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# A Decision Model Based on an Optimized Choquet Integral: Multifactor Prediction and Intelligent Agriculture Application

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## Abstract

Smart farming aims at improving agriculture production by using artificial intelligence and smart devices and, in continuity, farming optimization aims at supporting autonomous decision-making to maximize crop yield. In this context, the question of predicting the future days of growth stages transition of a plant is still a challenge, as existing automated predictions are not accurate nor reliable enough to be effectively used in the farming process. We propose here an approach based on Choquet integral, performing an aggregation of multiple imperfect predictions into a more accurate and reliable one, considering the specific relevance of various prediction sources as well as interactions, synergies, or redundancies between them. To identify the numerous parameter values defining the Choquet-based decision model, we propose a generic approach of optimization based on observed history, ensuring a reduced sensitivity to parameters, thanks to a principle of less specificity. Our proposal defines so an evaluation function assigning to any potential solution a predictive capability, quantifying the conformance of its outputs to evidence, as well as an associated optimization process based on the satisfaction degrees regarding a set of stated inequalities. The case study concerns an implemented prototype that enables, for a given culture and several input sources, to help farmers, providing them with better predictions of the growth stages. We also analyze the reliability of the process, enabling the assignment of an objective probabilistic criteria to any provided prediction. The experimental results are very encouraging, the predicted day remaining stable despite presence of noise and local errors.

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Keywords  
(separated by '-')

Choquet integral - Decision model - Parameters identification - Smart farming applications

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# A Decision Model Based on an Optimized Choquet Integral: Multifactor Prediction and Intelligent Agriculture Application

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**Abstract.** Smart farming aims at improving agriculture production by using artificial intelligence and smart devices and, in continuity, farming optimization aims at supporting autonomous decision-making to maximize crop yield. In this context, the question of predicting the future days of growth stages transition of a plant is still a challenge, as existing automated predictions are not accurate nor reliable enough to be effectively used in the farming process. We propose here an approach based on Choquet integral, performing an aggregation of multiple imperfect predictions into a more accurate and reliable one, considering the specific relevance of various prediction sources as well as interactions, synergies, or redundancies between them. To identify the numerous parameter values defining the Choquet-based decision model, we propose a generic approach of optimization based on observed history, ensuring a reduced sensitivity to parameters, thanks to a principle of less specificity. Our proposal defines so an evaluation function assigning to any potential solution a predictive capability, quantifying the conformance of its outputs to evidence, as well as an associated optimization process based on the satisfaction degrees regarding a set of stated inequalities. The case study concerns an implemented prototype that enables, for a given culture and several input sources, to help farmers, providing them with better predictions of the growth stages. We also analyze the reliability of the process, enabling the assignment of an objective probabilistic criteria to any provided prediction. The experimental results are very encouraging, the predicted day remaining stable despite presence of noise and local errors.

[AQ1](#)

**Keywords:** Choquet integral · Decision model · Parameters identification · Smart farming applications

## 1 Introduction

Rapid population growth has increased the demand for food production and hence the pressure among farmers to meet customer demand. We consider here the agricultural domain, a strategic area where, today, a major issue consists in increasing field productivity while respecting the natural environment and farms sustainability, requiring

the development of advanced decision support tools supporting the farmer's day-to-day decision process.

In this context, a key issue in maximizing crop efficiency is this of making reliable predictions regarding growth stage transitions of plants, especially dates of transitions from a present stage to the next one enabling the farmer to prepare and perform relevant actions at the best instant with a maximized impact on efficiency (e.g., triggering irrigation, adjusting water flow, adding precisely optimized quantity of intrants, ...). Such predictions should be based on various available sources of information, that deliver in practice more or less accurate and reliable information [9].

In this context, multi-criteria decision requires use of aggregation functions, such as fuzzy integrals, among those the well-known integral of Choquet. This latter is defined from a fuzzy measure representing not only the relative importance of the various criteria but also the possible interactions between them [2]. Choquet integral will be used here as an operator of aggregation, in charge of fusing multiple imperfect predictions into a more certain and accurate one, exploiting source diversity to get a more significant and reliable information for guiding decisions.

In this paper, we focus on the Choquet integral, proposing a parametric model for data aggregation. We put our interest on an application of smart farming, proposing a generic way to identify the parameters of a growth stage prediction model. We point out that in a previous work [7], we proposed a fuzzy decision support environment for smart farming ensuring better data structuration extracted from farms, and automated calculations, reducing the risk of missing operations.

Our aim here is to identify the parameters of a Choquet-based decision model using a training dataset including past data delivered by available sources jointed the corresponding observed evidence, proposing a set of optimized parameter values. Our proposal defines 1) a function enabling to evaluate the prediction capability of any potential solution based on a set of inequalities, standing for the evidence this model enables to satisfy, 2) an algorithm adapted from the classical gradient descent and providing a robust solution, 3) a way to objectively evaluate in operation the reliability of the made prediction in each case.

Our operator, based on Choquet integral, should apply ponderations to each information source, considering possible interactions, synergy complementarity, or, conversely, partial redundancy, between them. It will transform the input fuzzy sets delivered to sources into a new fuzzy set aggregating the input sources and delivering a global value of confidence based on several source-dependent inputs. The solution to our problem will be the optimum of our evaluation function, the obtained evaluation value quantifying both the ability of the solution to make right predictions and the robustness of this solution. This approach has been validated on data issued from a set of fields, on several years.

This paper focuses on a study in the agricultural domain aiming at improving smart technologies. The originality of our proposal relies on one hand on identifying a model only using conformance to stated inequalities, not requiring values of a function to be learnt, and on the other hand, on the robustness of the obtained solution due to its least specificity, reducing so the sensitivity to the defaults of training data.

The remainder of this paper is organized as follows. Section 2 presents the preliminaries and related works. Section 3 is dedicated to a formalization of our problem. Then, in Sect. 4, we detail our proposal and its main components, and present the algorithm enabling to identify the parameters of the Choquet integral. Section 5 presents a model of decision based on our approach and then, the obtained numerical results on the chosen case study and their interpretation are presented and discussed in Sect. 6. Finally, we conclude and present our future work in Sect. 7.

## 2 State of the Art and Motivations

In this section, we first present preliminaries, and an overview of the related research concerning Choquet integral and decision models along with our motivations and objectives, and we shall then consider related works.

### 2.1 Smart Farming Applications and Systems: Motivations

The present issue falls into the developments of Smart Applications and Systems, that aims at meeting the requirements of next generation agricultural processes, and more especially these of precision farming [13]. Smart farming approach consists in deploying computer-based systems enabling the control of all actions on a culture, based on information collected from terrain. The objective is to provide a smart support to the farmer, executing automatized actions at the right time (e.g., watering), or informing them of actions to be manually performed (e.g., adding a specified quantity of intrans).

In this context, a key aspect in smart farming is this of building estimations of present and future states of development of the controlled culture, based on available pieces of information, processing and consolidating them to obtain the best possible picture of reality at  $t$  instant, as well as projections into the future. These estimations should be reconsidered and rightly updated at any time with arrival of new pieces of information.

We consider here the case of surveillance of a culture with prediction of growth stages (*phenological stages*) of a plant [10]. The plant being in a stage  $S$ , the objective is to predict the future day of transition  $d$  toward the next stage  $S'$ . Each existing source delivering its own imperfect prediction, the system must build a relevant aggregation of the delivered data to deliver a better prediction, i.e., a more reliable and more accurate one.

The overall functional architecture of our prediction process is presented on the figure below. We have three levels, with 1) sensor and data acquisition level (including terrain sensors, aircraft and satellite images acquisition, external data collection through web services invocation), 2) the level of prediction sources, i.e. the various algorithms in charge of performing experimental or more complex predictions, and at last 3) the global aggregation level. The most general case being this of  $n$  sources of prediction, we shall consider here, as an illustrative example, the case of 3 input sources (cf. Fig. 1).

Commonly used prediction sources are:

- Empirical « Degree Days » calculation [13], based on a cumulation of daily observed min-max temperatures differences, quantity that should have to meet a given known threshold, specific to plant and to considered phase.

- NDVI (Normalized Difference Vegetation Index), processed from visible and Infrared reflectance extracted from satellite images.
- Interpolation from statistical tables.

