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▶ **To cite this version:**

Vivien Beuselinck, Jérôme Delobelle, Srdjan Vesic. A Principle-based Account of Self-attacking Arguments in Gradual Semantics. Journal of Logic and Computation, 2023, 33 (2), pp.230-256. 10.1093/logcom/exac093 . hal-04181223

HAL Id: hal-04181223
https://hal.science/hal-04181223v1

Submitted on 8 Nov 2023

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A Principle-Based Account of Self-Attacking Arguments in Gradual Semantics

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Abstract. The issue of how a semantics should deal with self-attacking arguments was always a subject of debate amongst argumentation scholars. A consensus exists for extension-based semantics because those arguments are always rejected (as soon as the semantics in question respects conflict-freeness). In case of gradual semantics, the question is more complex, since other criteria are taken into account. In this paper we check the impact of those arguments by using a principle-based approach. Principles like Self-Contradiction and Strong Self-Contradiction prescribe how to deal with self-attacking arguments. We show that they are incompatible with the well-known Equivalence principle (which is satisfied by almost all the existing gradual semantics), as well as with some other principles (e.g. Counting). This incompatibility was not studied until now and the class of semantics satisfying Self-Contradiction is under-explored. In the present paper, we explore that class of semantics. We show links and incompatibilities between several principles. We define a new general oriented argumentation semantics that satisfies (Strong) Self-Contradiction and a maximal number of compatible principles. We introduce an iterative algorithm to calculate our semantics and prove that it always converges. We also provide a characterisation of our semantics. Finally, we experimentally show that our semantics is computationally efficient.

Keywords: Abstract argumentation · Gradual semantics · Self-attack.[4]

1 Introduction

The computational argumentation theory [20] allows to model the reasoning and decision making based on exchange of arguments. The conflicts are represented by attacks between the arguments. Although in most cases a conflict occurs between two distinct arguments, sometimes an argument may conflict with itself. Such an argument is called a self-attacking argument. The self-attacking arguments seem anecdotal at first sight;[5] however, the discussion on how to deal with them is subject of debate amongst argumentation scholars. There exist examples in the literature attempting to formally represent

[4] This paper is an extended version of the paper published in the proceedings of 4th International Conference on Logic and Argumentation (CLAR'21). [12]

[5] Bodanza and Tohmé [13] claim that there is a lack of "indisputably sound examples" concerning this type of arguments

certain aspects with these arguments, such as the representation of the lottery paradox [27]. However, one quickly understands that the problem of representing the self-attacking arguments is mainly linked to the different choices made to formally represent an argument and the attacks between the arguments. This distinction can be seen, for example, when comparing the approaches used in deductive argumentation and in abstract argumentation. As mentioned by Baumann and Woltran [10], in classical logic-based frameworks, self-attacking arguments do not occur at all [11], while other argumentation systems like ASPIC [26] allow such arguments. Within the abstract setting, several methods have been defined by proposing to deal with them directly [13,9,8,18] or indirectly (e.g. when dealing with odd-length cycles because a self-attack is the smallest odd-length cycle) [7]. These methods essentially concern extension-based semantics.

In the context of ranking-based and gradual argumentation semantics [2,5], little research was conducted to find out how self-attacking arguments should be dealt with and what is the impact they have on the acceptability of other arguments. Existing studies are essentially done through the principle-based studies of these semantics. Indeed, defining and studying principles drew attention of many scholars in this area. Consider Equivalence, which is one of the well-known principles, stating that the acceptability degree of an argument should only depend on acceptability degrees of its direct attackers and observe the argumentation graph from Figure 1. Equivalence implies that a and b should be equally acceptable because a and b are both directly attacked by the same argument. However, this is debatable, since the intuition behind a self-attacking argument is that it is inconsistent in one way or another so we would tend to accept b being attacked by a (which is self-attacking) rather than accepting a.

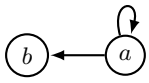


Fig. 1: An argumentation graph with two arguments (a attacks itself and b) showing that Equivalence and Self-Contradiction are incompatible.

Note that, under all semantics returning conflict-free extensions, a self-attacking argument is always rejected, i.e. it does not belong to any extension. Also, regarding the ranking-based and gradual semantics, it was pointed out that it would be natural to attach the worst possible rank to self-attacking arguments [25]. Furthermore, two principles were defined to formalise this intuition. The first one is called Strong Self-Contradiction, and was introduced by Matt and Toni [25]. It says that the acceptability degree of an argument must be minimal if and only if that argument is self-attacking. The second principle, called Self-Contradiction, was introduced by Bonzon et al. [14] and states that each self-attacking argument is strictly less acceptable than each non self-attacking argument. Consider the argumentation graph illustrated in Figure 1 again and note that, under every semantics that satisfies Self-Contradiction, b is strictly more

acceptable than a. This example shows that Equivalence and Self-Contradiction are not compatible, i.e. there exists no semantics that satisfies both of them.

To the best of our knowledge, there exists only one semantics (known as M&T) that satisfies Self-Contradiction and Strong Self-Contradiction. That semantics was introduced by Matt and Toni [25]. However, this semantics has a limitation that makes it inapplicable in practice. Namely, as noted by Matt and Toni themselves, as the space used to calculate the scores grows exponentially with the number of arguments, even with the optimisation techniques they used it did not scale to more than a dozen of arguments.

The research objective of the present paper is to study the under-explored family of semantics that satisfy Strong Self-Contradiction. Our goals are thus to identify which principles are (in)compatible with Strong Self-Contradiction and to define a new argumentation semantics, called `nsa` (no self-attacks), that satisfies Strong Self-Contradiction as well as a maximal number of compatible principles.

After introducing the formal setting and recalling the existing principles from the literature:

- We prove the incompatibilities between some of the principles, and identify a maximal set of principles that contains (Strong) Self-Contradiction;
- We introduce an iterative algorithm in order to define a new semantics and prove that it always converges. The acceptability degree of each argument with respect to `nsa` is then defined as the limit of the corresponding sequence;
- We provide a characterisation of `nsa`, i.e. a declarative (non-iterative) definition and show that the two are equivalent: each semantics satisfying the declarative definition coincides with `nsa`;
- We check which principles are satisfied by `nsa` and compare it with the M&T semantics [25] and the h-categorizer semantics [11] in terms of principle satisfaction;
- We formally prove that no semantics can satisfy a strict super-set of the set of principles satisfied by `nsa`;
- We experimentally show that `nsa` is computationally efficient and compare it with the M&T semantics and the h-categorizer semantics. The results confirm the hypothesis that the M&T semantics does not scale.

In order not to disrupt the reading of the paper, we have chosen to put the long proofs of propositions 8 and 9 in Appendix A.

2 Formal Setting and Existing Semantics

Dung's argumentation graph (AG) [20] is an abstract framework, in which there is no assumption on the nature of the elements it contains. More precisely, neither the structure nor the origin of the arguments are required. Then, an argumentation graph is composed of a finite set of arguments and of a relation of conflict between them.

