^\rho - s^\rho)^{m-\alpha-1} \left( s^{1-\rho} \frac{d}{ds} \right)^m f(s) ds, \quad \xi > a, \tag{3}$$

where  $\rho > 0, a \geq 0$ , and  $m - 1 < \alpha \leq m$

**Definition 4** ([38]). The Laplace transform of the Caputo fractional derivative of a function  $f(t)$  of order  $\alpha > 0$  is given by

$$\mathcal{L}({}^C D_0^\alpha f(t)) = s^\alpha f(s) - \sum_{k=0}^{n-1} f^{(k)} s^{\alpha-k-1}. \tag{4}$$

**Definition 5** ([39]). The Laplace transform of the function  $t^{\beta-1} E_{\alpha,\beta}(\pm \lambda t^\alpha)$  is defined as :

$$\mathcal{L}(t^{\beta-1} E_{\alpha,\beta}(\pm \lambda t^\alpha)) = \frac{s^{\alpha-\beta}}{s^\alpha \pm \lambda}, \tag{5}$$

where  $E_{\alpha,\beta}$  is the Mittag-Leffler function with two parameters with  $\alpha, \beta > 0$ . More, the following equation :

$$E_{\alpha,\beta}(f) = f E_{\alpha,\alpha+\beta}(f) + \frac{1}{\Gamma(\beta)}. \tag{6}$$

is satisfied by the Mittag-Leffler function.

**Property 1** (Linearity [26]). Assume that  $f, g : [a, b] \rightarrow \mathbb{R}$  be such that  ${}^C D_{t_0}^\alpha f(t)$  and  ${}^C D_{t_0}^\alpha g(t)$  exist almost everywhere and  $c_1, c_2$  are real numbers. Then  ${}^C D_{t_0}^\alpha (c_1 f(t) + c_2 g(t))$  exists almost everywhere and  ${}^C D_{t_0}^\alpha (c_1 f(t) + c_2 g(t)) = c_1 \times {}^C D_{t_0}^\alpha f(t) + c_2 \times {}^C D_{t_0}^\alpha g(t)$ .

**Lemma 1** ([40]). Assume that  $f(t) \in C([a, b])$  and  ${}^C D_{t_0}^\eta f(t) \in C([a, b])$  for  $0 < \eta \leq 1$ . Then,  $\forall t \in [a, b], \exists \tau \in ]a, t[$  satisfying  $f(t) = f(a) + \frac{1}{\Gamma(\eta)} {}^C D_{t_0}^\eta f(\tau)(t - a)^\eta$ .

**Lemma 2** ([41]). Let examine the fractional differential equation in the new generalized caputo sense

$$\begin{cases} {}^C D_0^{\beta,\theta} \xi(t) \in (0, Y), Y > 0 \\ \xi^{(k)}(0) = \xi_0^{(k)} \quad k = 0, 1, \dots, r - 1, \end{cases} \tag{7}$$

where  ${}^C D_0^{\beta,\theta}$  indicates the given new generalized Caputo derivative operator in Definition 3 with the corresponding Volterra integral equation

$$\xi(t) = \sum_{k=0}^{r-1} \frac{t^k}{k!} \xi_0^{(k)} + \frac{\theta^{1-\beta}}{\Gamma(\beta)} \int_0^t s^{\theta-1} (t^\theta - s^\theta)^{\beta-1} f(s, \xi) ds. \tag{8}$$

Suppose that  $\xi(t)$  is the solution to Eq. (8) and  $f(t, \xi)$  is bounded where  $h$  is sufficiently small. Then the following inequality holds

$$|\xi(t + h) - \xi(t)| \leq C_1 h^{\sigma(\beta)}, t \in [0, Y - h], \tag{9}$$

where constant  $C_1$  depends on  $h$ , and

$$\sigma(\beta) = \begin{cases} \beta & 0 < \beta < 1 \\ 1 & \beta \geq 1 \end{cases} \tag{10}$$

**Lemma 3** ([41]). Suppose that  $\xi(t)$  is the solution of (8) and  $f(t, \xi)$  is continuous and satisfies the Lipschitz condition with respect to  $\xi$  and  $t$  with a Lipschitz constant  $C_2$  for sufficiently small  $h$ . Then we have

$$\left| \int_0^{t^{\theta-1}} s^{\theta-1} (t^\theta - s^\theta)^{\beta-1} f(s, \xi) ds - \frac{\theta^{-\beta} h^\beta}{\Gamma(\beta)} \sum_{j=0}^m b_{j,m+1} f(t_j, \xi(t_j)) \right| \leq C_2 h^{\sigma(\beta)}. \tag{11}$$

**Remark 1.** Two important relations between these two fractional operators are specified as follows:

- 1 Given a continuous function  $f : [a, b] \rightarrow \mathbb{R}, {}^C D_{t_0}^\alpha I_{t_0}^\alpha f(t) = f(t)$

2 If  $f : [a, b] \rightarrow \mathbb{R}$  is a function of class  $C^n$ , then

$$I_{t_0}^\alpha {}^C D_{t_0}^\alpha f(t) = f(t) - \sum_{k=0}^{n-1} \frac{f^{(k)}(t_0)}{k!} (t - t_0)^k.$$

**Remark 2.** Consider that  $f(t) \in C([0, b])$  and  ${}^C D_t^\alpha f(t) \in C([0, b])$  for  $0 < \alpha \leq 1$ . From Lemma 1 it clear that the function  $f(t)$  is non-decreasing if  ${}^C D_t^\alpha f(t) \geq 0, \forall t \in (0, b]$  and it is non-increasing if  ${}^C D_t^\alpha f(t) \leq 0, \forall t \in (0, b]$

**Lemma 4 ([42]).** If  $0 < \varpi_0 < 1$  and  $\rho$  is an non-negative integer, then  $\exists C_{\varpi_0,1}, C_{\varpi_0,2} \in \mathbb{R}_+$  which depend on  $b_0$ , such that

$$(\rho + 1)^{\varpi_0} - \rho^{\varpi_0} \leq C_{\varpi_0,1}(\rho + 1)^{\varpi_0-1} \tag{12}$$

and

$$(2 + \rho)^{1+\varpi_0} - 2(1 + \rho)^{1+\varpi_0} + \rho^{1+b_0} \leq C_{\varpi_0,2}(1 + \rho)^{\varpi_0-1}. \tag{13}$$

**Lemma 5 ([42]).** Assume that  $d_{i,r} = (r - i)^{\varpi_0-1} (i = 1, 2, \dots, r - 1)$  and  $d_{i,r} = 0$  for  $i \geq r, rh \leq T, \varpi_0, h, \mathcal{M}, T > 0$  and  $r$  is a positive integer. Let  $\sum_{i=s}^{i=r} d_{i,r}|e_i| = 0$  for  $\rho > r \geq 1$ .

If  $|e_r| \leq \mathcal{M}\mathfrak{N}^{\varpi_0} \sum_{i=1}^{i=r-1} d_{i,r}|e_i| + |\beta_0|, r = 1, 2, \dots, s$ , then  $|e_s| \leq C|\beta_0|, s = 1, 2, \dots$  where  $C$  is a positive constant independent to  $\mathfrak{N}$  and  $s$ .

**Model formulation**

The model we consider is the one formulated in [18] and depicted in Fig. 1, which consists of a system of ten (10) ordinary differential equations. The populations involved are humans and vectors (mosquitoes). The human population is split into four compartments: susceptible humans denoted by  $S_h$ , exposed humans (infected but not infectious) denoted by  $E_h$ , infectious humans denoted by  $I_h$ , and recovered humans denoted by  $R_h$ . Due to the epidemiological knowledge of Chikungunya, the immunity conferred after being infectious is permanent. Since the aquatic phase plays a major role in the mosquito’s life, the dynamics of mosquitoes are composed of aquatic mosquitoes, which are divided into three sub-populations ( $E$  for Eggs,  $L$  for larvae, and  $P$  for pupae), and adult mosquitoes, which include the susceptible mosquitoes denoted by  $S_v$ , the exposed mosquitoes denoted by  $E_v$ , and the infectious mosquitoes denoted by  $I_v$ . Infected mosquitoes remain infected throughout their lives. Thus, the model considered in [18] is given as follows:

$$\dot{S}_h(t) = \Lambda_h + \rho N_h - \lambda_h(t)S_h(t) - (\mu_1 + \mu_2 N_h(t))S_h(t), \tag{14a}$$

$$\dot{E}_h(t) = \lambda_h(t)S_h(t) - \gamma_h E_h(t) - (\mu_1 + \mu_2 N_h(t))E_h(t), \tag{14b}$$

$$\dot{I}_h(t) = \gamma_h E_h(t) - \sigma_h I_h(t) - \delta_h I_h(t) - (\mu_1 + \mu_2 N_h(t))I_h(t), \tag{14c}$$

$$\dot{R}_h(t) = \sigma_h I_h(t) - (\mu_1 + \mu_2 N_h(t))R_h(t), \tag{14d}$$

$$\dot{S}_v(t) = \theta P(t) - \lambda_v(t)S_v(t) - (\mu_3 + \mu_4 N_v(t))S_v(t), \tag{14e}$$

$$\dot{E}_v(t) = \lambda_v(t)S_v(t) - \gamma_v E_v(t) - (\mu_3 + \mu_4 N_v(t))E_v(t), \tag{14f}$$

$$\dot{I}_v(t) = \gamma_v E_v(t) - (\mu_3 + \mu_4 N_v(t))I_v(t), \tag{14g}$$

$$\dot{E}(t) = \xi \left( 1 - \frac{E(t)}{K_E} \right) (I_v(t) + E_v(t) + S_v(t)) - (\mu_E + s)E(t), \tag{14h}$$

$$\dot{L}(t) = sE(t) \left( 1 - \frac{L(t)}{K_L} \right) - (\mu_L + l + \alpha L(t))L(t), \tag{14i}$$

$$\dot{P}(t) = lL(t) \left( 1 - \frac{P(t)}{K_P} \right) - (\mu_P + \theta + \beta L(t))P(t). \tag{14j}$$

In Eq. (14),  $\Lambda_h$  denotes the recruitment rate of susceptible individuals,  $\rho$  is the per capita birth rate of humans,  $\mu_i, i = 1, 2$ , represent the mortality rates  $\mu_1$  and  $\mu_2, \lambda_h(t)$  is the force of infection (FOI) given by

$$\lambda_h(t) = a\beta_h \frac{I_v(t)}{N_h(t)}, \tag{15}$$

where  $a$  and  $\beta_h$  are, respectively, the average numbers of mosquito bites and probability of infection transmission from an infectious vector to a susceptible human. The transition rate between compartments  $E_h$  and  $I_h$  is denoted by  $\gamma_h$  ( $1/\gamma_h$  is the average duration of the latent stage in infected human).  $\delta_h$  denotes the death rate induced by the disease, and  $\sigma_h$  is the recovered rate of infectious humans. For the adult mosquito dynamics,  $\theta$  is the rate at which pupae become adult females (susceptible mosquitoes),  $\mu_i, i = 3, 4$ , are the mortality rates of adult female mosquitoes,  $\gamma_v$  is the transition rate between compartments  $E_v$  and  $I_v$  ( $1/\gamma_v$  is the average duration of the latent stage in infected mosquito). For the aquatic mosquito dynamics,  $\xi$  represents the average number of eggs laid by each adult female, at each deposit;  $K_E, K_L, K_P$  are respectively the carrying capacity of eggs, larvae and pupae;  $s$  is the transition rate between  $E$  and  $L$  while  $l$  is the transition rate between  $L$  and  $P$ ;  $\mu_E, \mu_L$  and  $\mu_P$  are the natural death rates of eggs, larvae and pupae, respectively;  $\alpha$  and  $\beta$  are respectively the additional death rate of larvae and pupae, due to the density [43].

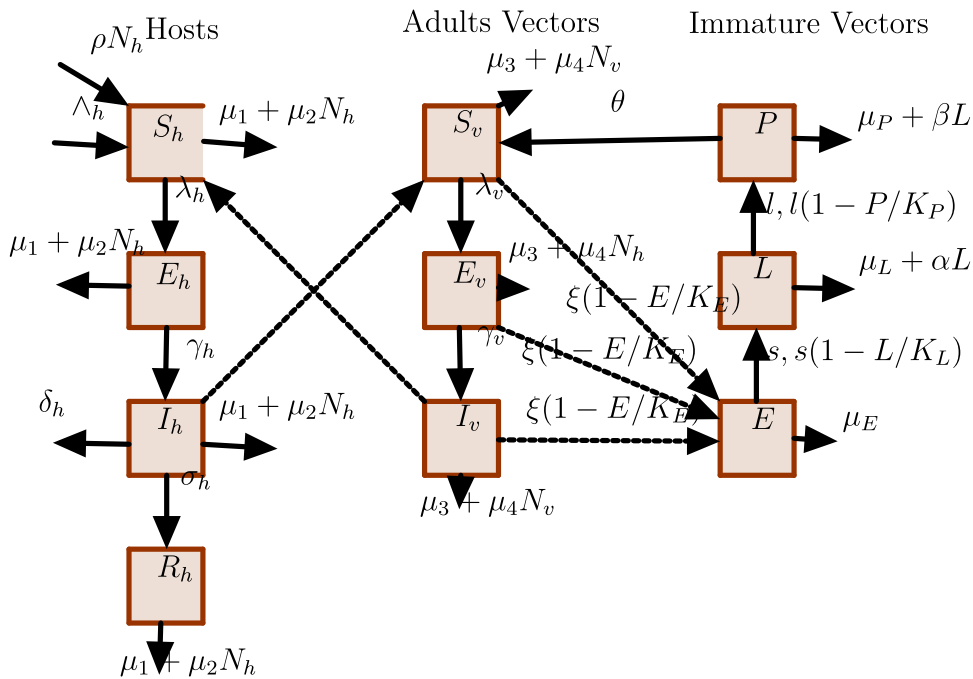


Fig. 1. Schematic of the mosquito-borne epidemic model with development stage of mosquitoes.

We denote the total population's sizes by  $N_h(t) = S_h(t) + E_h(t) + I_h(t) + R_h(t)$  and  $N_v(t) = S_v(t) + E_v(t) + I_v(t)$ .

The model (14) is defined in the following set  $\Sigma = \Sigma_h \times \Sigma_v$ , where  $\Sigma_h := \{(S_h, E_h, I_h, R_h) \in \mathbb{R}_+^4 : N_h > 0\}$  and  $\Sigma_v := \{(S_v, E_v, I_v, E, L, P) \in \mathbb{R}_+^6 : N_v \geq 0; E \geq 0; L \geq 0; P \geq 0\}$ .