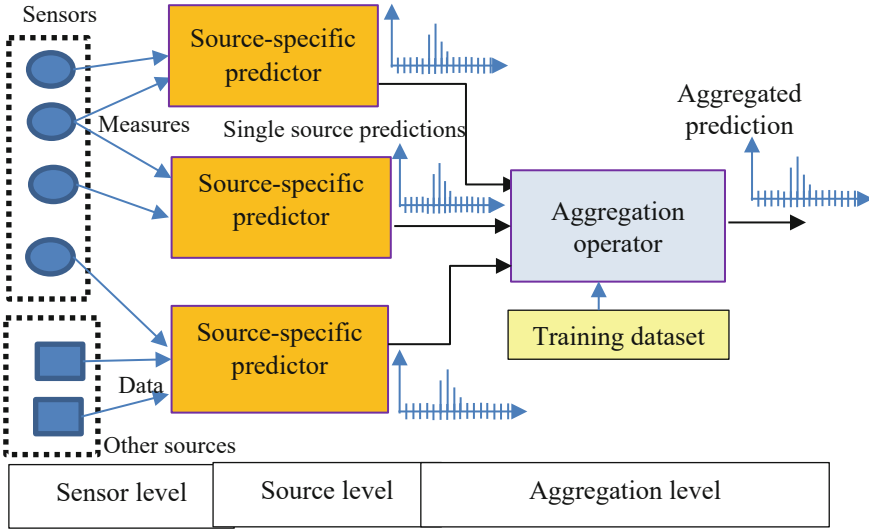


Fig. 1. Functional schema of growth stages prediction (from [9]).

An available dataset, containing evidence and past predictions, may be geometrically represented in of 2-dimensional factorial chart, thanks to a principal component analysis, giving typical results such as the simplified one (cf. Fig. 2).

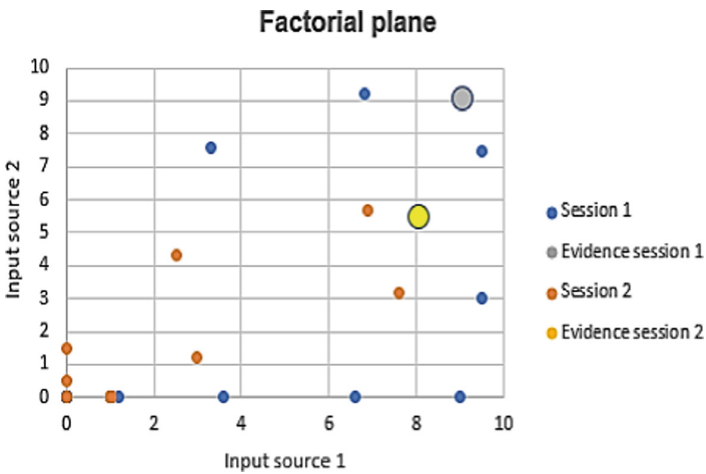


Fig. 2. Simplified representation of a Principal Component Analysis of a prediction dataset.

We can see there is no way to clearly separate transition and non-transition days by a simple hyperplane, or even by more complex classifiers, the reason being simply confidences delivered by sources are just relative values inside a given session of prediction (i.e., an instance of culture on a period), a level having no absolute quantitative meaning by itself.

## 2.2 Background: The Choquet Integral

Choquet integral is often adopted as multifactor aggregation operator, as it has an ability to properly model multiple types of interactions between sources, including partial redundancy and substitutability of some factors, with minimal hypothesis required on the nature of data. In addition, we must mention its simplicity of calculation to be performed at real time, as well as the human understandability of the various model parameters.

The notion of fuzzy integral, based on the concept of fuzzy measure [2], also called capacity, enables to assign a relative importance, not only to each individual decision criterion, but also for any subset of criteria. In the context of multi-criteria analysis, the weight or importance of the set of criteria association influences the entire combination of criteria, which could be also defined.

A capacity  $\mu$  on the set  $\mathcal{N} = \{1, \dots, n\}$  is a function from  $2^{\mathcal{N}}$  into  $\mathbb{R}^+$ , such as:

$$\mu(\emptyset) = 0 \quad (1)$$

$$\mu(\mathcal{N}) = 1 \quad (2)$$

$$\forall S \subseteq \mathcal{N}, \forall T \subseteq \mathcal{N}, S \subseteq T \Rightarrow \mu(S) \leq \mu(T) \text{ (monotony)} \quad (3)$$

The discrete Choquet integral related to the capacity  $\mu$  is then defined as the function that associates to any n-uple  $x = (x_1, \dots, x_n) \in \mathbb{R}^n$  a value  $C_\mu \in \mathbb{R}^+$ , defined by the following formula:

$$C_\mu(x_1, \dots, x_n) := \sum_{i=1}^n i = n(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i+1)})) \cdot x_{\sigma(i)} \quad (4)$$

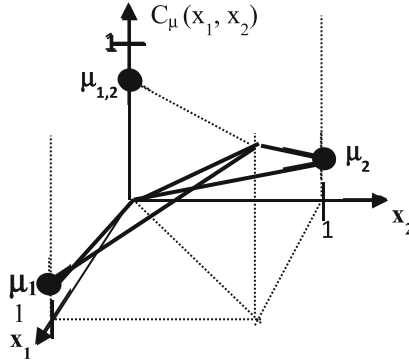
where  $\sigma$  is a permutation on  $\mathcal{N}$  such as  $X_{\sigma(1)} \leq \dots \leq X_{\sigma(n)}$ , and where  $A_{\sigma(i)} := \{\sigma(1) \dots, \sigma(i)\}$ ,  $\forall i \in \mathcal{N}$ , with, by convention,  $A_{\sigma(n+1)} := \emptyset$ . To simplify notations, we shall write, for  $i_1, \dots, i_p, p \leq n$ :

$$\begin{aligned} \mu(\{i\}) &= \mu_i \\ \mu(\{i, j\}) &= \mu_{i,j}, \quad (i \neq j) \\ \mu(\{i, j, k\}) &= \mu_{i,j,k} \quad (i \neq j, j \neq k, k \neq i) \end{aligned} \quad (5)$$

In a decision process,  $x_i$  inputs will be the values attached to considered evaluation criteria. With  $n = 1$ , i.e., with just two criteria, the Choquet integral formula (4) becomes simply:

$$\begin{aligned} x_1 \leq x_2 &\Rightarrow C_\mu(x_1, x_2) = (\mu_{1,2} - \mu_2) \cdot x_1 + \mu_2 \cdot x_2 \\ x_1 > x_2 &\Rightarrow C_\mu(x_1, x_2) = \mu_1 \cdot x_1 + (\mu_{1,2} - \mu_1) \cdot x_2 \end{aligned} \quad (6)$$





**Fig. 3.** Graph of a Choquet Integral, in the case of two variables on  $[0, 1]^2$  domain.

as illustrated in Fig. 3, in the case where  $x_i$  values belong to  $[0,1]$ .

Graph of a  $C_\mu$  function is always constituted of a set of hyperplanes sections defined on each convex polytope  $0 \leq X_{\sigma(1)} \leq X_{\sigma(2)} \cdots \leq X_{\sigma(n)}$ . The function  $C_\mu$  is continuous, and derivable except on some singularities, the discrete Choquet integral being so the unique linear interpolation of a function defined on the vertices of the unit hypercube  $[0,1]^n$ , i.e., on points  $M(1_S, 0_{N-S})$ ,  $\forall S \subseteq \mathcal{N} = \{1, \dots, n\}$ .

A particular case of Choquet integral is the simple weighted sum  $\sum_i \alpha_i \cdot x_i$  (with  $\sum_i \alpha_i = 1$ ). In the most general case, with any values of capacity  $\mu(S)$ ,  $S \subseteq \mathcal{N}$ , Choquet integral generalises a classical weight vector. Limit cases of Choquet integral are the Min and Max operators, that respectively corresponds to the limit capacities  $\mu_i = \mu_{i,j} = \dots = 1$  (Max operator) and  $\mu_i = \mu_{i,j} = 0, \dots$ , except  $\mu_{\mathcal{N}} = 1$  (Min operator):

$$\text{Min}(x_1, \dots, x_n) \leq C_\mu(x_1, \dots, x_n) \leq \text{Max}(x_1, \dots, x_n) \quad (7)$$

The cross terms  $\mu_{i,j}$ , and more generally  $\mu_{i,j,k}, \dots$ , enable to consider pairs, or more generally subsets of non-additive criteria. In the case of a pair of criteria, we may have so:

- Synergistic criteria:  $\mu_{i,j} > \mu_i + \mu_j$  (super-additive effect)
- Redundant criteria:  $\mu_{i,j} < \mu_i + \mu_j$  (sub-additive effect)
- Independent (complementary) criteria :  $\mu_{i,j} = \mu_i + \mu_j$  (additive effect)

The Choquet integral so enables the representation of non-additive criteria, i.e., with interactions between pairs or groups of more than two criteria. Its interest consists mainly in considering in decision possible relevant interactions between groups of factors.

