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Christine Chambris, Kalyanasundaram (ravi) Subramaniam

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Abstract:

Mathematicians have identified both problem-solving and theory-building as important for the development of mathematics. Recognizing that the latter is not prominent in school mathematics, some researchers have suggested the inclusion of theory-building practices in school mathematical learning. Inspired by this, we go further to suggest that learning school arithmetic may be seen as primarily a theory building activity. In this paper, we argue that a mathematical theory of quantity constitutes a unifying goal for arithmetic and provides a coherent basis for theory building practices, as well as applications of the theory to mathematical reasoning, which we suggest, enhances access to key mathematical practices in elementary mathematics classrooms. We present an outline of such a theory for the restricted case of whole numbers and addition. Numbers are seen as quantities and reasoning about numbers has underlying it a broader reasoning about quantities. In support, we point to some curricular approaches that emphasize numbers as quantities.

Keywords: Theory building, Theory of quantity, Coherence, School arithmetic, Japanese Curriculum, Davydov curriculum

Gowers (2000), distinguishes two broad kinds of interconnected mathematical practices – problem-solving, and building and understanding theories, and argues for the importance of both. The distinction has inspired researchers in mathematics education to inquire into the role of theory building in the school mathematics curriculum. Bass (2017), also a mathematician, defines

> theory-building practices to be creative acts of recognizing, articulating, and naming a mathematical concept or construct that is demonstrably common to a variety of apparently different mathematical situations (...) that, at least for those engaged in the work, might have had no prior conceptual existence. (p. 230)

Among theory-building practices in mathematics education, Bass includes cognitive processes of abstraction from a range of situations or problems, exhibiting mathematical connections between different representations or mathematical concepts, and investigating and identifying a common mathematical structure underlying apparently different problems.

The discussion above suggests that there are a variety of theory building practices that may be relevant to mathematics education. We are inspired to go further and suggest that theory building could be an important, if not central, part of learning elementary mathematics. Bass stresses the difference between the theory-building practices (the process) and the mathematical theories (the product). Further, if actions that involve "seeing connections, sensing structure,

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and abstracting commonalities" (p. 230) count as theory-building practices, one also expects to find practices that support the product through application of the theory to reasoning – deducing statements from other statements – which are central to both mathematics and the learning of mathematics. We suggest that in order to make possible the activity of theory building central in learning school arithmetic, a critical missing element is a reference mathematical theory that binds together these practices as a coherent whole. We inquire into what such a theory might be. But before that, we address an important related question.

WHY IS THEORY BUILDING NOT WIDELY RECOGNIZED AS AN IMPORTANT PART OF SCHOOL ARITHMETIC EDUCATION?

One of the major epistemological changes in mathematics in the 19th century was a change in the approach to axiomatization: axioms built on the idealisation of reality were replaced by axioms based on the no contradiction principle[3] that diverged from intuitions about reality. Simultaneously, sets replaced quantities as the basic objects of mathematics (Otte, 2007). Although set theory provides a common foundation for much of mathematics, it is extremely abstract, and not accessible to a child. The New Math reform (1955-1975) sought to introduce set theory as a foundation for mathematics education in many countries (Kilpatrick, 2012). Such attempts soon encountered strong criticisms and were eventually abandoned. Foundational or theoretical aspects of elementary mathematics and their axiomatic presentation were no longer seen as important and a largely application oriented problem-solving perspective became the dominant frame for school mathematics. This said, the foundation of the school arithmetic to be taught did not change again, and set theory remained the dominant, most often implicit, reference in many educational systems (Kilpatrick, 2012).