In the system, we substitute the Caputo fractional derivative of order  $\eta$  for the next ordinary derivative, where  $\eta$  is an arbitrary real in the interval  $(0, 1]$ . To resolve the irrelevancy of the dimensions between the left and right hand-sides of the equations of system (14), due to the substitution of ordinary derivatives with fractional derivatives in the model, we multiply the left-hand side of each equation by  $\chi^{\eta-1}$ , where  $\chi$  is a time constant used to compensate the units [44], after replacing the differential operators. Thus, the fractional system becomes :

$$\chi^{\eta-1} \times {}^C D_t^\eta S_h(t) = \Lambda_h + \rho N_h - \lambda_h(t) S_h(t) - (\mu_1 + \mu_2 N_h(t)) S_h(t), \tag{16a}$$

$$\chi^{\eta-1} \times {}^C D_t^\eta E_h(t) = \lambda_h(t) S_h(t) - \gamma_h E_h(t) - (\mu_1 + \mu_2 N_h(t)) E_h(t) \tag{16b}$$

$$\chi^{\eta-1} \times {}^C D_t^\eta I_h(t) = \gamma_h E_h(t) - \sigma_h I_h(t) - \delta_h I_h(t) - (\mu_1 + \mu_2 N_h(t)) I_h(t), \tag{16c}$$

$$\chi^{\eta-1} \times {}^C D_t^\eta R_h(t) = \sigma_h I_h(t) - (\mu_1 + \mu_2 N_h(t)) R_h(t), \tag{16d}$$

$$\chi^{\eta-1} \times {}^C D_t^\eta S_v(t) = \theta P(t) - \lambda_v(t) S_v(t) - (\mu_3 + \mu_4 N_v(t)) S_v(t), \tag{16e}$$

$$\chi^{\eta-1} \times {}^C D_t^\eta E_v(t) = \lambda_v(t) S_v(t) - \gamma_v E_v(t) - (\mu_3 + \mu_4 N_v(t)) E_v(t), \tag{16f}$$

$$\chi^{\eta-1} \times {}^C D_t^\eta I_v(t) = \gamma_v E_v(t) - (\mu_3 + \mu_4 N_v(t)) I_v(t), \tag{16g}$$

$$\chi^{\eta-1} \times {}^C D_t^\eta E(t) = \xi \left( 1 - \frac{E(t)}{K_E} \right) (I_v(t) + E_v(t) + S_v(t)) - (\mu_E + s) E(t), \tag{16h}$$

$$\chi^{\eta-1} \times {}^C D_t^\eta L(t) = s E(t) \left( 1 - \frac{L(t)}{K_L} \right) - (\mu_L + l + \alpha L(t)) L(t), \tag{16i}$$

$$\chi^{\eta-1} \times {}^C D_t^\eta P(t) = l L(t) \left( 1 - \frac{P(t)}{K_P} \right) - (\mu_P + \theta + \beta L(t)) P(t), \tag{16j}$$

with initial conditions Conditions:

$$\begin{aligned} S_h(t_0) &= S_{h0} > 0, E_h(t_0) = E_{h0} \geq 0, I_h(t_0) = I_{h0} \geq 0, R_h(t_0) = R_{h0} \geq 0, \\ S_v(t_0) &= S_{v0} \geq 0, E_v(t_0) = E_{v0} \geq 0, I_v(t_0) = I_{v0} \geq 0, \\ E(t_0) &= E_0 \geq 0, L(t_0) = L_0 \geq 0, P(t_0) = P_0 \geq 0. \end{aligned} \tag{17}$$

We also have

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta N_h(t) = \Lambda_h + \rho N_h(t) - (\mu_1 + \mu_2 N_h(t))N_h(t) - S_h(t)I_h(t), \tag{18a}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta N_v(t) = \theta P(t) - (\mu_3 + \mu_4 N_v(t))N_v(t). \tag{18b}$$

**Remark 3.** In the above systems,  ${}^C D_t^\eta$  denotes the Caputo fractional derivative of order  $\eta \in (0, 1]$ . For  $\eta = 1$ ,  ${}^C D_t^\eta \equiv \frac{d}{dt}(\bullet)$  which denotes derivative in the classical sense (integer derivative). The case  $\eta = 1$  was analysed in [18].

### Mathematical analysis of the fractional model

#### Dynamics analysis of mosquito population sub-model

Let us consider only the model for translating the mosquito dynamics. Adding the fifth, sixth, and seventh equations of (14) together with the eighth, ninth, and tenth equations of (14), we have:

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta E(t) = \xi \left( 1 - \frac{E(t)}{K_E} \right) N_v(t) - (\mu_E + s)E(t), \tag{19a}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta L(t) = sE(t) \left( 1 - \frac{L(t)}{K_L} \right) - (\mu_L + l + \alpha L(t))L(t), \tag{19b}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta P(t) = lL(t) \left( 1 - \frac{P(t)}{K_P} \right) - (\mu_P + \theta + \beta L(t))P(t), \tag{19c}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta N_v(t) = \theta P(t) - (\mu_3 + \mu_4 N_v(t))N_v(t). \tag{19d}$$

**Lemma 6.** Every solution  $\phi(t) = (E(t), L(t), P(t), N_v(t))$  of (19) with initial conditions  $\phi(0) = (E(0), L(0), P(0), N_v(0)) \in \mathbb{R}_+^4$  is defined and lies in  $\mathbb{R}_+^4$  for all  $t \geq 0$ .

**Proof.** Using the process discussed in the work of Lin [45] we can show the existence and uniqueness of the solution of system (19) on the time interval  $(0; \infty)$ . Next, we have to explain that the non-negative region  $\mathbb{R}_+^4$  is a positively invariant region. From (19), we find

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta E \Big|_{E=0, S_v, E_v, I_v \geq 0} = \xi N_v \geq 0, \tag{20a}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta L \Big|_{L=0, E \geq 0} = sE \geq 0, \tag{20b}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta P \Big|_{P=0, L \geq 0} = lL \geq 0, \tag{20c}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta N \Big|_{N_v=0, P \geq 0} = \theta P \geq 0. \tag{20d}$$

If  $\phi(0) \in \mathbb{R}_+^4$ , then according to (20) and Remark 2, the solution  $\phi(t)$  cannot escape from the hyper-planes  $E = 0, L = 0, P = 0$  and  $N_v = 0$ . Likewise on each hyper-plane bounding the non-negative orthan, the vector field points into  $\mathbb{R}_+^4$ , ie the domain  $\mathbb{R}_+^4$  is a positively invariant set.  $\square$

Let us consider the following set:

$$\mathbb{D} = \left\{ (E, L, P, N_v) \in \mathbb{R}_+^4 : E \leq K_E, L \leq K_L, P \leq K_P, N_v \leq \frac{\theta^\eta K_P}{\mu_3^\eta} \right\}. \tag{21}$$

Then the following result holds.

**Theorem 1.** For system (19), the region  $\mathbb{D}$  is a positive invariant set.

**Proof.** We have from system (19):  ${}^C D_t^\eta N_v(t) = \theta^\eta P(t) - (\mu_3^\eta + \mu_4^\eta N_v(t))N_v(t)$ .

This implies

$${}^C D_t^\eta N_v(t) \leq \theta^\eta P(t) - \mu_3^\eta N_v(t). \tag{22}$$

Using the Laplace transform to Eq. (22), we have

$$z^\eta \mathcal{L}(N_v) - z^{\eta-1} N_v(0) \leq \left( \frac{\theta^\eta K_P}{z} - \mu_3^\eta \mathcal{L}(N_v) \right) \tag{23}$$

which further gives

$$\mathcal{L}(N_v) \leq \left( \frac{z^{-1}}{z^\eta + \mu_3^\eta} \theta^\eta K_P + \frac{z^{\eta-1}}{z^\eta + \mu_3^\eta} N_v(0) \right). \tag{24}$$

From Eqs (4) and (5), we infer that if  $(E(0), L(0), P(0), N_v(0)) \in \mathbb{R}_+^4$ , then

$$N_v(t) \leq \theta^\eta K_P t^\eta \mathbb{E}_{\eta, \eta+1}(-\mu_3^\eta t^\eta) + \mathbb{E}_{\eta, 1}(-\mu_3^\eta t^\eta) N_v(0). \tag{25}$$

Using Eq. (6) we have for all  $t \geq 0$ ,

$$\begin{aligned} N_v(t) &\leq \theta^\eta K_P t^\eta \left( \frac{1}{-\mu_3^\eta t^\eta} \right) \left( \mathbb{E}_{\eta, 1}(-\mu_3^\eta t^\eta) - \frac{1}{\Gamma(1)} \right) + \mathbb{E}_{\eta, 1}(-\mu_3^\eta t^\eta) N_v(0), \\ &\leq \frac{\theta^\eta K_P}{\mu_3^\eta} + \left( N_v(0) - \frac{\theta K_P}{\mu_3^\eta} \right) \mathbb{E}_{\eta, 1}(-\mu_3^\eta t^\eta). \end{aligned} \tag{26}$$

Thus,  $\limsup_{t \rightarrow +\infty} N_v(t) \leq \frac{\theta^\eta K_P}{\mu_3^\eta}$ . Henceforth, the closed set  $\mathbb{D}$  is positively invariant for the fractional model (19).  $\square$

Let us set  $k_4 = \mu_E^\eta + s^\eta$ ,  $k_5 = \mu_L^\eta + l^\eta$  and  $k_6 = \mu_P^\eta + \theta^\eta$ . Before computing equilibrium points of system (19), we recall the following threshold

$$\mathbf{N} = \frac{\xi^\eta \theta^\eta s^\eta l^\eta}{\mu_3^\eta (s^\eta + \mu_E^\eta)(l^\eta + \mu_L^\eta)(\theta^\eta + \mu_P^\eta)} = \frac{\xi^\eta \theta^\eta s^\eta l^\eta}{\mu_3^\eta k_4 k_5 k_6}, \tag{27}$$

which represents the average number of new female mosquitoes produced by an adult female mosquito over its life (offspring reproductive number) [18]. Since the equilibrium point of the integer model (14) are the same for the fractional number (16), it follows from [18] that the fractional system (19) admits the following equilibrium points :  $\mathbb{E}_0 = (0, 0, 0, 0)$  and  $\mathbb{E}_1 = (E^*, L^*, P^*, N_v^*)$  where

$$P^* = \frac{(\mu_3^\eta + \mu_4^\eta N_v^*) N_v^*}{\theta^\eta}, E^* = \frac{\xi^\eta K_E + N_v^*}{\xi^\eta N_v^* + k_4 K_E}, L^* = \frac{k_6 K_P P^*}{l^\eta K_P - P^*(l^\eta + \beta^\eta K_P)}, \tag{28}$$

and  $N_v^*$  denotes a positive solution of the following equation

$$N_v^* [\omega_4 (N_v^*)^4 + \omega_3 (N_v^*)^3 + \omega_1 (N_v^*) + \omega_0] = 0,$$

with

$$\begin{aligned} \chi^{\eta-1} \omega_4 &= \mu_3 \mu_4^2 k_5 k_6 \mathbf{N} [K_E K_L l^2 s + 2K_E K_L K_P \beta l s + K_E K_P k_6 l s + K_E K_L K_P^2 \beta^2 s + K_E K_P^2 k_6 \beta s \\ &\quad + K_L K_P k_5 k_6 l + K_L K_P^2 k_6 (k_5 \beta - k_6 \alpha)], \\ \chi^{\eta-1} \omega_3 &= \mu_4 k_6 [K_E K_L K_P \mu_4 k_5 l^2 s \theta + K_E K_L K_P^2 \mu_4 l s \theta (k_5 \beta - k_6 \alpha) + 2K_E K_L \mu_3^2 k_5 \mathbf{N} l^2 s \\ &\quad + 4K_E K_L K_P \mu_3^2 k_5 \mathbf{N} \beta l s + 2K_E K_P \mu_3^2 k_5 k_6 \mathbf{N} l s + 2K_E K_L K_P^2 \mu_3^2 k_5 \mathbf{N} \beta^2 s \\ &\quad + 2K_E K_P^2 \mu_3^2 k_5 k_6 \mathbf{N} \beta s + 2K_L K_P \mu_3^2 k_5^2 k_6 \mathbf{N} l + 2K_L K_P^2 \mu_3^2 k_5 k_6 \mathbf{N} (k_5 \beta - k_6 \alpha)], \\ \chi^{\eta-1} \omega_2 &= -\mu_3 k_6 [2K_E K_L K_P \mu_4 k_5 l^2 s \theta (\mathbf{N} - 1) + 2K_E K_L K_P^2 \mu_4 k_5 \beta l s \theta (\mathbf{N} - 1) \\ &\quad + 2K_E K_L K_P^2 \mu_4 k_6 \alpha l s \theta + K_E K_P^2 \mu_4 k_5 k_6 \mathbf{N} l s \theta + K_L K_P^2 \mu_4 k_5^2 k_6 \mathbf{N} l \theta \\ &\quad - K_E K_L \mu_3^2 k_5 \mathbf{N} l^2 s - 2K_E K_L K_P \mu_3^2 k_5 \mathbf{N} \beta l s - K_E K_P \mu_3^2 k_5 k_6 \mathbf{N} l s - K_E K_L K_P^2 \mu_3^2 k_5 \mathbf{N} \beta^2 s \\ &\quad - K_E K_P^2 \mu_3^2 k_5 k_6 \mathbf{N} \beta s - K_L K_P \mu_3^2 k_5^2 k_6 \mathbf{N} l - K_L K_P^2 \mu_3^2 k_5^2 k_6 \mathbf{N} \beta + K_L K_P^2 \mu_3^2 k_5 k_6^2 \mathbf{N} \alpha], \\ \chi^{\eta-1} \omega_1 &= -K_P k_6 \theta [K_E K_L K_P \mu_4 k_5 l s \theta + K_E K_L \mu_3^2 k_5 l s (2\mathbf{N} - 1) + K_E K_L K_P \mu_3^2 k_5 \beta s (2\mathbf{N} - 1) \\ &\quad + K_E K_L K_P \mu_3^2 k_6 \alpha s + K_E K_P \mu_3^2 k_5 k_6 \mathbf{N} s + K_L K_P \mu_3^2 k_5^2 k_6 \mathbf{N}], \\ \chi^{\eta-1} \omega_0 &= K_E K_L K_P^2 \mu_3 k_5 k_6 l^2 s \theta^2 (\mathbf{N} - 1). \end{aligned}$$

To establish the local stability of equilibrium points, we apply the following fundamental theorem established by Matignon in [46].

**Theorem 2 ([46]).** *Let consider the following measured fractional order system:*

$${}^C_{t_0} D_t^\eta x(t) = f[t, x(t)]; x(t_0) = x_0, \tag{29}$$

where  ${}^C_{t_0} D_t^\eta$  denotes the Caputo derivative of order  $0 < \eta \leq 1$  and  $f[t, x(t)] : \mathbb{R}^+ \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a field of vectors. The equilibrium points of this system are locally asymptotically stable if the following condition:  $|\arg \lambda_i| > \frac{\pi}{2}$  is satisfied by all eigenvalues  $\lambda_i$  of the Jacobian matrix at the equilibrium point.