Definition 1 (Argumentation graph). *An argumentation graph (AG) is a directed graph $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ where $\mathcal{A}$ is a finite set of arguments and $\mathcal{R}$ a binary relation over*

$\mathcal{A}$, i.e. $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$. For $a, b \in \mathcal{A}$, $(a, b) \in \mathcal{R}$ means that a attacks b. The notation $\texttt{Att}_{\mathcal{F}}(a) = \{b \mid (b, a) \in \mathcal{R}\}$ represents the set of direct attackers of argument a. For two graphs $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$, we denote by $\mathcal{F} \otimes \mathcal{F}'$ the argumentation graph $\mathcal{F}'' = (\mathcal{A} \cup \mathcal{A}', \mathcal{R} \cup \mathcal{R}')$.

Dung's framework comes equipped with various types of semantics used to evaluate the arguments. These include:

- the *extension-based semantics* which return the sets of acceptable arguments that are coherent together (see [6] for an overview),
- the *labelling-based semantics* that assign a label to each argument. The label `in` indicates that the argument is explicitly accepted, the label `out` indicates that the argument is explicitly rejected, and the label `undec` indicates that the status of the argument is undecided, meaning that one abstains from a judgment whether the argument is accepted or rejected (see [16]),
- the *ranking-based semantics* that associate to any argumentation graph a ranking on the arguments from the most to the least acceptable ones (see [14] for an overview),
- the *gradual semantics* that assign a numerical acceptability degree to each argument.

We refer the reader to [15,1] for a complete overview of the existing families of semantics in abstract argumentation and the differences between these approaches (e.g., definition, outcome, application). In this paper, we focus on gradual semantics which assign to each argument in an argumentation graph a score, called *acceptability degree*. This degree belongs to the interval $[0, 1]$. Higher degrees correspond to stronger arguments. Note that this degree should not be confused with the weight, which is assigned to each argument of a weighted argumentation graph [21], and which comes from an external source.

Definition 2 (Gradual semantics). *A gradual semantics is a function $\mathcal{S}$ which associates to any argumentation graph $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ a function $Deg^{\mathcal{S}}_{\mathcal{F}} : \mathcal{A} \to [0, 1]$. Thus, $Deg^{\mathcal{S}}_{\mathcal{F}}(x)$ represents the acceptability degree of $x \in \mathcal{A}$.*

In the rest of the section we recall two gradual semantics. We first introduce h-categorizer, which is one of the most studied gradual semantics and also satisfies a maximal compatible set of principles from the literature.[6] Then we introduce M&T semantics which is the first gradual semantics (and the only one besides the one defined in this paper) to treat self-attacking and non-self-attacking arguments differently.

2.1 h-categorizer Semantics

The h-categorizer semantics [11,28] uses a categorizer function to assign a value to each argument by taking into account the strength of its attackers, which itself takes into account the strength of its attackers, and so on.

[6] formally: out of the principles from Section 3, no semantics satisfies a strict superset of the principles satisfied by h-categorizer.

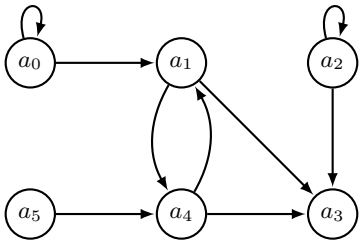


Fig. 2: An argumentation graph $\mathcal{F}$

Definition 3 (h-categorizer semantics). *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation graph. The h-categorizer semantics is a gradual semantics such that $\forall x \in \mathcal{A}$:*

$$Deg^{\mathtt{h}}_{\mathcal{F}}(x) = \frac{1}{1 + \sum_{y \in \mathtt{Att}_{\mathcal{F}}(x)} Deg^{\mathtt{h}}_{\mathcal{F}}(y)}$$

Formally, the acceptability degrees correspond to the solution of the non-linear system of equations with one equation per argument and can be computed via a fixed point technique for any argumentation framework.

Example 1 *Let us apply the h-categorizer semantics on the argumentation graph illustrated in Figure 2. We obtain the following acceptability degrees : $Deg^{\mathtt{h}}_{\mathcal{F}}(a_0) = 0.618$, $Deg^{\mathtt{h}}_{\mathcal{F}}(a_1) = 0.495$, $Deg^{\mathtt{h}}_{\mathcal{F}}(a_2) = 0.618$, $Deg^{\mathtt{h}}_{\mathcal{F}}(a_3) = 0.398$, $Deg^{\mathtt{h}}_{\mathcal{F}}(a_4) = 0.401$ and $Deg^{\mathtt{h}}_{\mathcal{F}}(a_5) = 1$.*

2.2 M&T Semantics

The gradual semantics introduced by Matt and Toni [25] computes the acceptability degree of an argument using a two-person zero-sum strategic game. For an AG $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and an argument $x \in \mathcal{A}$, the set of strategies for the proponent is the set of all subsets of arguments that contain x: $S_P(x) = \{P \mid P \subseteq \mathcal{A}, x \in P\}$ and for the opponent it is the set of all subsets of arguments: $S_O = \{O \mid O \subseteq \mathcal{A}\}$. Given two strategies $X, Y \subseteq \mathcal{A}$, the set of attacks from X to Y is defined by $Y^{\leftarrow X}_{\mathcal{F}} = \{(x, y) \in X \times Y \mid (x, y) \in \mathcal{R}\}$. Then, the notion of degree of acceptability of a set of arguments w.r.t. another one used to compute the reward of a proponent's strategy is defined.

Definition 4 (Reward). *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation graph, $x \in \mathcal{A}$ be an argument, $P \in S_P(x)$ be a strategy chosen by the proponent and $O \in S_O$ be a strategy chosen by the opponent. The degree of acceptability of P w.r.t. O is $\phi(P, O) = \frac{1}{2}\left[1 + f(|O^{\leftarrow P}_{\mathcal{F}}|) - f(|P^{\leftarrow O}_{\mathcal{F}}|)\right]$ with $f(n) = \frac{n}{n+1}$. The reward of P over O, denoted by $r_{\mathcal{F}}(P, O)$, is defined by:*

$$r_{\mathcal{F}}(P, O) = \begin{cases} 0 & \text{iff } P \text{ is not conflict-free} \\ 1 & \text{iff } P \text{ is conflict-free and} \\ & \quad |P^{\leftarrow O}_{\mathcal{F}}| = 0 \\ \phi(P, O) & \text{otherwise} \end{cases}$$

Proponent and opponent have the possibility of using a strategy according to some probability distributions, respectively $p = (p_1, p_2, \ldots, p_m)$ and $q = (q_1, q_2, \ldots, q_n)$, with $m = |S_P(x)|$ and $n = |S_O|$. For each argument $x \in \mathcal{A}$, the proponent's expected payoff $E(x, p, q)$ is then given by $E(x, p, q) = \sum_{j=1}^{n} \sum_{i=1}^{m} p_i q_j r_{i,j}$ with $r_{i,j} = r_{\mathcal{F}}(P_i, O_j)$ where P_i (respectively O_j) represents the i^{th} (respectively j^{th}) strategy of $S_P(x)$ (respectively S_O). The proponent can expect to get at least $\min_q E(x, p, q)$, where the minimum is taken over all the probability distributions q available to the opponent. Hence the proponent can choose a strategy which will guarantee her a reward of $\max_p \min_q E(x, p, q)$. The opposite is also true with $\min_q \max_p E(x, p, q)$.