### 2.3 Related Work

The Choquet integral is based on two fundamental concepts: (1) utility; a function which aims to model the preferences of the decision maker. Utility functions can be seen as making it possible to translate the values of the attribute into a satisfaction degree [4]. They are commensurable, monotonic and ascending because if an alternative  $a$  is preferred to  $b$  then  $u(a) \geq u(b)$  [8]; (2) capacity: models the fuzzy measure on which

the integral is based and summarizes the importance of the criteria (the weight vector traditionally used in additives) by aggregating utility functions. The learning ability of the Choquet integral has been demonstrated, mainly in [6]. Functions dealing with data mining issues such as least square and linear programming have been used in this context. Preference learning consists in observing and learning the preferences of an individual, precisely in particular when ordering a set of alternatives, to predict automatic scheduling of a new set of alternatives [7].

The Choquet integral learning function is based on a set of concepts that make it possible to leverage the consideration of user preferences (or decisions) and the interaction and/or synergy between the various criteria for data aggregation. Given a preferential ordering on a sample learning, the discrete Choquet integral enable to quantify, then learn, the relative weights of the different quality metrics.

In the literature, fuzzy integrals have been used for different purposes, for preferences or opinions fusion from a variety of sources, and several applications and extensions of fuzzy integrals have been developed. In [3], the authors have proven that the use of fuzzy neural network is more effective than the decision tree algorithms often used in the literature. The fuzzy neural network model allows precision improvement and less redundancy in decision-making.

In our previous work, we have proven that applying Choquet's integral to order data sources according to the user's preferences, is an interesting and challenging area of research and can lead to more relevant results [5]. One originality of the work described in this paper consists in the proposal of an evaluation function attaching to any potential solution a degree of acceptability, based on truth degrees of inequalities.

In [13] the importance of utilizing machine learning techniques in smart farming was demonstrated for end-users and experts in agriculture. The data mining techniques applied in smart farming problems offer numerous solutions to farmers in a variety of applications in the farming industry. The authors focus on yield forecasting, harvest time, crop information and best variety to plant next season, which can be provided to different end-users and experts. In [14], the authors distinguished the categories of cocoa beans and the differences between them. The proposal aims at adaptively accumulating contextual representations and introduces a contextual memory cell to progressively select contextual statistics. The authors simultaneously correlate contextual relationships to guide high-level representation retaining more detailed information, which helps to discriminate small variations in smart farming application task management. In [15] the authors proposed a Pythagorean fuzzy set (PFS) with Choquet Integral model integrated for vertical farming technology evaluation. They characterize the most feasible option from a group of vertical farming technologies considering a group of decision makers' opinions. The Vertical farming alternatives are assessed, and a suitable option is detected for the farm.

To meet the related works limitations, the originality of our proposal lies in answering famers' requirements by defining an approach of optimization, based on observation history, ensuring a reduced sensitivity to input data thanks to a principle of less specificity. A function assigning to any potential solution a predictive capability, quantifying the conformance of its outputs to evidence, as well as an associated optimisation process based on a degree of satisfaction regarding a set of stated inequalities.

### 3 Problem Statement

Considering a crop with  $n$  prediction sources, information delivered by a source will consist in a sequence of confidence levels  $x_i(d) \in [0, 1]$  attached to future days  $1, \dots, D$ , given a temporal horizon of  $D$  days.  $x_i(d)$  reflect the belief of the  $i^{\text{th}}$  source regarding the occurrence of transition from the present phenological stage to the next one exactly at  $d$  day (cf. Fig. 4). We have so as many functions  $d \in \{1, \dots, D\} \rightarrow x_i(d) \in [0, 1]$  as prediction sources  $i = 1, \dots, n$ , and  $x_i(d)$  can be seen as a membership function of a fuzzy subset of  $\{1, \dots, D\}$ , 0 meaning a null confidence, and 1 the maximum one.

No hypothesis is made on confidence levels semantics. One must point out these levels are not probabilities, only inequalities between two values issued from the same source at the same session of prediction being here significant. Note the case with several days having all a 1 value is possible, reflecting an inaccuracy of the prediction.

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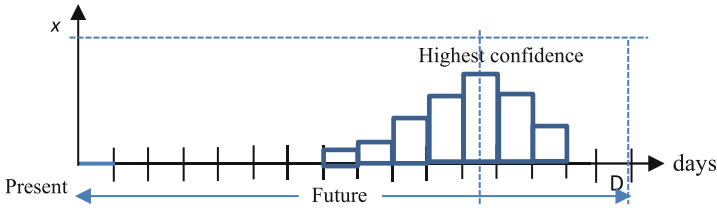


Fig. 4. A fuzzy prediction on a temporal horizon of  $D$  days (from [9]).

We use Choquet integral as an operator of aggregation, in charge of fusing the later  $n$  fuzzy predictions into a more certain and accurate one. It is expected from the combination of partially independent sources a more significant and reliable prediction for guiding decision. This operator will have to apply proper ponderations to each information source, considering possible, a priori unknown, interactions, synergy complementarity, or, at contrary, partial redundancy, between them. The Choquet integral will transform  $n$  fuzzy subsets into a result fuzzy subset, aggregating the  $n$  input sources. Our goal is to enable an automated estimation of Choquet integral coefficients based on a recorded history, i.e., on a set of past source prediction sessions in addition to the corresponding observed evidence at the same session.

$X = (x_1, \dots, x_n)$  denoting a confidence vector,  $n$  being the number of sources, the training dataset consists in  $P$  sessions regarding the same plant, a session being related to a field at a given time period. For each session, we have a sequence of  $X(d)$ , prediction vectors for days  $d = 1, \dots, D$ , in addition to  $d_{Tr}$ , the real day of transition, unknown at prediction time but a posteriori observed for this session.

Available information may be expressed with a set of  $R = (D-1) \cdot P$  inequalities:

$$C_\mu(X_k) < C_\mu(X_{evidence(k)}) \tag{8}$$

with  $k$  and  $evidence(k) \in [1, D \cdot P]$ ,  $evidence(k) \neq k$  being the  $d_{Tr}$  transition day for the session containing day  $k$ .

Our problem is to learn a  $\mu$  capacity, i.e. the values of the  $2^n - 2$  parameters  $\mu_i, \mu_{i,j}, \mu_{i,j,k}, \dots$  satisfying the above inequalities. Despite some of them may be trivially satisfied by any Choquet integral, i.e., for any  $\mu$ , we keep them as input data of our problem, intensities of differences being considered here as significant pieces of information. It is the same for inequalities implicitly satisfied by transitivity, e.g.,  $C_\mu(X) < C_\mu(Z)$ , if  $X < Y$  and  $C_\mu(Y) < C_\mu(Z)$ .

Based on Choquet integral definition (4), input information may be expressed under the form of R inequalities, applying on linear expressions:

$$a_1^k \cdot \mu_1 + \dots + a_n^k \mu_n + a_{1,1}^k \cdot \mu_{1,2} + \dots + a_{n-1,n}^k \cdot \mu_{n-1,n} + \dots + a_{1,\dots,n}^k \cdot \mu_{1,\dots,n} > 0, k = 1, \dots, R \quad (9)$$

That is, using a matrix notation:

$$[A]_{k:[\mu]} > 0, k = 1, \dots, R \quad (10)$$

where  $[A]_k$  is the  $(2^n - 1)$  row vector  $\left[ a^k, \dots, a_n^k, a_{1,1}^k, \dots, a_{n-1,n}^k, \dots, a_{1,\dots,n}^k \right]$  and  $[\mu]$  the  $(2^n - 1)$  column vector  $\left[ \mu_1, \dots, \mu_n, \mu_{1,1}, \dots, \mu_{n-1,n}, \dots, \mu_{1,\dots,n} \right]$

That we may denote:

$$[A] \cdot [\mu] > 0 \quad (11)$$

$[A]$  being the rectangular matrix build with rows  $[A]_k$ .

And coefficients  $\mu_i, \mu_{i,j}, \mu_{i,j,k}$ , etc., satisfying the minimal set of constraints:

$$\begin{aligned} \mu_i &\leq \mu_{i,j}, \forall i, j; i \neq j \\ \mu_{i,j} &\leq \mu_{i,j,k}, \forall i, j, k; i \neq j, j \neq k, k \neq i \\ &\dots \\ \text{With } \mu_i &\geq 0, \forall i, \text{ and } \mu_{1,\dots,n} = 1 \end{aligned} \quad (12)$$

that may be more concisely expressed by:

$$\begin{aligned} \mu(S) &\leq \mu(S'); |S'| = |S| + 1, S \subset S' \\ \text{with } \mu(\emptyset) &= 0 \text{ and } \mu_{1,\dots,n} = \mu(2^N) \leq 1 \end{aligned} \quad (13)$$

We have here no values regarding a function to be learnt, but only a set of statements regarding inequalities. So, a direct identification method of the Choquet integral is not applicable. For building the solution, we are expecting 1) a scalable algorithm, i.e., that will be efficient for a huge training dataset with a time of execution linearly increasing with respect to the number of sessions P. In addition, 2) we consider data as potentially inaccurate, the solution having to be robust in case of conflicting examples, i.e., able to tolerate some local “nearly satisfied” inequalities. At last, 3) we are expecting a solution easily improvable by increments when new data are acquired.