Following the theorem, to prove the local stability of equilibrium points  $\mathbb{E}_i$ ,  $i = 0, 1$ , it is sufficient to show that all eigenvalues of the Jacobian matrix of (29) evaluated at  $\mathbb{E}_i$  have non positive real parts.

**Theorem 3.** *If  $\mathbf{N} < 1$ , then the trivial equilibrium  $\mathbb{E}_0 = (0, 0, 0, 0)$  is locally asymptotically stable in  $\mathbb{D}$ .*

**Proof.** The Jacobian matrix of the system (19) evaluated at the trivial equilibrium is given by :



$$J(\mathbb{E}_0) = \begin{pmatrix} -k_4 & 0 & 0 & \xi^\eta \\ s^\eta & -k_5 & 0 & 0 \\ 0 & l^\eta & -k_6 & 0 \\ 0 & 0 & \theta^\eta & -\mu_3^\eta \end{pmatrix}.$$

Its characteristic polynomial is given by :  $P(X) = X^4 + A_1X^3 + A_2X^2 + A_3X + A_4$  where  $A_0 = 1; A_1 = k_6 + k_5 + k_4 + \mu_3^\eta; A_2 = (k_5 + k_4 + \mu_3^\eta)k_6 + (k_4 + \mu_3^\eta)k_5 + \mu_3^\eta k_4; A_3 = ((k_4 + \mu_3^\eta)k_5 + \mu_3^\eta k_4)k_6 + \mu_3^\eta k_4 k_5$  and  $A_4 = \mu_3^\eta k_4 k_5 k_6 (1 - N)$ . We have  $A_0 > 0, A_1 > 0, A_3 > 0$  and  $A_4 > 0$  if and only if  $N < 1$ . From [18], the following Routh Hurwitz conditions are satisfied:  $\mathbb{H}_1 = A_1 > 0, \mathbb{H}_2 = A_1 A_2 - A_3 > 0, \mathbb{H}_3 = A_1 A_2 A_3 - A_1^2 A_4 - A_3^2 > 0$ . It follows that  $J(\mathbb{E}_0)$  has all its eigenvalues with negative real part and these eigenvalues satisfy the condition of Theorem 2. Hence  $\mathbb{E}_0 = (0, 0, 0, 0)$  is locally asymptotically stable whenever  $N < 1$ . □

**Theorem 4 ([18]).**

1. If  $N < 1$ , then  $\mathbb{E}_0 = (0, 0, 0, 0)$  is globally asymptotically stable (GAS) in  $\mathbb{D}$ ;
2. If  $N \geq 1$ , then  $\mathbb{E}_0 = (0, 0, 0, 0)$  becomes unstable and the co-existence equilibrium point  $\mathbb{E}_1 = (E^*, L^*, P^*, N_v^*)$  is GAS in  $\mathbb{D}$ .

*Dynamics analysis of the vector-host transmission model*

Let us consider  $\Sigma_1 = \mathbb{R}_+^{10} \setminus (\{N_v = E = L = P = 0\} \cup \{N_h = 0\})$  which is the set on which vectors and humans populations size are not equal to zero.

We consider the fractional-order Chikungunya of vector-host transmission model. From Theorem 4,  $\mathbb{E}_1$  is GAS on  $\mathbb{D}$ , it follows that  $N_v^* = N_v^0$ . It is thus sufficient to consider the following reduced system

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta S_h(t) = \Lambda_h + \rho N_h - \lambda_h(t)S_h(t) - (\mu_1 + \mu_2 N_h(t))S_h(t), \tag{30a}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta E_h(t) = \lambda_h(t)S_h(t) - \gamma_h E_h(t) - (\mu_1 + \mu_2 N_h(t))E_h(t) \tag{30b}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta I_h(t) = \gamma_h E_h(t) - \sigma_h I_h(t) - \delta_h I_h(t) - (\mu_1 + \mu_2 N_h(t))I_h(t), \tag{30c}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta R_h(t) = \sigma_h I_h(t) - (\mu_1 + \mu_2 N_h(t))R_h(t), \tag{30d}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta S_v(t) = \theta P^* - \lambda_v(t)S_v(t) - (\mu_3 + \mu_4 N_v(t))S_v(t), \tag{30e}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta E_v(t) = \lambda_v(t)S_v(t) - \gamma_v E_v(t) - (\mu_3 + \mu_4 N_v(t))E_v(t), \tag{30f}$$

$$\chi^{\eta-1} \times_{t_0}^C D_t^\eta I_v(t) = \gamma_v E_v(t) - (\mu_3 + \mu_4 N_v(t))I_v(t), \tag{30g}$$

with initial conditions:

$$\begin{aligned} S_h(t_0) &= S_{h0} > 0, E_h(t_0) = E_{h0} \geq 0, I_h(t_0) = I_{h0} \geq 0, R_h(t_0) = R_{h0} \geq 0, \\ S_v(t_0) &= S_{v0} \geq 0, E_v(t_0) = E_{v0} \geq 0, I_v(t_0) = I_{v0} \geq 0. \end{aligned} \tag{31}$$

The system (30) without disease, ie,  $I_h = I_v = 0$  admits two equilibrium points:  $\mathbf{Q}_1 = (N_h^0, 0, 0, 0, 0, 0, 0)$  which always exists, and  $\mathbf{Q}_2 = (N_h^0, 0, 0, 0, N_v^0, 0, 0)$  which exists if  $N > 1$ .

For the fractional model (30), we define the fractional basic reproductive number

$$R_0 = \sqrt{R_{vh} R_{hv}}, \tag{32}$$

where  $R_{vh} = a^\eta \beta_v^\eta \left( \frac{N_v^0}{N_h^0} \right) \left( \frac{\gamma_h^\eta}{\gamma_h^\eta + \mu_1^\eta + \mu_2^\eta N_h^0} \right) \left( \frac{1}{\mu_1^\eta + \delta_h^\eta + \sigma_h^\eta + \mu_2^\eta N_h^0} \right)$  denotes the number of female mosquitoes that one human infects through his/her infectious period, and

$R_{hv} = a^\eta \beta_h^\eta \left( \frac{\gamma_h^\eta}{\gamma_v^\eta + \mu_3^\eta + \mu_4^\eta N_v^0} \right) \left( \frac{1}{\mu_3^\eta + \mu_4^\eta N_v^0} \right)$  denotes the number of humans infected by a infected female mosquito during the period of time that it can transmit the pathogen until her death.

We thus claim the following result (we omit the proof which is similar to the one from [18]):

**Theorem 5 ([18]).**

1. If  $N \leq 1$  then  $\mathbf{Q}_1$  is globally asymptotically stable on  $\Sigma_1$  and unstable otherwise;
2. If  $N > 1$  and  $R_0 \leq 1$  then  $\mathbf{Q}_2$  is locally asymptotically stable on  $\Sigma_1$  and unstable otherwise;
3. The fractional model (16) admits at least one endemic equilibrium whenever  $R_0 > 1$ , and the backward bifurcation can occur in the case ( $N > 1$  and  $R_0 \leq 1$ ).

Biological implication of Theorem 4 is that the population of mosquitoes can be eradicated if one can apply control measures which able to decrease the offspring number. Although the complete eradication of mosquitoes in tropical areas is still an option for many countries in sub-Saharan Africa, the maintenance of vector control measures will permit to reduce the mosquito population, and thus reduce the likelihood of contact mosquito–Human. Similarly, it follows from Theorem 5 that the control measures aimed at protecting humans against mosquito bites, the prose in charge of the sick with the administration of an effective treatment, and

the preventive measures, will make it possible to decrease the basic reproduction number at its minimal value. Indeed, the epidemic period corresponds to  $R_0 > 1$ . So in most cases, if we manage to get  $R_0 < 1$ , then we can end the epidemic.

*Existence, uniqueness and uniform stability of the solutions of the fractional model*

This subsection provides the fractional model solution's existence and uniqueness (16).

Let  $X(t) = (S_h(t), E_h(t), I_h(t), R_h(t), S_v(t), E_v(t), I_v(t), E(t), L(t), P(t))$  and  $\mathcal{H}(t, X(t)) = (\phi_i)$ ,  $i \in [1, 10] \cap \mathbb{N}$ , where the  $\phi_i$  represent the right hand-sides of system (16), respectively. Then the dynamical system (16) can be written as

$$\begin{cases} D_t^\eta X(t) = \mathcal{H}(t, X(t)), \\ X(0) = X_0 \geq 0, \quad t \in [0, T], \quad 0 < \eta \leq 1. \end{cases} \tag{33}$$

Solution of Eq. (33) is written as:

$$X(t) = X_0 + I_0^\eta (\mathcal{H}(t, X(t))) = X_0 + \frac{1}{\Gamma(\eta)} \int_0^t (t - \tau)^{\eta-1} \mathcal{H}(\tau, X(\tau)) d\tau. \tag{34}$$

Next, through the above integral representation, we shall analyse model (16). For that goal, let  $\varepsilon = C([0, T], \mathbb{R})$  be the Banach space of all continuous functions from  $[0, T]$  to  $\mathbb{R}$  endowed with the norm

$$\|X\|_\varepsilon = \sup_{t \in [0, T]} \{|X(t)|\},$$

where  $|X(t)| = \sum_{i=1}^{10} \|X_i\|$ . Note that for  $i = 1, 2, \dots, 10$ ,  $X_i \in C([0, T], \mathbb{R})$ . Moreover, we define the operator  $A : \varepsilon \rightarrow \varepsilon$  by

$$(AX)(t) = X_0 + \frac{1}{\Gamma(\eta)} \int_0^t (t - \tau)^{\eta-1} \mathcal{H}(\tau, X(\tau)) d\tau \tag{35}$$

Note that operator  $A$  is well defined due to the obvious continuity of  $\mathcal{H}$ .

*Existence and uniqueness of solutions*

Here, we establish the existence and uniqueness of solutions of model (16). We claim the following preliminary result.

**Lemma 7.** Let  $\bar{X} = (\bar{S}_h, \bar{E}_h, \bar{I}_h, \bar{R}_h, \bar{S}_v, \bar{E}_v, \bar{I}_v, \bar{E}, \bar{L}, \bar{P})'$ . The function  $\mathcal{H} = (\phi_i)$  defined above satisfies

$$\|\mathcal{H}(t, X(t)) - \mathcal{H}(t, \bar{X}(t))\| \leq L_{\mathcal{H}} \|X - \bar{X}\|_\varepsilon \tag{36}$$

for some  $L_{\mathcal{H}} > 0$ .

**Proof.** From the first component of  $\mathcal{H}$ , we have

$$\begin{aligned} & \left| \phi_1(t, X(t)) - \phi_1(t, \bar{X}(t)) \right| \\ &= \left| \rho^\eta \left[ S_h(t) + E_h(t) + I_h(t) + R_h(t) - (\bar{S}_h(t) + \bar{E}_h(t) + \bar{I}_h(t) + \bar{R}_h(t)) \right] \right. \\ & \quad \left. - a^\eta \beta_h^\eta \left( \frac{I_v(t)S_h(t)}{S_h(t) + E_h(t) + I_h(t) + R_h(t)} - \frac{\bar{I}_v(t)\bar{S}_h(t)}{\bar{S}_h(t) + \bar{E}_h(t) + \bar{I}_h(t) + \bar{R}_h(t)} \right) \right. \\ & \quad \left. - \mu_1^\eta (S_h - \bar{S}_h) \right. \\ & \quad \left. - \mu_2^\eta [(S_h(t) + E_h(t) + I_h(t) + R_h(t))S_h(t) - (\bar{S}_h(t) + \bar{E}_h(t) + \bar{I}_h(t) + \bar{R}_h(t))\bar{S}_h(t)] \right| \\ & \leq \left| \rho^\eta - \mu_1^\eta \right| |S_h(t) - \bar{S}_h(t)| + \rho^\eta \left( |E_h(t) - \bar{E}_h(t)| + |I_h(t) - \bar{I}_h(t)| + |R_h(t) - \bar{R}_h(t)| \right) \\ & \quad + a^\eta \beta_h^\eta \left| \frac{I_v(t)S_h(t)}{S_h(t) + E_h(t) + I_h(t) + R_h(t)} - \frac{\bar{I}_v(t)\bar{S}_h(t)}{\bar{S}_h(t) + \bar{E}_h(t) + \bar{I}_h(t) + \bar{R}_h(t)} \right| \\ & \quad + \mu_2^\eta |(S_h(t) + E_h(t) + I_h(t) + R_h(t))S_h(t) - (\bar{S}_h(t) + \bar{E}_h(t) + \bar{I}_h(t) + \bar{R}_h(t))\bar{S}_h(t)| \end{aligned} \tag{37}$$

However

$$\begin{aligned} & \left| \frac{I_v(t)S_h(t)}{S_h(t) + E_h(t) + I_h(t) + R_h(t)} - \frac{\bar{I}_v(t)\bar{S}_h(t)}{\bar{S}_h(t) + \bar{E}_h(t) + \bar{I}_h(t) + \bar{R}_h(t)} \right| \\ &= \left| \frac{I_v(t)S_h(t)(\bar{S}_h(t) + \bar{E}_h(t) + \bar{I}_h(t) + \bar{R}_h(t)) - \bar{I}_v(t)\bar{S}_h(t)(S_h(t) + E_h(t) + I_h(t) + R_h(t))}{(S_h(t) + E_h(t) + I_h(t) + R_h(t))(\bar{S}_h(t) + \bar{E}_h(t) + \bar{I}_h(t) + \bar{R}_h(t))} \right| \\ & \leq f_1(t) \left( |S_h(t) - \bar{S}_h(t)| + |E_h(t) - \bar{E}_h(t)| + |I_h(t) - \bar{I}_h(t)| + |R_h(t) - \bar{R}_h(t)| \right) \\ & \quad + f_2(t) |S_h(t) - \bar{S}_h(t)| + f_3(t) |I_h(t) - \bar{I}_h(t)|, \end{aligned} \tag{38}$$

where  $f_1(t) = |I_v(t)S_h(t)|$ ,  $f_2(t) = |I_h(t)(\bar{S}_h(t) + \bar{E}_h(t) + \bar{I}_h(t) + \bar{R}_h(t))|$  and

$$f_3(t) = \left| \overline{S}_h(t)(\overline{S}_h(t) + \overline{E}_h(t) + \overline{I}_h(t) + \overline{R}_h(t)) \right|.$$

Similarly, we obtain

$$\begin{aligned} & |(S_h(t) + E_h(t) + I_h(t) + R_h(t))S_h(t) - (\overline{S}_h(t) + \overline{E}_h(t) + \overline{I}_h(t) + \overline{R}_h(t))\overline{S}_h(t)| \\ &= |S_h(t) \left[ (S_h(t) - \overline{S}_h(t)) + (E_h(t) - \overline{E}_h(t)) + (I_h(t) - \overline{I}_h(t)) + (R_h(t) - \overline{R}_h(t)) \right]| \\ &+ |(\overline{S}_h(t) + \overline{E}_h(t) + \overline{I}_h(t) + \overline{R}_h(t))(S_h(t) - \overline{S}_h(t))| \\ &\leq g_1(t) \left( |S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| + |R_h(t) - \overline{R}_h(t)| \right) \\ &+ g_2(t) |S_h(t) - \overline{S}_h(t)|, \end{aligned} \tag{39}$$

where  $g_1(t) = |S_h(t)|$  and  $g_2(t) = |\overline{S}_h(t) + \overline{E}_h(t) + \overline{I}_h(t) + \overline{R}_h(t)|$ .

