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A distributed scheduling method for demand response in energy communities with distributed generation and storage

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Abstract

This paper presents a distributed optimization method for energy. Communities have distributed renewable generation and storage units. We explain how the resulting optimization problem can be cast as a bi-level optimization problem where the followers solve mixed-integer linear programs. Given the difficulty of these problems, we develop a heuristic where incentives are sent to the followers. Specifically, to maximize the collective-self consumption rate, we exploit the notion of allocation key traditionally used a posteriori for economic gain sharing to build an a priori incentive to Demand response. The proposed distributed management method returns high-quality solutions to the optimal centralized real-world benchmark instances constructed over a month with accurate historical data from our demonstrator in south France.

Keywords— Distributed management, Demand response, Collective self-consumption, Bilevel optimization.

1 Introduction

Global economic and demographic growth is driving a significant increase in energy demand. Against this backdrop, governments and communities strive to increase the production and consumption of renewable energies. States are developing regulations to promote access to production and storage tools on the one hand Clean energy for all Europeans package and Directive (EU) 2018/2001, and surplus trading on the other hands: Code de l'énergie, Décret n° 2017-676 and PUCA. Thus, collective self-consumption appeared in France in 2015, enabling renewable energy generators to exchange their surpluses with specific individuals. The French Energy Code

defines collective self-consumption as an operation linking one or more producers to one or more consumers using the public electricity distribution network for their energy exchanges. In addition to the social benefits, the advantages of collective self-consumption include participating in the energy transition, saving money on energy bills, reducing fuel poverty, and minimizing energy generation surpluses. These energy and economic challenges require optimization tools throughout the energy community lifecycle from the planning point of view.

Moret et al. (2020); Sani et al. (2022) address long-term planning. Sani et al. (2022) study a power system extension model. The goal is to determine the investment to expand a multi-commodity power system. To do so, the authors consider Demand Response as a capacity reserve. The resulting problem is uncertain since DR is uncertain even over short-term planning. While considering uncertainty in energy systems' management, many kinds of uncertain parameters make robust models untenable given the systems' size; Moret et al. (2020) presents a robust optimization framework to address this issue.

In medium-term planning, van Ackooij et al. (2018) presents a bilevel problem of a single leader and several followers. The leader interacts with each follower via a contract detailing the parties' exchange prices. The leader's problem is choosing the contract to propose to each follower to maximize his profit, given that the members may or may not accept them. If a follower refuses the leader's contract, she can exchange with one of the leader's competitors. Each follower's objective is to choose the least expensive contract.

Finally, Ströhle and Flath (2016) belongs in short-term planning; the authors deal with the decentralized coordination of flexible loads and uncertain renewable generation. The authors rely on demand-side incentives to attract low-cost flexibility in a decentralized and uncertain context to balance supply and demand in each period.

Two optimization approaches to energy system planning exist in the literature: centralized and distributed optimization. Centralized optimization methods, which consist of a central manager collecting information and preferences from all members, solving an optimization problem then sending signals to each community member, are widely used in the literature (Conti et al., 2012; Sangaré et al., 2023; Tsikalakis and Hatziargyriou, 2008) as well as the aforementioned (Sani et al., 2022; van Ackooij et al., 2018). While they can return optimal solutions, they have some drawbacks:

- The large computational capacity requirement for the centralized manager.
- The information bottlenecks that lead to long transfer times.

- The question of social acceptance, since people may be reluctant to accept instructions from a third party regarding using their devices Shafie-Khah and Siano (2018).

Therefore, distributed optimization is a relevant alternative for large electrical systems; distributed optimization refers to each member locally determining their loads. Thus, the coordinator does not bear the computational effort since each member locally solves a local optimization problem. The local optimization problems are often some linear program (LP). Some of the most used distributed optimization methods are: the alternating direction method of multipliers (ADMM) (Du et al., 2019; Li et al., 2021; Lilla et al., 2020; Liu et al., 2017; Stephant et al., 2021), dual decomposition (Molzahn et al., 2017) and Consensus + innovations (Hug et al., 2015; Kar and Hug, 2012).

Regarding ADMM, Lilla et al. (2020) proposes a method for day-ahead scheduling and compares the centralized and distributed approaches. Reference Li et al. (2021) presents a distributed model for multi-microgrids that cooperate to reduce the aggregated operational cost. Stephant et al. (2021) presents a community that cannot inject the surplus in the distribution network and an ADMM-based method to maximize the community’s utility. The authors use game theory to model members’ preferences. Each member solves an optimization problem to maximize the local utility. Thus, global problems are solved under global constraints, such as the battery storage system (BSS) management constraints. On a ring network composed of several agents, the next agent i starts to calculate and update only after receiving information from the previous one, which is problematic for complex problems. Thus Du et al. (2019) present a parallel and distributed method for energy management to increase ADMM’s convergence rate.

Turning to Consensus + innovation, Hug et al. (2015) present an approach to coordinate the loads, energy generation, and storage for a set of auto-sufficient agents of a microgrid. Each agent interacts with a defined neighborhood to find a consensus on the power supply price while ensuring the total generation and load are equal. Kar and Hug (2012) provide a relaxed consensus + innovation-based management to coordinate the power exchanges and prices among the microgrid and the smart grid and study the uncertainty impact on such system’s performance.

Finally, the ADMM works in a cascade, making it ineffective for large systems. Wu et al. (2020) present a distributed Lagrangian dual method that rapidly converges based on Nesterov accelerated gradient and dynamic set-sizes. Li (2019) generalizes the Lagrangians to a broader class of functions that satisfies the strong duality between primal and dual problems then, derives the generalized Karush–Kuhn–Tucker conditions for this generalized Lagrange multiplier

method which is helpful to provide consensus protocols.

In this paper, we introduce a bilevel formulation to manage a community where the members do not directly interact. Thus, no peer-to-peer exchanges, no interaction between community’s members. The problem’s generic form is:

$$x_L^* = \min_{x_L \in X_L} f_L(x_L, x_F) \tag{1a}$$

$$\text{s.t. } x_F \in \arg \max_{x_F \in X_F} g_F(x_L, x_F), \tag{1b}$$

where x_L and x_F are the leader’s and followers’ decision variables, and f_L is the leader’s objective function, which in our case is the energy coming from the primary grid. Each follower’s objective function is the gain, which is negative if the member’s global need is greater than the energy availability; it is positive otherwise. The bilevel formulation is hardly solvable due to integer variables in the followers’ problem and the problem’s size. Thus, we introduce a heuristic where CC provides the surplus allocation rule called allocation key as an incentive, which is the main methodological contribution of this paper. The notion of allocation key is a well-known concept used (a posterior) to share the economic gain between the members who share their generation surplus Roy et al. (2023). In this paper, we use the concept of allocation key to share the energy injected by prosumers in the community for day-ahead scheduling. Thus, we define the allocation key as an algorithm that returns the maximum amount of energy a given member can collect from the community at each period. After receiving the incentive, each member determines their load, need, withdraws, and/or storage and injection schedules. Then, she sends her energy needs and availability information to CC, which updates its incentives. Since there are no direct information links between each pair of members, prosumer i does not know which part of the injection is consumed in the community. If the total periodic injection exceeds the total needed, the remaining energy is injected into the grid; otherwise, the remaining required energy is collected from the primary grid. The main aim is to consume local generation as much as possible locally.

The rest of the paper is structured as follows: Section 2 presents the description of the problem we study. Section 3 details the formulation of each level of formulation (1). Section 4 presents an iterative heuristic, returning the efficient solution. Section 5 presents the experimental results on a realistic instance, in Section 6 report more results on real world data. The paper concludes with a conclusion in Section 7.

2 Problem description

We consider a set of $|N|$ individuals involved in a collective self-consumption project, each having different asset possession characteristics as schematized in Figure 1. Figure 1 is presented as an indication that there are different profiles of individuals in the community. In this work, an energy community refers to individuals connected to the same distribution network that produces, consumes, and exchanges energy. We specifically consider renewable energy communities as defined by the package Clean energy for all Europeans. The members who generate energy can store their surpluses and exchange them within the community or the main grid. We assume the local exchange prices are more attractive than those with the main grid. Thus, local producers prioritize local exchange over main grid injection, maximizing their profit. However, in such a community, no additional links exist between members apart from being in the same community, i.e., they do not directly interact. A community coordinator (CC) is the intermediary between members to ensure a given management objective. For example, CC ensures the community's functioning in terms of collective self-consumption. She interacts with each member, collecting specific information and sharing incentives to demand response (DR) to ensure the whole community's management. DR is a load profile modulation granted by a member in return for a certain amount of consideration. Each member remains directly connected to the primary grid and collects energy when needed. Our partners have a margin of each kWh energy locally exchanged in the community. An aggregator collects each producer's surpluses; for simplicity, we assume there is a unique aggregator for all community members.

