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Appendix of Reachability Analysis of a Class of Hybrid Gene Regulatory Networks

Honglu Sun¹[0000-0002-8265-0984], Maxime Folschette²[0000-0002-3727-2320], and Morgan Magnin¹[0000-0001-5443-0506]

¹ Nantes Université, École Centrale Nantes, CNRS, LS2N, UMR 6004, F-44000 Nantes, France

² Univ. Lille, CNRS, Centrale Lille, UMR 9189 CRISTAL, F-59000 Lille, France

1 Simulation of HGRNs

This appendix formally describes the simulation of HGRNs.

Consider a state $h = (\pi, d_s)$ and a trajectory τ which reaches h at $t > 0$.

- If h does not belong to any boundary, then $\frac{d\tau(t)}{dt} = c_s$ (the temporal derivative of a hybrid state $h = (\pi, d_s)$ is defined as $\frac{dh}{dt} = \frac{d\pi}{dt}$).
- If h only belongs to one boundary e , let us consider that e is the upper boundary in i^{th} dimension (the result is easily adapted when e is the lower boundary). In case d_s^i is not the maximal discrete level of i^{th} gene, the discrete state on the other side of e is noted as d_r , where $d_s^k = d_r^k$ for all $k \neq i$, and $d_s^i + 1 = d_r^i$. There are four possible cases:
 - If $c_s^i < 0$, then the trajectory from the current state will enter the interior of the current discrete state. e called an *input boundary* of d_s . $\frac{d\tau(t^+)}{dt} = c_s$, $\frac{d\tau(t^-)}{dt} = c_r$ and $\tau(t) = ((\pi', d_r), (\pi, d_s))$, where $\pi'^k = \pi^k$ for all $k \neq i$, and $\pi'^i = 0$, which means that there is an instant transition from (π', d_r) to (π, d_s) at t .
 - If $c_s^i = 0$, then the trajectory from the current state will slide along the boundary e , which is then called a *neutral boundary* of d_s . $\frac{d\tau(t^+)}{dt} = \frac{d\tau(t^-)}{dt} = c_s$ and $\tau(t) = (\pi, d_s)$.
 - If $c_s^i > 0$, and either d_s^i is the maximal discrete level of the i^{th} gene, or d_s^i is not the maximal discrete level of the i^{th} gene but the i^{th} component of c_r is negative, then the trajectory from the current state will slide along the boundary e , which is called an *attractive boundary* of d_s . If τ reaches e at t , then: $\frac{d\tau(t^+)}{dt} = c_s^k$ for all $k \neq i$, $\frac{d\tau(t^+)}{dt} = 0$, $\frac{d\tau(t^-)}{dt} = c_s$ and $\tau(t) = (\pi, d_s)$. If τ reaches e at $t_0 < t$, then: $\frac{d\tau(t)}{dt} = c_s^k$ for all $k \neq i$, $\frac{d\tau(t)}{dt} = 0$, and $\tau(t) = (\pi, d_s)$.
 - If $c_s^i > 0$, d_s^i is not the maximal discrete level of the i^{th} gene, and the i^{th} component of c_r is positive, then the trajectory from the current state will cross instantly the boundary e and enter the discrete state d_r . e is called an *output boundary* of d_s . $\frac{d\tau(t^+)}{dt} = c_r$, $\frac{d\tau(t^-)}{dt} = c_s$ and $\tau(t) = ((\pi, d_s), (\pi', d_r))$, where $\pi'^k = \pi^k$ for all $k \neq i$, and $\pi'^i = 0$.

- If h belongs to several boundaries, then the previous cases can be mixed:
 - If in these boundaries there is no output boundary, then the trajectory from the current state will exit all input boundaries and slide along all attractive or neutral boundaries.
 - If in these boundaries there is only one output boundary, then the trajectory from the current state will cross this output boundary.
 - If in these boundaries there are several output boundaries, then the trajectory from the current state can cross any of them, but can only cross one boundary at one time, which causes non-deterministic behavior.