Plugging (38) and (39) in (37) finally gives

$$\begin{aligned} & \left| \phi_1(t, X(t)) - \phi_1(t, \overline{X}(t)) \right| \\ & \leq \left[ \rho^n - \mu_1^n + a^n \beta_h^n (f_1(t) + f_2(t)) + \mu_2^n (g_1(t) + g_2(t)) \right] |S_h(t) - \overline{S}_h(t)| \\ & \quad + (\rho^n + a^n \beta_h^n f_1(t) + \mu_2^n g_1(t)) |E_h(t) - \overline{E}_h(t)| \\ & \quad + \left[ \rho^n + a^n \beta_h^n (f_1(t) + f_3(t)) + \mu_2^n g_1(t) \right] |I_h(t) - \overline{I}_h(t)| \\ & \quad + \left[ \rho^n + a^n \beta_h^n f_1(t) + \mu_2^n g_1(t) \right] |R_h(t) - \overline{R}_h(t)| \\ & \leq \Delta \left( |S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| + |R_h(t) - \overline{R}_h(t)| \right) \\ & \quad + \mu_1^n |S_h(t) - \overline{S}_h(t)| \\ & \leq \Delta \left[ |S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| + |R_h(t) - \overline{R}_h(t)| \right] \\ & \quad + \mu_1^n \left( |S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| + |R_h(t) - \overline{R}_h(t)| \right) \\ & \leq L_1 \left[ |S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| + |R_h(t) - \overline{R}_h(t)| \right] \end{aligned} \tag{40}$$

where  $L_1 = \Delta + \mu_1^n$  with  $\Delta = 4\rho^n + \max_{t \in [0, T]} \{4a^n \beta_h^n f_1(t) + f_2(t) + \mu_2^n g_1(t) + f_3(t) + 3\mu_2^n g_1(t) g_2(t)\}$ .

In a similar manner, we obtain

$$\begin{aligned} \left| \phi_2(t, X(t)) - \phi_2(t, \overline{X}(t)) \right| & \leq L_2 \left[ |S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| \right. \\ & \quad \left. + |R_h(t) - \overline{R}_h(t)| \right], \end{aligned} \tag{41}$$

where  $L_2 = \gamma^n + \mu_1^n + \max_{t \in [0, T]} \{4a^n \beta_h^n f_1(t) + f_2(t) + f_3(t) + g_2(t) + 4g_4(t)\}$  with  $g_4(t) = |E_h(t)|$ .

For the remaining components of  $\mathcal{H}$ , it holds that

$$\begin{aligned} \left| \phi_3(t, X(t)) - \phi_3(t, \overline{X}(t)) \right| & \leq L_3 \left[ |S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| \right. \\ & \quad \left. + |R_h(t) - \overline{R}_h(t)| \right], \\ \left| \phi_4(t, X(t)) - \phi_4(t, \overline{X}(t)) \right| & \leq L_4 \left[ |S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| \right. \\ & \quad \left. + |R_h(t) - \overline{R}_h(t)| \right], \\ \left| \phi_5(t, X(t)) - \phi_5(t, \overline{X}(t)) \right| & \leq L_5 (|S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| \\ & \quad + |R_h(t) - \overline{R}_h(t)| + |S_v(t) - \overline{S}_v(t)| + |E_v(t) - \overline{E}_v(t)| \\ & \quad + |I_v(t) - \overline{I}_v(t)| + |P(t) - \overline{P}(t)|), \\ \left| \phi_6(t, X(t)) - \phi_6(t, \overline{X}(t)) \right| & \leq L_6 (|S_h(t) - \overline{S}_h(t)| + |E_h(t) - \overline{E}_h(t)| + |I_h(t) - \overline{I}_h(t)| \\ & \quad + |R_h(t) - \overline{R}_h(t)| + |S_v(t) - \overline{S}_v(t)| + |E_v(t) - \overline{E}_v(t)| \\ & \quad + |I_v(t) - \overline{I}_v(t)|), \\ \left| \phi_7(t, X(t)) - \phi_7(t, \overline{X}(t)) \right| & \leq L_7 (|S_v(t) - \overline{S}_v(t)| + |E_v(t) - \overline{E}_v(t)| + |I_v(t) - \overline{I}_v(t)|), \\ \left| \phi_8(t, X(t)) - \phi_8(t, \overline{X}(t)) \right| & \leq L_8 (|S_v(t) - \overline{S}_v(t)| + |E_v(t) - \overline{E}_v(t)| + |I_v(t) - \overline{I}_v(t)| \\ & \quad + |E(t) + \overline{E}(t)|), \\ \left| \phi_9(t, X(t)) - \phi_9(t, \overline{X}(t)) \right| & \leq L_9 (|E(t) + \overline{E}(t)| + |L(t) + \overline{L}(t)|), \\ \left| \phi_{10}(t, X(t)) - \phi_{10}(t, \overline{X}(t)) \right| & \leq L_{10} (|L(t) + \overline{L}(t)| + |P(t) + \overline{P}(t)|), \end{aligned} \tag{42}$$

where  $f_4(t) = |I_h(t)|$ ,  $f_5(t) = |S_v(t)I_h(t)|$ ,  $f_6(t) = |S_v(t) \| S_h(t) + E_h(t) + I_h(t) + R_h(t)|$ ,  $f_7(t) = |\overline{I}_h(t)| \| S_h(t) + E_h(t) + I_h(t) + R_h(t)|$ ,  $g_3(t) = |R_h(t)|$ ,  $g_4(t) = |S_v(t)|$ ,  $g_5(t) = |\overline{S}_v(t) + \overline{E}_v(t) + \overline{I}_v(t)|$ ,  $g_6(t) = |E_v(t)|$ ,  $g_7(t) = |I_v(t)|$ ,  $g_8(t) = |E(t)|$ ,  $g_9(t) = |L(t)|$ ,  $g_{10}(t) = |\overline{E}(t)|$ ,

$g_{11}(t) = |L(t) + \bar{L}(t)|$ ,  $g_{12} = |\bar{P}(t)|$ , and

$$\begin{aligned}
 L_3 &= \gamma_h^n + \delta_h^n + \sigma_h^n + \mu_1^n + \max_{t \in [0, T]} \{4\mu_2^n f_4(t) + \mu_2^n g_2(t)\}, \\
 L_4 &= \sigma_h^n + \mu_1^n + \max_{t \in [0, T]} \{4\mu_2^n g_3(t) + \mu_2^n g_2(t)\}, \\
 L_5 &= \mu_3^n + \theta^n + a^n \beta_h^n + \max_{t \in [0, T]} \{a^n \beta_v^n (3f_5(t) + f_6(t) + f_7(t)) + 3\mu_4^n g_4(t) + \mu_4^n g_5(t)\}, \\
 L_6 &= \gamma_v^n + \mu_3^n + \max_{t \in [0, T]} \{a^n \beta_v^n (4f_5(t) + f_6(t) + f_7(t)) + \mu_4^n (g_5(t) + 3g_6(t))\}, \\
 L_7 &= \gamma_v^n + \mu_3^n + \max_{t \in [0, T]} \{3\mu_4^n g_7(t) + \mu_4^n g_5(t)\}, \\
 L_8 &= 3\xi^n + \mu_E^n + s^n + \max_{t \in [0, T]} \left\{ \frac{s^n}{K_E} g_5(t) + 3 \frac{\xi^n}{K_E} g_8(t) \right\}, \\
 L_9 &= s^n + \mu_L^n + l^n + \max_{t \in [0, T]} \left\{ \frac{s^n}{K_E} (g_9(t) + g_{10}(t)) + \alpha^n g_{11}(t) \right\}, \\
 L_{10} &= l^n + \mu_P^n + \theta^n + \max_{t \in [0, T]} \left\{ \left( \beta^n + \frac{l^n}{K_P} \right) (g_9(t) + g_{12}(t)) \right\}.
 \end{aligned}$$

Consequently

$$\|\mathcal{H}(t, X(t)) - \mathcal{H}(t, \bar{X}(t))\|_\epsilon = \sup_{t \in [0, T]} \sum_{i=1}^{10} |\phi(t, X(t)) - \phi(t, \bar{X}(t))| \leq \overbrace{\left( \sum_{i=1}^{10} L_i \right)}^{L_H} \|X(t) - \bar{X}(t)\|_\epsilon. \quad \square \tag{43}$$

Let  $a_k(t)$ ,  $t \in I = [0, T]$ ,  $k = 0, 1, 2, \dots$ , are given functions. We consider here the initial value problem of the nonlinear fractional-order differential equation

$$D^\alpha x(t) = \sum_{k=0}^n a_k(t) f_k(x(t)) \tag{44}$$

with the initial data

$$x(0) = x_0 \tag{45}$$

The initial value problem (44)–(45) is a general case of the initial value problem

$$D^\alpha x(t) = \sum_{k=0}^n a_k(t) x^k(t), t > 0 \text{ and } x(0) = x_0 \tag{46}$$

**Theorem 6.** Assume that the result of Lemma 7 holds and  $\Omega = \frac{T^\eta}{\Gamma(\eta + 1)}$ . If  $\Omega L_H < 1$ , then there exists a unique solution of (16) on  $[0, T]$ .

**Proof.** It is clear that the function  $\mathcal{H} : [0, T] \times \mathbb{R}^{10} \rightarrow \mathbb{R}_+^{10}$  is continuous on its domain. So, (16) is the existence of solution from [47, Theorem 3.1]. We will use the Banach contraction mapping principle on the operator  $A$  defined above to show the uniqueness. For this reason, we want to show that  $A$  is both a self-map and a contraction. Firstly, by definition  $\sup_{t \in [0, T]} \|\mathcal{H}(t, 0)\| = A_h^n$ .

We define  $\mathcal{K} > \frac{\|X_0 + \Omega A_h^n\|}{1 - \Omega L_H}$  and a closed convex set  $B_{\mathcal{K}} = \{X \in \mathcal{E}; \|X\|_\epsilon \leq \mathcal{K}\}$ .

Therefore for self map property, we can just show that  $AB_{\mathcal{K}} \subseteq B_{\mathcal{K}}$ . Thus let  $X \in B_{\mathcal{K}}$ , then

$$\begin{aligned}
 \|AX\|_\epsilon &= \sup_{t \in [0, T]} \left\{ |X_0 + \frac{1}{\Gamma(\eta)} \int_0^t (t - \tau)^{\eta-1} \mathcal{H}(\tau, X(\tau)) d\tau| \right\} \\
 &\leq |X_0| + \frac{1}{\Gamma(\eta)} \sup_{t \in [0, T]} \left\{ \int_0^t (t - \tau)^{\eta-1} (|\mathcal{H}(\tau, X(\tau)) - \mathcal{H}(\tau, 0)| + |\mathcal{H}(\tau, 0)|) d\tau \right\} \\
 &\leq |X_0| + \frac{1}{\Gamma(\eta)} \sup_{t \in [0, T]} \left\{ \int_0^t (t - \tau)^{\eta-1} (\|\mathcal{H}(\tau, X(\tau)) - \mathcal{H}(\tau, 0)\|_\epsilon + \|\mathcal{H}(\tau, 0)\|_\epsilon) d\tau \right\} \\
 &\leq |X_0| + \frac{L_H \|X\|_\epsilon + A_h^n}{\Gamma(\eta)} \sup_{t \in [0, T]} \left\{ \int_0^t (t - \tau)^{\eta-1} d\tau \right\} \\
 &\leq |X_0| + \frac{L_H \mathcal{K} + A_h^n}{\Gamma(\eta)} \sup_{t \in [0, T]} \left\{ \int_0^t (t - \tau)^{\eta-1} d\tau \right\} \\
 &\leq |X_0| + \frac{L_H \mathcal{K} + A_h^n}{\Gamma(\eta + 1)} T^\eta = |X_0| + \Omega(L_H \mathcal{K} + A_h^n) \\
 &\leq \mathcal{K}.
 \end{aligned} \tag{47}$$

Then it follows that  $AX \subseteq B_{\mathcal{K}}$  and  $A$  is indeed a self-map. It stays at this point to prove that  $A$  is a contraction. Let  $X$  and  $\tilde{X} \in \varepsilon$ . Then again using the result of Lemma 7, we have

$$\begin{aligned} \|AX - A\tilde{X}\|_{\varepsilon} &= \sup_{t \in [0, T]} \{|(AX)(t) - (A\tilde{X})(t)|\} \\ &= \frac{1}{\Gamma(\eta)} \sup_{t \in [0, T]} \left\{ \int_0^t (t - \tau)^{\eta-1} |\mathcal{H}(\tau, X(\tau)) - \mathcal{H}(\tau, \tilde{X}(\tau))| d\tau \right\} \\ &\leq \frac{L_{\mathcal{H}}}{\Gamma(\eta)} \sup_{t \in [0, T]} \left\{ \int_0^t (t - \tau)^{\eta-1} |X(\tau) - \tilde{X}(\tau)| d\tau \right\} \\ &\leq \Omega L_{\mathcal{H}} \|X - \tilde{X}\|_{\varepsilon}. \end{aligned} \tag{48}$$

Thus, if  $\Omega L_{\mathcal{H}} < 1$  then  $A$  is a contraction mapping and consequently by the Banach contraction mapping,  $A$  has a unique fixed point on  $[0, T]$  which is solution of (16).  $\square$

**Global stability**

Before proving the uniform Lyapunov stability, let us recall the following result.

**Definition 6** ([48]). The solution of problem (44)–(45) is stable if,  $\forall \varepsilon > 0$  and  $t_0 > 0$  there exists  $\delta(\varepsilon, t_0) > 0$  such that for  $t \geq t_0$

$$\|x_0 - x_0^*\| < \delta(\varepsilon, t_0) \Rightarrow \|x(t) - x^*(t)\| < \varepsilon. \tag{49}$$

If  $\delta$  depends only on  $\varepsilon$ , then the solution is uniformly stable where  $x^*(t)$  is the solution of the initial value problem  $D^{\alpha}x(t) = \sum_{k=0}^n a_k(t)f_k(x(t))$  and  $x(0) = x_0^*$ .

Then, we claim what follows.