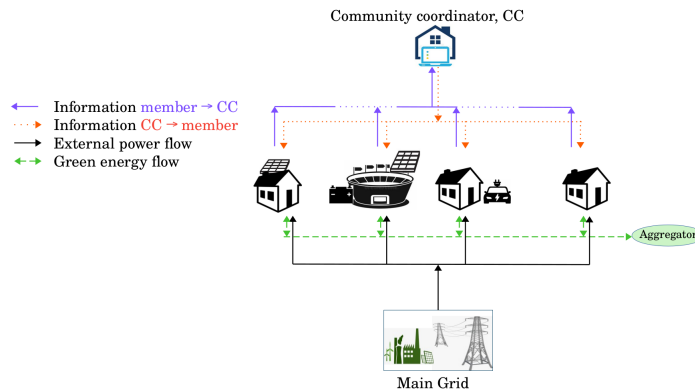


Figure 1: Community scheme.

The coordinator fixes the periodic local energy exchange prices. However, to preserve trust in the community, these prices are not differentiated, i.e., the prices are the same for each member. Thus, more than the price-based incentive is required in such a system. We do

not address the pricing in this paper. CC must build some energy-based incentives to maximize the collective self-consumption rate since there is no price differentiation. Thus, CC receives each member's hourly energy demand and prosumers' energy availability to compute the energy-based incentives. These incentives aim to limit the periodic energy exchanges with the aggregator and the main grid, which is equivalent to maximizing collective self-consumption. They also incentivize prosumers having batteries to store and inject when needed during planning horizon to serve the community or the primary grid. Notice the more green energy is exchanged in the community, the more the coordinator realizes the economic gain as they are correlated; this allows CC to help such a community unleash its potential fully (Fioriti et al. (2021)).

As shown in Figure 1, a member can be a traditional consumer, a prosumer with a battery storage system (BSS), or a consumer owning BSS. Following Tsikalakis and Hatziargyriou (2008), we assume each member has the required control and exchange equipment for this study. Each member must perform some tasks that we classify into three classes according to their characteristics. The first class contains tasks that allow modulation of specific environments' temperatures (house room, water heater). We call this type A class. Each member must decide the periodic power consumption to reach the desired temperature at the expected time. We use function θ_j to evaluate the temperature in member i 's room where task $j \in J_{iA}$ is performed.

- The temperature variation function derived from the Newton cooling law is:

$$\theta_1(p_h) = \theta_1(p_{h-1}) + \frac{\Delta}{C_r}(p_h - U(\theta_1(p_{h-1}) - T_h^{\text{out}})),$$

for house rooms. Where C_r is heat capacity (J/K), parameter U designates the heat loss coefficient of a room in (W/K), and Δ is the heating time in second. T_h^{out} is the external temperature at period h . Notice that θ_1 work for both heating and cooling.

- For a water heater, using the method from Shen et al. (2021), the temperature variation function θ_2 is given by

$$\theta_2(p_h) = \frac{\Delta_h}{Mc_p}(-\frac{S}{R}[\theta_2(p_{h-1}) - T_h^{\text{room}}] + vp_h - 1000M_w c_p[\theta_2(p_{h-1}) - T_{in}]) + \theta_2(p_{h-1}),$$

where c_p is the isobaric specific heat capacity of water (kcal/kg \cdot °C), v is the efficiency of the electricity-to-heat transformation, M is the weight of the water (kg). The data S designates the exchange surface of the water container with the external area, R is the

thermal resistance of the tank insulation in $m^2 \cdot ^\circ\text{C}/\text{W}$ and M_w is the average hot water demand rate during the time interval, which we assume to be equal to zero since we don't have that information. Parameter T_{in} is the supply domestic cold water temperature, and T^{room} is the temperature of the ambient environment. Finally, Δ_h heating time in seconds.

The schedules start at period 1. We have the initial period $h = 0$ where no decision is required, i.e., $p_0 = 0$. In addition, each task $j \in J_{iA}$ that is performed in room k of member i has a power $\bar{p}_{ijk} > 0$, expressed in kW. In this work, we discretize $[0, \bar{p}_{ijk}]$ by considering only the integer values in $[1, \bar{p}_{ijk}]$ (0 is not considered as in that case, the device is turned off). Our numerical experiments consider devices with $\bar{p}_{ijk} = 2$.

The second class, called type B, contains the tasks that must be performed during defined periods under some energy consumption level constraint (charging the electric vehicle or washer). For example, each member i must decide a schedule in a set of feasible schedules S_{ij} to perform task j . The last class, called type C class, contains uncontrollable tasks. Type C tasks' periodic energy consumption is estimated for each member during the planning horizon. The question we address in this paper is: how to efficiently perform the tasks in a community to maximize the collective self-consumption rate?

3 Mathematical formulation

In this section, we detail the bilevel formulation (1). CC is the leader and each member in the community is a follower. Let us first detail the input data and followers' variables, provided in Tables 1 and 2. In addition leader's variables are the maximum amount of energy a given member can take from the community. We detail next the main components of (1).

3.1 Leader's problem

CC determines the periodic day-ahead the incentive to maximize the collective self-consumption rate. This incentive that we note F_{ih}^{RCom} is the maximum amount of energy member i can draw from the community at period h . The leader shares the energy between members via F^{RCom} , and the remaining energy at period h is denoted I_h , which is injected into the main grid. Recall f_L is the leader's objective function, which designates the total energy collected from the main grid. Minimizing f_L is equivalent to maximizing the collective self-consumption rate. The leader's

Sets	
N, N^{BSS}	set of community's members, owing a storage unit
J_{iA}	set of member i 's type A tasks
J_{iB}	set of member i 's type B tasks
H	set of the planning horizon's periods
K_{ij}^A	set of rooms of member i where task $j \in J_{iA}$ can be performed
S_{ij}	set of schedules given by $i \in N$ for task $j \in J_{iB}$
P_{ijk}^A	set of the power levels available to perform task $j \in J_{iA}$ in room $k \in K_{ij}^A$ of $i \in N$
Parameters	
Δ_h	length of period h (<i>hour</i>)
π_i	member i 's subscribed power level
Γ_i	member's i 's BSS's capacity
ξ_i	initial amount of electricity in member i 's BSS $i \in N$
η_i	member i 's BSS's automatic discharge rate of battery $i \in N^{\text{BSS}}$
ϕ_i	member i 's BSS's maximum cycles during horizon H $i \in N^{\text{BSS}}$
β	maximum spending degradation threshold allowed (%)
Ω_i	equal to 1 if member i is allowed to exchange electricity, 0 otherwise
d_i, c_i	discharge and charging efficiencies of i 's BSS $i \in N^{\text{BSS}}$ (%)
ν_{ijk}	equal 1 if task $j \in J_{iA}$ is executed by member i in room $k \in K_{ij}^A$, 0 otherwise
$\theta_j(p)$	temperature variation function when performing task $j \in J_{iA}$ according to power p
$[t_{ijk}^{\text{low}}, t_{ijk}^{\text{up}}]$	temperature targeted by member i when performing task $j \in J_{iA}$ in room k ($^{\circ}\text{C}$)
$[h_{ijk}^{\text{low}}, h_{ijk}^{\text{up}}]$	time where member i 's confort zone must be reached when performing task $j \in J_{iA}$
$\bar{T}_{i1k}^{\text{room}}$	initial temperature of room $k \in K_{i1}^A$ where member i wants to perform task $j = 1$
$\bar{T}_{i2k}^{\text{water}}$	initial water's temperature in heater $k \in K_{i2}^A$ where member i wants to perform task $j = 2$
$\bar{T}_{ijk}^{\text{all}}$	$\bar{T}_{ijk}^{\text{all}} = \bar{T}_{i1k}^{\text{room}}$ if $j = 1$, $\bar{T}_{ijk}^{\text{all}} = \bar{T}_{i2k}^{\text{water}}$ if $j = 2$
T_h^{out}	outside temperature at time period $h \in H$ ($^{\circ}\text{C}$)
T_{kh}^{room}	ambient temperature in the room where is placed water heater $k \in K_{i2}^A$ at period h ($^{\circ}\text{C}$)
P_{ih}^{Gen}	energy production of member i at period h (kW)
P_{ijhs}^{B}	consumption of task $j \in J_{iB}$ of member i at period h in the schedule $s \in S_{ij}$ (kW)
P_{ih}^{Gen}	energy production of member i at period h (kW)
P_{ijhs}^{B}	consumption of task $j \in J_{iB}$ of member i at period h in the schedule $s \in S_{ij}$ (kW)
$P_i^{\text{in}}, P_i^{\text{out}}$	charging and discharging power of member i 's BSS (kW)
p_{ih}^{C}	cumulative power consumption of type C tasks at time period $h \in H$
G_i	gain of member $i \in N$ when operating outside a community (€)
v_h^{MG}	unit purchase price of electricity from the primary grid at period $h \in H$ (€/kWh)
\tilde{v}_h^{MG}	unit sale price of electricity to the primary grid during period $h \in H$ (€/kWh)
v_h^{Com}	unit purchase price of electricity in the community during period $h \in H$ (€/kWh)
\tilde{v}_h^{Com}	unit sale price in the community during period $h \in H$ (€/kWh)
v_h^{GES}	unit purchase price to the green energy supplier at period $h \in H$ (€/kWh)
\tilde{v}_h^{GES}	unit sale price to the aggregator at period $h \in H$ (€/kWh)