2 Proof of Theorem 1

Proof. For a trajectory τ attracted by a cycle of discrete domains, $\exists t_0$ such that after t_0 , τ always stays in a cycle of discrete domains $\mathcal{C}_\tau = (\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_p, \mathcal{D}_0)$ and before t_0 , τ has already crossed at least one discrete state. τ can be separated on two trajectories by t_0 : the restriction of τ on time interval $[0, t_0]$, noted by $\tau_{[0, t_0]}$, and the restriction of τ on time interval $[t_0, \infty]$, noted by $\tau_{[t_0, \infty]}$.

For the trajectory $\tau_{[0, t_0]}$, whether it reaches R_2 is decidable. If it reaches R_2 , then it is reachable. And if it does not reach R_2 , then we need to consider $\tau_{[t_0, \infty]}$.

For the trajectory $\tau_{[t_0, \infty]}$, in case that $\tau_{[0, t_0]}$ does not reach R_2 , whether $\tau_{[t_0, \infty]}$ reaches R_2 is equivalent to whether it reaches $Pre_{d_{s_2}}(R_2) = \bigcup_{i \in \{1, 2, \dots, q\}} Z_i$. If $\forall i \in \{0, 1, 2, \dots, p\}$, $\forall j \in \{1, 2, 3, \dots, q\}$, $Z_j \not\subseteq \mathcal{D}_i$, then $Pre_{d_{s_2}}(R_2)$ is not reachable. Otherwise, if $\exists i_0 \in \{0, 1, 2, \dots, p\}$ and $\exists j_0 \in \{1, 2, 3, \dots, q\}$, such that $Z_{j_0} \subseteq \mathcal{D}_{i_0}$, then to verify if Z_{j_0} is reachable, we need to consider the relation between the intersection points of $\tau_{[t_0, \infty]}$ with \mathcal{D}_{i_0} noted by a infinite sequence $(h_{p_1}, h_{p_2}, \dots)$ and Z_{j_0} as following. (Need to mention that when a trajectory reaches a discrete domain, it might slide inside this discrete domain, in this case, based on the general meaning of "intersection", the intersection points of this trajectory and this discrete domain are not singular points. In this paper, we define that the intersection points between a trajectory and a discrete domain are the points where the trajectory just reaches the discrete domain.)

Since τ always stays inside this cycle of discrete domains, the sequence $(h_{p_1}, h_{p_2}, \dots)$ converges asymptotically to a state or reaches a state which belongs to the closure of the compatible zone of \mathcal{C}_τ . For both cases, this particular state is noted by h_∞ .

If $h_\infty \in Z_{j_0}$, then $Pre_{d_{s_2}}(R_2)$ is reachable. Need to mention that, in case that h_∞ is on the boundary of Z_{j_0} and the sequence $(h_{p_1}, h_{p_2}, \dots)$ converges asymptotically to h_∞ without reaching directly Z_{j_0} , Z_{j_0} is also considered as reachable, which in fact is reached after an infinite times of intersections.

If $h_\infty \notin Z_{j_0}$, then we can find a small neighborhood of h_∞ inside \mathcal{D}_{i_0} , noted by \mathcal{N}_{h_∞} , such that $\forall h \in \mathcal{N}_{h_\infty}$, $h \notin Z_{j_0}$. Since the sequence $(h_{p_1}, h_{p_2}, \dots)$ converges to h_∞ or finally reaches h_∞ , we can find n_0 such that $\forall n > n_0$, $h_{pn} \in \mathcal{N}_{h_\infty}$. This means that after that $\tau_{[t_0, \infty]}$ reaches h_{pn_0} , it will never reach Z_{j_0} . So, in

this case, to verify if Z_{j_0} is reached, we only need to verify if the finite sequence $(h_{p1}, h_{p2}, \dots, h_{pn_0})$ ever reaches Z_{j_0} , which is decidable.