## 4 Proposed Approach

Except in singular cases, inequalities (9) and (12) have either zero or an infinity of solutions. In practice, as numerous  $x_i$ , provided by sources are not perfect values, local violations of a inequalities (2) should be accepted. So, we do not consider only exact solutions, but all potential solutions with a  $\mu$  vector satisfying the only strict inequalities (5) related to a regular capacity definition. On this domain of potential solutions, we shall optimize an evaluation function reflecting the expected characteristics considered above, to get the best solution.

### 4.1 Evaluation Function

Considering a given  $\mu$  capacity, the empirical distribution of  $[A]_k.[\mu]$  values on  $[-1, 1]$ , associated to a dataset, looks as the following histogram (cf. Fig. 5). We must choose as solution the  $\mu$  capacity such as this distribution has the less as possible negative values (i.e., the most possible number of correct predictions), and so the greatest as possible positive values, according to a principle of lowest specificity of the expected solution. This is illustrated on the Fig. 5, case 2 being better than case 1, ideal case being this where all  $[A]_k.[\mu]$  are exactly equal to 1.

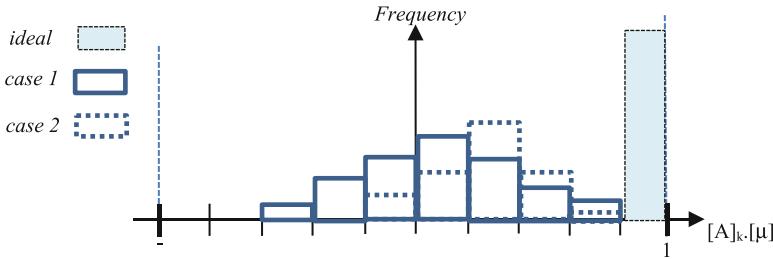


Fig. 5. A fuzzy prediction on a temporal horizon of D days (from [9]).

To meet such an expected distribution of  $[A]_k.[\mu]$  values, we choose to minimize an additive cost function having the following form:

$$\Phi(\mu) = \sum_{k=1, \dots, R} \varphi([A]_k[\mu]) \quad (14)$$

where  $\varphi$  is a function :  $[-1, 1]^m \rightarrow \mathbb{R}^+$  continue, strictly decreasing, and C1 class, with:

- $\varphi(1) = 0$  (case of a perfect compliance to required inequality).
- $\varphi(x)$  being maximum for  $x = -1$  (worst ordering).
- $\varphi$  being in addition strictly convex.

The fact that  $\forall \delta x > 0, \varphi(-\delta x) + \varphi(\delta x) > 2.\varphi(0)$ , resulting from convexity, will give us the guarantee that a local ordering defect corresponding to  $-\delta x$  is not compensated by a positive margin of same intensity  $\delta x$ .

So, we can choose a local cost function:

$$\varphi(x) = -L \cdot \log_2(1/2 \cdot (x+1)), \text{ with } L \in \mathbb{R}^+ \quad (15)$$

Normalizing the expression of  $\varphi(x)$  with respect to  $R$ , number of inequality statements, we get:

$$\Phi(\mu) = -(1/R) \cdot \sum_{k=1, \dots, R} \log_2(1/2 \cdot ([A]_k \cdot [\mu] + 1)) \quad (16)$$

This quantity quantifies the average “degree of order” with respect to evidence present in results delivered by  $C_\mu$  integral, expressing thus the predictive capability of  $C_\mu$  with the chosen  $\mu$  capacity. This order reflects both the crispness of the aggregated prediction and its degree of matching with observed reality.

We can remark  $\varphi$  is infinite for  $x = -1$ , i.e.:

$$\lim_{\delta \rightarrow -1} \varphi(\delta) = +\infty \quad (17)$$

corresponding to the case of a worst ordering, Fig. 6 showing the graphical representation of  $\varphi$  function.

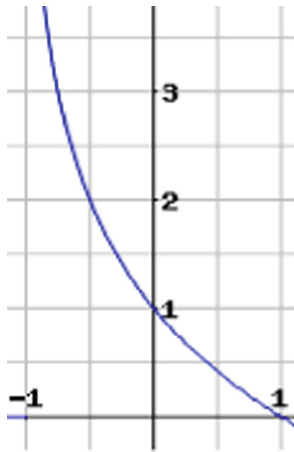
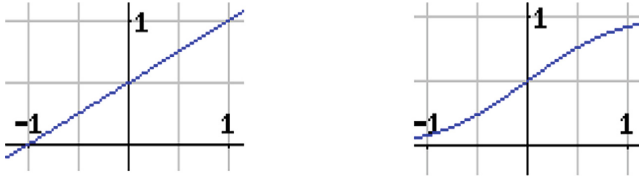


Fig. 6. Graphical representation of  $\varphi(x) = \log_2(1/2 \cdot (x + 1))$ .

This definition of  $\Phi$  appears as a particular case of a more general family of functions:

$$\Phi_v(\mu) = -(1/R) \cdot \sum_{k=1, \dots, R} \log_2(v([A]_k \cdot [\mu])) \quad (18)$$

$v(\delta)$  being a fuzzy comparator, i.e. a non-decreasing function associating to any  $\delta = [A]_k \cdot [\mu] \in [-1, 1]$  a value  $\in [0,1]$ , standing for the degree of truth of an assertion  $[C_\mu(X_d) < C_\mu(X_{Evidence(d)})]$ .  $v$  may be, for example,  $v(\delta) = 1/2 \cdot \lambda \cdot (1 + \text{erf}((\sqrt{2} \cdot \delta)))$ , where erf is the Gauss error function, enabling us to take into account a known inaccuracy of input values  $x$ . Our first expression corresponds just to the case where  $v$  is just the



Linear comparator  $v_1(\delta) = \frac{1}{2} \cdot (\delta + 1)$       Comparator  $v_2(\delta) = \frac{1}{2} \cdot \lambda \cdot (1 + \text{erf}(\sqrt{2} \cdot \delta))$

**Fig. 7.** Graphical representation of two comparators.

linear comparator  $v(\delta) = 1/2 \cdot (\delta + 1)$ , Fig. 7 giving a comparison of the two definitions of  $v$  comparator:

Note that another choice for  $\Phi$  would be an energy-like functions having the following definition.

$$\Phi(\mu) = \sum_{k=1, \dots, R} (\text{Exp}(-K \cdot 1/2 \cdot ([A]_k \cdot [\mu] + 1)) - \text{Exp}(-K)) \quad (19)$$

Note also that these functions may be approximated by a family of simple quadratic functions such as, for example:

$$\Phi(\mu) = \sum_{k=1, \dots, R} ([A]_k \cdot [\mu] - 1) \cdot ((2 - 4 \cdot \alpha) \cdot ([A]_k \cdot [\mu] - 1)) \quad (20)$$

where  $\alpha$  is the expected value of the function  $\varphi$  at  $x = 0.5$ .

## 4.2 A Measure of Order

More generally, one can associate to any fuzzy prediction  $y$ ,  $y(d)$ ,  $d = 1, \dots, D$ , either coming from a single source, either issued from a multisource aggregation process, a quantity  $S$  reflecting its actual lack of information in comparison to independently observed evidence, value standing for a quantity of disorder. It is given by the formula (15), evaluated on the only considered session of prediction, i.e., on the fuzzy subset  $y$  given as input:

$$S(y) = (1/(D - 1)) \cdot \sum_{d=1, \dots, D, d \neq d_p} \varphi(y(d_p) - y(d)) \quad (21)$$

where  $y(d_p)$  is the prediction regarding the day of transition  $d_p$ , and where  $\varphi(\delta y) = -\log_2(1/2 \cdot (\delta y + 1))$ . We can equivalently consider a quality factor defined by:

$$Q(y) = 2^{-S(y)} \in [0, 1] \quad (22)$$

$Q(y) = 1$  standing for a perfect prediction and  $Q(y) = 0$  for the worst possible one.

Let's define  $d_0(y)$ , the day index maximizing  $y(d)$ , i.e.,  $d_0(y) = \text{Arg}(\text{Sup}\{y(d), d = 1, D\})$ . We have:

- If  $0 \leq S(y) \leq 1/(D - 1)$ , i.e., if  $Q(y) \in [2^{1/(D-1)}, 1] = [Q_2(D), 1]$ , then the real transition day  $d =$  evidence necessary correspond to  $d_0$ , and the prediction evidence  $= d_0(y)$  is necessarily right.