Table 1: Data.

problem is:

$$x_L^* = \min_{x_L \in X_L} f_L(x_L, x_F), \quad (2)$$

$x_{ijkhp}^A \in \{0, 1\}$	is equal to 1 if and only if task $j \in J_{iA}$ is at progress at period h in member i 's room $k \in K_{ij}^A$ and the device is on power level p ,
$x_{ij}^s \in \{0, 1\}$	is equal to 1 if and only if schedule $s \in S_{ij}$ is chosen for member i 's task $j \in J_{iB}$,
$z_{ih} \in \{0, 1\}$	is equal to 1 if and only if i 's BSS is charging in time period h ,
$w_{ih} \in \{0, 1\}$	is equal to 1 if and only if i 's BSS's state changes from discharging or inactive to charging at period h ,
$T_{ijkh} \geq 0$	the temperature in room k of i where task $j \in J_{iA}$ is performed at period h ,
$q_{ih} \in \mathbb{R}$	is the amount of energy injected into/out i 's BSS at period h , with $q_{bih} \leq 0$ if b discharges, $q_{bih} \geq 0$ if b charges, and $q_{bih} = 0$ if b is inactive,
$E_{bih} \geq 0$	the amount of electricity in battery $b \in B_i$ of i at the end of period h ,
$F_{ih}^{\text{out}} \geq 0$	is the amount of energy injected by member i at period h ,
$F_{ih}^{\text{in}} \geq 0$	is the amount of energy drawn from the community by i at period h ,
$C_{ih} \geq 0$	is the amount of energy withdrawn from the grid by $i \in N$ at period h ,
$\tilde{G}_i \in \mathbb{R}$	is the gain of member i when operating in the community.

Table 2: Each member's variables.

where x_F represent followers' response to leader's decision, and $x_L = (F^{\text{RCom}}, I)$. More explicitly, the leader's problem is:

$$\min \sum_{i \in N} \sum_{h \in H} C_{ih} \quad (3a)$$

$$\text{s.t.} \quad \sum_{i \in N} F_{ih}^{\text{RCom}} + I_h = \sum_{i \in N} F_{ih}^{\text{out}} \quad h \in H \quad (3b)$$

$$F_{ih}^{\text{RCom}}, I_h \geq 0 \quad i \in N, h \in H. \quad (3c)$$

The leader shares the injection between members via F^{RCom} and sends the remaining energy I_h $h \in H$ to the grid in constraint (3b).

3.2 The followers

Each follower i 's objective is to determine her consumption, injection, and storage schedules to maximize her gain g_F . Let X_F be the feasible region of the lower level's decision variable x_F . Thus:

$$x_F^* = \max_{x_F \in X_F} g_F(x_L, x_F). \quad (4)$$

The detailed lower level problem is presented in what follows. Each follower i has individual constraints related to the tasks to perform. These are listed below and commented subsequently.

$$\sum_{s \in S_{ij}} x_{ij}^s = 1 \quad j \in J_{iB} \quad (5a)$$

$$\forall j \in J_{iA}, k \in K_{ij}^A,$$

$$\sum_{p \in P_{ijk}^A} x_{ijkhp}^A \leq 1 \quad h \in H \quad (5b)$$

$$T_{ijkh} = \theta_j \left(\sum_{p \in P_{ijk}^A} p \cdot x_{ijkhp}^A \right) \quad h \in H \quad (5c)$$

$$t_{ijk}^{\text{low}} \leq T_{ijkh} \leq t_{ijk}^{\text{up}} \quad h \in [h_{ijk}^{\text{low}}, h_{ijk}^{\text{up}}] \text{ if } \nu_{ijk} = 1 \quad (5d)$$

$$T_{ijk0} = \bar{T}_{ijk}^{\text{all}} \quad \text{if } \nu_{ijk} = 1. \quad (5e)$$

Recall that a type A task is performed to regulate the temperature in certain rooms (room or water heating), and a type B task must be performed under time window and consumption level constraints. Constraint (5a) refers to type B tasks. Indeed, exactly one schedule must be selected for each type B task. Constraints (5b)–(5e) are related to type A tasks. At each period, for each type A task, at most, one power level is selected (5b). Constraint (5c) ensures the temperature calculation at each period. Constraint (5d) imposes the temperature in the desired comfort zone, and (5e) sets the initial rooms' temperature. Another individual constraint concerns the BSS management constraint $\forall i \in N^{\text{BSS}}$:

$$\Delta_h d_i P_i^{\text{out}} (z_{ih} - 1) \leq q_{ih} \quad h \in H \quad (6a)$$

$$q_{ih} \leq \Delta_h c_i P_i^{\text{in}} z_{ih} \quad h \in H \quad (6b)$$

$$z_{ih} - z_{i(h-1)} \leq w_{i(h-1)} \quad h \in H \setminus \{1\} \quad (6c)$$

$$\frac{E_{ih} - q_{i(h-1)}}{E_{i(h-1)}} = \eta_i \quad h \in H \quad (6d)$$

$$\sum_{h \in H} w_{ih} \leq \phi_i \quad (6e)$$

$$E_{ih} \leq \Gamma_i \quad h \in H \quad (6f)$$

$$E_{i0} = \xi_i. \quad (6g)$$

Constraints (6a) and (6b) avoid the simultaneous BSS's charge and discharge. Variable q_{ih} is the amount injected if positive. Otherwise, it is the amount discharged from BSS. (6d) updates the BSS's state at each period, and (6e) limits the number of cycles during the planning horizon.

Constraint (6f) enforces the amount of energy in the BSS to be less than its capacity, and (6g) sets the BSS's initial state.

Each member i maximizes her gain \tilde{G}_i . Constraint (7a) calculates member i 's gain that must not be deteriorated by more than $\beta\%$ of i 's state when she does not exchange energy (7b). Prosumers gain also depends on the amount injected in the grid; we denote τ_{ih} as a parameter that designates member i 's contribution to this amount at period h .

$$\tilde{G}_i = \sum_{h \in H} [v_h^{\text{Com}} F_{ih}^{\text{out}} - \tilde{v}_h^{\text{Com}} F_{ih}^{\text{in}} + \tau_{ih} v_h^{\text{GES}} I_h - \tilde{v}_h^{\text{MG}} C_{ih}] \Delta_h \quad (7a)$$

$$\frac{\tilde{G}_i - G_i}{|G_i|} \geq \beta. \quad (7b)$$

In addition, each member i has some global operational constraints at $h \in H$ which are:

$$\sum_{j \in J_{iA}} \sum_{k \in K_{ij}^A} \sum_{p \in P_{ijk}^A} p \cdot x_{ijkhp}^A + \sum_{j \in J_{iB}} \sum_{s \in S_{ij}} P_{ijhs}^B x_{ijs}^s + p_{ih}^C + \frac{q_{ih}}{\Delta_h} = P_{ih}^{\text{Gen}} + C_{ih} + F_{ih}^{\text{in}} - F_{ih}^{\text{out}} \quad (8a)$$

$$F_{ih}^{\text{out}} \leq P_{ih}^{\text{Gen}} + \frac{E_{ih}}{\Delta_h} \Omega_i \quad (8b)$$

$$F_{ih}^{\text{in}} + C_{ih} \leq \pi_i \quad (8c)$$

$$F_{ih}^{\text{in}} \leq F_{ih}^{\text{RCom}} \quad (8d)$$

$$F_{ih}^{\text{out}} \leq \pi_i. \quad (8e)$$

Constraint (8a) ensures energy conservation. A member can inject if she is a prosumer or a prosumer owning BSS (8b). Parameter Ω_i equals one if member i can inject energy, i.e., if i is a prosumer or holds a BSS, 0 otherwise. Constraints (8c), and (8e) enforce each member's energy flows to be less than her subscribed power. Constraint (8d) imposes F_{ih}^{RCom} as the upper bound on F_{ih}^{in} for $i \in N, h \in H$.