Definition 5 (M&T semantics). *The semantics M&T is a gradual semantics that assigns a score to each argument $x \in \mathcal{A}$ in $\mathcal{F}$ as follows:*

$$Deg^{\mathtt{MT}}_{\mathcal{F}}(x) = \max_p \min_q E(x, p, q) = \min_q \max_p E(x, p, q)$$

Example 1 (cont.) *Let us apply the semantics M&T on the argumentation graph illustrated in Figure 2. We obtain the following acceptability degrees : $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_0) = 0$, $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_1) = 0.25$, $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_2) = 0$, $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_3) = 0.167$, $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_4) = 0.25$ and $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_5) = 1$.*

3 Principles for Gradual Semantics

Principles have been introduced in [4] in order to better understand the behavior of the gradual semantics, choose a semantics for a particular application, guide the search for new semantics, compare semantics with each other, etc. We do not claim that all of these principles are mandatory (we will see later that some of them are incompatible). At first, after being introduced, those principles where compared and studied from a theoretical point of view (e.g. the links between the principles). Then, scholars took interest in studying more practical aspects, such as whether and under which conditions humans comply with those principles [29]. In the rest of this section, we introduce these principles.[7]

The first one, called Anonymity, states that the name of an argument should not impact its acceptability degree.

Principle 1 (Anonymity) *A semantics S satisfies Anonymity iff for any two AGs $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ for any isomorphism γ from $\mathcal{F}$ to $\mathcal{F}'$, $\forall a \in \mathcal{A}, Deg^S_{\mathcal{F}}(a) = Deg^S_{\mathcal{F}'}(\gamma(a))$.*

Independence says that the acceptability degree of an argument should be independent of unconnected arguments.

Principle 2 (Independence) *A semantics S satisfies Independence iff, for any two AGs $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$, $\forall a \in \mathcal{A}$, $Deg^S_{\mathcal{F}}(a) = Deg^S_{\mathcal{F} \otimes \mathcal{F}'}(a)$.*

[7] We do not include the Proportionality principle since it is only applicable when arguments are attached intrinsic weights.

Directionality states that the acceptability of argument x can depend on y only if there is a path from y to x.

Principle 3 (Directionality) *A semantics S satisfies Directionality iff, for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ *and* $\mathcal{F}' = (\mathcal{A}, \mathcal{R}')$ *such that* $a, b \in \mathcal{A}$, $\mathcal{R}' = \mathcal{R} \cup \{(a, b)\}$ *it holds that :* $\forall x \in \mathcal{A}$, *if there is no path from b to x, then* $Deg^S_{\mathcal{F}}(x) = Deg^S_{\mathcal{F}'}(x)$.

Neutrality states that an argument with an acceptability degree of 0 should have no impact on the arguments it attacks.

Principle 4 (Neutrality) *A semantics S satisfies Neutrality iff, for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ *if* $\forall a, b \in \mathcal{A}$, $\texttt{Att}_{\mathcal{F}}(b) = \texttt{Att}_{\mathcal{F}}(a) \cup \{x\}$ *with* $x \in \mathcal{A} \setminus \texttt{Att}_{\mathcal{F}}(a)$ *and* $Deg^S_{\mathcal{F}}(x) = 0$ *then* $Deg^S_{\mathcal{F}}(a) = Deg^S_{\mathcal{F}}(b)$.

Equivalence says that if two arguments have the same attackers, or more generally attackers of the same strength, they should have the same acceptability degree.

Principle 5 (Equivalence) *A semantics S satisfies Equivalence iff, for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, $\forall a, b \in \mathcal{A}$, *if there exists a bijective function f from* $\texttt{Att}_{\mathcal{F}}(a)$ *to* $\texttt{Att}_{\mathcal{F}}(b)$ *s.t.* $\forall x \in \texttt{Att}_{\mathcal{F}}(a)$, $Deg^S_{\mathcal{F}}(x) = Deg^S_{\mathcal{F}}(f(x))$ *then* $Deg^S_{\mathcal{F}}(a) = Deg^S_{\mathcal{F}}(b)$.

Maximality states that a non-attacked argument should have the highest acceptability degree.

Principle 6 (Maximality) *A semantics S satisfies Maximality iff, for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, $\forall a \in \mathcal{A}$, *if* $\texttt{Att}_{\mathcal{F}}(a) = \emptyset$ *then* $Deg^S_{\mathcal{F}}(a) = 1$.

Counting states that a non-zero degree attacker should impact the acceptability of the attacked argument.

Principle 7 (Counting) *A semantics S satisfies Counting iff for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, $\forall a, b \in \mathcal{A}$, *if i)* $Deg^S_{\mathcal{F}}(a) > 0$ *and ii)* $\texttt{Att}_{\mathcal{F}}(b) = \texttt{Att}_{\mathcal{F}}(a) \cup \{y\}$ *with* $y \in \mathcal{A} \setminus \texttt{Att}_{\mathcal{F}}(a)$ *and* $Deg^S_{\mathcal{F}}(y) > 0$ *then* $Deg^S_{\mathcal{F}}(a) > Deg^S_{\mathcal{F}}(b)$.

Weakening says that the acceptability of an argument should be strictly lower than 1 if it has at least one attacker with a non-zero acceptability degree.

Principle 8 (Weakening) *A semantics S satisfies Weakening iff for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, $\forall a \in \mathcal{A}$, *if* $\exists b \in \texttt{Att}_{\mathcal{F}}(a)$ *s.t.* $Deg^S_{\mathcal{F}}(b) > 0$, *then* $Deg^S_{\mathcal{F}}(a) < 1$.

Weakening Soundness states that if the acceptability degree of an argument is not maximal, it must be that it is attacked by at least one non-zero degree attacker.

Principle 9 (Weakening Soundness) *A semantics S satisfies Weakening Soundness iff, for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, $\forall a \in \mathcal{A}$, *if* $Deg^S_{\mathcal{F}}(a) < 1$ *then* $\exists b \in \texttt{Att}_{\mathcal{F}}(a)$ *such that* $Deg^S_{\mathcal{F}}(b) > 0$.

Reinforcement states that the acceptability degree increases if the acceptability degrees of attackers decrease.

Principle 10 (Reinforcement) *A semantics S satisfies Reinforcement iff for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$*,* $\forall a, b \in \mathcal{A}$*, if i)* $Deg^S_\mathcal{F}(a) > 0$ *or* $Deg^S_\mathcal{F}(b) > 0$*, ii)* $\texttt{Att}_\mathcal{F}(a) \backslash \texttt{Att}_\mathcal{F}(b) = \{x\}$*, iii)* $\texttt{Att}_\mathcal{F}(b) \backslash \texttt{Att}_\mathcal{F}(a) = \{y\}$ *and iv)* $Deg^S_\mathcal{F}(y) > Deg^S_\mathcal{F}(x)$*, then* $Deg^S_\mathcal{F}(a) > Deg^S_\mathcal{F}(b)$*.*

Resilience states that no argument in an argumentation graph can have an acceptability degree of 0. It is certainly not a mandatory principle.