Now we see that, for a certain Z_i , whether $\tau_{[t_0, \infty]}$ reaches Z_i or not is decidable. Since $Pre_{d_{s_2}}(R_2)$ has a finite number of such Z_i , whether $\tau_{[t_0, \infty]}$ reaches $Pre_{d_{s_2}}(R_2)$ is also decidable. \square

3 Proof of Theorem 2

Proof. Firstly, we prove that it is a sufficient condition:

In the case that a trajectory τ is attracted by a cycle of discrete domains, then after certain moment, τ will always stay in the same cycle of discrete domains, noted by $\mathcal{C}_\tau = (\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_p, \mathcal{D}_0)$.

We consider at first the case that there is one \mathcal{D}_i of \mathcal{C}_τ such that \mathcal{D}_i has at least one free dimension (meaning that there exists one dimension such that the boundaries are not reached in this dimension), without loss of generality we assume that \mathcal{D}_0 has at least one free dimension, so the first condition is satisfied.

A priori, we cannot ensure the second condition, because the decomposition of $r(h_0) - r_\infty$ in the directions of certain eigenvectors of the reduction matrix of \mathcal{C}_τ can be zero and these eigenvectors happen to be associated to eigenvalues of which the absolute values are greater than 1, however this is extremely rare. So in this paper we ignore this case, and based on this assumption, we must have that for any eigenvalue λ_i , $|\lambda_i| \leq 1$, otherwise we cannot ensure that τ always stays inside this cycle, because if there exists an eigenvalue such that $|\lambda_i| > 1$, then r_∞ is infinitely far away from $r(h_0)$. If there exists $\lambda_i = -1$, we can double this cycle of discrete domains (the doubled cycle of discrete domains is $(\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_p, \mathcal{D}_0, \mathcal{D}_1, \dots, \mathcal{D}_p, \mathcal{D}_0)$) to eliminate this eigenvalue. So the second condition is satisfied.

The intersection points of τ with \mathcal{D}_0 will converge to or reach certain state $h_\infty = (\pi_\infty, d_{s_0})$ which belongs to the closure of the compatible zone. We have that r_∞ is the short version of π_∞ by only considering the dimensions where boundaries are not reached in \mathcal{D}_0 . Based on Assumption 3, the reduction compatible zone can be described by $\{x \mid Wx > c\}$. If h_∞ belongs to the compatible zone, then we have $\forall i \in \{1, 2, \dots, n_0\}$, $W_i r_\infty > c_i$, else if h_∞ does not belong to the compatible zone but belongs to the closure of the compatible zone, then we have that for some $i \in \{1, 2, \dots, n_0\}$, $W_i r_\infty = c_i$, meaning that some "boundaries" of the reduction compatible zone are reached, and for other $i \in \{1, 2, \dots, n_0\}$, we have $W_i r_\infty > c_i$. So the third condition is satisfied.

We note the intersection states of τ and \mathcal{D}_0 as the sequence (h_0, h_1, h_2, \dots) . We have that $r(h_n) = r_\infty + \sum_{i=1}^{n_1} \lambda_i^n \alpha_i v_i$ where v_i are eigenvectors of the reduction matrix of \mathcal{C}_τ and $r(h_0) - r_\infty = \sum_{i=1}^{n_1} \alpha_i v_i$, by choosing λ_1 as the eigenvalue with maximum absolute eigenvalue among the eigenvalues that differ from 1, if $\lambda_1 \neq 0$, then we have: $\lim_{n \rightarrow \infty} |\lambda_1^n \alpha_1| \gg |\lambda_i^n \alpha_i|$ where $i \neq 1$, and $\lim_{n \rightarrow \infty} \sum_{i=1}^{n_1} \lambda_i^n \alpha_i v_i \rightarrow 0$. This proves that the fifth and the sixth conditions are satisfied.