- If  $1/(D-1) \leq S(y) \leq (D-2)/(D-1)$ , i.e., if  $Q(y) \in [2^{(D-2)/(D-1)}, 2^{1/(D-1)}] = [Q_1(D), Q_2(D)]$ , then the transition day may correspond to  $d_0$ .
- If  $(D-2)/(D-1) \leq S(y) \leq 1$ , i.e., if  $Q(y) \in [0, 2^{(D-2)/(D-1)}] = [0, Q_1(D)]$ , then the transition day cannot correspond to  $d_0$ , and the prediction *evidence* =  $d_0(y)$  is necessarily wrong.

$\Phi(\mu)$  appears as the average value of  $S(y)$  on the given training dataset. It represents the expected value of  $S$  related for any new prediction, quantify thus the performance of the aggregation operator prediction.

The function  $\Phi(\mu)$  being strictly convex, as a sum of strictly convex functions, and so, having a unique minimum, we use at first a simple gradient descent method to minimize it. We combine this basic method with the use of a penalization function to keep candidate solutions strictly inside the limits of the domain defined by the canonical constraints (5).

### 4.3 Gradient Descent

The solution is a  $(2^n-1)$ -dimension vector  $[\mu] = [\mu_1, \dots, \mu_n, \mu_{1,1}, \dots, \mu_{n-1,n}, \dots, \mu_{1,\dots,n}]$ , denoted here  $[m] = [m_1, m_2, \dots, m_{2^n-1}]$ , minimizing  $\Phi$ .  $\Phi$  may be expressed as:

$$\Phi(m) = -L \cdot \sum_{k=1, \dots, R} \log\left(1 + \sum_{i=1, \dots, 2^n-1} a_{k,i} \cdot m_i\right) \quad (23)$$

where  $a_{k,i}$  is the  $i^{\text{th}}$  component of  $[A]_k$ , and where  $L = 1/(R \cdot \text{Log}(2))$

The  $j^{\text{th}}$  component of the gradient being:

$$\partial\Phi/\partial m_j(m) = -L \cdot \sum_{k=1, \dots, R} a_{k,j} / \left(1 + \sum_{i=1, \dots, 2^n-1} a_{k,i} \cdot m_i\right) \quad (24)$$

First, we calculate, for each statement of inequality  $k$ , the  $2^n-1$  values  $a_{i,k}$ . Then, a loop calculates successive iterations  $m_p$  of the  $m$  vector, according to the formula:

$$m_{p+1} = m_p - \varepsilon_p \cdot \text{grad}(m_p) + \Psi(m_p) \quad (25)$$

where  $\text{grad}(m_p)$  is the gradient vector  $[\partial\Phi/\partial m_j(m_p)]$ , and where  $\Psi(m_p)$  represents a penalizations related to domain frontiers associated to the constraints  $\mu_i \geq 0$ ,  $\mu_{i1, i2} \geq \mu_{i1}$ ,  $\mu_{i1, i2, i3} \geq \mu_{i1, i2}$ ,  $\dots$ , and  $\varepsilon_p$  being a step size with an initial value  $\varepsilon_0$ , and possibly updated at each iteration. The iteration loop stops when  $\|m_{p+1} - m_p\| < \eta$ , where  $\eta$  is a predefined value.



#### 4.4 Penalization and Projected Gradient

We consider the frontiers of solution domain with the use of a penalization function  $m \rightarrow \Psi(m)$ , defined as:

$$\psi(m) = \sum_{i=1, \dots, n} \theta(\mu_i) + \sum_{S \subseteq N, S=\emptyset, S \neq N, \{i\} \cap S = \emptyset} \theta(\mu_{S \cup \{i\}} - \mu_S) \quad (26)$$

where  $\theta$  is continuous and derivable,  $\theta(x) \approx 0$  for  $x > 0$ ,  $\theta(x)$  being large positive for  $x < 0$ . E.g., for  $n = 2$ , to express the required constraints  $\mu_1 \geq 0$ ,  $\mu_2 \geq 0$ ,  $\mu_1 \leq \mu_{1,2}$ ,  $\mu_2 \leq \mu_{1,2}$  and  $\mu_{1,2} \leq 1$  we shall have:

$$\Psi([\mu_1, \mu_2, \mu_{1,2}]) = \theta(\mu_1) + \theta(\mu_2) + \theta(\mu_{1,2} - \mu_1) + \theta(\mu_{1,2} - \mu_2) + \theta(1 - \mu_{1,2}) \quad (27)$$

We use here a simple exterior penalization  $\theta(x) = \min(x, 0)^2 / (2 \cdot \gamma)$ ,  $\gamma$  being a parameter in relationship with the expected result accuracy (e.g.,  $\eta = 0.01$ ).

As convergence process may be long, especially when the optimum solution is on the domain frontier, i.e. on one of the canonical hyperplanes  $\mu_i = 0, \dots, \mu_{i_1, \dots, i_p} = \mu_{i_1, \dots, i_p, i_{p+1}}, \dots, \mu_{i_1, \dots, i_n} = 1$ , instead of a penalization, we decided to use an adaptation of projected gradient method, that is simple in our case where domain is a convex polytope closed by a set of canonical hyperplanes.

At each iteration, we evaluate the quantities:

$$\begin{aligned} \omega_i(\mu) &= \mu_i, \dots \\ \omega_{i_1, \dots, i_p, i_{p+1}}(\mu) &= \mu_{i_1, \dots, i_p, i_{p+1}} - \mu_{i_1}, \dots, \mu_{i_p}, \dots \\ \omega_{i_1, \dots, i_n}(\mu) &= 1 - \mu_{i_1, \dots, i_n}, \dots \end{aligned} \quad (28)$$

a negative  $\omega$  value meaning that the candidate solution vector  $m_p$  is out of the domain. In this case, the actual step  $\varepsilon_p$  is adapted in such a way that candidate solution is put just on the frontier. Then, at the next iteration, gradient  $\text{grad}(m_p)$  is replaced by  $\text{grad}(m_p)^{\text{proj}}$ , orthogonal projection of  $\text{grad}(m_p)^{\text{proj}}$ , on the considered hyperplane, ensuring that the candidate solution will remain inside the domain.

## 5 Decision Model

Once the training achieved on the available dataset and having got, as a result, the values of our  $\mu$  capacity, the decision model will be used in operation. For each new session, i.e., for each new multisources  $n$ -vector input data  $X$ ,  $X(d)$ ,  $d = 1, \dots, D$ , the Choquet integral will deliver a fuzzy dataset defined by the membership function  $y(d) = C_\mu(X(d))$ ,  $d = 1, \dots, D$ , and our decision model will propose as awaited transition day this of maximum confidence:

$$d_{\text{Pred}}(X) = \text{Arg}\left(\text{Sup}\{C_\mu(X(d)), d = 1, \dots, D\}\right) \quad (29)$$

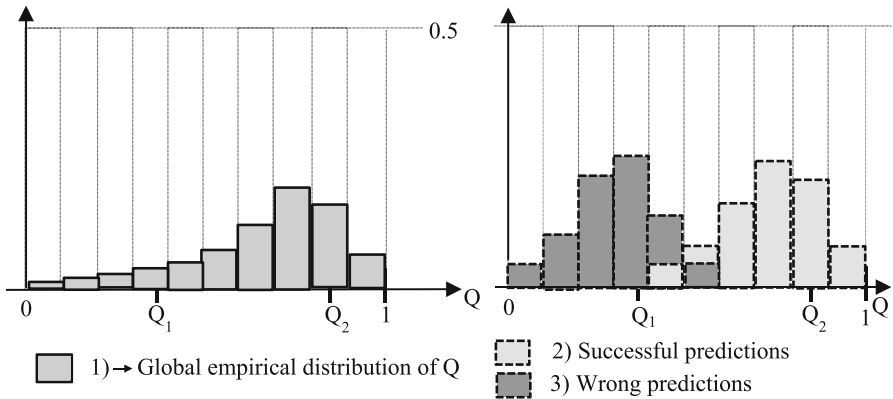
The user having no idea about the reliability of this result, and the actual quality of it depending on the statistical distribution of  $Q$  on the dataset, the problem is now to

provide the user with a quantity that will objectively reflect the confidence they can have in the proposed day.

Figure 8 shows a typical statistical distribution of the Q quantity on a training dataset, with 1) the global distribution of inputs on Q values, 2) the distribution of successful predictions and 3) the distribution of wrong predictions (presented here in the case of  $D = 11$  days).