Principle 11 (Resilience) *A semantics S satisfies Resilience if for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$*,* $\forall a \in \mathcal{A}$*,* $Deg^S_\mathcal{F}(a) > 0$*.*

The last three principles are incompatible with each other. The first principle, called Cardinality Precedence states, roughly speaking, that the greater the number of direct attackers of an argument, the lower its acceptability degree.

Principle 12 (Cardinality Precedence) *A semantics S satisfies Cardinality Precedence iff for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$*,* $\forall a, b \in \mathcal{A}$*, if i)* $Deg^S_\mathcal{F}(b) > 0$*, and ii)* $|\{x \in \texttt{Att}_\mathcal{F}(a)$ *s.t.* $Deg^S_\mathcal{F}(x) > 0\}| > |\{y \in \texttt{Att}_\mathcal{F}(b)\ s.t.\ Deg^S_\mathcal{F}(y) > 0\}|$ *then* $Deg^S_\mathcal{F}(a) < Deg^S_\mathcal{F}(b)$*.*

Quality Precedence states, roughly speaking, that the greater the acceptability degree of the strongest attacker of an argument, the lower its acceptability degree.

Principle 13 (Quality Precedence) *A semantics S satisfies Quality Precedence if for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$*,* $\forall a, b \in \mathcal{A}$*, if i)* $Deg^S_\mathcal{F}(a) > 0$ *and ii)* $\exists y \in \texttt{Att}_\mathcal{F}(b)$ *s.t.* $\forall x \in \texttt{Att}_\mathcal{F}(a)$*,* $Deg^S_\mathcal{F}(y) > Deg^S_\mathcal{F}(x)$ *then* $Deg^S_\mathcal{F}(a) > Deg^S_\mathcal{F}(b)$*.*

Compensation states that several attacks from arguments with a low acceptability degree may compensate one attack from an argument with high acceptability degree. [8]

Principle 14 (Compensation) *A semantics S satisfies Compensation iff both Cardinality Precedence and Quality Precedence are not satisfied.*

In the literature, two principles directly refer to the self-attacking arguments. The first one, called Self-Contradiction, was introduced by Bonzon et al. [14] and states that the degree of a self-attacking argument should be strictly lower than the degree of an argument that does not attack itself.

Principle 15 (Self-Contradiction) *A semantics S satisfies Self-Contradiction iff, for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ *with two arguments* $a, b \in \mathcal{A}$*, if* $(a, a) \in \mathcal{R}$ *and* $(b, b) \notin \mathcal{R}$ *then* $Deg^S_\mathcal{F}(b) > Deg^S_\mathcal{F}(a)$*.*

The second principle was introduced by Matt and Toni [25]. Its original name was "Self-contradiction must be avoided". We rename it for clarity reasons, namely in order to avoid the confusion with the name of Principle 15. This principle states that the self-attacking arguments are the only arguments with the minimum acceptability degree (i.e. having the degree 0).

Principle 16 (Strong Self-Contradiction) *A semantics S satisfies Strong Self-Contradiction iff, for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ *with* $a \in \mathcal{A}$*,* $Deg^S_\mathcal{F}(a) = 0$ *iff* $(a, a) \in \mathcal{R}$*.*

[8] There are several version of this principle. We use the version that allows to clearly distinguish between the three cases (CP, QP, Compensation). Namely, each semantics satisfies *exactly* one of the three principles.

4 Analysis of Principles and Links Between Them

In this section we analyse the links between principles defined in the previous section. Let us first recall the links between principles 1-14.

Proposition 1 ([4]). *The three following properties hold.*

- *Cardinality Precedence, Quality Precedence and Compensation are pairwise incompatible.*
- *Independence, Directionality, Equivalence, Resilience, Reinforcement, Maximality and Quality Precedence are incompatible.*
- *Cardinality Precedence (respectively Compensation) is compatible with all principles 1–11.*

Let us now focus on the relationship between the principles dealing with self-attacking arguments (both with each other and with the other principles). The first observation is that Strong Self-Contradiction implies Self-Contradiction. The next proposition follows directly from the definitions of the respective principles.

Proposition 2. *If a gradual semantics $\mathcal{S}$ satisfies Strong Self-Contradiction, it satisfies Self-Contradiction.*

Proof. Let us suppose that Strong Self-Contradiction is satisfied by $\mathcal{S}$. This means that those and only those arguments that have the minimum score are the self-attacking arguments ($\forall a \in \mathcal{A}$, $Deg^{\mathcal{S}}_{\mathcal{F}}(a) = 0$ iff $(a, a) \in \mathcal{R}$). This implies that all arguments that do not attack themselves have an acceptability degree greater than 0. Formally, $\forall b \in \mathcal{A}$, $Deg^{\mathcal{S}}_{\mathcal{F}}(b) > 0$ iff $(b, b) \notin \mathcal{R}$. Consequently, for two arguments $a, b \in \mathcal{A}$, if $(a, a) \in \mathcal{R}$ and $(b, b) \notin \mathcal{R}$ then $Deg^{\mathcal{S}}_{\mathcal{F}}(b) > Deg^{\mathcal{S}}_{\mathcal{F}}(a) = 0$. □

As discussed in the introduction, the next result shows that Equivalence and Self-Contradiction are incompatible.

Proposition 3. *There exists no gradual semantics $\mathcal{S}$ that satisfies both Equivalence and Self-Contradiction.*

Proof. We provide a proof by contradiction. Let us suppose that a gradual semantics S satisfies both Equivalence and Self-Contradiction and consider the argumentation graph from Figure 1 on page 2. From Self-Contradiction, we have $Deg^{\mathcal{S}}_{\mathcal{F}}(a) < Deg^{\mathcal{S}}_{\mathcal{F}}(b)$ because $(a, a) \in \mathcal{R}$ and $(b, b) \notin \mathcal{R}$. From Equivalence, we have $Deg^{\mathcal{S}}_{\mathcal{F}}(a) = Deg^{\mathcal{S}}_{\mathcal{F}}(b)$ because $\texttt{Att}_{\mathcal{F}}(a) = \{a\}$ and $\texttt{Att}_{\mathcal{F}}(b) = \{a\}$ (and by using the identity function as the bijection from Definition 5).
Contradiction. Hence, S does not satisfy both Equivalence and Self-Contradiction.
Since S was arbitrary, we conclude that there exists no semantics that satisfies both Equivalence and Self-Contradiction. □

However, the Equivalence principle is not the only one incompatible with Strong Self-Contradiction. Some other incompatibilities exist mainly because self-attacking arguments are treated differently from other arguments. Indeed, according to Strong Self-Contradiction, self-attacking arguments are directly classified as the worst arguments, whereas the other principles just consider a self-attack as an attack like any other (i.e. an attack between two distinct arguments).