Supposing that $\lambda_1 \neq 0$. Since the sequence (h_0, h_1, h_2, \dots) is inside the compatible zone, if I_e is not empty, then for any $j \in I_e$, we should have $\forall n \in \mathbb{N}, W_j \sum_{i=1}^{n_1} \lambda_i^n \alpha_i v_i > 0$. When n is sufficiently big, for any $j \in I_e$, the sign of $W_j \sum_{i=1}^{n_1} \lambda_i^n \alpha_i v_i$ is dominated only by $W_j \lambda_1^n \alpha_1 v_1$, meaning that $W_j \sum_{i=1}^{n_1} \lambda_i^n \alpha_i v_i$ and $W_j \lambda_1^n \alpha_1 v_1$ have the same sign, in fact, when n is sufficiently big, we have $|W_j \lambda_1^n \alpha_1 v_1| > \sum_{i=2}^{n_1} |W_j \lambda_i^n \alpha_i v_i|$, this means that adding $\sum_{i=2}^{n_1} W_j \lambda_i^n \alpha_i v_i$ to $W_j \lambda_1^n \alpha_1 v_1$ does not change the sign of $W_j \lambda_1^n \alpha_1 v_1$. So the sign of λ_1 is positive, otherwise we can not ensure that $\forall j \in I_e, \forall n \in \mathbb{N}, W_j \sum_{i=1}^{n_1} \lambda_i^n \alpha_i v_i > 0$, which is equivalent to $\forall j \in I_e, \forall n \in \mathbb{N}, W_j \lambda_1^n \alpha_1 v_1 > 0$ when n is sufficiently big. This proves that the forth condition is satisfied.

In the case that any \mathcal{D}_i of $\mathcal{C}_{\mathcal{T}}$ has no free dimension, we can easily see that it is a sufficient condition.

Secondly, we prove that it is a necessary condition.

If \mathcal{D}_0 has no free dimension, then τ reaches a limit cycle which is a special case of that τ is attracted by a cycle of discrete domains.

If these six conditions are satisfied, to prove that τ is attracted by a cycle of discrete domains, we only need to prove that τ always returns to the compatible zone of $\mathcal{C}_{\mathcal{T}}$. This can be easily proved if $\lambda_1 = 0$. From now on, we suppose the $\lambda_1 \neq 0$. We note that trajectory from h_0 returns to \mathcal{D}_0 at h_1 . We have that $r(h_1) - r_\infty = \sum_{i=1}^{n_1} \lambda_i \alpha_i v_i$. A sufficient condition for " h_0 belongs to the compatible zone" is $(\forall j \in I_e, W_j \sum_{i=1}^{n_1} \alpha_i v_i > 0) \wedge (\forall j \in I_n, \max_{\beta \in \{-1, 1\}^{n_1}} \|\sum_{i=1}^{n_1} \beta_i \alpha_i v_i\|_2 < \frac{W_j r_\infty - c_i}{\|W_j\|_2})$ which is satisfied in this case. In fact, the first part of this sufficient condition is a necessary condition for that h_0 belongs to the compatible zone and the second part is ensured by the sixth condition. Now we want to prove that this sufficient condition is also satisfied for h_1 . The fifth condition indicates that for any $j \in I_e$ the sign of $W_j \sum_{i=1}^{n_1} \alpha_i v_i$ is dominated by the sign of $W_j \alpha_1 v_1$, so $W_j \alpha_1 v_1$ is also positive. Since $\forall i \neq 1$, we have either $|\lambda_1| > |\lambda_i|$ or $|\lambda_1| < |\lambda_i| \wedge \alpha_i = 0$, so we have $\forall j \in I_e, \forall i \in \{2, \dots, n_1\}, |W_j v_1 \lambda_1 \alpha_1| > n_1 |W_j v_i \lambda_i \alpha_i|$. This means that for any $j \in I_e$ the sign of $W_j \sum_{i=1}^{n_1} \alpha_i \lambda_i v_i$ is also dominated by the sign of $W_j \alpha_1 \lambda_1 v_1$. Since λ_1 is positive if I_e is not empty, we have $\forall j \in I_e, W_j \sum_{i=1}^{n_1} \lambda_i \alpha_i v_i > 0$, because the sign of $W_j \sum_{i=1}^{n_1} \lambda_i \alpha_i v_i$ is same as the sign of $W_j \alpha_1 \lambda_1 v_1$ which is same as the sign of $W_j \alpha_1 v_1$ of which the sign is positive. Since $\forall \lambda_i, |\lambda_i| \leq 1$, we have $\max_{\beta \in \{-1, 1\}^{n_1}} \|\sum_{i=1}^{n_1} \beta_i \lambda_i \alpha_i v_i\|_2 \leq \max_{\beta \in \{-1, 1\}^{n_1}} \|\sum_{i=1}^{n_1} \beta_i \alpha_i v_i\|_2$. So we also have $\forall j \in I_n, \max_{\beta \in \{-1, 1\}^{n_1}} \|\sum_{i=1}^{n_1} \beta_i \lambda_i \alpha_i v_i\|_2 < \frac{W_j r_\infty - c_i}{\|W_j\|_2}$. By now we prove that h_1 also belongs to the compatible zone. By mathematical induction, we can prove that τ always returns to the compatible zone of $\mathcal{C}_{\mathcal{T}}$. So τ is attracted by a cycle of discrete domains. \square