Finally, we have the variables domains defined as follows: $\forall i \in N, j \in J_{iA}, h \in H, p \in P_{ijk}^A, k \in K_{ij}^A, j' \in J_{iB}, s \in S_{ij}$,

$$x_{ijkhp}^A \in \{0, 1\} \quad (9a)$$

$$x_{ijs}^s \in \{0, 1\} \quad i \in N \quad (9b)$$

$$z_{ih}, w_{ih} \in \{0, 1\} \quad (9c)$$

$$F_{ih}^{\text{out}}, F_{ih}^{\text{in}}, C_{ih}, E_{ih} \geq 0, q_{ih} \in \mathbb{R}. \quad (9d)$$

Overall, the lower level problem is:

$$\begin{aligned} \max \quad & \tilde{G}_i \\ \text{s.t.} \quad & (5), (6), (7), (8), (9). \end{aligned} \tag{10a}$$

3.3 The full bilevel problem

Let us specify again the overall bilevel problem we are facing as:

$$\min_{x_L \in X_L} f_L(x_F^1) \tag{11a}$$

$$\text{s.t.} \quad g_L(x_L, x_F^1) \leq 0 \tag{11b}$$

$$x_F = (x_F^1, x_F^2) \in \arg \max_{x_F \in X_F} g_F(x_F), \tag{11c}$$

where $X_L \subset \mathbb{R}^{n_\ell}$ and $X_F = \{(x_F^1, x_F^2) \in \mathbb{R}^{n_f^1} \times \{0, 1\}^{n_f^2} : g_F^1(x_F^1) \leq x_L, g_F^2(x_F^1, x_F^2) \leq 0\}$, $n_\ell, n_{x_f^1}, n_{x_f^2} \in \mathbb{N}$. We have seen in the previous section that the follower’s problem contains integer variables. Therefore, classical approaches like KKT reformulation cannot be used.

Approach	Paper	Larger instance’s size	Assumptions	Conflict
Branch & bound	Moore and Bard (1990)	40/16	No coupling constraint in the upper-level.	X (3b)
	Xu and Wang (2014)	920/184	All leader’s variables are required to be integral.	X (3c)
Branch & cut	DeNegre and Ralphs (2009)	14/16	No continuous variables in the leader’s problem.	X (3c)
	Dempe and Kue (2017)	- ¹	Followers’ variables do not affect the leader’s constraints.	X (3b)
	Fischetti et al. (2018)	80000/5000	No leader’s continuous variables in followers’ problem.	X (8d)

¹ No computational experiments.

Table 3: Brief state of art.

Type	Leader	Followers	Number
Continuous variables	864	3456	4128
Binary variables	0	288	288
Constraints	144	5091	5235

Table 4: Problem’s size when there are no type A and B tasks.

There are not so many algorithms available in the literature that can handle bilevel problems with integer variables in the follower problems. Table 3 presents those that we are aware of, where the column “largest instance size” provides the numbers of variables/constraints for the largest instances considered in each of these papers. We further present in Table 4 the dimensions of our problem in the simpler case where there are no type A and B tasks; notice that Smart Lou Quila contains to date $|N| = 17$ members including three BSS owners, if the time horizon is sliced into periods of 30 minutes, $|H| = 48$ for one day. Comparing the two

tables, we see that only the algorithm of Fischetti et al. (2018) has been tested on instances of similar dimensions. Unfortunately, a strong assumption of that algorithm is that there be no leader’s continuous variables in followers’ problem, as mentioned in the column “Assumption” of Table 3. Thus, their algorithm is not applicable to our problem, and we further see on the table that each other algorithm also requires an assumption that is violated by our model.

In addition to this, we should note that even solving the followers’ problem can be difficult on its own, as we sometime have issues to reach convergence. Therefore, it is not reasonable to hope for an exact algorithm for the overall bilevel problem. Instead, we present in the next section an ad-hoc math-heuristic algorithm adapted to the problem we face in this paper.

4 Iterative heuristic

Since the bilevel program (1) can hardly be solved exactly for the large instances, we introduce a heuristic algorithm. Each member determines local schedules and sends information about power requirements and availability to CC, the intermediary between members. Before presenting our distributed framework, we present the local scheduling model solved by each member. The local scheduling model for each member $i \in N$ is the following:

$$\begin{aligned} \max \quad & \tilde{G} \\ \text{s.t.} \quad & (5), (6), (7a), (8), (9). \end{aligned} \tag{12a}$$

Model (12) does not contain constraint (7b) since the prosumer’s gain depends on the ability to exchange energy. The members do not have information on the community’s state in the distributed scheme. As a result, consumers do not know when to consume, and who sends the received energy. Thus, at each iteration, CC will provide the information F^{RCom} to incentivize the members to shift their consumption to period $h \in H$ where F_{ih}^{RCom} exceeds zero.

4.1 Allocation keys

Allocation keys are traditionally used as an after-the-fact means of distributing financial gains among members who inject their surplus during one month. Used a posteriori, allocation key is not an incentive since members cannot decide at this point. In this paper, we use allocation keys for day-ahead scheduling. We use this notion to incentivize members to modulate their consumption during their local day-ahead planning. Therefore, in this paper, the allocation

key is an algorithm that returns values that indicate to each member the maximum amount of energy she can collect from the community.

Definition 1. *An allocation key is an algorithm \mathbf{K} that, given C_{ih} and F^{out} , computes F_{ih}^{RCom} for $i \in N, h \in H$.*

The key allows CC to indicate the energy availability state to each member without sharing private information with the members. Several a posteriori keys exist in the literature Roy et al. (2023), some currently used in the French electrical power system. Among the keys in the literature, we have the allocation in prorate to consumption, production, investment, etc. The a posteriori key used in Smart Lou Quila inspires **K1**. We introduce a second key, denoted **K2**, that fits well with day-ahead scheduling.

- **Key K1.** Presented in Algorithm 3 for $l = 1$, this key shares the energy according to members' demand. Lines 7–10 in Algorithm 3 address the case where the total injection $\sum_{i \in N} F_{ih}^{\text{out}}$ exceeds the demand $\sum_{i \in N} C_{ih}$ for $h \in H$. Each member i receives C_{ih} at period h . In lines 14–33, we consider the case where each member can receive the minimum energy required in the community C^{min} at a given period h , i.e., if $|N| \cdot C^{\text{min}} \leq \text{residual}$. In that case, we allocate C^{min} to members. We remove the satisfied members and share the *residual* energy among the remaining members. In lines 31–33, the residual is insufficient to serve each member, i.e., $u \cdot C^{\text{min}} \geq \text{residual}$, where u is the number of unsatisfied members, so we share the *residual* uniformly among the remaining members. In lines 36–37, we handle cases where the energy available is insufficient to satisfy the total need and not sufficient to allocate C^{min} to each member, so we share the energy available uniformly between members. Finally, in lines 40–42, value F_{ih}^{RCom} equals zero for each member i if the total injection equals zero at period h .
- **Key K2.** This key consists in determining the candidates for the energy reception and then sharing (uniformly) the power between these members. Member i is a candidate at period h if she does not inject energy, i.e., if $F_{ih}^{\text{out}} = 0$. After receiving all members' information, CC determines at each period who are the individuals likely to receive energy and shares the energy available uniformly among these members as detailed in Algorithm 1 where σ_{ih} is equal to 1 if member $i \in N$ is a candidate to receive energy from other members at period $h \in H$, in other words, if $F_{ih}^{\text{out}} = 0$. Value σ_h^{Tot} is the number of candidates to receive energy from the community at period $h \in H$.