Using the values  $C_{\mu}(X(d))$  delivered by the Choquet integral, we first calculate a quality of the prediction, evaluated with regards to the predicted day  $d_{pred}$ :

$$Q^*(y) = 2^{-S^*(y)}, \text{ with } S^*(y) = 1/(D - 1) \cdot \sum_{d=1, \dots, D, d \neq d_{Pred}} \varphi(y(d_{Pred}) - y(d)) \quad (30)$$



**Fig. 8.** Empirical distribution of Q quantity on a dataset.

We consider then the probability density  $p(q)$  of Q, the conditional probability densities of successful and wrong predictions  $p_s(q)$  and  $p_w(q)$ , and, at last, the global probabilities of successful and wrong predictions  $P_s$  and  $P_w$ .

Using Bayes' theorem, the conditional probability of a successful prediction given the known present value  $q$  of  $Q^*$  is immediately given by:

$$P(\text{success} | q) = p_s(q) \cdot P_s / (p_s(q) \cdot P_s + p_w(q) \cdot P_w) \quad (31)$$

That give the probability of the system estimation to be the real still unknow day of transition. Here, the required densities of probabilities are simply estimated by intervals of values  $[q_i, q_i + 1]$ , after solution delivery at the last step of the algorithm.

## 6 Case Study and Experimental Evaluation

Our goal is to deliver to farmers estimated future states of their culture regarding its development, maintaining and improving this estimation over time. More specifically, evaluation of the proposed approach is led in the context of crop monitoring, aiming at predicting the growth stages of a plant. So, in each stage, we propose to predict the transition day regarding the next development stage of a plant. As already pointed out, knowing with a sufficient advance the day of transition from a growth stage to the next one is a very important issue for farmers, who have to plan the sequence of actions to be taken with the right timing (e.g. fertilization, watering, other treatments, ...).

### 6.1 Experimental Setup

As an illustrative example, we base our experiment on three sources of input, which are the classical empirical calculation, a statistical model of the plant, and an observation by digital image processing [13].

The case study concerns cultivation of winter wheat in Normandy, which is a region with a temperate oceanic climate in France. This case is based on data issued from data recorded over several years mixed with data issued from simulation models.

In the considered example, we are at the beginning of stage 8 (maturity) and we are trying to predict the end of ripening and therefore the beginning of stage 9 (senescence), in order to harvest wheat at the best moment. The prototype was developed in Python version 3.9.12 programming language.

To assess the robustness of our algorithm, we placed as inputs sensor confidence data, parametrized as follows:

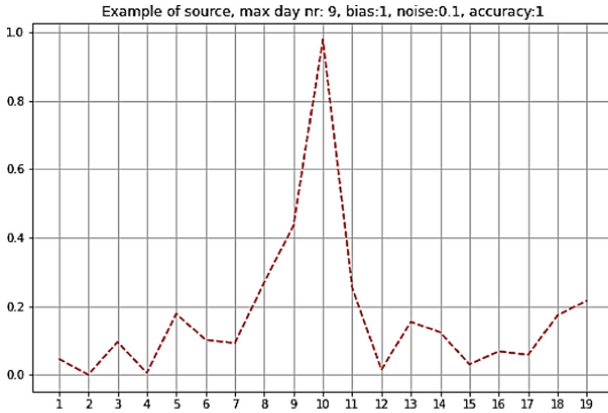
- The day of maximum confidence, drawn pseudo-randomly according to a Gaussian centred on the day in question and with a sigma standard deviation called bias.
- The spread of the source over adjacent days (accuracy of the source), which is calculated using the function  $f$  which associates  $x$  with:

$$f(x) = e^{-k \cdot |x - \text{day}|} \quad (33)$$

With:

- Day: day this of maximum confidence
- $k$ : accuracy (“spreading coefficient”) of the function. The larger  $k$  is, the more accuracy the source (the spread is less important).
- Noise: variation of the confidence drawn randomly according to a Gaussian of standard deviation fixed in advance. For each day, the confidence value corresponds to the starting confidence, to which we add the noise, which is drawn randomly until we obtain a positive value.

The first three parameters (day of maximum confidence, bias and accuracy) correspond to the configuration of the sources along the  $x$  axis (knowing that the triangular shape of the source, in  $y$ , is also fixed by these parameters). Finally, the last parameter (noise) corresponds to the variation in  $y$  of the source (noise). Figure 9 illustrates an example of data source.



**Fig. 9.** Example of data source.

## 6.2 Simulation Protocol

As a control, before of each simulation set, we first performed the algorithm on zero bias and non-noisy sources. The spread is previously chosen and fixed for each of the following simulation set.

For comparison and justification for using the Choquet integral, we also applied our algorithm to a simple weighted sum (calculation of weights  $\mu_1, \mu_2, \mu_3$ ), for identical noisy or non-noisy sources. We performed simulations with noisy inputs 50 times per fixed bias/accuracy/noise values. For each simulation set, we evaluated performance with the following descriptive statistics about the predicted day of passage (cf. Table 1).

**Table 1.** Computed indicators.

Indicator	Control data source or simulations	Description
Type	Both	Weighted sum or Choquet integral
Day target (without noise)	Control data sources	Day predicted by the algorithm with unbiased and noise-free control data sources
Max value (without noise)	Control data sources	Maximum confidence value with unbiased and noise-free control data sources
Avg predicted day	Simulations	Average predicted day
Std deviation pred day	Simulations	Average standard deviation of predicted day

(continued)

**Table 1.** (continued)

Indicator	Control data source or simulations	Description
Min pred day, Max pred day	Simulations	Resp. Minimum and maximum predicted days by the algorithm during the simulations
Avg max integral value	Simulations	Average maximum confidence of predicted days
Avg evaluation	Simulations	Average evaluation function value (please refer to 4.1). We chose an evaluation function in the form: $\Phi(\mu) = 2^{-\sum_{k=1, \dots, R} \log 2(1/2 \cdot ([A]^{k \cdot [\mu] + 1}))} \quad (34)$ The indicator is between 0 and 1. The closer this indicator is to 1, the better the prediction quality. On the contrary, the closer the indicator is to 0, the worse the quality
Avg source 1 quality, Avg source 2 quality, Avg source 3 quality, Avg result model quality	Simulations	The average quality resp. For each source and for the computed result model (Choquet or Weighted sum). Please refer to part 6
Source 1 ratio, Source 2 ratio, Source 3 ratio, Result model ratio	Simulations	Ratio of right predicted day/ number of simulations resp. For each source and for the computed result model (Choquet or Weighted sum). Please refer to part 6

In summary, the simulation protocol is set as follows:

- A precision value is fixed (identical for each source).
- We set a transition day.
- We generate the three data sources.
- We run the program for calculating the coefficients:
  - for the Choquet integral.
  - for the weighted sum.
- We fix a bias and a noise.
- We perform the following 50 simulations:
  - We regenerate the 3 sources of biased and noisy data.
  - We run the program for calculating the coefficients:
    - for the Choquet Integral.

– for the weighted sum.

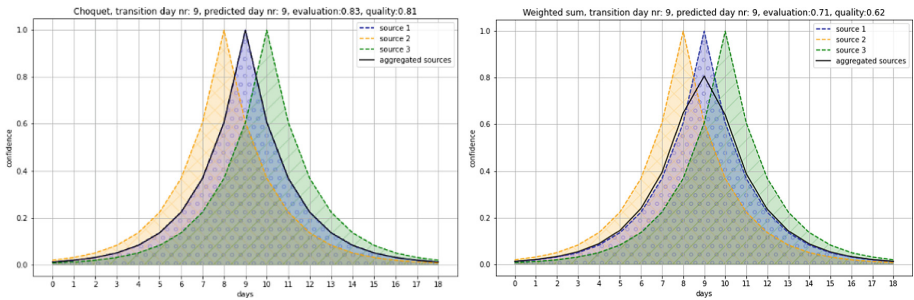
- The descriptive statistics are calculated on the 50 simulations carried out.

### 6.3 Results

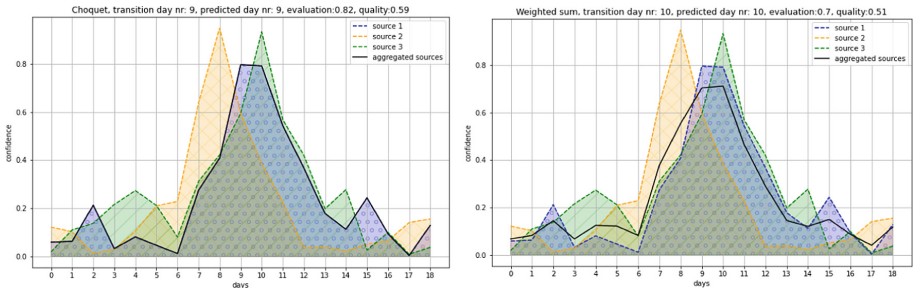
In this section, we present the results obtained under the form of graphs and summary tables. We performed simulations with noisy inputs 50 times per fixed bias/accuracy/noise values.