4 Details about the function *Stop_condition*

The details of the function *Stop_condition* are presented in Algorithm 1. Its objective is, knowing that a trajectory is attracted by a cycle of discrete domains, to determine if the trajectory can reach R_2 after an infinite number of transitions (see Fig 3 right), in this case, return "Reached", or if from the current moment,

there is no more chance to reach R_2 (see Fig 3 middle), in this case, return “Yes”, otherwise return “No”. The cases other than “No” mean that the main algorithm for reachability analysis can be stopped.

In order to describe the Non-intersection condition in Algorithm 1, we note the predecessor in the same discrete state of R_2 as $Pre_{d_{s_2}}(R_2) = \bigcup_{i \in \{1, 2, \dots, q\}} Z_i$. We note $r(Z_i)$ as the set of the reduction vectors of all states of Z_i .

This Non-intersection condition (its formal definition will be given later) is a sufficient condition for that $\forall j \in \{1, 2, \dots, q\}$, if $Z_j \subset \mathcal{D}_i$, then $\{x \in \mathcal{S}_r(\mathcal{D}_i) \mid \|x - r(h_i^\infty)\|_2 \leq \max_{\beta \in \{-1, 1\}^{n_1}} \|\sum_{k=1}^{n_1} \beta_k \alpha_k v_k\|_2\} \cap r(Z_j) = \emptyset$ where $\mathcal{S}_r(\mathcal{D}_i)$ is the reduction compatible zone of this cycle inside \mathcal{D}_i , n_1 is the dimension of $\mathcal{S}_r(\mathcal{D}_i)$, $r(h_i) - r(h_i^\infty) = \sum_{k=1}^{n_1} \alpha_k v_k$, v_k are the eigenvectors of the reduction matrix of this cycle $(\mathcal{D}_i, \dots, \mathcal{D}_i)$.

The idea of the Non-intersection condition is illustrated by Fig 1. Firstly, we want to verify $\forall j \in \{1, 2, \dots, q\}$, if $Z_j \subset \mathcal{D}_i$, then $\{x \in \mathcal{S}_r(\mathcal{D}_i) \mid \|x - r(h_i^\infty)\|_2 \leq \max_{\beta \in \{-1, 1\}^{n_1}} \|\sum_{k=1}^{n_1} \beta_k \alpha_k v_k\|_2\} \cap r(Z_j) = \emptyset$ where $\mathcal{S}_r(\mathcal{D}_i)$ is the reduction compatible zone of this cycle inside \mathcal{D}_i . This is equivalent to that the black box does not intersect the blue boxes in Fig 1 left. The reason why we want to verify this condition, noted here as Condition1, is that once it is satisfied, there is no more chance that the trajectory can reach R_2 from \mathcal{D}_i inside this discrete state. However, it is complicated to compute directly the intersection of these sets. So, firstly we overestimate all states in this discrete state which can be reached by $\{h \in \mathcal{S}(\mathcal{D}_i) \mid \|r(h) - r(h_i^\infty)\|_2 \leq \max_{\beta \in \{-1, 1\}^{n_1}} \|\sum_{k=1}^{n_1} \beta_k \alpha_k v_k\|_2\}$ (black box) by the red zone in Fig 1 middle (need to mention that Fig 1 is just a illustration, the shape of the real red zone is slightly different from the one in this figure). If this red zone does not intersect the blue rectangle, then Condition1 is satisfied. However, this intersection is still difficult to compute. So secondly, we move the "thickness" of the red zone to the blue rectangle in Fig 1 right. Now to verify if Condition1 is satisfied, we only need to verify if the red trajectory in Fig 1 right reaches the blue and red zone, which can be done automatically and is called Non-intersection condition.