Algorithm 1: Key K2

Input : F^{out}
Output: initial F^{RCom}
 $\sigma_h^{\text{Tot}} = |N|$; $\sigma_{ih} \leftarrow 1$ for
 $i \in N, h \in H$
for $h \in H$ **do**
 for $i \in N$ **do**
 if $F_{ih}^{\text{out}} > 0$ **then**
 $\sigma_{ih} \leftarrow 0$
 $\sigma_h^{\text{Tot}} \leftarrow \sigma_h^{\text{Tot}} - 1$
 for $i \in N$ **do**
 $F_{ih}^{\text{RCom}} \leftarrow \sigma_{ih} \sum_{i' \in N} F_{i'h}^{\text{out}} / \sigma_h^{\text{Tot}}$

Algorithm 2: Update K2.

Input : $F^{\text{in}}, F^{\text{out}}$ and F^{RCom}
Output: updated F^{RCom}
 $\sigma_h^{\text{Tot}} = |N|$; $\sigma_{ih} \leftarrow 1$ for
 $i \in N, h \in H$
for $h \in H$ **do**
 for $i \in N$ **do**
 if $F_{ih}^{\text{RCom}} - F_{ih}^{\text{in}} > 0$ **then**
 $\sigma_{ih} \leftarrow 0$; $F_{ih}^{\text{RCom}} \leftarrow F_{ih}^{\text{in}}$
 $\sigma_h^{\text{Tot}} \leftarrow \sigma_h^{\text{Tot}} - 1$
 for $i \in N$ **do**
 $F_{ih}^{\text{RCom}} \leftarrow$
 $F_{ih}^{\text{RCom}} + \sigma_{ih}(F_h^{\text{out}} - F_h^{\text{in}}) / \sigma_h^{\text{Tot}}$

We set $F_h^{\text{in}} = \sum_{i \in N} F_{ih}^{\text{in}}$ and $F_h^{\text{out}} = \sum_{i \in N} F_{ih}^{\text{out}}$ for $h \in H$ for simplicity in Algorithm 2.

- **Key K3.** Key **K1** allocates energy even to members who inject energy since it is based on consumption, and there is no test on prosumers' injections. Thus, a prosumer can simultaneously receive and inject energy at certain periods. To avoid simultaneous injection and extraction, we combine **K1** and **K2** in a key called **K3**. Key **K3** is presented in Algorithm 3 for input $l = 3$. We add some lines into **K1** that allow determining the candidates as defined previously. Thus, **K3** allocates energy according to consumption to members who need energy.
- **Key K4.** This key consists of sharing the surplus to promote consumption between needy members. Thus,

$$F_{ih}^{\text{RCom}} = \frac{\sigma_{ih} C_{ih}}{\sum_{i' \in N: \sigma_{i'h} > 0} C_{i'h}} \sum_{i' \in N} F_{i'h}^{\text{out}},$$

where σ_{ih} equals one if member i is candidate to receive energy as defined in **K2**.

- **Key K5.** This key consists in sharing the surplus between the members who need energy in prorate to their investment in storage and generation tools. Thus, let us note Σ_i as member i 's total investment to acquire some tools,

$$F_{ih}^{\text{RCom}} = \frac{\sigma_{ih} \Sigma_i}{\sum_{i' \in N: \sigma_{i'h} > 0} \Sigma_{i'}} \sum_{i' \in N} F_{i'h}^{\text{out}}.$$

Notice that **K1** and **K3** are fair in the sense of Joseph et al. (2016), i.e., the member who

Algorithm 3: key K1 and K3 .

Input : C, F^{out} and $l \in \{1, 3\}$
Output : F^{RCom}

- 1 **if** $l = 3$ **then** $\sigma_h^{\text{Tot}} = |N|$ for $h \in H$; $\sigma_{ih} = 1$ for $i \in N, h \in H$
- 2 **for** $h \in H$ **do**
- 3 **if** $\sum_{i \in N} F_{ih}^{\text{out}} > 0$ **then**
- 4 **for** $i \in N$ **do**
- 5 **if** $F_{ih}^{\text{out}} > 0$ **and** $l = 3$ **then**
- 6 $\sigma_{ih} \leftarrow 0$; $\sigma_h^{\text{Tot}} \leftarrow \sigma_h^{\text{Tot}} - 1$; $C_{ih} \leftarrow +\infty$
- 7 **if** $\sum_{i \in N} F_{ih}^{\text{out}} \geq \sum_{i \in N} C_{ih}$ **then**
- 8 **for** $i \in N$ **do**
- 9 **if** $l = 1$ **then**
- 10 $F_{ih}^{\text{RCom}} \leftarrow C_{ih}$
- 11 **else**
- 12 $F_{ih}^{\text{RCom}} \leftarrow C_{ih} \cdot \sigma_{ih}$
- 13 **else**
- 14 $C^{\min} \leftarrow \min_{i \in N}(C_{ih})$
- 15 **if** $l = 1$ **then** $u \leftarrow |N|$ **else** $u \leftarrow \sigma_h^{\text{Tot}}$
- 16 **if** $u \cdot C^{\min} \leq \sum_{i \in N} F_{ih}^{\text{out}}$ **then**
- 17 **for** $i \in N$ **do**
- 18 $\text{if } l = 1 \text{ then } idx_i \leftarrow i \text{ else } idx_i \leftarrow i \cdot \sigma_{ih}$
- 19 $residual \leftarrow \sum_{i \in N} F_{ih}^{\text{out}}$
- 20 **while** $u \cdot C^{\min} \leq residual$ **do**
- 21 $m \leftarrow 0$
- 22 **for** $i \in N$ **do**
- 23 **if** $idx_i \neq 0$ **then**
- 24 $F_{ih}^{\text{RCom}} \leftarrow F_{ih}^{\text{RCom}} + C^{\min}$
- 25 **if** $C_{ih} = C^{\min}$ **then**
- 26 $m \leftarrow m + 1$; $idx_i \leftarrow 0$; $C_{ih} \leftarrow +\infty$
- 27 **else**
- 28 $C_{ih} \leftarrow +C_{ih} - C^{\min}$
- 29 $residual \leftarrow residual - u \cdot C^{\min}$
- 30 $u \leftarrow u - m$; $C^{\min} \leftarrow \min_{i \in N}(C_{ih})$
- 31 **for** $i \in N$ **do**
- 32 **if** $idx_i \neq 0$ **then**
- 33 $F_{ih}^{\text{RCom}} \leftarrow F_{ih}^{\text{RCom}} + (residual/u)$
- 34 **else**
- 35 **for** $i \in N$ **do**
- 36 **if** $l = 1$ **then**
- 37 $F_{ih}^{\text{RCom}} \leftarrow (\sum_{i \in N} F_{ih}^{\text{out}} / u)$
- 38 **else**
- 39 $F_{ih}^{\text{RCom}} \leftarrow (\sum_{i \in N} F_{ih}^{\text{out}} / u) \cdot \sigma_{ih}$
- 40 **else**
- 41 **for** $i \in N$ **do**
- 42 $F_{ih}^{\text{RCom}} \leftarrow 0$

consumes the less is favored to the one who consumes the most while sharing the surplus. Note that by using **K2**, some members needing energy might receive more than required at certain times since the surplus is allocated uniformly, hence the need to update values F_{ih}^{RCom} at each iteration to ensure more efficient management using Algorithm 2. Indeed, CC must reallocate F_h^{rest} . We update **K2** as presented in Algorithm 2. Member i will not receive more energy than F_{ih}^{in} at period h if $F_{ih}^{\text{RCom}} - F_{ih}^{\text{in}} > 0$. Let us denote $F_h^{\text{out}} = \sum_{i \in N} F_{ih}^{\text{out}}$ $h \in H$ and $F_h^{\text{in}} = \sum_{i \in N} F_{ih}^{\text{in}}$ $h \in H$ for each period $h \in H$. Thus, $F_h^{\text{rest}} = F_h^{\text{out}} - F_h^{\text{in}}$, is the amount of remaining energy after each iteration using **K2**.

4.2 The overall algorithm

Each member initially solves the load scheduling MILP with no energy exchanges between the members, i.e., $F_{ih}^{\text{RCom}} = 0$ for $i \in N, h \in H$. That allows members to determine their initial periodic energy needs and availability. The members then send their injection and need information to CC, who shares the energy available using the allocation key, sends values F_i^{RCom} to each member who updates the local schedules, and so on until the community reaches an acceptable level for the day's operations. Algorithm 4 presents the distributed management framework.

Algorithm 4: Distributed management model.