Proposition 4. *There exists no gradual semantics $\mathcal{S}$ that satisfies both Strong Self-Contradiction and Resilience.*

Proof. We provide a proof by contradiction. Let us suppose that a gradual semantics S satisfies both Strong Self-Contradiction and Resilience, and consider the argumentation graph $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ where $\mathcal{A} = \{a\}$ and $R = \{(a, a)\}$.
From Strong Self-Contradiction, we have $Deg^{\mathcal{S}}_{\mathcal{F}}(a) = 0$, while from Resilience, we have $Deg^{\mathcal{S}}_{\mathcal{F}}(a) > 0$.
Contradiction. Hence, S does not satisfy both Strong Self-Contradiction and Resilience. Since S was arbitrary, there exists no semantics that satisfies both Resilience and Strong Self-Contradiction. □

Proposition 5. *There exists no gradual semantics $\mathcal{S}$ that satisfies both Strong Self-Contradiction and Weakening Soundness.*

Proof. We provide a proof by contradiction. Let us suppose that a gradual semantics S satisfies both Strong Self-Contradiction and Weakening Soundness, and consider the argumentation graph $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ where $\mathcal{A} = \{a\}$ and $R = \{(a, a)\}$.
From Strong Self-Contradiction, we have $Deg^{\mathcal{S}}_{\mathcal{F}}(a) = 0$, while from Weakening Soundness, we have $Deg^{\mathcal{S}}_{\mathcal{F}}(a) > 0$ because a is the only attacker of a and $Deg^{\mathcal{S}}_{\mathcal{F}}(a) = 0$.
Contradiction. Hence, S does not satisfy both Strong Self-Contradiction and Weakening Soundness. Since S was arbitrary, there exists no semantics that satisfies both Strong Self-Contradiction and Weakening Soundness. □

Proposition 6. *There exists no gradual semantics $\mathcal{S}$ that satisfies both Strong Self-Contradiction and Reinforcement.*

Proof. We provide a proof by contradiction. Let us suppose that a gradual semantics S satisfies both Strong Self-Contradiction and Reinforcement, and consider the argumentation graph $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ represented in Figure 3.

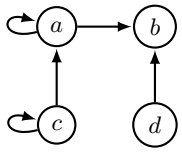


Fig. 3: AG showing that Reinforcement and Strong Self-Contradiction are incompatible.

From Strong Self-Contradiction, we have $0 = Deg^{\mathcal{S}}_{\mathcal{F}}(a) < Deg^{\mathcal{S}}_{\mathcal{F}}(b)$. From Reinforcement, we have $Deg^{\mathcal{S}}_{\mathcal{F}}(a) > Deg^{\mathcal{S}}_{\mathcal{F}}(b)$ because i) $Deg^{\mathcal{S}}_{\mathcal{F}}(b) > 0$, ii) $\texttt{Att}_{\mathcal{F}}(a) \backslash \texttt{Att}_{\mathcal{F}}(b) = \{c\}$, iii) $\texttt{Att}_{\mathcal{F}}(b) \backslash \texttt{Att}_{\mathcal{F}}(a) = \{d\}$, and iv) $Deg^{\mathcal{S}}_{\mathcal{F}}(d) > Deg^{\mathcal{S}}_{\mathcal{F}}(c)$.
Contradiction. Hence, S does not satisfy both Strong Self-Contradiction and Reinforcement. Since S was arbitrary, there exists no semantics that satisfies both Strong Self-Contradiction and Reinforcement. □

Proposition 7. *There exists no gradual semantics $\mathcal{S}$ that satisfies both Strong Self-Contradiction and Neutrality.*

Proof. We provide a proof by contradiction. Let us suppose that a gradual semantics S satisfies both Strong Self-Contradiction and Neutrality, and consider the argumentation graph $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ represented in Figure 4.

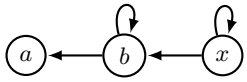


Fig. 4: AG showing that Neutrality and Strong Self-Contradiction are incompatible.

From Strong Self-Contradiction, we have $0 = Deg^{\mathcal{S}}_{\mathcal{F}}(b) < Deg^{\mathcal{S}}_{\mathcal{F}}(a)$. From Neutrality, we have $Deg^{\mathcal{S}}_{\mathcal{F}}(a) = Deg^{\mathcal{S}}_{\mathcal{F}}(b)$ because $\texttt{Att}_{\mathcal{F}}(b) = \texttt{Att}_{\mathcal{F}}(a) \cup \{x\}$ with $Deg^{\mathcal{S}}_{\mathcal{F}}(x) = 0$. Contradiction. Hence, S does not satisfy both Strong Self-Contradiction and Neutrality. Since S was arbitrary, there exists no semantics that satisfies both Strong Self-Contradiction and Neutrality. □

Taking these incompatibilities into account, our goal is now to study two maximal compatible sets of principles we are interested in. A compatible set of principles is a set of principles such that two principles belonging to this set are not incompatible. In other words, a compatible set of principles is a set of principles that can be jointly satisfied by a semantics. In order to capture the idea of a maximal compatible sets of principles, let us define the notion of dominance. A semantics S dominates a semantics S' on the set of principles P if the subset of principles from P satisfied by S is a strict superset of the subset of principles from P satisfied by S'. In the rest of the discussion, we suppose that P is the set of all principles studied in Section 3. Note that if a semantics S satisfies a maximal for set inclusion set of principles, it is not dominated by any semantics.

A first maximal (for set inclusion) compatible set of principles has been identified by [4] and is a direct consequence of their Proposition 1. We define this set of principles as $P_{CREW} = \{$Anonymity, Independence, Directionality, Neutrality, Equivalence, Maximality, Weakening, Counting, Weakening Soundness, Reinforcement, Resilience and Compensation$\}$.

Theorem 1 ([4]). *P_{CREW} is a maximal compatible for set inclusion set of principles.*

We can formally show that there is a unique maximal compatible set of principles that includes Compensation, Resilience, Equivalence and Weakening Soundness.