Since school arithmetic had only so abstract a starting point as set theory, it is not surprising that the practice of theory building by students was seen as unlikely and found little place in how the learning of school mathematics was imagined. Yet, unlikely does not mean impossible or unnecessary. We are guided by Kolmogorov's (1938/1960) assertion that "divorcing mathematical concepts [numbers] from their origins, in teaching, results in a course with a complete absence of principles and with defective logic" (p.10, quoted by Davydov, 1975, p. 120). What mathematical theory do we expect learners to build in the early years of schooling? We argue that such a theory must be built on children's intuitions about quantities. Our position is that arithmetic, as learned in elementary school, is about quantities. Numbers can be seen as operators on quantities, hence inherit the properties of quantities (Steiner, 1969) and can be treated as quantities. Arithmetic and quantities therefore do not belong to two different domains – the mathematical and the physical – as some researchers have assumed (e.g., Nunes & Bryant, 2022).

3This does not mean that mathematics was contradictory before!

In the following sections, we provide an outline of a mathematical theory of quantity (restricted to the addition operation for reasons of space), whose aim is to provide a sound and coherent basis for modeling knowledge required for reasoning in school arithmetic. Applications could be to form a reference for school arithmetic and to guide theory building practices in instructional settings. The theory formalises certain ideas that may be abstracted from experience. For explanatory support, we present a possible set of these experiences, reconstructed with attention to logical coherence rather than actual learning trajectories. Deriving an instructional sequence from these experiences is an empirical research question that we do not address here. We also do not investigate how the abstract ideas emerge from experience or how difficult or easy it is for learners to abstract these ideas but merely point to the considerable literature that does so (e.g., Davydov, 1999; Steffe & Olive, 2010).

BUILDING NUMBER THEORY (PART 1): QUANTITIES

Historically speaking, numbers were founded on the measurement of quantities (e.g., Bourbaki, 1984). Even though this viewpoint was abandoned, prominent mathematicians in the 20th Century, concerned about issues of teaching and learning mathematics, developed theories that constructed numbers from quantities. We start this section with naive ideas on quantities, which can be abstracted from experience and form the basis for developing a mathematical understanding of quantity. It is to be noted that these ideas do not include any notion of number or counting. In the subsequent section, we sketch the outline of a mathematical theory of quantity in correspondence with these naive ideas.

Towards the abstraction of an intuitive idea of the *quantity*, and of the idea of a *size*

One can imagine carrying or weighing diverse objects, thereby forming an idea of “heaviness”, related to one’s senses, i.e., a specific sensation. We highlight the following ideas associated with such experiences: A) The imagined sensation of heaviness will include the idea of being more or less heavy. B) At times, it may not be possible to say which “heaviness” is greater. In this case, the two “heavinesses” will be said to be similar or equal. C) One can form an idea of combining two “heavinesses”, perhaps by imagining holding two objects, one for each of the two, in one hand. From their combination, one can form the idea of a third heaviness (the heaviness of the combination). D) Perhaps after some experiences of this kind, one will be convinced that the combined third heaviness is greater than each of the first two. In other words, the heaviness increases in combination. E) Another question arises when one chooses two heavinesses, one big and one small: Is it possible to find a third one (the difference) such that when the small and the third are combined, it forms the bigger one? This would require several attempts, but the conclusion should be positive. F) Let us consider the combination of a first heaviness and a second one. The first could itself be the combination of two heavinesses, which we may call “parts” of the first heaviness. We can now combine these three heavinesses in other ways: Combine one of the two parts of the first with the second, then the resulting heaviness with the remaining part of the first heaviness. One should get convinced that these different ways of combining finally result in the same heaviness. We leave to the reader to *reason* through the previous points A-F to solve the following G and H tasks. G) Imagine three heavinesses, the first bigger than the second, and the second bigger than the third. Is the first bigger than the third? H) Imagine three heavinesses, the first bigger than the second, and the second bigger than the third. Is the difference of the second and the first smaller or greater than the difference of the third and the first? The experiences described in I and J serve as an introduction to the idea of composition by repeating a certain quantity. I) Imagine two heavinesses and wanting to know whether one needs a big or a small amount of times the small one to get the big one. (This could be useful for example to anticipate if one needs many or few round trips to transport a big heap of sand, in a barrow that has a limited capacity, in terms of heaviness.) J) One may also notice that if the barrow is not filled to capacity in each trip, there will be more trips required.