Input : $\mathbf{K} \in \{\mathbf{K1}, \mathbf{K2}, \mathbf{K3}, \mathbf{K4}, \mathbf{K5}\}$

Output: F^{RCom} and the local best schedules

Step 0: $F_{ih}^{\text{RCom}} \leftarrow 0$ for $i \in N, h \in H, \text{iter} \leftarrow 1$

Each member i solves (12) (in parallel) and sends (F_i^{out}, C) to CC; $F_h^{\text{rest}} \leftarrow F_h^{\text{out}}$

Step 1: CC determines F^{RCom} using key \mathbf{K} and sends F_i^{RCom} to member i who solves (12) for $i \in N, \text{iter} \leftarrow \text{iter} + 1$

if $\mathbf{K} = \mathbf{K2}$ **then**

while $\text{iter} \leq \text{maxIter}$ **do**

CC updates F^{RCom} using Algorithm 2 and sends F_i^{RCom} to member $i \in N$

Each member i solves (12) and sends $(F_i^{\text{out}}, F_i^{\text{in}})$ to CC, $\text{iter} \leftarrow \text{iter} + 1$

Step 2: Manage F^{rest} which corresponds to variable I , determine τ for gain sharing.

At the end of the second step, if there exists a period h where $F_h^{\text{rest}} \neq 0$, the amount F_h^{rest} is injected into the grid, and the economic gain associated to F^{rest} is shared between the prosumers, **Step 2** refers to that in Algorithm 4. We do not address gain-sharing aspect in this paper. Observe that, for $\mathbf{K} = \mathbf{K2}$, the members' local solutions do not change after at most $\text{maxIter} = |N| + 1$ if each local problem is optimally solved at each iteration. Indeed, at

each iteration, a group of $l \in \{1, \dots, |N|\}$ members is satisfied and removed from the set of candidates to receive energy all over the horizon. These members will not receive more energy over the horizon: their solutions stay the same, and F_h^{rest} is allocated to the candidates $h \in H$ who optimally solve their problem. The worst case occurs if $l = 1$ at each iteration; hence $\text{maxIter} = |N| + 1$ iterations with the initial iteration.

Remark 1. *As each member solves the optimization problem (12) after receiving the leader's incentive, the heuristic's solution is feasible for the bilevel formulation (1).*

4.3 The remaining energy and storage

The members that own BSS should be encouraged to store energy at certain periods of the planning horizon. Indeed, since CC maximizes the consumption of locally produced energy and the member the local profit, the prosumers can inject more energy than required to satisfy the global energy need. If $\sum_{i \in N} F_{ih}^{\text{out}}$ is greater than the energy requirement at period h , a portion of it can be consumed in the community, and the other is injected into the main grid (MG) or stored. Being decentralized, prosumers and CC cannot foresee what portion is consumed locally. Therefore, CC can only provide storage incentives when needed. Thus, members owning a BSS are encouraged to store if the need is less than the energy available or if CC wants the community to provide energy to the main network. The storage incentive must prioritize local consumption; hence, there is interest in setting limits to the maximum a member who owns a battery can store. Let us introduce ρ_{ih} as the value which limits member $i \in N$ injection at period $h \in H$. We assume that ρ_{ih} is proportional to quantity depending on member $i \in N$'s physical characteristic in terms of asset owing plus a variable a quantity depending on the member i 's participation in the global production, which multiplies the global need.

$$\rho_{ih} = \delta_i + \frac{P_{ih}^{\text{Gen}}}{\sum_{i' \in N} P_{i'h}^{\text{Gen}}} C_h \quad h \in H. \quad (13)$$

We set ρ_{ih} for each $i \in N, h \in H$ such that it is proportional to the energy need and inversely proportional to the community's generation as presented in constraint (13), where δ_i is an ad-hoc constant determined by trial and error. This makes sense since we assume that the self-discharge rate of the batteries is low. We add constraints (14) to the members' model (12).

$$F_h^{\text{out}} \leq \rho_{ih} \zeta_i + \pi_i (1 - \zeta_i) \quad h \in H, \quad (14)$$

where ζ_i is equal to 1 if member $i \in N$ owns a storage unit, 0 otherwise and δ_i a physical parameter related to member i 's generation and storage systems' size. Figure 5 presents the periodic F^{rest} and $Need$ before and after adding constraints (14).

5 Numerical experiments

We use a realistic instance based on Smart Lou Quila's demonstrator. The planning horizon consists of a day sliced into 24 equal-length periods. The first six members' real asset-owing characteristics are presented in Table 5. The last member does not have any assets. Members subscribed power are respectively $\pi = (36\ 9\ 6\ 9\ 9\ 6\ 9)$ kVa.

Member 1		Member 2	
Photovoltaic (PV)	Yes	PV	Yes
PV Capacity	6.12kWp	PV Capacity	3.2kWp
Battery	Yes	Battery	Yes
Number	1 unit	Number	1 unit
Capacity	9.8kWh	Capacity	9.8kWh
Initial state of charge	4.5kWh	Initial state of charge	4.5kWh
Efficiency	97.5%	Efficiency	97.5%
Power	5kW	Power	3.7kW
Periodic discharge rate	1%	Periodic discharge rate	1%
Member 3		Member 4	
PV	No	PV	Yes
PV Capacity	0kWp	PV Capacity	3.2kWp
Battery	yes	Battery	No
Number	1 unit	Number	0 unit
Capacity	9.8kWh	Capacity	0kWh
Initial state of charge	4.5kWh	Initial state of charge	0kWh
Efficiency	97.5%	Efficiency	0%
Power	3.7kW	Power	0kW
Periodic discharge rate	1%	Periodic discharge rate	0%
Member 5		Member 6	
PV	Yes	PV	Yes
PV Capacity	3.2kWp	PV Capacity	3.2kWp
Battery	No	Battery	No

Table 5: Production and storage assets description in the community.

We use the energy generation data collected on July 23th 2022. Figure 2 presents each prosumer's generation and the community's total periodic production. The members' loads are artificially generated. In other words, we generate loads of the different types: A, B, and C. The following array ν shows each member's requested type A tasks. We consider two different type A tasks, i.e., $|J_{iA}| = 2 \forall i \in N; \forall j \in J_{iA}, j = 1$ means the corresponding task is room

temperature regulation and $j = 2$ means that the requested task is water heating. Finally, ν_{ijk} is equal to 1 if task $j \in J_{iA}$ is executed by member i in room k , 0 otherwise. We refer to Sangaré et al. (2023) for more details:

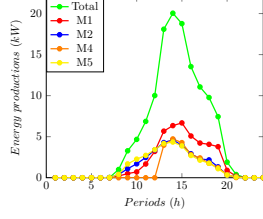


Figure 2: Generation on July 23th 2022.

$$\nu_1 = \begin{pmatrix} \mathbf{1} & \mathbf{1} \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \quad \nu_2 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} \quad \nu_3 = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} .$$

Table 6 presents members 2 to 7 houserroom characteristics. For simplicity we assume that each room k of each member has an initial temperature equal to 22°C and that the temperature of these rooms must lie in the interval $[t_{ik}^{\text{low}}, t_{ik}^{\text{up}}]$ during the time interval $[34, 48]$.

$$t^{\text{low}} = \begin{pmatrix} 20 & 19 & 20 & 20 & 21 & 20 & 19 \\ 20 & 19 & 20 & 20 & 20 & 20 & 19 \\ 20 & 19 & 20 & 20 & 20 & 20 & 19 \end{pmatrix} \quad t^{\text{up}} = \begin{pmatrix} 25 & 25 & 25 & 25 & 25 & 25 & 25 \\ 25 & 25 & 25 & 25 & 25 & 25 & 25 \\ 25 & 25 & 25 & 25 & 25 & 25 & 25 \end{pmatrix}$$

Each member has a single water heater, and Table 7 presents their physical characteristics. Parameter $c_p = 1$ is the specific heat capacity of water. Column M shows the weight of the water (kg), column S designates the exchange surface of the water container with the external area, K is the exchange coefficient (kcal/h·m²·°C). Finally, column \bar{T}^{water} presents the initial temperature of each member's water heater. The members' water must reach 60°C at period 16. The efficiency of the electricity-to-heat transformation $v = Mc_p / (Mc_p r + \Delta SKr)$. Also, for simplicity, we assume that p_{ih}^c which is member $i \in N$ non-controllable consumption at period h , is equal to one for each member at each period.

Member	room	Surface	C_r	U
Member 2	room 1	$9m^2$	297	12
	room 2	$15m^2$	495	20
	room 3	$18m^2$	594	24
Member 3	room 1	$18m^2$	594	24
	room 2	$9m^2$	297	12
	room 3	$9m^2$	297	12
Member 4	room 1	$12m^2$	396	16
	room 2	$20m^2$	660	26.6
	room 3	$12m^2$	396	16
Member 5	room 1	$25m^2$	825	33.3
	room 2	$10m^2$	330	13.3
	room 3	$12m^2$	396	16
Member 6	room 1	$18.5m^2$	610.5	24.6
	room 2	$9m^2$	297	12
	room 3	$10m^2$	330	13.3
Member 7	room 1	$15m^2$	495	20
	room 2	$10m^2$	330	13.3
	room 3	$14m^2$	462	18.6

Table 6: Rooms physical characteristics.