Theorem 2. *Let P be the set of all principles defined in Section 3 (Principles 1-16). Let $\mathcal{S}$ be a gradual semantics that satisfies Compensation, Resilience, Equivalence and Weakening Soundness. If $\mathcal{S}$ is not dominated w.r.t. P, then $\mathcal{S}$ satisfies exactly the principles from P_{CREW}.*

Proof. On one hand, we know from the work by [4] that h-categorizer satisfies all the principles from P_{CREW}. On the other hand, it is clear from the incompatibility results between the principles that $\mathcal{S}$ cannot satisfy Strong Self-Contradiction which is incompatible with Resilience (see Proposition 4), Self-Contradiction which is incompatible with Equivalence (see Proposition 3), Cardinality/Quality Precedence which are both incompatible with Compensation (see [4]). Thus, in order not to be dominated by h-categorizer, $\mathcal{S}$ must satisfy all the principles from P_{CREW}; due to the incompatibilities, $\mathcal{S}$ cannot satisfy any more principles. □

In this paper we choose to explore the space of principles compatible with Strong Self-Contradiction (which is not in P_{CREW}). One naturally wants to maximise the set of satisfied principles. Can we satisfy Strong Self-Contradiction and all the other principles? The answer is negative (see Propositions 3-7). First, one has to choose between Cardinality Precedence, Quality Precedence and Compensation. In this paper, we explore the possibility of satisfying Compensation. This choice is based on the fact that this principle is satisfied by virtually all semantics, as showed by Amgoud et al. [4]. Indeed, Cardinality Precedence and Quality Precedence represent, roughly speaking, *drastic* or *extreme* cases and are satisfied only by the semantics specifically designed to satisfy them, like max-based semantics and card-based semantics [4] or by semantics having other specificities. For instance, iterative schema [22], which satisfies Quality Precedence, is a discrete semantics (it takes only three possible values). This yields another maximal compatible set of principles which includes those two principles. We define this set of principles as P_{2S2C} = {Anonymity, Independence, Directionality, Maximality, Weakening, Counting, Compensation, Self-Contradiction, Strong Self-Contradiction}.

Theorem 3. *P_{2S2C} is a maximal compatible for set inclusion set of principles.*

Proof. Note first that in this proof, we mention the `nsa` semantics, which is formally introduced in Definition 6 (see below). Firstly, all the principles in P_{2S2C} are compatible because `nsa` satisfies all of them (see Proposition 8 below). Secondly, P_{2S2C} is maximal because for each remaining principle $p \in$ {Equivalence, Weakening Soundness, Neutrality, Reinforcement, Cardinality Precedence, Quality Precedence and Resilience}, there exists (at least) one principle in P_{2S2C} which is incompatible with p:

- Equivalence and Self-Contradiction are incompatible (see Proposition 3);
- Neutrality and Strong Self-Contradiction are incompatible (see Proposition 7);
- Reinforcement and Strong Self-Contradiction are incompatible (see Proposition 6);
- Weakening Soundness and Strong Self-Contradiction are incompatible (see Proposition 5);
- Cardinality Precedence and Compensation are incompatible (see [4]);
- Quality Precedence and Compensation are incompatible (see [4]);
- Resilience and Strong Self-Contradiction are incompatible (see Proposition 4).

□

We now show that there is a unique maximal compatible set of principles that includes Strong Self-Contradiction and Compensation. This follows from the fact that

if a semantics satisfies Strong Self-Contradiction, this semantics cannot satisfy some existing principles (see the incompatibilities identified in Propositions 3-7).

Theorem 4. *Let P be the set of all principles defined in Section 3 (Principles 1-16). Let $\mathcal{S}$ be a gradual semantics that satisfies Strong Self-Contradiction and Compensation. If $\mathcal{S}$ is not dominated w.r.t. P, then $\mathcal{S}$ satisfies exactly the principles from P_{2S2C}.*

Proof. It is clear that from the incompatibility results between different principles, $\mathcal{S}$ cannot satisfy (i) Resilience, Equivalence and Weakening Soundness which are incompatible with Strong Self-Contradiction (or Self-Contradiction), and (ii) Cardinality Precedence and Quality Precedence which are both incompatible with Compensation. The set of remaining principles corresponds exactly to P_{2S2C} which is a maximal for set inclusion set of principles. However, $\mathcal{S}$ cannot satisfy exactly a subset of P_{2S2C} because, in this case, $\mathcal{S}$ will be dominated by a semantics that satisfies the principles of P_{2S2C}. Consequently, when $\mathcal{S}$ satisfies Strong Self-Contradiction and Compensation, the only way to ensure that $\mathcal{S}$ is not dominated is when $\mathcal{S}$ satisfies exactly the principles from P_{2S2C}. □

To the best of our knowledge, no semantics that satisfies all the principles from P_{2S2C} has been presented in the literature. In the next section, we define a semantics that satisfies this set of principles.

Before doing that, let us comment on the non satisfaction of some principles. It is tempting to change the principles in order to treat the self-attacks in another way, and consequently make the principles fit some definitions or theorems. We argue that it is better to start by having a full picture of what happens with *existing* principles. Indeed, the principles should be the most stable part of a theory. We are not against the introduction of new principles (or changing the existing ones). This might be part of future work.

5 No Self-Attack h-categorizer Semantics

In this section, we define a new gradual semantics, called no self-attack h-categorizer (nsa) semantics, inspired by the h-categorizer semantics. The main difference is that we assign degree 0 to the self-attacking arguments while the acceptability degrees of the other arguments, i.e. those that are not self-attacking, are calculated using the formula from h-categorizer semantics.

Definition 6. *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AG. We define $f^{\mathcal{F},i}_{\texttt{nsa}} : \mathcal{A} \to [0, +\infty]$ as follows : for every argument $a \in \mathcal{A}$ for $i \in \{0, 1, 2, ..\}$,*

$$f^{\mathcal{F},i}_{\texttt{nsa}}(a) = \begin{cases} 0 & \text{if } (a,a) \in \mathcal{R} \\ 1 & \text{if } (a,a) \notin \mathcal{R} \text{ and } i = 0 \\ \dfrac{1}{1 + \sum_{b \in \texttt{Att}_{\mathcal{F}}(a)} f^{\mathcal{F},i-1}_{\texttt{nsa}}(b)} & \text{if } (a,a) \notin \mathcal{R} \text{ and } i > 0 \end{cases} \quad (1)$$

By convention, if $\texttt{Att}_{\mathcal{F}}(a) = \emptyset$, $\sum_{b \in \texttt{Att}_{\mathcal{F}}(a)} f^{\mathcal{F},i-1}_{\texttt{nsa}}(b) = 0$.

Although nsa is inspired by the h-categorizer semantics, the modifications made change the result obtained requiring the verification that nsa also converges to a unique result. Thus, in the next result, we show that for every argumentation graph $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, for every argument $a \in \mathcal{A}$, $f_{\texttt{nsa}}^{\mathcal{F},i}(a)$ converges as i approaches infinity. Roughly speaking, the goal of the next theorem is to formally check that assigning zero values to self-attacking arguments does not impact the convergence of the scores. Thus, applying nsa to the original argumentation graph $\mathcal{F}$ provides the same result as when the h-categorizer semantics is applied on a restricted version of $\mathcal{F}$ where the self-attacking arguments are deleted.

Theorem 5. *For every argumentation graph $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, for every $a \in \mathcal{A}$, if $(a, a) \notin \mathcal{R}$, we have $\lim_{i \to \infty} f_{\texttt{nsa}}^{\mathcal{F},i}(a) = Deg_{\mathcal{F}'}^{\texttt{h}}(a)$ where $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ with the set of arguments $\mathcal{A}' = \{x \in \mathcal{A} \mid (x, x) \notin \mathcal{R}\}$ and $\mathcal{R}' = \{(x, y) \in \mathcal{R} \mid x \in A' \text{ and } y \in A'\}$.*

Proof. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AG and $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ be an AG such that $\mathcal{A}' = \{x \in \mathcal{A} | (x, x) \notin \mathcal{R}\}$ and $\mathcal{R}' = \{(x, y) \in \mathcal{R} \mid x \in A' \text{ and } y \in A'\}$. Without loss of generality, let us denote $\mathcal{A} = \{a_0, a_1, \ldots, a_n\}$.