**Theorem 7.** Let the results of Lemma 7 holds and  $1 - L_{\mathcal{H}}\Omega > 0$ . Then, the fractional order model (16) is uniformly Lyapunov stable.

**Proof.** To prove the uniform Lyapunov stability, let  $X(t)$  and  $X^*(t)$  are the solutions of the problem (16). Then we have:

$$\begin{aligned} \|X(t) - X^*(t)\| &= \|X_0 + \frac{1}{\Gamma(\eta)} \int_0^t (t - \tau)^{\eta-1} \mathcal{H}(\tau, X(\tau)) d\tau - X_0^* - \frac{1}{\Gamma(\eta)} \int_0^t (t - \tau)^{\eta-1} \mathcal{H}(\tau, X^*(\tau)) d\tau\| \\ &= \|X_0 - X_0^* + \frac{1}{\Gamma(\eta)} \int_0^t (t - \tau)^{\eta-1} \{\mathcal{H}(\tau, X(\tau)) - \mathcal{H}(\tau, X^*(\tau))\} d\tau\| \\ &\leq \|X_0 - X_0^*\| + \frac{1}{\Gamma(\eta)} \int_0^t (t - \tau)^{\eta-1} \|\mathcal{H}(\tau, X(\tau)) - \mathcal{H}(\tau, X^*(\tau))\| d\tau \\ &\leq \|X_0 - X_0^*\| + \frac{L_{\mathcal{H}}\|X - X^*\|}{\Gamma(\eta)} \sup_{t \in [0, T]} \left\{ \int_0^t (t - \tau)^{\eta-1} d\tau \right\} \\ &\leq \|X_0 - X_0^*\| + L_{\mathcal{H}} \overbrace{\left( \frac{T^{\eta}}{\Gamma(\eta + 1)} \right)}^{\Omega} \|X(t) - X^*(t)\|. \end{aligned}$$

We thus obtain

$$\|X(t) - X^*(t)\| \leq \|X_0 - X_0^*\| + L_{\mathcal{H}}\Omega \|X(t) - X^*(t)\| \Rightarrow \|X(t) - X^*(t)\| \leq \frac{1}{(1 - L_{\mathcal{H}}\Omega)} \|X_0 - X_0^*\|.$$

From Definition 6, we deduce that the solution of (16) is uniformly stable.  $\square$

**Numerical scheme and simulation results**

To solve fractional differential equations (FDEs), numerous analytical and numerical methods have been used, the majority of which do not have exact analytic solutions. For numerical solution of the system (16), we will apply the *Generalized Fractional Euler Method* (GFEM) [49].

**Numerical scheme**

Here, we construct a numerical scheme to simulate our fractional-order Chikungunya model (16). We use the new generalized fractional Euler method described in [42]. For the reader’s convenience, we resume the description of this method before adapting it to our model.

Let us consider the following new generalized Caputo-type initial value problem :

$$\begin{cases} {}^C D_t^{\varphi, \eta} X(t) = \mathcal{H}(t, X(t)), \\ X(0) = X_0. \end{cases} \quad 0 < \varphi \leq 1, b < t \leq \mathcal{T} \tag{50}$$

Note that if  $\varphi = 1$ , then the above system represents an initial value problem with the classical Caputo-type derivative.

Let us consider the time interval  $[b, \mathcal{T}]$  which is discretized on a set of points as follows:

$$\begin{cases} t_0 = b \\ t_{i+1} = (t_i^\eta + \aleph)^{1/\eta}, \quad i = 0, 1, \dots, N - 1 \end{cases} \tag{51}$$

where  $\aleph = \frac{\mathcal{T}^\eta - t_0^\eta}{N}$ . This formulation does that the new proposed grid is non-uniform.

The following sequence  $X_i, i = 0, 1, \dots, N$ , such that  $X_k \simeq X(t_k)$  for  $k = 1, 2, \dots, i$  is used to approximate the solution of the initial value problem (50).

The exact solution of (50) can be written in terms of the Volterra integral equation as

$$X(t) = X(t_0) + \frac{\eta^{1-\varphi}}{\Gamma(\varphi)} \int_{t_0}^t \tau^{\eta-1} (t^\eta - \tau^\eta)^{\varphi-1} \mathcal{H}(\mu, X) d\tau. \tag{52}$$

As consequences, we can write

$$X(t_{i+1}) = X(t_0) + \frac{\eta^{1-\varphi}}{\Gamma(\varphi)} \sum_{k=0}^i \int_{t_k^\eta}^{t_{k+1}^\eta} \tau^{\eta-1} (t_{i+1}^\eta - \tau^\eta)^{\varphi-1} \mathcal{H}(\mu, X(s)) d\tau. \tag{53}$$

Substituting  $z = \tau^\eta$ , we get

$$X(t_{i+1}) = X(t_0) + \frac{\eta^{1-\varphi}}{\Gamma(\varphi)} \sum_{k=0}^i \int_{t_k^\eta}^{t_{k+1}^\eta} (t_{i+1}^\eta - z)^{\varphi-1} \mathcal{H}(z^{1/\eta}, X(z^{1/\eta})) dz. \tag{54}$$

Using the composite rectangle rule in each of the sub-intervals, and approximating, we obtain the following formulation, which represents the numerical algorithm to obtain the solution of the IVP (50).

$$\begin{aligned} \frac{\eta^{-\varphi}}{\Gamma(\varphi)} \sum_{k=0}^i \int_{t_k^\eta}^{t_{k+1}^\eta} (t_{i+1}^\eta - z)^{\varphi-1} \mathcal{H}(t_k, X(t_k)) dz &= \frac{\eta^{-\varphi}}{\Gamma(\varphi)} \sum_{k=0}^i \mathcal{H}(t_k, X(t_k)) \int_{t_k^\eta}^{t_{k+1}^\eta} (t_{i+1}^\eta - z)^{\varphi-1} dz, \\ &= \frac{\eta^{-\varphi}}{\Gamma(\varphi)} \sum_{k=0}^i \mathcal{H}(t_k, X(t_k)) \left[ -\frac{1}{\varphi} (t_{i+1}^\eta - z)^\varphi \Big|_{t_k^\eta}^{t_{k+1}^\eta} \right], \\ &= \frac{\eta^{-\varphi}}{\Gamma(\varphi + 1)} \sum_{k=0}^i \mathcal{H}(t_k, X(t_k)) \left( (t_{i+1}^\eta - t_k^\eta)^\varphi - (t_{i+1}^\eta - t_{k+1}^\eta)^\varphi \right). \end{aligned} \tag{55}$$

From (55), we finally obtain the following explicit formula

$$X_{k+1} = X_0 + \frac{\eta^{-\varphi} \aleph^\varphi}{\Gamma(\varphi + 1)} \sum_{i=0}^k a_{i,k+1} \mathcal{H}(t_i, X_i), \tag{56}$$

where  $a_{i,k+1} = (k + 1 - i)^\varphi - (k - i)^\varphi$ .

**Theorem 8.** Assume that the results of Lemma 7 hold, which means that  $\mathcal{H}(t, X)$  satisfies Lipschitz property. Let  $X_j (j = 1, 2, \dots, r + 1)$  be the solutions of the algorithm (55). Then, numerical scheme (55) is conditionally stable.

**Proof.** Let  $\tilde{X}_0$  and  $\tilde{X}_j (j = 0, \dots, k + 1)$  be perturbations of  $X_0$  and  $X_j$  respectively. By simulating

$$X_{k+1} = X_0 + \frac{\eta^{-\varphi} \aleph^\varphi}{\Gamma(\varphi + 1)} \sum_{i=0}^k a_{i,k+1} \mathcal{H}(t_i, X_i), \tag{57}$$

we obtain

$$X_{k+1} + \tilde{X}_{k+1} = X_0 + \tilde{X}_0 + \frac{\eta^{-\varphi} \aleph^\varphi}{\Gamma(\varphi + 1)} \sum_{k=0}^i a_{k,i+1} \mathcal{H}(t_k, X_k + \tilde{X}_k), \tag{58}$$

where  $a_{k,i+1} = (i + 1 - k)^\varphi - (i - k)^\varphi$ .

Combining (57) and (58), we have

$$|\tilde{X}_{k+1}| = \left| \tilde{X}_0 + \frac{\eta^{-\varphi} \aleph^\varphi}{\Gamma(\varphi + 1)} \sum_{k=0}^i a_{k,i+1} (\mathcal{H}(t_k, X_k + \tilde{X}_k) - \mathcal{H}(t_k, X_k)) \right| \tag{59}$$

Using the triangle inequality and Lipschitz property, we get

$$|\tilde{X}_{k+1}| \leq \varsigma_0 + \frac{\eta^{-\varphi} \aleph^\varphi m_1 c_\varphi}{\Gamma(\varphi + 2)} \left( \sum_{k=1}^i a_{k,i+1} |\tilde{X}_k| \right) \tag{60}$$

where  $\varsigma_0 = \max_{0 \leq k \leq N} \left\{ |\tilde{X}_0| + \frac{\eta^{-\varphi} \aleph^\varphi m_1 a_{k,0}}{\Gamma(\varphi + 2)} |\tilde{X}_k| \right\}$ .

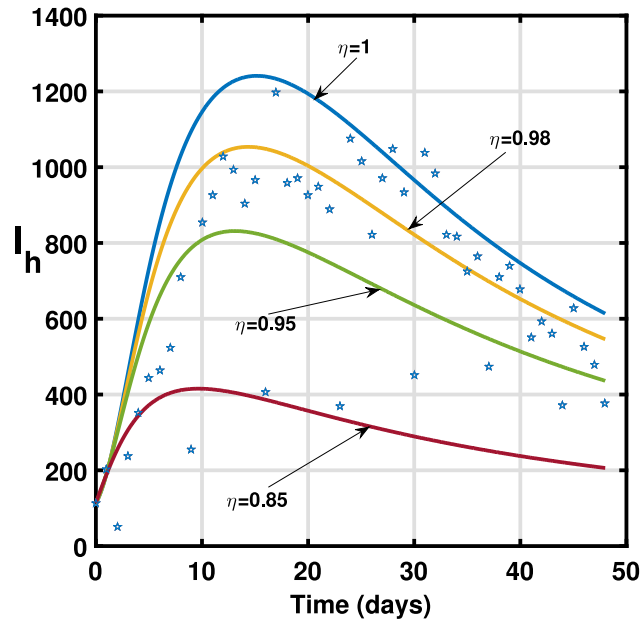


Fig. 2. Daily cases reported cases (in blue stars) of Chikungunya in Chad compared to the model simulations for different values of the fractional-order parameter  $\eta$ .

In (60),  $c_\varphi$  is a positive constant that only depends on  $\varphi$  (Lemma 4) and  $\aleph$  is supposed to be small enough. Lemma 5 gives  $|\tilde{X}_{k+1}| \leq c\zeta_0$ , where  $c$  is an independent positive constant independent of  $k$  and  $\aleph$ . This ends the proof.  $\square$

**Theorem 9.** For the numerical method (56), we have

$$|X(t_{k+1}) - X_{k+1}| \leq C\aleph^\varphi, k = 0, \dots, N - 1 \tag{61}$$

where  $C$  is a positive constant which does not depend on  $\aleph$  and  $k$ .

**Proof.** For the projected numerical scheme (56), we express the following relation by assuming  $X(t_0) = X_0$  and let  $e_k = X(t_k) - X_k$ . Using Volterra Eq. (8) of new generalized Caputo sense and employing Eq. (50), we get the following error equation

$$X(t_{k+1}) - X_{k+1} = \frac{\eta^{1-\varphi}}{\Gamma(\varphi)} \int_0^t \chi^{\eta-1} (t^\eta - \chi^\eta)^{\varphi-1} \mathcal{H}(\chi, X) d\chi - \frac{\eta^{-\varphi}\aleph^\varphi}{\Gamma(\varphi+1)} \sum_{i=0}^k a_{i,k+1} \mathcal{H}(t_i, X_i). \tag{62}$$

Therefore, using the Lipschitz condition as well as Lemmas 2 and 4, we finally obtain

$$\begin{aligned} |X(t_{k+1}) - X_{k+1}| &= \left| \frac{\eta^{1-\varphi}}{\Gamma(\varphi)} \int_0^t \chi^{\eta-1} (t^\eta - \chi^\eta)^{\varphi-1} \mathcal{H}(\chi, X) d\chi - \frac{\eta^{-\varphi}\aleph^\varphi}{\Gamma(\varphi+1)} \sum_{i=0}^k a_{i,k+1} \mathcal{H}(t_i, X_i) \right| \\ &\leq \left| \frac{\eta^{1-\varphi}}{\Gamma(\varphi)} \int_0^t \chi^{\eta-1} (t^\eta - \chi^\eta)^{\varphi-1} \mathcal{H}(\chi, X) d\chi - \frac{\eta^{-\varphi}\aleph^\varphi}{\Gamma(\varphi+1)} \sum_{i=0}^k a_{i,k+1} \mathcal{H}(t_i, X_i) \right| \\ &\quad + \frac{\eta^{-\varphi}\aleph^\varphi}{\Gamma(\varphi+1)} \sum_{i=0}^k a_{i,k+1} |\mathcal{H}(t_i, X(t_i)) - \mathcal{H}(t_i, X_i)| \\ &\leq \frac{c_2}{\Gamma(\varphi)} \aleph^{\sigma(\varphi)} + \frac{Lc_{\varphi,1}\eta^{-\varphi}\aleph^\varphi}{\Gamma(\varphi+1)} \sum_{i=1}^k (k+1-i)^{\varphi-1} |e_i| \end{aligned} \tag{63}$$

Inequality (61) is obtained by applying Lemma 5. This ends the proof.  $\square$

According to the above described method for some  $\mathcal{T}$ , the approximate solutions of

$X = (S_h, E_h, I_h, R_h, S_v, E_v, I_v, E, L, P)$  denoted by

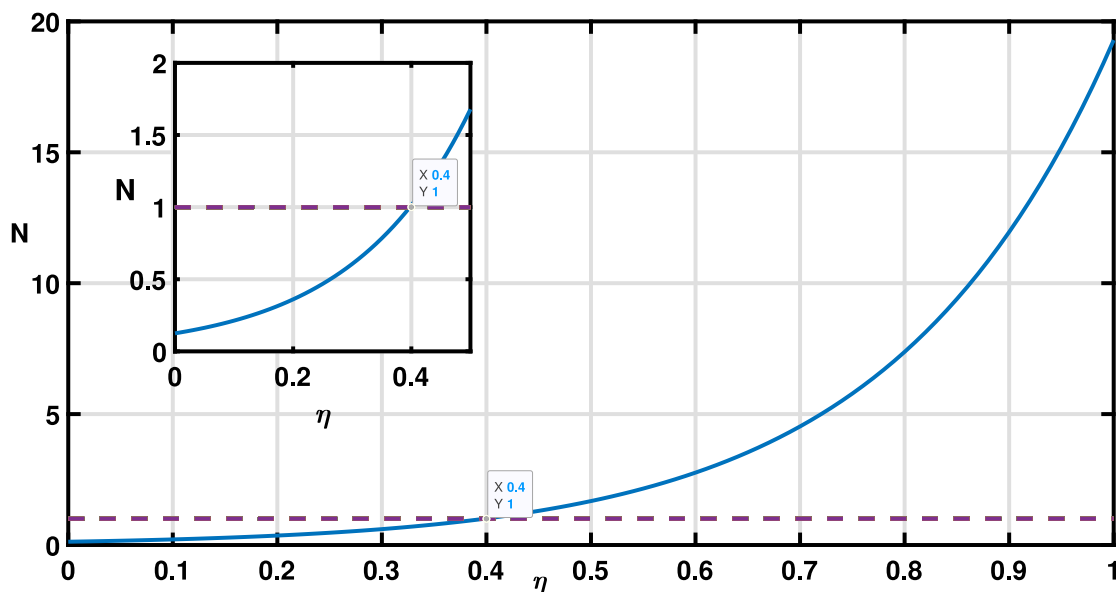


Fig. 3. Plot of  $N$  in terms of the fractional-order  $\eta$ . The following parameter values are used:  $\mu_3 = 0.033$ ,  $\mu_4 = 0.00002$ ,  $\xi = 12$ ,  $\theta = 0.08$ ,  $s = 0.7$ ,  $l = 0.4$ ,  $\mu_E = 0.4$ ;  $\mu_L = 0.4$ ;  $\mu_P = 0.4$ ,  $\alpha = 2 \times 10^{-4} = \beta = 2 \times 10^{-4}$ .