Member	S	M	\bar{T}^{water}	K	r
1	7	350	15	1	1.2
2	1.5	75	17	1	1.2
3	2	100	18	1	1.2
4	3.75	200	19	1	1.2
5	2.4	150	10	1	1.2
6	2	100	10	1	1.2
7	2.6	150	8	1	1.2

Table 7: Water heaters characteristics.

5.1 Keys comparison

We present our numerical results in this section. First, we compare the key calculation methods and an illustration of the need for CC to control storage. Then, we show the impact of this control on the distributed solution. Each member’s local scheduling problem (12) is reformulated using Special Ordered Set variables as presented in Section 7.1 of Sangaré et al. (2023) with a time limit `time_limit = 20` seconds. Table 8 presents the distributed solution for our instance. We do not update **K2**. In this table, Column **Avail** is the amount of photovoltaic energy that can be shared in the community or injected into the grid, and column **Loss** is part of **Avail** that needs to be injected into the grid. Table 8 shows that the distributed management with keys **K2**

Key	Distributed solutions			
	obj kWh	CPU	Available kWh	Loss kWh
K1	125.62	0.20	27.97	13.78
K2	115.50	0.20	27.97	3.65
K3	117.96	0.20	27.97	6.11
K4	115.50	0.25	27.97	3.65
K5	119.30	0.25	27.97	7.44

Table 8: Key comparison.

and **K4** returns the best solutions, i.e., ensures more local consumption of green energy than the other keys. Since initial consumption is determined without information, consumers consume randomly to meet their initial optimum. Therefore, a sharing scheme based on consumption will not incentivize these members to shift the load. On the other hand, sharing based on consumption leads to allocating energy to members who do not need it. Figure 3 shows the different keys' consumption shifts during the planning horizon after $|N| + 1$ iterations for key **K2**, and two iterations for **K1** and **K3**. Curves **Available** and **Used** represent respectively

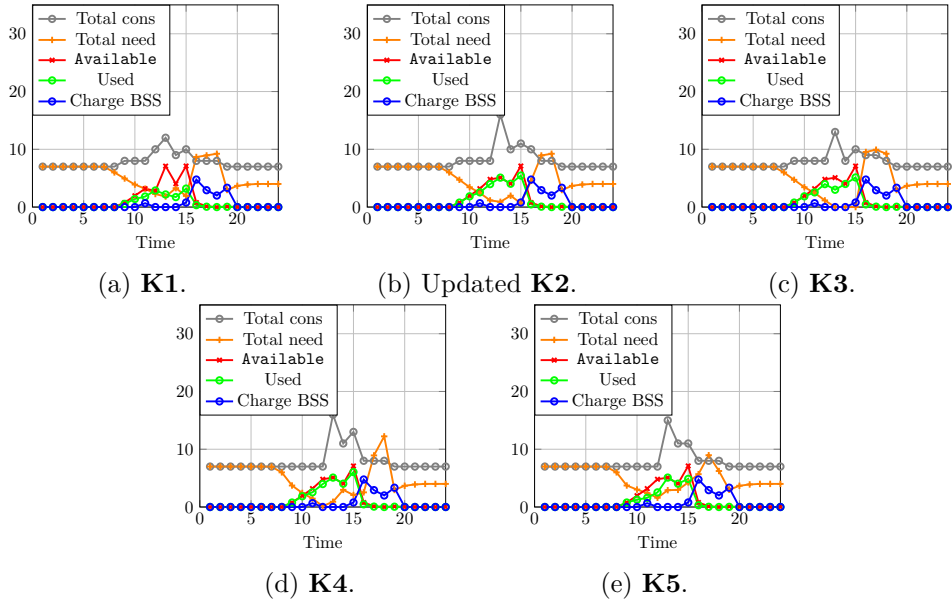


Figure 3: Demand response illustration.

the amount of green energy injected by prosumers and the amount of green energy available that is locally consumed. **Total need** represents the total energy needed in the community for direct consumption or to be stored for ulterior consumption. The area between curves **Available** and **Used** is the energy injected into the main grid. It corresponds to column **Loss** in Table 8. Allocating energy to members who need it reduces the area between available and Used curves, leading to a good distributed solution.

Table 9 presents the solution instance using updated **K2**. Iteration 1 in column 1 corresponds to the case where $F^{\text{RCom}} = 0$. Each column shows each member’s total energy collected from the grid during the planning horizon, one row per member. Row **Sum** presents the energy collection for the whole community during the planning horizon.

iteration \ Members	1	2	3	4	5	6	7	8
1	8.71	8.13	8.13	8.13	8.13	8.13	8.13	8.13
2	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6
3	12	9.86	9.69	9.64	9.64	9.63	9.63	9.63
4	13.52	12.39	12.39	12.39	12.39	12.39	12.39	12.39
5	34	26.84	26.21	25.75	25.66	25.58	25.56	25.54
6	31	23.84	23.21	22.75	22.66	22.58	22.56	22.54
7	34	26.84	26.27	26.09	26.06	26.04	26.03	26.03
Sum	139.83	115.5	113.5	112.35	112.14	111.94	111.9	111.86

Table 9: Distributed management solution.

Updating keys provides good solutions for distributed management since it leads to better allocation. Consequently, consumers shift more loads at available energy periods leading to a good solution since the objective value of the centralized optimum solution is 111.16 kWh. However, there is a need to incentive storage if the energy to be shared is important. Indeed, since the prosumers maximize their profit, they inject more at periods where the prices lead to a maximum profit. This results in too much injection than required at certain periods. For example, if we increase the total production $\sum_{i \in N} P_{ih}^{\text{Gen}}$ for $h \in H$ by 50%, heuristic leads to a solution with 79.34 kWh versus 64.02 kWh the optimal centralized solution’s objective value, which is an increase of 24%. The remaining energy F^{rest} is presented by Figure 4. We notice a lot of green energy available in periods 10 to 17 while no energy is needed in the whole community (the opposite situation occurs after period 17). That is due to the lack of global perception of the community’s state for prosumers. CC could have incentive members owning batteries to store energy during periods 10 to 17 for ulterior use.

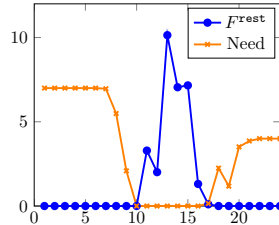


Figure 4: Storage requirement illustration.

5.2 Storage control

As shown in Figure 4, there is a need to incentivize the prosumers owing a BSS to store energy at certain periods. We express this incentive as a limitation of the periodic injection of these members. Indeed, if they cannot inject, they will store for the next periods. We introduce constraint (14) to achieve it. Then, at each iteration, CC calculates ρ_{ih} for $i \in N, h \in H$. In this paper, we set δ_i proportional to the size of member $i \in N$ compared to the other generating members who own the BSS. Each member i owing a BSS has an initial $\delta_i = 1$ kWh. Suppose member i' produces twice as much as the other members, $\delta_{i'} = 2$ kWh. Three members own the BSS, namely members 1, 2, and 3, according to the previous affirmation $\delta = (2 \ 1 \ 0 \ 0 \ 0 \ 0)$. We set each member's local scheduling time limit to 50 seconds at each iteration.



(a) Before storage control: objective is 79.37 kWh, energy injected into MG is 31.10 kWh.

(b) After storage control: objective 68.13 kWh, energy injected into MG is 14.65 kWh.

Figure 5: Storage control's impact.

Since each member maximizes the profit, without injection limitation, the prosumers will directly inject their surplus. That leads to important energy injected into the main grid (the area between the red and green curves, which corresponds to the curve F^{rest} respectively in Figures 5a and 5b). Limiting prosumers owing BSS makes these members stock to reinject at other times, allowing better solutions, as displayed in Figure 5b.

6 Case study

This section presents the results of accurate historical data from Smart Lou Quila. Since we do not know the actual load classification, we assume each member's loads belong to the type C, i.e., no consumption modulation is expected. We illustrate the impact of having a storage unit in such a community by comparing the results with and without storage units. When there is no storage, each member solves (12) with all scheduling variables fixed except $C_{ih}, F_{nh}^{\text{out}}$ and F_{nh}^{in} for $n \in N, h \in H$. Otherwise, the variables related to storage are not fixed in addition to these previous variables.

