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Analysing interaction and localization dynamics in modulation instability via data-driven dominant balance

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Abstract. We report the first application of the Machine Learning technique of data-driven dominant balance to optical fiber noise-driven Modulation Instability, with the aim to automatically identify local regions of dispersive and nonlinear interactions governing the dynamics. We first consider the analytical solutions of Nonlinear Schrödinger Equation – solitons on finite background – where it is shown that dominant balance distinguishes two particularly different dynamical regimes: one where the nonlinear process is dominating the dispersive propagation, and one where nonlinearity and second order dispersion act together driving the localization of breathers. By means of numerical simulations, we then analyse the spatio-temporal dynamics of noise-driven Modulation Instability and demonstrate that data-driven dominant balance can successfully identify the associated dominating physical regimes even within the turbulent dynamics.

1 Introduction

We present an application of a fundamentally new approach for interpreting noise-driven Modulation Instability (MI) dynamics in optical fiber propagation via the Machine Learning (ML) technique of data-driven dominant balance [1]. We aim to automate the problem of identifying which particular physical processes drive propagation in different regimes by means of unsupervised learning algorithms, a task usually performed using intuition and/or asymptotic limits [2]. Here we apply the first use of dominant balance to interpret continuous wave MI in the Nonlinear Schrödinger Equation (NLSE), allowing us to readily associate different interactions with various structures seen during the evolution. By using analytic results for MI-related breather solutions, we automatically distinguish purely nonlinear propagation from regions where nonlinearity and dispersion combine to drive localization. We then use numerical simulations to apply the technique to the more complex case of noise-driven chaotic MI evolution.

2 Methods

MI describes the breakup of a continuous wave injected into optical fiber in the anomalous dispersion regime. The governing NLSE is written in normalized form as:

$$iu_{\xi} + u_{\tau\tau} + |u|^2u = 0, \quad (1)$$

where ξ is the distance, τ is comoving time and u is the field envelope. It has been recently demonstrated that the data-driven dominant balance technique based on unsupervised learning algorithms can be used in the identification of dominating physical regimes. Generally, this involves the following steps: (i) determining the evolution map of $u(\xi, \tau)$ by means of numerical simulations of Eq. 1 or using the analytical solutions; (ii) analysing the obtained map in the associated “equation space,” applying the cluster detection algorithm to identify regions where combinations of the NLSE terms (iu_{ξ} , $u_{\tau\tau}$, $|u|^2u$) dominate the interaction, i.e. selecting ensembles of points in the three-dimensional equation space that have a significantly reduced variance with respect to some of its directions; (iii) finally, re-mapping these identified clusters back onto the (ξ, τ) domain allows us to directly compare the selected subregions associated with the dominant physical processes with the initial spatio-temporal field distribution.

3 Results

3.1. Solitons on finite background

Figures 1(a-c) illustrate the application of the technique to the analytic solutions for the Peregrine soliton and the Akhmediev and Kuznetsov-Ma breathers - strongly localized structures known to emerge from noise-driven MI [3]. For each case, subfigure (i) shows the corresponding spatio-temporal distribution of the $|u(\xi, \tau)|^2$, subfigure (ii) illustrates the equation space cluster

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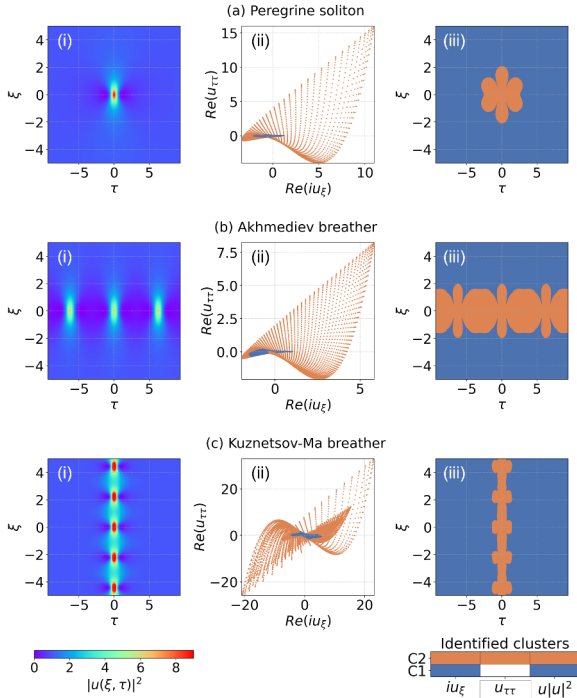