We now take another example (Fig. 1). Imagine two heaps of identical buttons. A) One has only a few buttons (H1), the other has many (H2). This is sensible. B) In some cases, it would be difficult to say which heap is bigger (H2 and H3). To solve this problem, one can use a shovel that resembles the 'pelle à grosse'. (Such shovels were actually used, in a more sophisticated way, by foremen in a shell button factory in North-France to measure the work done in a day). Sometimes the shovel may not work, when the heaps both have less buttons than the shovel can contain (Sh0), or both have more (Sh1). One will then have to find another suitable shovel (Sh2). If the amounts of the two heaps perfectly fit the same shovel, one can say, it is the same amount of buttons. (The two amounts of buttons are similar or equal.) C) One can combine two amounts of buttons. For instance considering two amounts of buttons (H1 and H2) and associated shovels (shovels that perfectly fit the heaps, Sh1 and Sh2), one can find a shovel that perfectly fits the combined quantity (Sh4). We let the reader imagine further realistic situations and problems with shovels that would correspond to the letters D to J discussed above with regard to heaviness.

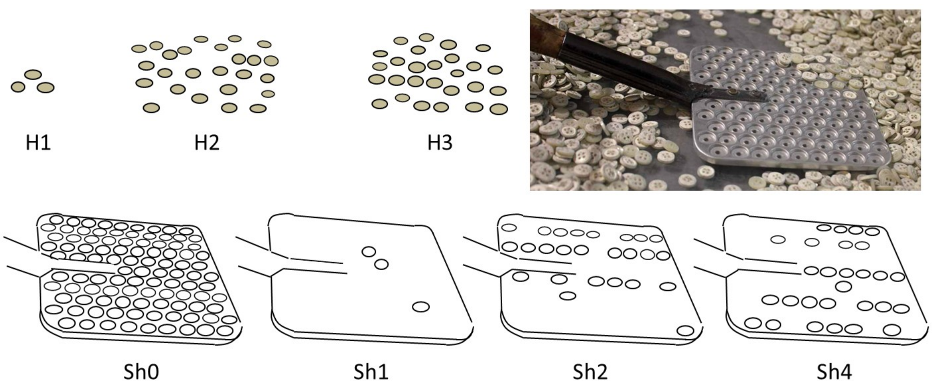

Figure 1: heaps of buttons (H1, H2, H3), *pelle* http://ouvragesdedames.canalblog.com,

drawings of shovels (Sh0 – copy of original; Sh1, Sh2, Sh4 – *ad hoc* shovels)

These descriptions of experiences involve discrete quantities. Experiencing these as quantities without the aid of counting is uncommon since counting starts very early in most cultures. Nevertheless, it is possible to treat these as quantities even without the cultural resource of a counting number system. We remind the reader however, that these descriptions do not carry any recommendations for designing instruction.

According to Bass, as mentioned earlier, an important aspect of theory building practices is abstraction of commonalities. In the discussion above, we notice two moves that can be characterised in this way. One of them is to be able to replace "in the mind" the sensation (in terms of heaviness or the amount of buttons) by the idea (of the heaviness, the idea of the amount of buttons), we call the *size*, and the other to be able to imagine comparisons, and combinations of several sizes, based on the concrete realization of only some of them. We also suggest that the use of the words "more" and "less" participate in this abstraction.