**Simulation 1:** Transition day: 9, runs: 50, bias: 0.8, noise: 0.1, accuracy: 0.5, deviation between the confidence maximums of each source: 1 (cf. Figs. 10, 11, and Table 2).

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**Fig. 10.** Graphs with control data sources and both Choquet/weighted sources results.

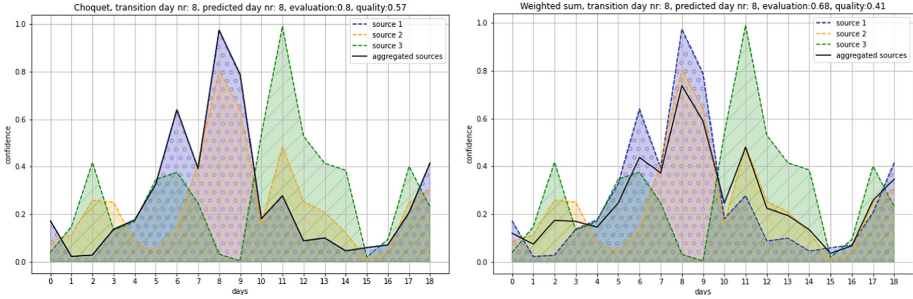


**Fig. 11.** Example of simulation result (nr 2 out of 50) with biased/noisy data.

**Table 2.** Experimental results (1<sup>st</sup> simulation set).

Type	Control data sources		Predicted day (calculated on 50 simulations)				Avg max integral value	Avg evaluation	Avg quality				Ratio of right day prediction			
	Day target (without noise)	Max value (without noise)	Avg	Std dev	Min	Max			Source 1	Source 2	Source 3	Result model	Source 1	Source 2	Source 3	Result model
Weighted sum	9	0.81	9.04	0.69	8	10	0.75	<b>0.69</b>	0.6	0.48	0.45	<b>0.5</b>	0.56	0.22	0.1	<b>0.52</b>
Choquet	9	1.0	8.94	0.65	8	10	0.88	<b>0.83</b>	0.6	0.48	0.45	<b>0.59</b>	0.56	0.22	0.1	<b>0.58</b>

**Simulation 2:** Transition day: 9, runs: 50, bias: 0.8, *noise*: 0.2, accuracy: 0.5, deviation between the confidence maximums of each source: 1. Graphs with control data sources and both Choquet and weighted sources results appear as identical to those of the previous simulation (cf. Fig. 12 and Table 3).

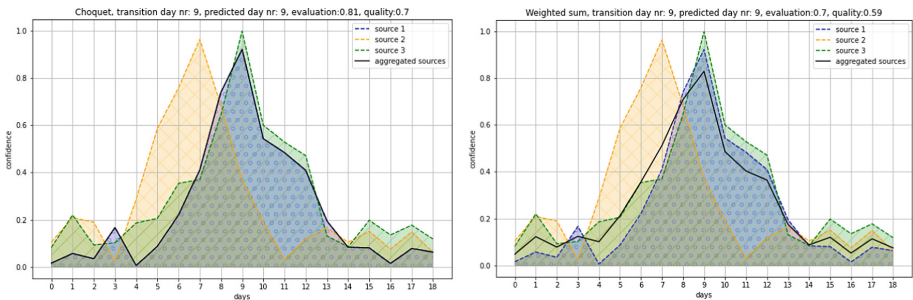


**Fig. 12.** Example of simulation result (nr 8 out of 50) with biased/noisy data.

**Table 3.** Experimental results (1<sup>st</sup> simulation set).

Type	Control data sources		Predicted day (calculated on 50 simulations)				Avg max integral value	Avg evaluation	Avg quality				Ratio of right day prediction			
	Day target (without noise)	Max value (without noise)	Avg	Std dev	Min	Max			Source 1	Source 2	Source 3	Result model	Source 1	Source 2	Source 3	Result model
Weighted sum	9	0.81	8.9	1.14	7	13	0.73	<b>0.67</b>	0.48	0.45	0.44	<b>0.43</b>	0.36	0.16	0.26	<b>0.36</b>
Choquet	9	1.0	8.78	1.17	7	13	0.85	<b>0.79</b>	0.48	0.45	0.44	<b>0.48</b>	0.36	0.16	0.26	<b>0.36</b>

**Simulation 3:** Transition day: 9, runs: 50, *bias*: 1.2, noise: 0.1, accuracy: 0.5, deviation between the confidence maximums of each source: 1. Graphs with control data sources and both Choquet/weighted sources results are identical to those of the previous simulation (cf. Fig. 13 and Table 4).

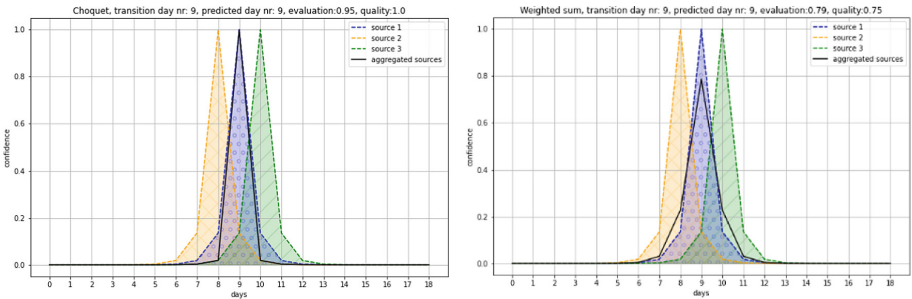


**Fig. 13.** Example of simulation result (nr 16 out of 50) with biased/noisy data.

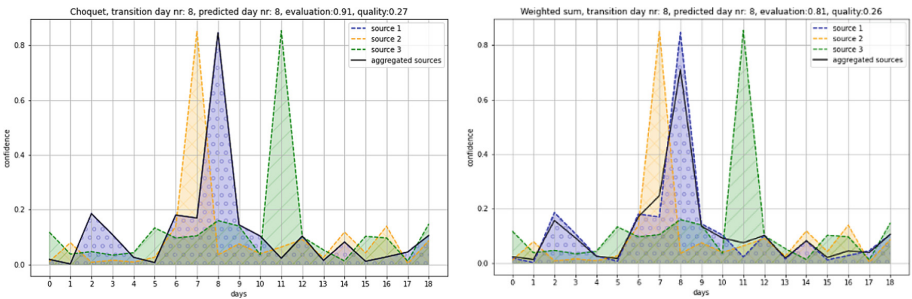
**Table 4.** Experimental results (1<sup>st</sup> simulation set).

Type	Control data sources		Predicted day (calculated on 50 simulations)				Avg max integral value	Avg evaluation	Avg quality				Ratio of right day prediction			
	Day target (without noise)	Max value (without noise)	Avg	Std dev	Min	Max			Source 1	Source 2	Source 3	Result model	Source 1	Source 2	Source 3	Result model
Weighted sum	9	0.81	8.92	1.11	6	12	0.74	0.7	0.53	0.47	0.5	0.47	0.38	0.24	0.28	0.38
Choquet	9	1.0	8.82	1.13	6	12	0.91	0.82	0.53	0.47	0.5	0.53	0.38	0.24	0.28	0.38

**Simulation 4:** Transition day: 9, runs: 50, bias: 0.8, noise: 0.1, *accuracy*: 2, deviation between the confidence maximums of each source: 1 (cf. Figs. 14, 15 and Table 5).



**Fig. 14.** Graphs with control data sources and both Choquet/weighted sources results.



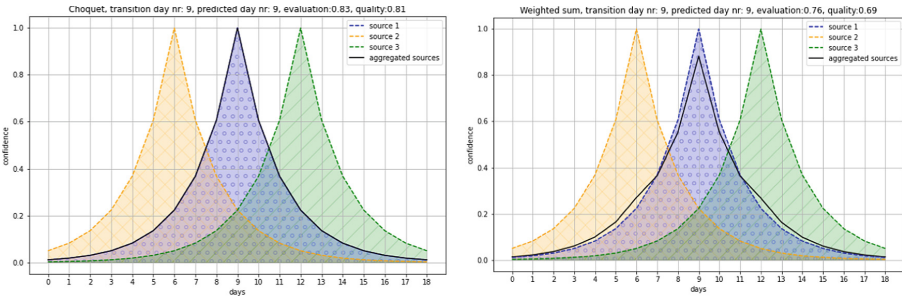
**Fig. 15.** Example of simulation result (nr 4 out of 50) with biased/noisy data.