We collect the generation and load data on a monthly basis for each instance. Specifically, we build an instance for each month from January to September 2023. The data are collected every 30 minutes over the month. Figure 7a illustrates Smart Lou Quila’s total generation and consumption over August 2023. We have three BSS owners, as shown in Table 2. We set the maximum cycle to $\phi = 10$ for each BSS owner. The data and models presented in the paper are available on GitHub. Each local MILP is solved with `time_limit = 50s` at each iteration.

Figure 6 presents the objective function for distributed and centralized management. We compute the centralized solution using the MILP of Sangaré et al. (2023) with `time_limit = t_l = 3600s`. The distributed management heuristic returns high-quality solutions compared to the centralized solution. Table 10 presents more detail on the solutions; in this table, columns Month and $|N|$ present the time horizon and Smart Lou Quila’s demonstrator’s size over the horizons. Notice that Smart Lou Quila contains five members who generates energy in 2023. In the same line, column P^{Gen} and Demand present respectively the total renewable generation and energy consumption over each horizon; Figure 7a presents the generation and consumption profiles over May 2023. Values Obj give the total amount of energy collected from the grid over the horizon. Finally, column GAP presents the gap between the best-distributed value and the centralized solution with storage units.

Remark 2. *A centralized solution is a solution of the problem where the coordinator solves a single MILP with all members’ variables and constraints, and peer-to-peer exchanges are allowed.*

Table 10: Solutions comparison (period length: 0.5h).

Month	$ N $	P^{Gen} (kW)	Demand(kW)	With storage units						Without storage units	
				Centralized solutions			Distributed solutions			Distributed solution	
				Obj(kWh)	gap(%)	CPU	K1	K2	GAP%	K1	GAP%
Jan	8	397	33016	16310	0	2	16315	16310	0	16310	0
Fev	8	752	26989	13118	0	2	13157	13122	0.03	13155	0.28
Mar	8	1493	33016	15761	0	2	15797	15769	0.05	15810	0.31
Avr	8	2354	11522	4679	0.86	t_l	4924	4807	2.72	5028	7.46
May	8	2363	8795	3396	3.58	t_l	3605	3533	4.04	3741	10.16
Jun	13	1944	12997	5529	0	71	5632	5560	0.57	5659	2.35
Jul	17	2017	16027	7009	0	10	7091	7031	0.32	7117	1.54
Aug	17	1781	16094	7160	0	96	7246	7191	0.44	7273	1.58
Sep	17	1581	15143	6782	0	11	6841	6803	0.32	6863	1.19

From Table 10, **K2** returns the best solutions while operating with BSS. This because it allows storage for ulterior usage. Indeed, **K2** shares uniformly, if a member i owning a BSS receives at period h an amount F_{ih}^{RCOM} greater than her consumption, i.e., $F_{ih}^{\text{RCOM}} \geq C_{ih}$, she can store the excess for her ulterior usage; the sum of the periodic state of charge over May 2023

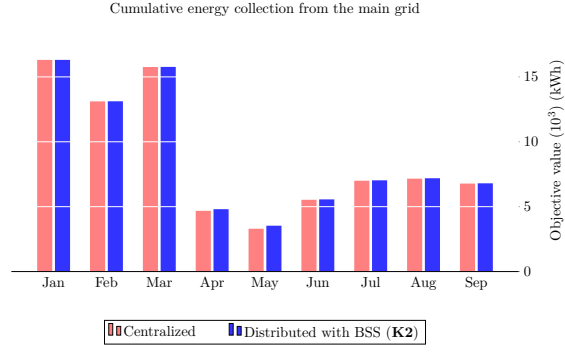
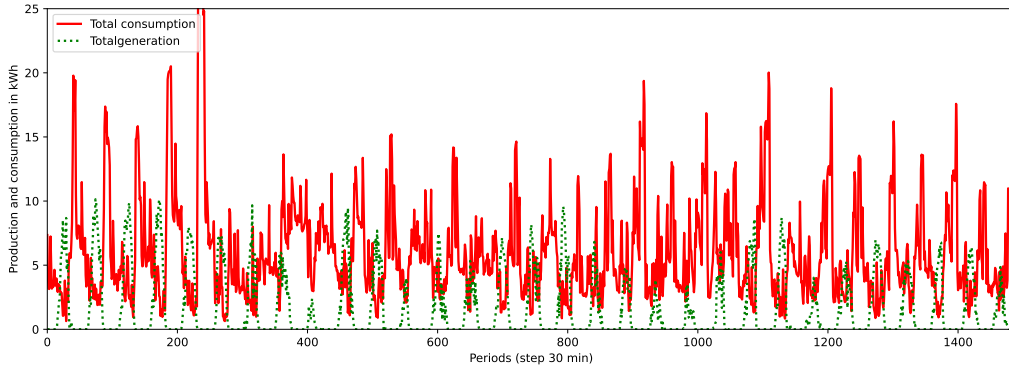
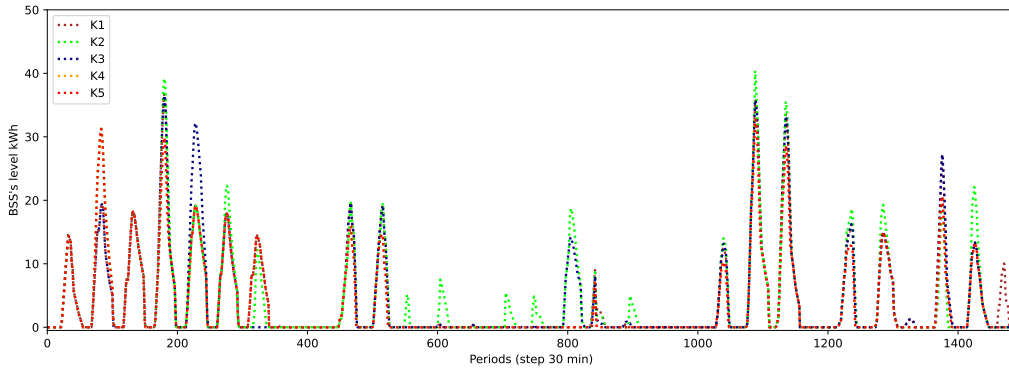


Figure 6: Distributed management solution's quality.



(a) Inputs



(b) Total BSSs' level

Figure 7: Instance for may 2023.

per key is: (5557, 6154, 4994, 5107, 5943) kWh.

Experience has shown that, unlike the conventional approach, proactive management can effectively establish DR in a community. However, its effectiveness depends on the surplus allocation rules. We recall that fairness in this work consists in prioritizing the member who consumes the least when sharing the energy surplus. In this case, the keys **K2**, **K4**, **K5** are not

fair in the sens of Joseph et al. (2016).

Notice that we do not address the pricing issue in this work, i.e., we do not consider how prices \tilde{v}_h^{Com} and v_h^{Com} for $h \in H$ are set. CC determines these prices for the Smart Lou Quila scheme. The aim is to maximize local consumption of locally produced energy correlated to CC's margin. Therefore, the exchange prices in the community must be more attractive, i.e., the selling in the community must be higher than the price at which the aggregator takes the excess, and the buying prices in the community must be lower than the buying price from MG. That is why we use the following prices in euros per kWh:

$$\begin{aligned} v_h^{\text{MG}} = v_h^{\text{GES}} &= 0.1685, \tilde{v}_h^{\text{GES}} = 0.065 \\ v_h^{\text{Com}} &= 0.1400, \tilde{v}_h^{\text{Com}} = 0.12. \end{aligned}$$

where \tilde{v}^{GES} is the price of selling the surplus to the aggregator. These prices must also be sufficient to amortize the marginal cost of using the BSS if we limit the injections.

7 Conclusion

In this paper, we present a distributed management method for large energy communities, which leads to some reasonable solutions compared to centralized solutions. Furthermore, we introduce an efficient allocation key for a priori management, which leads to high quality solutions compared the optimal centralized solutions on real world data.

Some critical perspectives of this work are electric mobility and grid services. Concerning mobility, for example, we would like to charge the community's electric vehicles with the quantities injected into the grid, considering the travel schedule. As for grid services, we can imagine a form of collaboration between CC and the main grid, which would enable the community to supply neighboring households or communities, depending on the state of the overall grid. Finally, the system includes uncertainty at several levels; we have handled the deterministic case for simplicity's sake. Future work may consider them.

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