Another aspect of theory-building is to identify the common structure between the different types of sizes (for example, heaviness and amount of buttons). Above, when imagining situations D to J involving buttons, the reader may have identified such a structure. Although it predates the major reorganization of mathematics, Bezout's introductory words of his arithmetic treatise (1764/1821) can be interpreted as a theoretical statement that summarizes a structure and an abstraction of commonalities: "In general, everything that is susceptible of increase or decrease [structure] is called quantity. Extent, duration, weight, etc., are quantities [quantity is the abstraction of extent, duration, weight]". (p.1)

The quantity as a mathematical structure

Here we present more precisely stated propositions about quantity. They are inspired notably by Kolmogorov's theory (1979) and our naive ideas on quantities presented in the previous section that guided our own understanding of the theory. We have chosen the axioms aligned with the basic hypothetical experiences described above. Although we do not assume commutative property, one could choose it to be part of the axioms. Indeed, various choices are possible for the axioms or starting points of the theory. The choice could be made on the basis of the cumulative experience of the learners and the confidence with which they reason about certain relationships.

(a) The first axiom is that of trichotomy. There is a relation: Given two quantities a, b, one and only one of the three holds: $a>b$, $b>a$, $a=b$.

(c) There is a composition law: $a*b=c$. This means, given two quantities a and b, a can be composed (or combined) with b to form a third quantity c.

An important aspect of the composition law is its link with the relation. Indeed, when a quantity is composed with another, it increases. In other words, (d)

Given a, b, $a*b>a$. This axiom (monotonicity of *) has a reciprocal facet (e): existence of complement. Given a, b, $a>b$, the quantity c exists such that $b*c=a$.

To remain close to F above, we choose the following axiom as a fundamental property of the composition law: (f) For any a, b, c, $(a*b)*c=(a*c)*b$ (f1) and $(a*b)*c=a*(b*c)$ (f2). From (e) and (f) we can deduce the relation > is transitive, and that > is a strict total order.

Properties of the composition law: The law is 1) associative, 2) commutative (the law will then be noted (+), as per usual convention) and 3) any element is cancellative ($a+b=a+c \Rightarrow b=c$). We now prove these properties.

1) The associative property is (f2), part of (f) in the description above.

2) Proof of commutative property: Given a, b, c, $b*c=a$, we have: $b*b=b*b \Rightarrow (b*b)*c=(b*b)*c \Rightarrow (b*b)*c=(b*c)*b$, and $(b*b)*c=b*(b*c) \Rightarrow a*b=b*a$.

3) Proof of cancellation property: If $b>c$, then there is d such that $c+d=b$ and $a+b=a+(c+d)=(a+c)+d$, then $a+b>a+c$. If $b=c$, then $a+b=a+c$. If $b<c$, then $a+b<a+c$. Consequently: $a+b=a+c \Rightarrow b=c$.

A Quantity is a structure $(Q,+)$ such that $a>b \Leftrightarrow$ there exists c such that $b+c=a$.

(g) A consequence of 3 is that the complement is unique. This allows one to define subtraction. Given a, b, $a>b$, we denote the complement as $a-b$. (h) Given a, b, c, such that $a>b>c$, then, $a-b<a-c$. The proof based on the facts: $\underline{a}=\underline{b+d}$, $b=c+e$ (e), $\underline{a}=(c+e)+d=\underline{c+(e+d)}$ (f). Informally: one difference is a part of the other.

We have presented and in some cases proved, in formal or natural language, some properties of quantities. We see the latter language as a means to support and guide teachers' work. The former notably supports education research about mathematical coherence. Other properties were omitted for reasons of place.

From quantities to numbers

The brief sketch of the mathematical notion of quantity presented above does not involve the notion of a number. In the theories of quantity developed by several mathematicians, it is clear that the "amounts of times" in "the amount of times a given quantity is iterated" forms the idea of a number (as suggested in our I and J above). The number is seen as a number of times, whether these times are a whole number of times or fractional times[4]. This is a bridge between quantities and numbers that can be formulated in different ways. For instance, Whitney (1968) says that the iterations (of an element) can be seen as a set of operators, and that this set satisfies the Peano axioms. Steiner (1969) assumes that the set of whole numbers is already available and lets it operate on the set of quantities. Defining a general set of operators, he shows that this set is

4 "Fractional times" requires an additional axiom: Be n whole number, a a quantity (i.e., a size), there exist b a quantity (i.e., a size), such that $a = n.b$, where $n.b$ means n iterations of b. Then b is the "n-th of a".

isomorphic to the set of numbers. In this way, the properties of quantities are transferred to numbers. This enables one to obtain order and composition law on the sets of numbers (that are actually the same as common order and addition of numbers) (Steiner, 1969).