Let us recall the iterative version of h-categorizer, that can be used to calculate the scores of arguments [28]: for every a, for $i \in \mathbb{N}$

$$f_h^{\mathcal{F},i}(a) = \begin{cases} 1 & \text{if } i = 0 \\ \dfrac{1}{1 + \sum_{b \in \texttt{Att}_{\mathcal{F}}(a)} f_h^{\mathcal{F},i-1}(b)} & \text{if } i > 0 \end{cases} \tag{2}$$

We prove by induction on i that for each $a \in \mathcal{A}'$:

$$f_{\texttt{nsa}}^{\mathcal{F},i}(a) = f_h^{\mathcal{F}',i}(a)$$

Base: Let $i = 0$. From the formal definition of nsa (Definition 6) and equation (2), we have $f_{\texttt{nsa}}^{\mathcal{F},0}(a) = f_h^{\mathcal{F}',0}(a) = 1$. Thus, the inductive base holds.

Step: Let us suppose that the inductive hypothesis is true for every $k \in \{0, 1, \ldots i\}$ and let us show that it is true for $i + 1$. We need to prove :

$$f_{\texttt{nsa}}^{\mathcal{F},i+1}(a) = f_h^{\mathcal{F}',i+1}(a)$$

From the inductive hypothesis, we know that for each argument $a \in A'$, $f_{\texttt{nsa}}^{\mathcal{F},i}(a) = f_h^{\mathcal{F}',i}(a)$. Thus, from equation (1), we have:

$$f_{\texttt{nsa}}^{\mathcal{F},i+1}(a) = \frac{1}{1 + \sum_{b \in \texttt{Att}_{\mathcal{F}}(a)} f_{\texttt{nsa}}^{\mathcal{F},i}(b)}$$

From equation (2), we have

$$f_h^{\mathcal{F}',i+1}(a) = \frac{1}{1 + \sum_{b \in \texttt{Att}_{\mathcal{F}'}(a)} f_h^{\mathcal{F}',i}(b)}$$

Let us note $\texttt{Att}_{\mathcal{F}}(a) = \texttt{Att}_{\mathcal{F}'}(a) \cup \{b_0, \ldots, b_m\}$ with $m \geq 0$ and remark that $\forall b \in \{b_0, \ldots, b_m\}$, we have $(b, b) \in \mathcal{R}$. According to equation (1), $\forall b \in \{b_0, \ldots, b_m\}$, $f_{\texttt{nsa}}^{\mathcal{F},i}(b) = 0$. Consequently, as 0 is the neutral element of the addition, we have $\forall a \in \mathcal{A}'$, $f_{\texttt{nsa}}^{\mathcal{F},i+1}(a) = f_h^{\mathcal{F}',i+1}(a)$.

By induction, we conclude that for every $i \in \mathbb{N}$ and for every $a \in \mathcal{A}'$

$$f_{\texttt{nsa}}^{\mathcal{F},i}(a) = f_h^{\mathcal{F}',i}(a)$$

Since f_h converges when $i \to \infty$ and $f_{\texttt{nsa}}$ coincides with f_h for every argument of $\mathcal{A}'$, we conclude that $f_{\texttt{nsa}}$ converges too. Formally, $\forall a \in A'$,

$$\lim_{i\to\infty} f_{\texttt{nsa}}^{\mathcal{F},i}(a) = \lim_{i\to\infty} f_{\texttt{h}}^{\mathcal{F},i}(a) = Deg_{\mathcal{F}'}^{\texttt{h}}(a)$$

□

We can now introduce the formal definition of nsa.

Definition 7 (nsa semantics). *The no self-attack h-categorizer semantics is a function* nsa *which associates to any argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ a function $Deg_{\mathcal{F}}^{\texttt{nsa}}(a) : A \to [0, 1]$ as follows: $Deg_{\mathcal{F}}^{\texttt{nsa}}(a) = \lim_{i\to\infty} f_{nsa}^{\mathcal{F},i}(a)$.*

We can now show that the acceptability degrees attributed to arguments by nsa satisfy the equation from Definition 6 (naturally, not taking into account the second line of the equation, since it considers the case $i = 0$).

Theorem 6. *For any $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, for any $a \in \mathcal{A}$,*

$$Deg_{\mathcal{F}}^{\texttt{nsa}}(a) = \begin{cases} 0 & \text{if } (a, a) \in \mathcal{R} \\ \dfrac{1}{1 + \sum_{b \in \texttt{Att}_{\mathcal{F}}(a)} Deg_{\mathcal{F}}^{\texttt{nsa}}(b)} & \text{otherwise} \end{cases}$$

Proof. Let $\mathcal{F} = (A, R)$ be an argumentation graph and $a \in A$.
The case where a is a self-attacking argument is trivial.
In the rest of the proof we consider the case where a is not a self-attacking argument. Letting $\lim_{i\to\infty}$ in the following equality

$$f_{\texttt{nsa}}^{i+1}(a) = \frac{1}{1 + \sum_{b \in \texttt{Att}_{\mathcal{F}}(a)} f_{\texttt{nsa}}^{i}(b)}$$

and using the fact that arithmetical operations and sum are continuous functions, we obtain :

$$\lim_{i\to\infty} f_{\texttt{nsa}}^{i+1}(a) = \frac{1}{1 + \sum_{b \in \texttt{Att}_{\mathcal{F}}(a)} \lim_{i\to\infty} f_{\texttt{nsa}}^{i}(b)}$$

then

$$Deg_{\mathcal{F}}^{\texttt{nsa}}(a) = \frac{1}{1 + \sum_{b \in \texttt{Att}_{\mathcal{F}}(a)} Deg_{\mathcal{F}}^{\texttt{nsa}}(a)}$$

□

We now show that the equation from Theorem 6 is not only satisfied by nsa, but is also its characterization. More precisely, the next result proves that if an arbitrary semantics D satisfies that equation, it must be that D coincides with nsa.

Theorem 7. *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AG with $a \in \mathcal{A}$ and $D : \mathcal{A} \to [0,1]$ be a function with the following formula:*

$$D(a) = \begin{cases} 0 & \text{if } (a,a) \in \mathcal{R} \\ \dfrac{1}{1 + \sum_{b \in Att_{\mathcal{F}}(a)} D(b)} & \text{otherwise} \end{cases} \tag{3}$$

then $D \equiv Deg^{\mathtt{nsa}}_{\mathcal{F}}$.

Proof. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AG and suppose that $D : \mathcal{A} \to [0,1]$ is the function from equation (3).