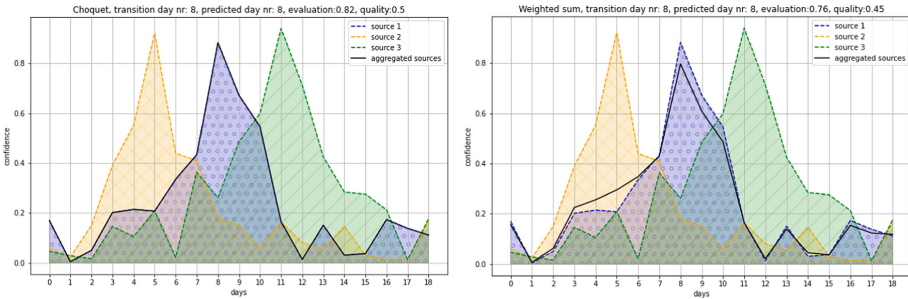
**Table 5.** Experimental results (1<sup>st</sup> simulation set).

Type	Control data sources		Predicted day (calculated on 50 simulations)				Avg max integral value	Avg evaluation	Avg quality				Ratio of right day prediction			
	Day target (without noise)	Max value (without noise)	Avg	Std dev	Min	Max			Source 1	Source 2	Source 3	Result model	Source 1	Source 2	Source 3	Result model
Weighted sum	9	0.79	9.15	0.73	8	11	0.82	<b>0.83</b>	0.6	0.44	0.46	<b>0.57</b>	0.6	0.3	0.35	<b>0.6</b>
Choquet	9	1.0	9.15	0.73	8	11	0.88	<b>0.9</b>	0.6	0.44	0.46	<b>0.6</b>	0.6	0.3	0.35	<b>0.6</b>

**Simulation 5:** Transition day: 9, runs: 50, bias: 0.8, noise: 0.1, accuracy: 0.5, deviation between the confidence maximums of each source: 3 (cf. Figs. 16, 17, Table 6).



**Fig. 16.** Graphs with control data sources and both Choquet/weighted sources results.

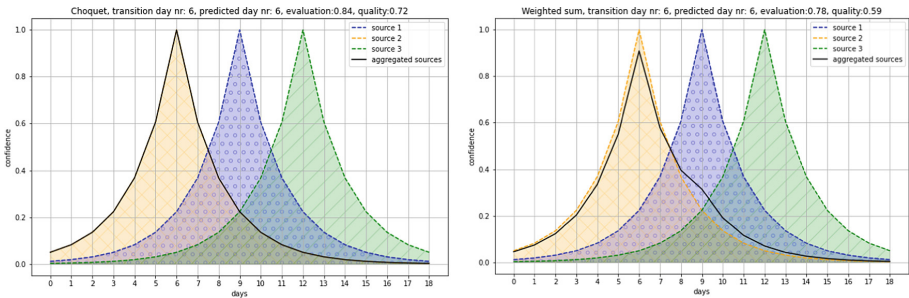


**Fig. 17.** Example of simulation result (nr 5 out of 50) with biased/noisy data.

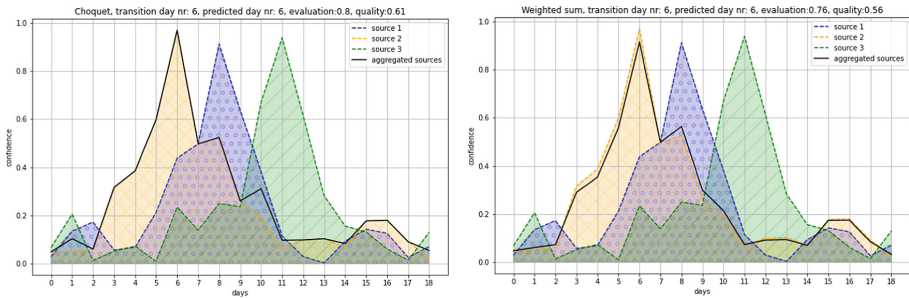
**Table 6.** Experimental results (1<sup>st</sup> simulation set).

Type	Control data sources		Predicted day (calculated on 50 simulations)				Avg max integral value	Avg evaluation	Avg quality				Ratio of right day prediction			
	Day target (without noise)	Max value (without noise)	Avg	Std dev	Min	Max			Source 1	Source 2	Source 3	Result model	Source 1	Source 2	Source 3	Result model
Weighted sum	9	0.88	9.22	0.81	8	11	0.88	<b>0.79</b>	0.57	0.32	0.33	<b>0.54</b>	0.48	0.0	0.0	<b>0.48</b>
Choquet	9	1.0	9.14	0.92	6	11	0.89	<b>0.81</b>	0.57	0.32	0.33	<b>0.56</b>	0.48	0.0	0.0	<b>0.48</b>

**Simulation 6:** Transition day: 6, runs: 50, bias: 0.8, noise: 0.1, accuracy: 0.5, deviation between the confidence maximums of each source: 3 (cf. Figs. 18, 19, and Table 7).



**Fig. 18.** Graphs with control data sources and both Choquet/weighted sources results.



**Fig. 19.** Example of simulation result (nr 16 out of 50) with biased/noisy data.

**Table 7.** Experimental results (1<sup>st</sup> simulation set).

Type	Control data sources		Predicted day (calculated on 50 simulations)				Avg max integral value	Avg evaluation	Avg quality				Ratio of right day prediction			
	Day target (without noise)	Max value (without noise)	Avg	Std dev	Min	Max			Source 1	Source 2	Source 3	Result model	Source 1	Source 2	Source 3	Result model
Weighted sum	6	0.91	5.86	0.89	4	8	0.91	<b>0.8</b>	0.17	0.4	0.15	<b>0.38</b>	0.0	0.42	0.0	<b>0.42</b>
Choquet	6	1.0	5.86	0.89	4	8	0.93	<b>0.82</b>	0.17	0.4	0.15	<b>0.39</b>	0.0	0.42	0.0	<b>0.42</b>

## 6.4 Discussion

Experimental results show the mean prediction quality for the Choquet integral is always higher than this of a classical weighted sum, independently from input values, validating the choice of the Choquet integral as aggregation operator. The prediction rate seems low at first, but this is explained by the bias attached to the day of maximum confidence of each source, which is different at each stage of the simulation.

Finally, the quality of prediction as well as the ratio of right day prediction of our Choquet-based decision model are systematically superior or equal (the rare times it is not the case can be weighed against the significance of certain calculations) to those of the different sources separately considered, that tends to validate the interest of our approach.

According to the calculations, the noise and then the bias that have the greatest impact on the quality of prediction and the ratio of right day prediction. However, the descriptive statistics seem to show a certain robustness of the algorithm, even with noisy signal. The average predicted day does not seem to vary significantly.

## 7 Conclusion and Future Work

In this paper, we proposed a Choquet-based decision model associated to an original parameter identification process, enabling to consider possible non-additivity such as partial redundancy and the synergies between various input criteria. Using our approach, we are able to identify the values of the decision model parameters, considering an input training dataset from which we derive a set of preference constraints the model has to conform with. A measure enabling to evaluate a prediction capability, attaching to any potential solution a degree of order, have been detailed. The results of experiments are promising, as they provide better right day prediction ratios than those obtained by both classical weighted sum approach and use of one data source considered separately. Our approach provides in addition a useful quality of robustness due to the principle on which the evaluation function used in our identification process relies.

The case study concerns smart farming, and the implemented prototype allows, for a given culture and several input sources, to assist farmers, predicting growth stages of a culture. This case study concerns the cultivation of winter wheat in Normandy. Indeed, for a given culture, several sources of input are considered, mainly the classical empirical calculation called “growing degree-days”, a statistical model of the plant and

digital image processing. The experimental results are very encouraging, the predicted days being stable despite the variation of the noise attached to inputs.

The proposed algorithm is currently extended to integrate more sophisticated entropy criteria in the evaluation function, future work including an extension of the proposal supporting the bi-capacities, which emerge as a natural generalization of capacities in such context and could be interesting to integrate information that may go to against a phase transition over a given period.

The number of inputs to be considered being potentially high, another extension of our work concerns, among those, an algebraic approach enabling to a priori reduce the dimension of the problem, and so reduce the number of Choquet parameters to be identified, enabling so to consider larger source sets with less calculations.

At last, another extension of this work is this of considering additional inputs in decision model, taking into account context specificities about a field or a region (e.g. climate, quality of soil, ...). The goal is to dispose of models with larger domains of validity, i.e., applicable to larger sets of fields and cultures, one issue being the impossibility of a direct integration of such variables as input of the integral due to nonlinear effects, requiring the study of non-linear and fuzzy extensions of our Choquet-based decisions model.

□

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