In the following sections, we discuss two different curricular contexts, that reflect elements of the theory that we have sketched above.

THE EL'KONIN-DAVYDOV (ED) CURRICULUM: THE STATUS OF QUANTITIES

The ED curriculum is generally mentioned for its features of early algebra. Interestingly, Kolmogorov's theory (1979) forms the foundation for the ED curriculum (Davydov, 1975). Today, this feature of the curriculum is rarely made explicit. For instance, it is not mentioned in the ESM special issue on Davydov (106, 2021) that a mathematical quantity theory shapes the curriculum.

Are there visible features of the curriculum that recall the theory? Davydov (1975, p. 135-138) summarizes the grade 1 curriculum. It is organized in a series of six topics. Topic I is "Comparing and assembling objects (according to length, volume, weight, composition, and other parameters)". Topics II and III focus on comparison and signs >, <, =. Topic IV, "The operation of addition (and subtraction)", starts with "Observations of changes in objects in one or another parameter (such as volume, weight, length, or time)" and mentions "increase" and "decrease". Later, Topic V is titled: "The shift from an inequality of the type $A < B$ to equality through addition (or Subtraction)". It includes "Writing formulas of the type: if $A < B$, then $A+x = B$; if $A > B$, then $A-x = B$".

Topic I recalls "steps" A and B of our intuitive approach of the theory, while topic IV includes step "D", and topic V recalls "step" E. In Davydov's progression, the operations of addition and subtraction are first considered independently, each in its relation with order in topic IV (a kind of step D for both, in other words). They are linked in topic V. Investigating the curriculum, we understand that the successive topics enable the students to state, one after the other, expressions of the axioms with letters and signs (that represent quantities -thus sizes- and their relationships students dealt with in their experiments); to engage in thinking with the meaning, and relationships of letters and signs, thanks to the realistic problems students have to solve. Students explore quantities of different types, simultaneously. This thus suggests the abstraction of the notion of quantity, and of its structure. We think our approach sheds light on the internal mathematical coherence of the curriculum, a coherence that is possible only thanks to the presence of quantities at the starting point of the theory, a feature that is, according to us not often stressed in articles that focus on the ED curriculum. One can also notice that our intuitive approach did not include any visible algebraic feature. This said, a perhaps invisible feature of our intuitive presentation is the use of language to describe relations between quantities, an important means to abstract from realistic contexts.

Davydov makes a case for the study of quantities in the following quote:

> Real number is based on positive scalar quantities, the concept of which is defined by all ten of the properties [listed by Kolmogorov]. (…) It is striking that natural numbers, fractions (rational numbers), and real numbers themselves can be represented as quantities (both Kagan and Kolmogorov mention this).
>
> It may be concluded from the material cited above that natural and real numbers are equally closely related to quantities and certain of their essential characteristics (properties 1 to 7 [equivalent to our (a) to (f)]). **Might not the child study these and the other properties as a special topic before he is introduced to the numerical form for describing the relationship between quantities**? (Davydov, 1975, p. 133, underlined as in original, bold is ours)

THE JAPANESE CURRICULUM: THE GENERAL ASPECT OF NUMBER

Fujii (2015) describes Japanese teachers' focus on quasi-variables, suggesting to us that it might be a feature of the Japanese curriculum:

> Close attention to the specific numbers does not mean that teachers are sticking to a concrete level of thinking and encouraging students to think about things concretely. On the contrary: teachers consider the general aspect of the number – its quasi-variable aspects (p.14).