Fig. 1 Application of dominant balance analysis for the (a) Peregrine soliton, (b) Akhmediev breather and (c) Kuznetsov-Ma breather: (i) represents the spatio-temporal map of $|u(\xi, \tau)|^2$, (ii) shows an example of the cluster assignment algorithm performed in the equation space, and (iii) depicts the clusters mapped back to the (ξ, τ) coordinates. The colormaps on the bottom apply to all figures. Blue subregions represent the dominance of nonlinearity, while orange shows subregions where dispersion and nonlinearity contribute equally.

detection by showing the identified cluster in $(iu_\xi, u_{\tau\tau})$ -plane – one of the planes of three-dimensional equation space, – and subfigure (iii) shows these clusters remapped back to (ξ, τ) enabling us to directly compare it with the initial evolution map. The color scales are on the bottom, and are identical for all figures. The key physical insight that appears from the subfigures (iii) is that the blue regions are associated only with dominant $(iu_\xi, |u|^2u)$ nonlinear interactions, whilst the orange region are prescribed to the interplay of all three NLSE terms $(iu_\xi, u_{\tau\tau}, |u|^2u)$. These results clearly indicate that in the blue "continuous wave" region, where temporal localization is absent, dispersive effects are not playing the dominant role. Conversely, in the orange region where there is significant temporal localization, it is shown that both dispersion and nonlinearity contribute equally.

3.2. Noise-driven MI

Figure 2 shows more complex evolution for chaotic MI driven by an input continuous wave with low amplitude noise (corresponding to a one photon per mode noise background in dimensional terms). Yet even for this highly random dynamics, data-driven dominant balance can successfully distinguish the spatio-temporal regions where only the nonlinear (blue) and combined nonlinear

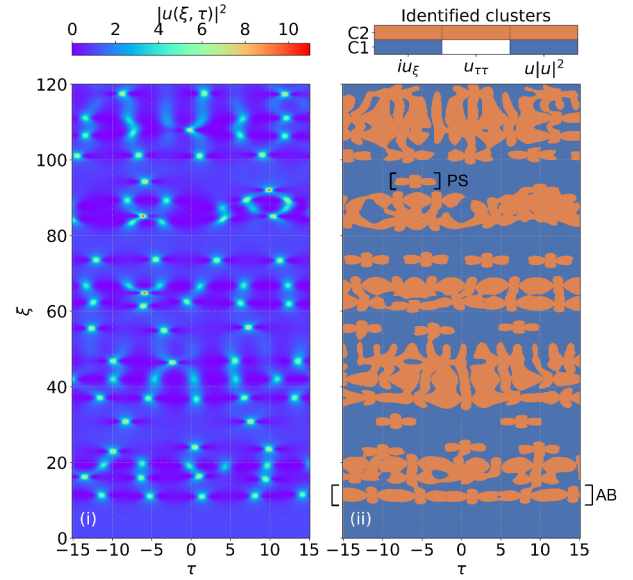


Fig. 2 (i) the evolution map and (ii) the remapped clustering results for noise-driven MI. Features PS and AB in plot (ii) show signatures of the Akhmediev breather and Peregrine soliton.

and dispersive (orange) effects are playing the dominant role. Moreover, we readily see how dominant balance identifies the characteristic interaction signatures of Peregrine solitons and Akhmediev breathers that we saw earlier from the analytic results in Fig. 1(a) and (b). There are many such signatures visible in the figure, and two are shown as examples as marked features PS and AB respectively in Fig. 2(ii).

3 Discussion & Conclusion

Taken together, these results show the ability of data-driven dominant balance techniques to automatically attribute contributing physical processes to specific stages of MI evolution and are another example of how machine learning can complement traditional approaches to analysis of nonlinear dynamics. While the current results were obtained using analytic results and simulations, advancements in experimental full-field characterization techniques suggest that future research may explore the application of dominant balance to laboratory data in order to discover underlying physical principles. [4].

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