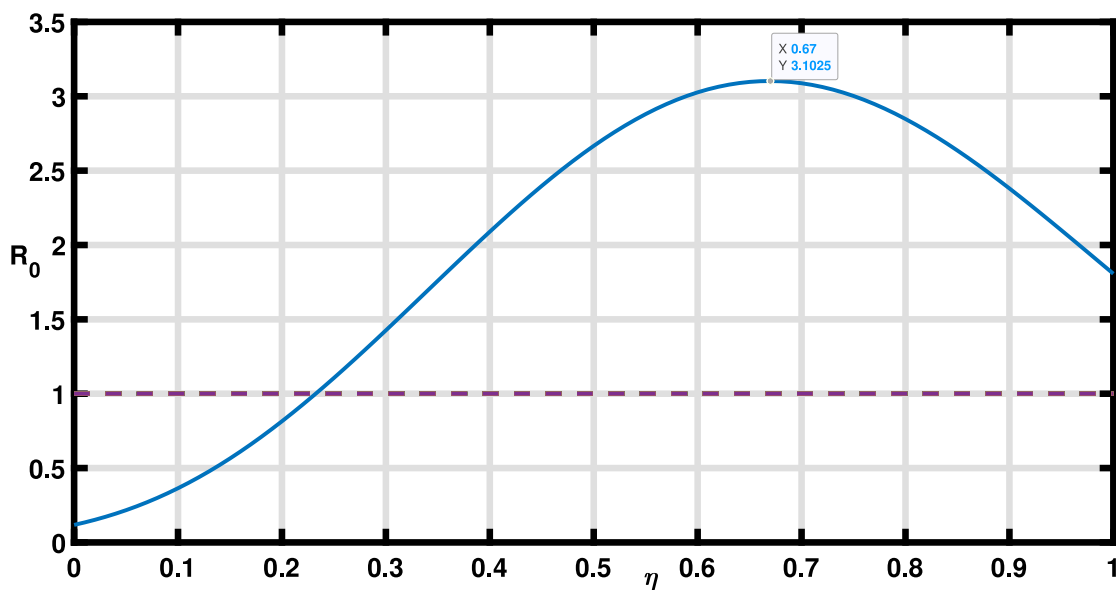


Fig. 4. Plot of  $R_0$  in terms of the fractional-order  $\eta$ . The following parameter values are used:  $\mu_3 = 0.033$ ,  $\mu_4 = 0.00002$ ,  $\xi = 12$ ,  $\theta = 0.08$ ,  $s = 0.7$ ,  $l = 0.4$ ,  $\mu_E = 0.4$ ;  $\mu_L = 0.4$ ;  $\mu_P = 0.4$ ,  $\alpha = 2 \times 10^{-4} = \beta = 2 \times 10^{-4}$ .



$$\begin{aligned}
 X_{k+1} &= (S_{h(k+1)}, E_{h(k+1)}, I_{h(k+1)}, R_{h(k+1)}, S_{v(k+1)}, E_{v(k+1)}, I_{v(k+1)}, E_{k+1}, L_{k+1}, P_{k+1}), \text{ for } k \in \mathbb{N} \text{ and } \mathcal{T} > 0 \text{ is given by} \\
 S_{h(k+1)} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} \left( A_h^\eta + \rho^\eta N_{h(j)} - \frac{a^\eta \beta_h^\eta I_{v(j)} S_{h(j)}}{N_{h(j)}} - (\mu_1^\eta + \mu_2^\eta N_{h(j)}) S_{h(j)} \right), \\
 E_{h(k+1)} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} \left( \frac{a^\eta \beta_h^\eta I_{v(j)} S_{h(j)}}{N_{h(j)}} - \gamma_h^\eta E_{h(j)} - (\mu_1^\eta + \mu_2^\eta N_{h(j)}) E_{h(j)} \right), \\
 I_{h(k+1)} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} (\gamma_h^\eta E_{h(j)} - \sigma_h^\eta I_{h(j)} - \delta_h^\eta I_{h(j)} - (\mu_1^\eta + \mu_2^\eta N_{h(j)}) I_{h(j)}), \\
 R_{h(k+1)} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} (\sigma_h^\eta I_{h(j)} - (\mu_1^\eta + \mu_2^\eta N_{h(j)}) R_{h(j)}), \\
 S_{v(k+1)} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} \left( \theta^\eta P_j - \frac{a^\eta \beta_v^\eta I_{h(j)} S_{v(j)}}{N_{h(j)}} - (\mu_3^\eta + \mu_4^\eta N_{v(j)}) S_{v(j)} \right), \\
 E_{v(k+1)} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} \left( \frac{a^\eta \beta_v^\eta I_{h(j)} S_{v(j)}}{N_{h(j)}} - \gamma_v^\eta E_{v(j)} - (\mu_3^\eta + \mu_4^\eta N_{v(j)}) E_{v(j)} \right), \\
 I_{v(k+1)} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} (\gamma_v^\eta E_{v(j)} - (\mu_3^\eta + \mu_4^\eta N_{v(j)}) I_{v(j)}), \\
 E_{k+1} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} \left( \xi \left( 1 - \frac{E_j}{K_E} \right) N_{v(j)} - (\mu_E^\eta + s^\eta) E_j \right), \\
 L_{k+1} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} \left( s^\eta E_j \left( 1 - \frac{L_j}{K_L} \right) - (\mu_L^\eta + l^\eta + \alpha^\eta L_j) L_j \right), \\
 P_{k+1} &\approx \frac{\eta^{-\varphi} \mathfrak{N}^\varphi}{\Gamma(\varphi + 1)} \sum_{j=0}^k \pi_{j,k+1} \left( l^\eta L_j \left( 1 - \frac{P_j}{K_P} \right) - (\mu_P^\eta + \theta^\eta + \beta^\eta L_j) P_j \right).
 \end{aligned} \tag{64}$$

where  $\pi_{j,k+1} = (k + 1 - j)^\varphi - (k - j)^\varphi$ ,  $N_{h(j)} = S_{h(j)} + E_{h(j)} + I_{h(j)} + R_{h(j)}$ , and  $N_{v(j)} = S_{v(j)} + E_{v(j)} + I_{v(j)}$ .

**Numerical results**

In this section, we perform several numerical simulations to firstly validate our analytical results and secondly to compare the dynamics of both models. The following parameter values are used:  $\mu_3 = 0.033$ ,  $\mu_4 = 0.00034$ ,  $\xi = 12$ ,  $\theta = 0.08$ ,  $s = 0.7$ ,  $l = 0.4$ ,  $\mu_E = 0.4$ ;  $\mu_L = 0.4$ ;  $\mu_P = 0.4$ ,  $\alpha = \beta = 2 \times 10^{-4}$ ,  $\Lambda_h = 1$ ,  $\rho = 2.7 \times 10^{-4}$ ,  $\mu_1 = 1.00038 \times 10^{-10}$ ,  $\mu_2 = 10^{-8}$ ,  $\beta_h = 0.9999$ ;  $a = 2.4676$ ;  $\gamma_h = 0.3333$ ,  $\sigma_h = 0.1499$ ,  $\delta_h = 9.999 \times 10^{-4}$ ,  $\beta_v = 0.600$ ,  $\gamma_v = 0.4999$  [18].

In Fig. 2, we depict reported cases of the 2019 Chikungunya epidemic in Chad versus model predictions for different values of the fractional-order  $\eta$ . We see that for  $\eta \in (0.85; 1)$ , daily detected cases are closer to those the model predict.

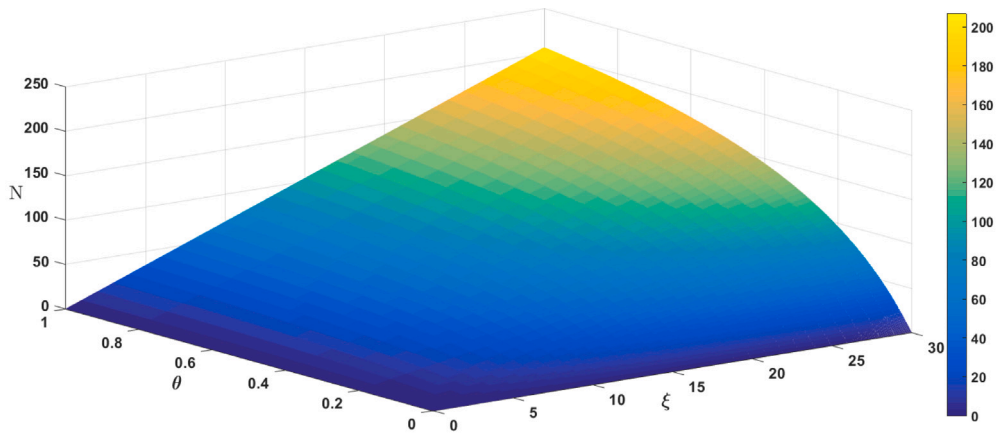
We begin by representing the two thresholds  $N$  and  $R_0$  which govern the dynamics of both models as function of the fractional-order parameter  $\eta$ . The results are depicted on Figs. 3 & 4. We clearly see that the offspring reproduction number  $N$  is a increasing function of  $\eta$  (Fig. 3). However, the basic reproduction number  $R_0$  increases for the fractional-order parameter  $\eta \in (0; 0.67)$ , and decrease for  $\eta \in [0.67; 1]$  (Fig. 4).

We then plot the two important thresholds  $N$  and  $R_0$  in term of some model parameters. The results are depicted in Fig. 5. The offspring number  $N$  increases with the increases of  $\xi$  and  $\theta$ , while it decreases when the density-independent mortality rate of mosquitoes  $\mu_3$  increases. This suggests that control measures which result to the decrease of total number of female mosquitoes are the best way to fight the disease. Indeed, decrease  $N$  implies to the eradication of mosquito population. We see that  $R_0$  is a increasing function of  $\beta_h$  and  $\beta_v$ , while it decreases when the treatment rate  $\sigma_h$  increases.

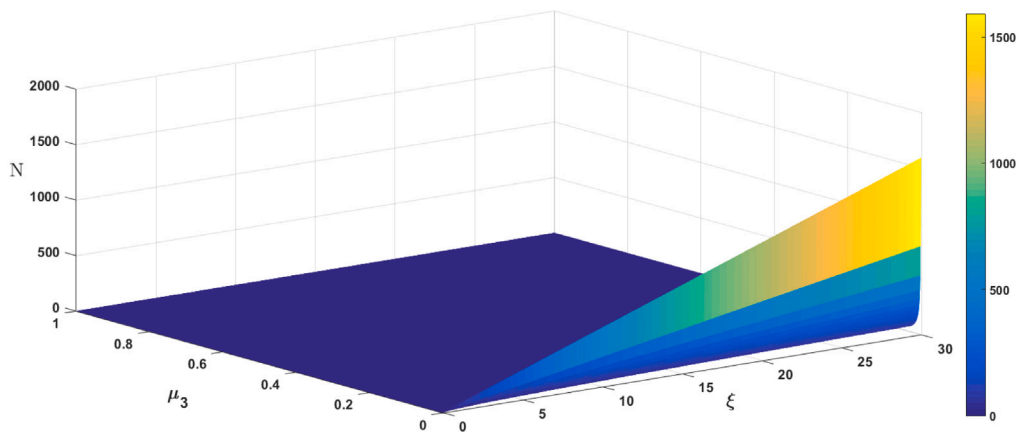
Figs. 6 and 7 validate the results claimed in Theorems 3 and 4. We see that for  $N < 1$  (Fig. 6), populations of aquatic vectors decrease until the trivial point  $(0, 0, 0, 0)$ , and thus proving its global asymptotic stability. However, when  $N > 1$  (Fig. 7), all trajectories of state variables of the vector dynamics model (19), independently of the fractional-order parameter value, tend to the non-trivial equilibrium  $(E^*, L^*, P^*, N_v^*)$ , proving thus its global asymptotic stability.

The results depicted in Fig. 7 confirm that the non-trivial equilibrium is globally asymptotically stable whenever  $N > 1$ . On a quantitative viewpoint, Fig. 6 shows that the obtained numerical solutions which are Eggs ( $E$ ) Larvae ( $L$ ), Purpae ( $P$ ) and Adult vectors ( $N_v$ ) decrease according to the increase of fractional order  $\eta$ . In Fig. 7, Eggs ( $E$ ) Larvae ( $L$ ), Purpae ( $P$ ) and Adult vectors ( $N_v$ ) decrease when  $\eta$  decreases. This means that vector control measures which result to the decrease of the offspring reproduction number  $N$  are appropriates to decrease the mosquito population. Thus, if one could applied all vector control measures which permits to decrease the proliferation of mosquitoes, then the probability that a infected mosquito bites a susceptible human will be reduce. Indeed, without mosquito, Chikungunya virus cannot spread in human population.

Setting the offspring reproduction number  $N$  greater than one ( $N > 1$ ), we then simulate the reduce model given by Eq. (30). The results are depicted in Figs. 8 and 9. Note that Fig. 8 corresponds to the case  $R_0 < 1$  while Fig. 9 corresponds to the case  $R_0 > 1$ .