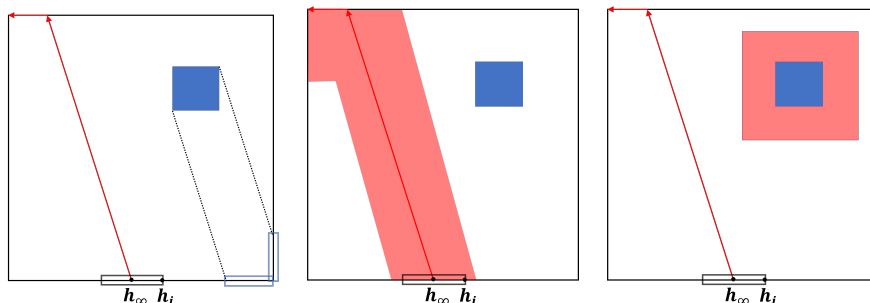


Fig. 1: Illustration of the Non-intersection condition in Algorithm 1.

The Non-intersection condition is defined formally in the following.

Algorithm 1 Stop condition

Input 1: A list of hybrid states $Cycle_h = (h_0, h_1, h_2, \dots, h_p, h'_0)$ **which describes a simulation of a model**

Input 2: A list of discrete domains $Cycle_{\mathcal{D}} = (\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_p, \mathcal{D}_0)$ **such that** $\forall i \in \{0, 1, 2, \dots, p\}, h_i \in \mathcal{D}_i, h'_0 \in \mathcal{D}_0$ **and the trajectory from** h_0 **is attracted by** $(\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_p, \mathcal{D}_0)$

Input 3: A region $R_2 = \{(\pi, d_{s_2}) \mid \pi^i \in [a_i, b_i], i \in \{1, 2, \dots, N\}\}$

Output (three possibilities): Yes, No, and Reached

- 1: Compute the states $h_0^\infty, h_1^\infty, h_2^\infty, \dots, h_p^\infty$ such that the intersection states between this trajectory and $\mathcal{D}_0, \mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_p$ converge to or reach these states when $t \rightarrow \infty$.
 - 2: $var_stop \leftarrow True$
 - 3: **for** $i \in \{0, 1, 2, \dots, p\}$ **do**
 - 4: **if** \mathcal{D}_i is on an input boundary of d_{s_2} **then**
 - 5: **if** Trajectory from h_i^∞ reaches R_2 inside d_{s_2} **then**
 - 6: **return** “Reached”
 - 7: **else if** Non-intersection condition is satisfied **then**
 - 8: $var_stop \leftarrow var_stop \wedge True$
 - 9: **else if** Non-intersection condition is not satisfied **then**
 - 10: $var_stop \leftarrow var_stop \wedge False$
 - 11: **end if**
 - 12: **end if**
 - 13: **end for**
 - 14: **if** $var_stop = True$ **then**
 - 15: **return** “Yes”
 - 16: **else if** $var_stop = False$ **then**
 - 17: **return** “No”
 - 18: **end if**
-