Fujii does not mention quantities. Yet, we find his words an interesting echo of Davydov's words. Here are some examples involving quasi-variables:

> The reason why 13 – 9 or 12 – 9 is the first task is that the minuend 9 is close to ten, and it is easier for the student to separate 13 into 10 and 3 and subtract 9 from 10 and then add the difference to 3. 13 – 9=(10 + 3) – 9=(10 – 9) + 3. (p.15)

Fujii (2015) describes a procedure in terms of "subtracting-adding", combined with place value knowledge. We note instead this is a subtraction version of our property (f). The quantity 13 is seen as composed of 10 and 3 (based on place value). It is then equivalent to remove a quantity from it or from any of its parts.

In a design study with Australian and Japanese students, Fujii (2003) reported some examples of students' reasoning about subtraction problems such as 32-5=32-10+5: "The bigger the number you are subtracting, the smaller the number you are pulsing [plusing]. They all make a ten together." (p.61) [Tim] … "whatever number he is taking away" (Tim), "whatever the number is you are taking away" (Zoe), "for any number you are taking away" (Alan), "there is always a number to make ten" (Adam)." (p.61)

Focusing on the generality of the students' reasoning Fujii interprets these as instances of "algebraic thinking" (p. 61) in students. Looking closely at the students' arguments, we identify that some of them are *quantitative* (those underlined by us). We suggest the generality stems from the fact that the reasoning is about the notion of size that has been abstracted and that its general properties involving order and addition [combination] became available in students. For instance, we interpret the sentence "there is always a number to make ten" as: from a given number (a size, smaller than 10), *there is always a number* (a size) that can be added, to form a greater number (a greater size, 10). This is property (e). We also notice that Tim's reasoning expresses property (h). In other words, we suggest these students express their sense of the structure of the quantity that they have recognised in numbers.

We argue that the properties embedded in the "general aspect of number" are those of the "quantity". Are there reasons why Japanese teachers foster quantitative reasoning? We also ask: do the quantities play a specific role in the Japanese curriculum? Several scholars have highlighted a specific focus on measurement in the Japanese curriculum (Batteau, 2019, Watanabe, 2007). More recently, scholars (Karagöz Akar et al., 2022) speculated about a relationship between high level of tasks in multiplicative reasoning and measurement in the Japanese curriculum. Chambris and Batteau (2021) have suggested a connection between measurement and the arithmetic curriculum.

With similar arguments to Davydov, we make the following hypothesis**: there is a transfer of the properties of quantities (those of order and of the composition law), i.e., of a structure, to the set of operators / numbers, that enable to "understand" the written / oral number as a quantity, i.e., a similar structure.** Can Japanese school arithmetic be seen as fostering theory building practices based on the notion of quantities? Answering this question requires a further investigation of the curriculum with this question in mind.

CONCLUDING REMARKS

The mathematician Wu (2011) argues about the lack of coherence in some curricula. We suggest that this might be a consequence of the invisible constraint caused by the level of abstraction in the starting points available for the theory. The mathematical and curricular analyses provided in this paper suggest that basing an arithmetic curriculum on quantities might provide the required coherence for theory building practices. This is because quantity provides a starting point of the curriculum based in intuition, but is defined as a size that has specific properties, that in turn may be investigated and used both to solve meaningful quantitative problems, and to create new mathematical abstractions. Thus, a mathematical theory of quantity, of the sort outlined in this paper, could provide a reference theory that supports theory-building in school arithmetic. Such theory building would lead to a recognition that the mathematical structure of quantity underlies that of number. Elaborating such a structure would not only provide coherence in understanding number and number relationships, but could support students' in confidently making deductions and providing explanations, which we suggest, enhances access to key mathematical practices in elementary mathematics classrooms.

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