(a)  $N = f(\xi, \theta)$

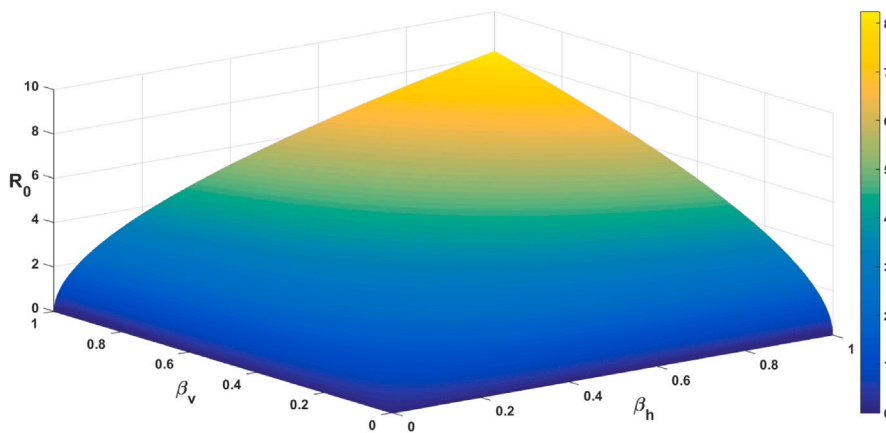


(b)  $N = f(\xi, \mu_3)$

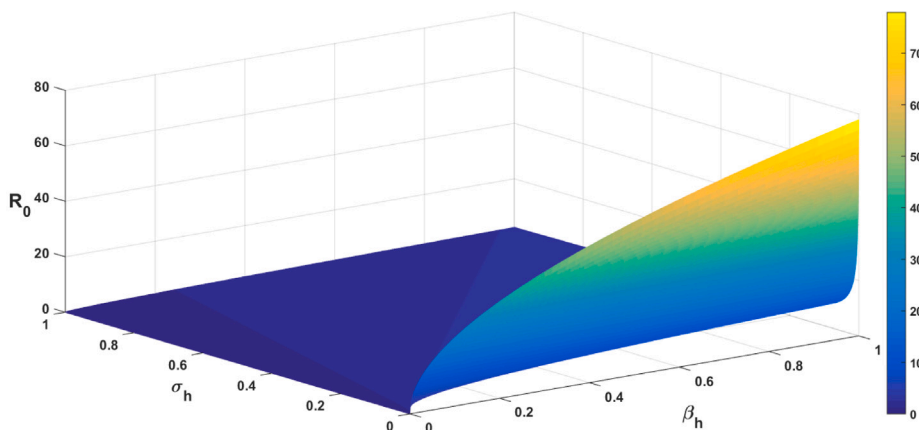
Fig. 5. 3-D Plots of  $N$  and  $R_0$  in term of some model parameters with the fractional-order  $\eta$  equal to one.

Figs. 8 shows that Susceptible humans ( $S_h$ ) (resp. removed humans  $R_h$ , susceptible vectors  $S_v$ ) increases when  $\eta$  increases. The Infected humans in the latent stage  $E_h$  (resp. infectious humans  $I_h$ , infected vectors  $E_v$  and  $I_v$ ) decrease according to the increase of  $\eta$ . Fig. 8 shows that the Infectious and Symptomatic humans ( $I_h$ ) and the Recovered humans ( $R_h$ ) increase when  $\eta$  increases. The same reasoning is done in Fig. 9. In a control point of view, if the population of mosquito cannot be eradicated, it would better to apply others existing control measures as individual protection (Wearing suitable clothes), quarantined of infectious persons as well as their rapid management with administration of appropriate treatments, to reduce as much as possible the basic reproduction number  $R_0$  below unity.

Also, numerical simulation results show that fractional model provides various flexibility in taking decision concerning disease controls.



(c)  $R_0 = f(\beta_h, \beta_v)$



(d)  $R_0 = f(\beta_h, \sigma_h)$

Fig. 5. (continued).

**Conclusion**

In this work, a fractional mathematical model for Chikungunya transmission dynamics has been investigated. We began by recalling some preliminary material on fractional calculus, and we presented the fractional-order Chikungunya model in the Caputo sense. Then, we firstly analysed the mosquito model dynamics and computed the offspring reproductive number  $N$  as well as the trivial and non-trivial equilibrium points. We proved that these two equilibrium points are globally asymptotically stable depending of the value of  $N$ . Indeed, if  $N < 1$ , then the trivial equilibrium point is GAS, while if  $N > 1$ , then the non trivial equilibrium is GAS. After that, we analysed the full model by computing the basic reproduction number  $R_0$ , the equilibrium points of the full model as well as the stability analysis of these equilibrium points. Indeed, we proved that the trivial equilibrium of the full model is globally asymptotically stable whenever  $N < 1$ , while for  $N > 1$ , the trivial equilibrium point becomes unstable and the biological meaningful disease-free equilibrium is stable locally whenever  $R_0 < 1$ . As in the case of the integer model, the fractional model admits at least one endemic equilibrium point whenever the basic reproduction number is greater than one.

We proved the existence and uniqueness of solutions using the Banach contraction theorem. The global stability of the fractional model is obtained by means of the Ulam–Hyers method. Indeed, we proved that under certain conditions, the fractional model is uniformly Lyapunov stable. The Generalized Fractional Euler Method is used to construct a numerical scheme. We performed several simulations to validate our theoretical results, as well as see the impact of the fractional-order in the model dynamics. Numerical

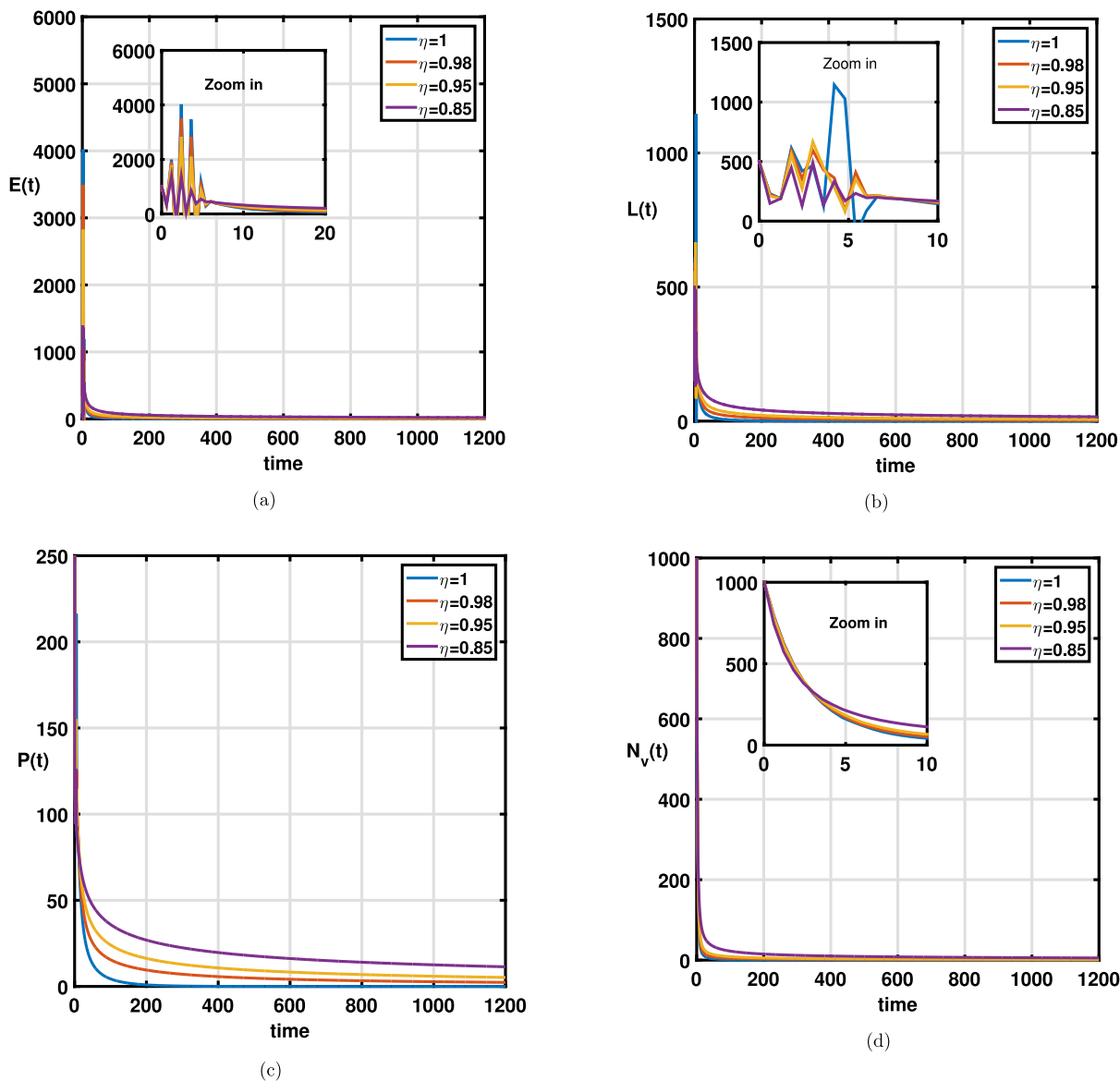
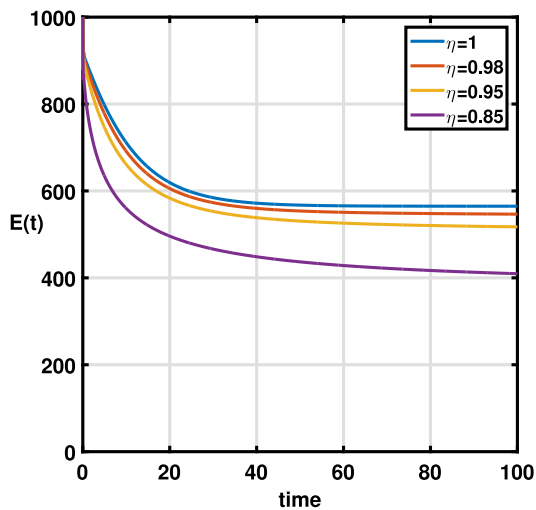


Fig. 6. Time-series of  $E$ ,  $L$ ,  $P$  and  $N_v$  for different values of  $\eta$  whenever the net reproduction number  $N$  is less than one ( $N < 1$ ). The following parameter values are used:  $\mu_3 = 0.35$ ,  $\mu_4 = 0.00002$ ,  $\xi = 6$ ,  $\theta = 0.08$ ,  $s = 0.7$ ,  $l = 0.4$ ,  $\mu_E = 0.4$ ;  $\mu_L = 0.4$ ;  $\mu_P = 0.4$ ,  $K_E = 1000$ ,  $K_L = 500$ ,  $K_P = 250$ ,  $\alpha = 2 \times 10^{-4}$ , and  $\beta = 2 \times 10^{-4}$ .

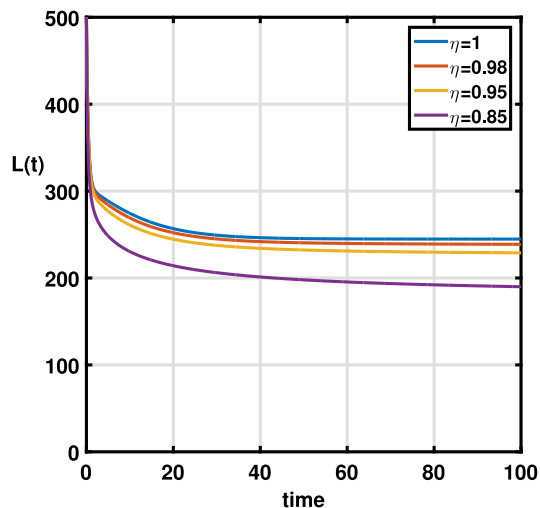
simulation results permit to conclude that mosquito control measures permit to reduce the population of mosquito. Although it is not possible to eradicate the mosquito population (indeed the case  $N < 1$  is not biologically feasible in Sub-Saharan African countries), one can reduce as well as possible the basic reproduction number until obtained  $R_0 < 1$ . The biological implication of these observations is that it is necessary to combine mosquito control measures with other control measures like individual protection, efficient treatment, and possible vaccination if this last is available. Indeed, in [18], it has been proved that combination of all available control measures is the best ways to fight vector-borne disease like Chikungunya.

Even though both models (classical and fractional derivatives) have the same qualitative dynamics, we discovered through numerical simulations that the fractional model is a better approach from a quantitative standpoint because it better fits the Chikungunya reported cases, and thus provides more flexibility in making disease control decisions.

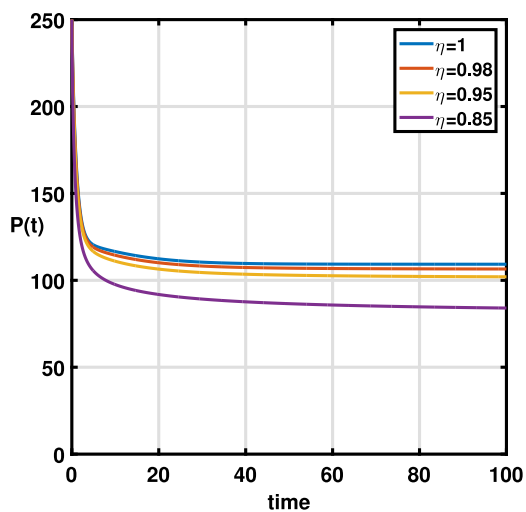
In this work, we did not determine the exact value of the fractional-order parameter  $\eta$  which better fits the data. Also, numerous fractional derivative exist and must be taking into account. The direct perspective of this work can be resumed to the determination of the fractional-order parameter  $\eta$  and the type of fractional derivative which better predict Chikungunya epidemic in Chad.



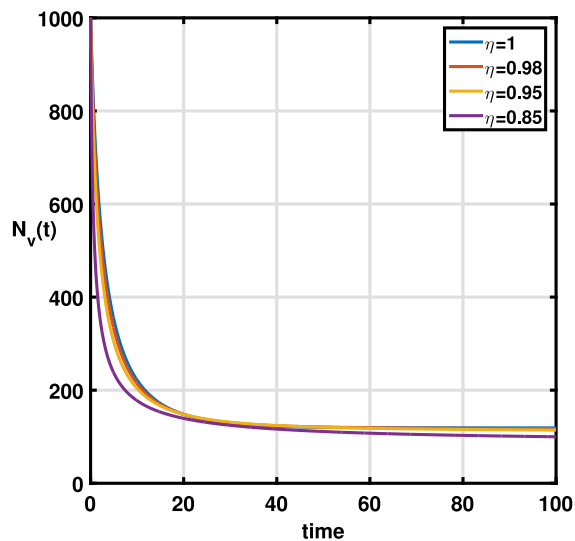
(a)



(b)



(c)



(d)

Fig. 7. Time-series of  $E$ ,  $L$ ,  $P$  and  $N_v$  for different values of  $\eta$  whenever the net reproduction number  $N$  is greater than one ( $N > 1$ ). The parameters values are the same as ones used in Fig. 6 except  $\xi = 30$ ,  $\mu_3 = 0.035$ ,  $\mu_E = 0.2$ ;  $\mu_L = 0.2$ ;  $\mu_P = 0.2$ , to maintain  $N > 1$  whatever the value of the fractional-order parameter  $\eta$ .