The trajectory from h_i^∞ can be represented by $h_{i,0}^\infty = h_{i,0}^\infty \rightarrow h_{i,1}^\infty \rightarrow \dots \rightarrow h_{i,K}^\infty$ where $\{h_{i,j}^\infty, j \in \{1, 2, \dots, K\}\}$ represents all states at which this trajectory reaches new boundaries and $h_{i,K}^\infty$ represents the state at which this trajectory reaches an output boundary of d_{s_2} for the first time. We note $u_j, j \in \{1, 2, \dots, K\}$ as the left derivative of state $h_{i,j}^\infty$. We use $w_j \in \{0, 1\}^N, j \in \{1, 2, \dots, K\}$ to describe the dimension where new boundary is reached in each elementary transition, for the transition $h_{i,j-1}^\infty \rightarrow h_{i,j}^\infty, j \in \{1, 2, \dots, K\}$, if the new boundary is reached in m_j^{th} dimension, then $w_j^{m_j} = 1$ and $w_j^k = 0, k \in \{1, 2, \dots, N\} \setminus \{m_j\}$. We associate a distance $l_j, j \in \{1, 2, \dots, K, K+1\}$ to each transition, which is computed as following: $l_1 = \max_{\beta \in \{-1, 1\}^{n_1}} \|\sum_{k=1}^{n_1} \beta_k \alpha_k v_k\|_2$ and $\forall j \in \{2, 3, \dots, K, K+1\}, l_j = \frac{l_{j-1}}{\frac{|u_j \cdot w_j|}{\|u_j\|_2}}$ where $u_j \cdot w_j$ is the inner product of u_j and w_j . The Non-intersection condition is satisfied if the trajectory from $h_{i,0}^\infty$ does not reach the region

$R'_2 = \{(\pi, d_{s_2}) \mid \pi^i \in [\max(a_i - l_{K+1}, 0), \min(b_i + l_{K+1}, 1)], i \in \{1, 2, \dots, N\}\}$ inside the discrete state d_{s_2} .

We note this trajectory $h_i^\infty = h_{i,0}^\infty \rightarrow h_{i,1}^\infty \rightarrow \dots \rightarrow h_{i,K}^\infty$ which begins from $h_{i,0}^\infty$ and finally reaches $h_{i,K}^\infty$ as τ_i . If the Non-intersection condition is satisfied, then for any state h_x which belongs to R_2 and for any state h_y which belongs to τ_i , the distance between h_x and h_y is greater than l_{K+1} . On the other hand, for any trajectory τ_x which is inside this discrete state and begins from the region $\{h \in \mathcal{S}(\mathcal{D}_i) \mid \|r(h) - r(h_i^\infty)\|_2 \leq \max_{\beta \in \{-1,1\}^{n_1}} \|\sum_{k=1}^{n_1} \beta_k \alpha_k v_k\|_2\}$ where $\mathcal{S}(\mathcal{D}_i)$ is the compatible zone of this cycle inside \mathcal{D}_i , for any state h'_x on τ_x , we can find a state h'_y on τ_i , such that the distance between h'_x and h'_y is smaller than l_{K+1} . If τ_x crosses R_2 , then there exists a state h''_y which is on τ_x and also belongs to R_2 , such that $\exists h''_x$ on τ_i , the distance between h''_x and h''_y is smaller than l_{K+1} which is not possible as h''_y belongs to R_2 . This proves that $\forall j \in \{1, 2, \dots, q\}$, if $Z_j \subset \mathcal{D}_i$, then $\{x \in \mathcal{S}_r(\mathcal{D}_i) \mid \|x - r(h_i^\infty)\|_2 \leq \max_{\beta \in \{-1,1\}^{n_1}} \|\sum_{k=1}^{n_1} \beta_k \alpha_k v_k\|_2\} \cap r(Z_j) = \emptyset$.

In fact, if h_i is sufficiently close to h_i^∞ , then the Non-intersection condition must be satisfied, because L_{K+1} is sufficiently close to 0 in this case. Based on this fact, it can be proved that if trajectory from h_1 is attracted by a cycle of discrete domains and does not reach R_2 , then the Non-intersection condition must be verified in finite time for all discrete domains which belong to an input boundary of the discrete state of R_2 and belong to the attractive cycle of discrete domains (the cycle of discrete domains which attracts the trajectory from h_1). In this case, this algorithm returns "Yes", meaning that the trajectory from h_1 has no more chance to reach R_2 .