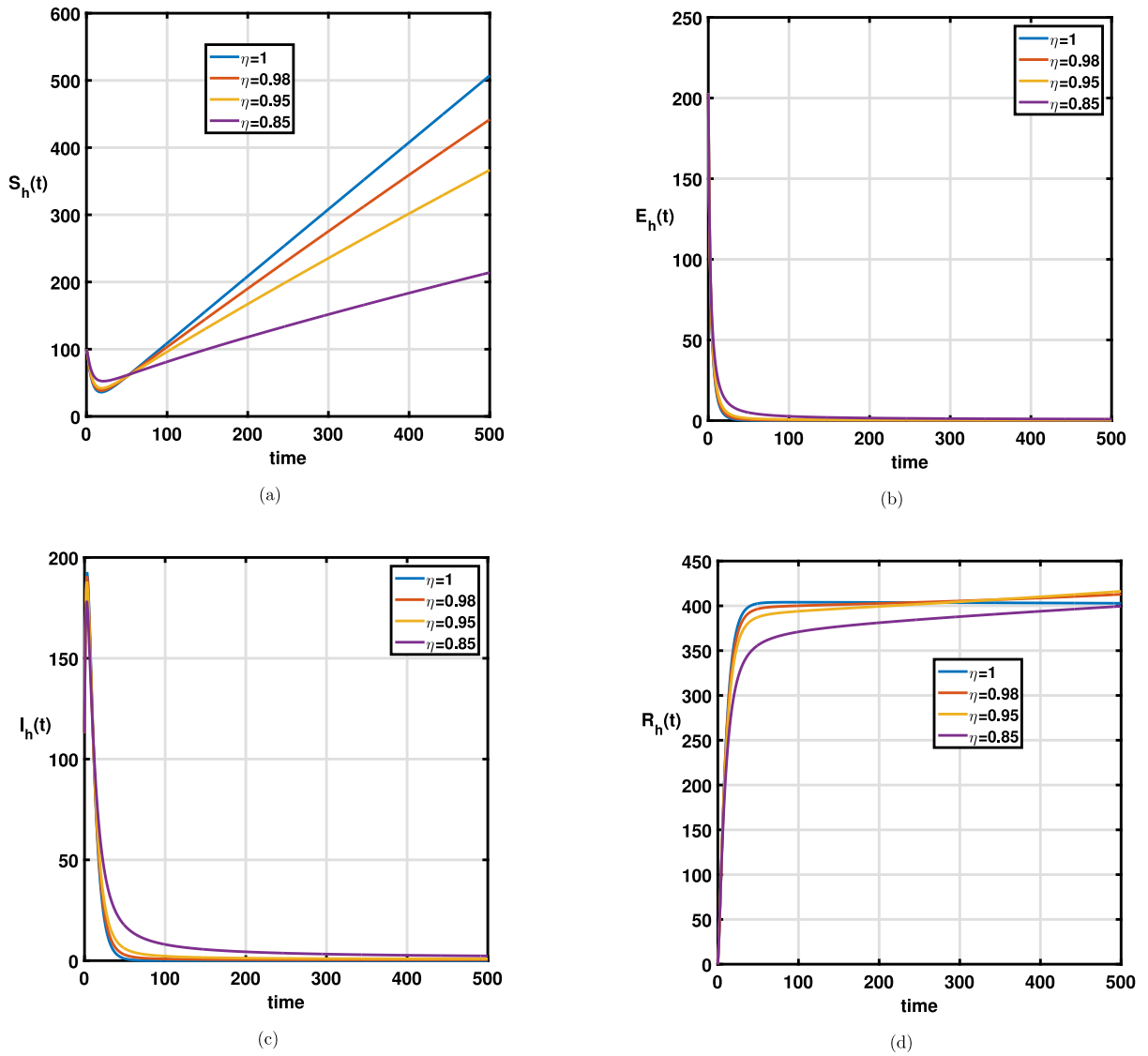
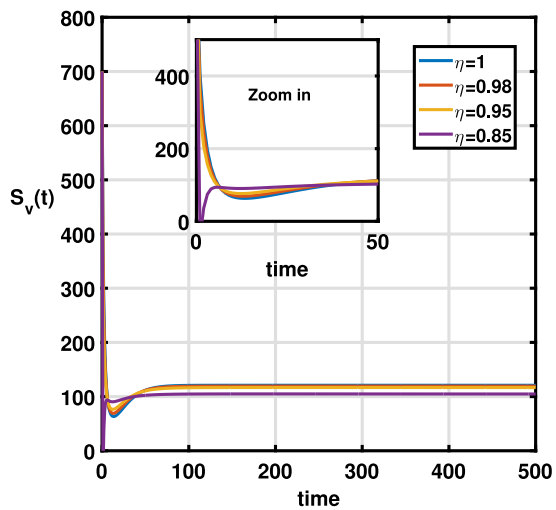
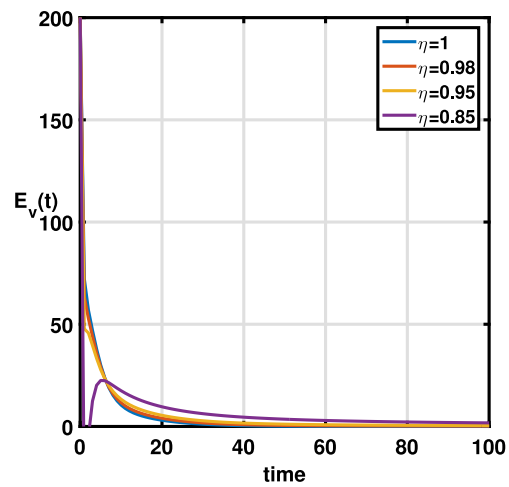


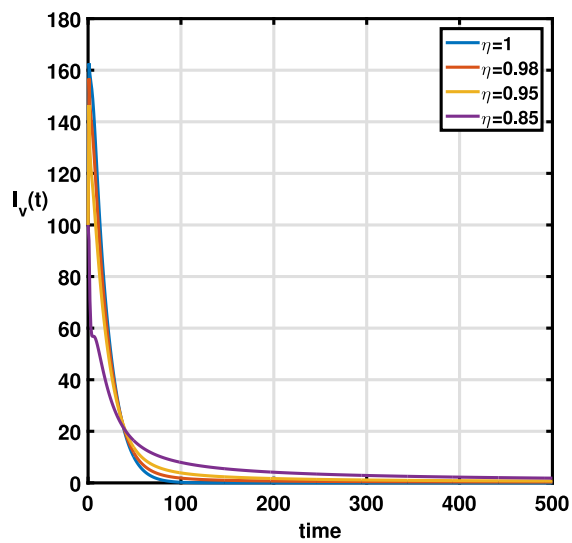
Fig. 8. Time-series of  $S_h$ ,  $E_h$ ,  $I_h$ ,  $R_h$ ,  $S_v$ ,  $E_v$  and  $I_v$  for different values of  $\eta$  when  $N > 1$  and  $R_0 < 1$ .



(e)

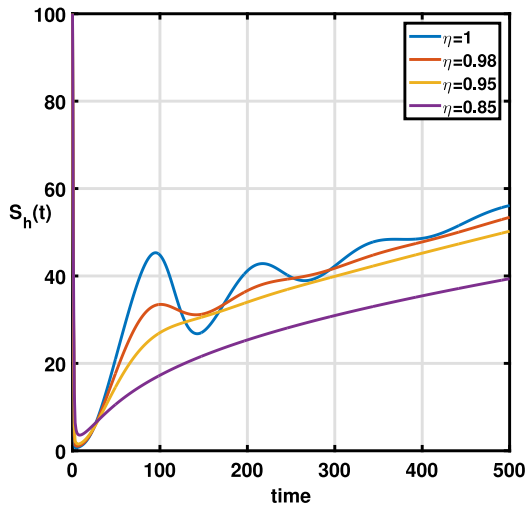


(f)

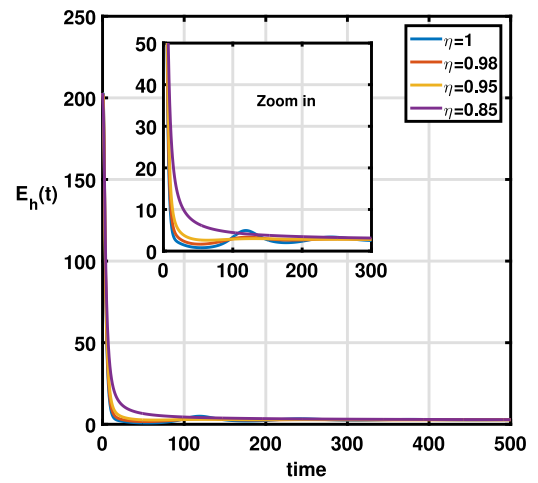


(g)  $I_v$

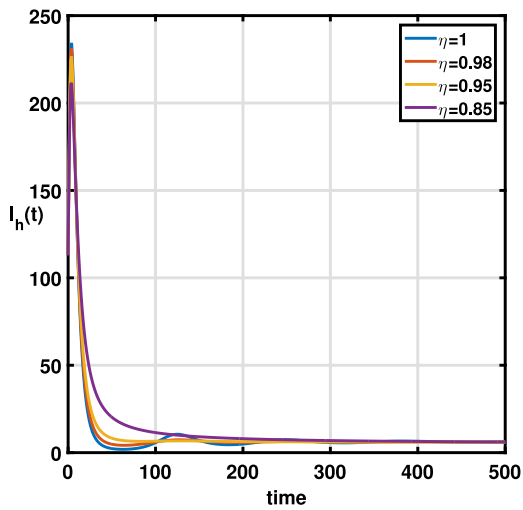
Fig. 8. (continued).



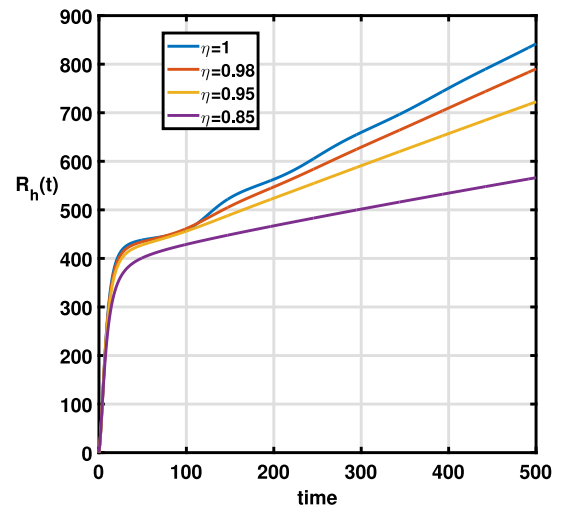
(a)



(b)



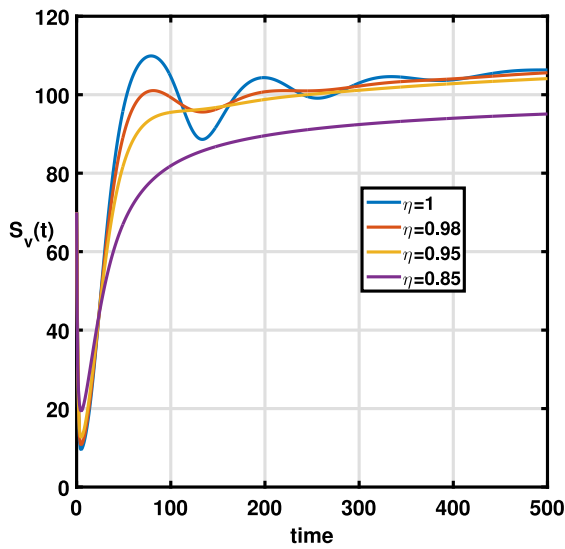
(c)



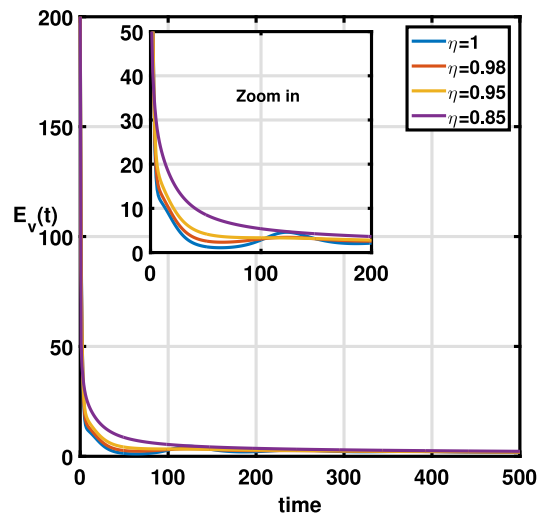
(d)

Fig. 9. Time-series of  $S_h$ ,  $E_h$ ,  $I_h$ ,  $R_h$ ,  $S_v$ ,  $E_v$  and  $I_v$  for different values of  $\eta$  when  $N > 1$  and  $R_0 > 1$ .

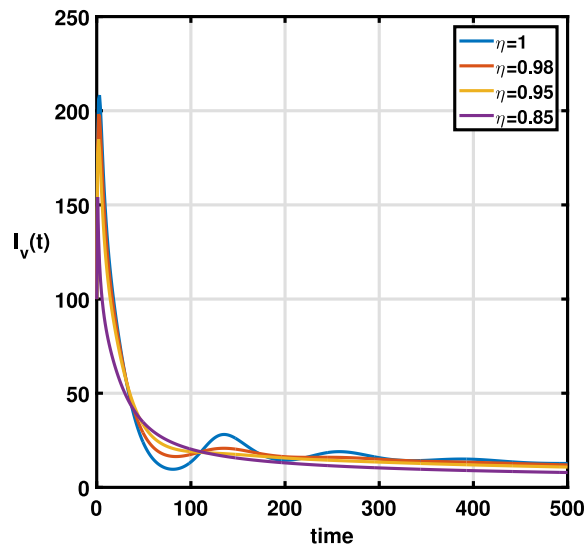




(e)



(f)



(g)

Fig. 9. (continued).

**CRedit authorship contribution statement**

**Joseph Yangla:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Writing – original draft. **Hamadjam Abboubakar:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Supervision, Writing – original draft, Writing – review & editing. **Ezekiel Dangbe:** Project administration, Resources, Supervision. **Richard Yankoulo:** Data curation, Formal analysis, Investigation, Methodology, Software. **Ado Adamou Abba Ari:** Project administration, Supervision, Writing – review & editing. **Irépran Damakoa:** Project administration, Supervision. **Kottakaran Soopy Nisar:** Conceptualization, Methodology, Project administration, Resources, Writing – review & editing.

**Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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