Let $A = \{a_1, .., a_n\}$ and let $F : [0,1]^n \to [0,1]^n$ be the function such that $F(x_1, .., x_n) = (F_1(x_1, .., x_n), ..., F_n(x_1, ..., x_n))$ where the functions F_i are defined by the following equality:

$$F_i(x_1, \ldots, x_n) = \begin{cases} 0 & \text{if } (a_i, a_i) \in \mathcal{R} \\ \dfrac{1}{1 + \sum\limits_{j : a_j \in Att_{\mathcal{F}}(a_i)} x_j} & \text{otherwise} \end{cases} \tag{4}$$

We also define the partial order $\leq$ on $\mathbb{R}^n$ in the following way: if $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ then $x \leq y$ iff for every i it holds that $x_i \leq y_i$.

Thus, from Equation (3), it follows that

$$F(D(a_1), ..., D(a_n)) = (D(a_1), ..., D(a_n)).$$

Observe that F is a non-increasing function and that $G = F \circ F$ is a non-decreasing function, and that :

$$(f^{i+1}_{\mathtt{nsa}}(a_1), ..., f^{i+1}_{\mathtt{nsa}}(a_n)) = F((f^{i}_{\mathtt{nsa}}(a_1), ..., f^{i}_{\mathtt{nsa}}(a_n)))$$

for every $i \in \mathbb{N}$. Since $(f^0_{\mathtt{nsa}}(a_1), ..., f^0_{\mathtt{nsa}}(a_n)) \in [0,1]^n$ with $f^0_{\mathtt{nsa}}(a_i) = 0$ iff $(a_i, a_i) \in \mathcal{R}$ and $f^0_{\mathtt{nsa}}(a_i) = 1$ otherwise, by the inequalities, we obtain

$$(f^0_{\mathtt{nsa}}(a_1), ..., f^0_{\mathtt{nsa}}(a_n)) \geq (D(a_1), ..., D(a_n)) \tag{5}$$

From (5), and since F is non-increasing, we have:

$$(f^1_{\mathtt{nsa}}(a_1), ..., f^1_{\mathtt{nsa}}(a_n)) \leq (D(a_1), ..., D(a_n)) \tag{6}$$

From (6), and since $G = F \circ F$ is non-decreasing, we have:

$$(f^{2i}_{\mathtt{nsa}}(a_1), ..., f^{2i}_{\mathtt{nsa}}(a_n)) \geq (D(a_1), ..., D(a_n)) \tag{7}$$

and

$$(f^{2i+1}_{\texttt{nsa}}(a_1), ..., f^{2i+1}_{\texttt{nsa}}(a_n)) \leq (D(a_1), ..., D(a_n)) \quad (8)$$

for every $i \in \mathbb{N}$.
Since all f^i converge, from (7) and (8) we obtain

$$(Deg^{\texttt{nsa}}_{\mathcal{F}}(a_1), \ldots, Deg^{\texttt{nsa}}_{\mathcal{F}}(a_n)) \geq (D(a_1), ..., D(a_n))$$

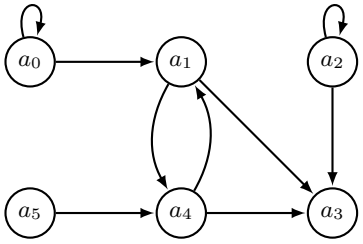
and

$$(Deg^{\texttt{nsa}}_{\mathcal{F}}(a_1), \ldots, Deg^{\texttt{nsa}}_{\mathcal{F}}(a_n)) \leq (D(a_1), ..., D(a_n))$$

and thus $\forall a \in \mathcal{A}, Deg^{\texttt{nsa}}_{\mathcal{F}}(a) = D(a)$. □

Below is an example of the nsa semantics applied on an argumentation graph.

Example 1 (cont.) *Let us apply the no self-attack h-categorizer semantics (*nsa*) on the argumentation graph illustrated in Figure 5. By definition, the self-attacking arguments*



$Deg^{\mathcal{S}}_{\mathcal{F}}$	nsa	h	MT
a_0	0	0.618	0
a_1	0.732	0.495	0.25
a_2	0	0.618	0
a_3	0.477	0.398	0.167
a_4	0.399	0.401	0.25
a_5	1	1	1

Fig. 5: On the left, an argumentation graph $\mathcal{F}$ and, on the right, the table containing the degrees of acceptability of each argument of $\mathcal{F}$ w.r.t. the no self-attack h-categorizer semantics (nsa), the h-categorizer semantics (h) and the semantics M&T (MT).

have an acceptability degree of 0 : $Deg^{\texttt{nsa}}_{\mathcal{F}}(a_0) = Deg^{\texttt{nsa}}_{\mathcal{F}}(a_2) = 0$*. The non-attacked arguments or the arguments only attacked by self-attacking arguments have, by definition, the maximum score:* $Deg^{\texttt{nsa}}_{\mathcal{F}}(a_5) = 1$*. Applying the formula from Theorem 6, we obtain the following acceptability degrees for* a_1 *and* a_4 *:* $Deg^{\texttt{nsa}}_{\mathcal{F}}(a_1) = 0.732$ *and* $Deg^{\texttt{nsa}}_{\mathcal{F}}(a_4) = 0.399$*. Finally, following the same method, here are the details concerning* a_3 *:*

$$\begin{aligned} Deg^{\texttt{nsa}}_{\mathcal{F}}(a_3) &= \frac{1}{1 + Deg^{\texttt{nsa}}_{\mathcal{F}}(a_1) + Deg^{\texttt{nsa}}_{\mathcal{F}}(a_2) + Deg^{\texttt{nsa}}_{\mathcal{F}}(a_4)} \\ &= \frac{1}{1 + 0.732 + 0 + 0.399} \\ &= 0.477 \end{aligned}$$

In order to have an overview of the difference between nsa *and the gradual semantics introduced in Section 2, the degrees of acceptability of arguments w.r.t. the h-categorizer semantics and the M&T semantics have also been added in the table of Figure 5. This comparison clearly shows that nullifying the impact of self-attacking arguments (i.e.* a_0 *and* a_2*) more or less significantly changes the degree of acceptability of other arguments (e.g.* a_1*,* a_3 *and* a_4*).*

6 Principle-Based Evaluation of Semantics

In this section we evaluate the nsa semantics with respect to principle compliance listed in Section 3, and compare the results with two existing semantics, namely M&T and h-categorizer. We first show that nsa satisfies all the principles from P_{2S2C}, and thus cannot be dominated by any semantics. Let us recall that proofs of propositions 8 and 9 can be found in Appendix A. The results are reported in Table 1.

Proposition 8. *The gradual semantics* nsa *satisfies all the principles from* P_{2S2C}*. The other principles are not satisfied.*

In order to axiomatically compare nsa with the two other gradual semantics, let us check for the principles studied in this paper those that are satisfied by M&T and recall those satisfied by the h-categorizer semantics.

Proposition 9. *The gradual semantics M&T satisfies Anonymity, Independence, Directionality, Maximality, Weakening, Compensation, Self-Contradiction and Strong Self-Contradiction. The other principles are not satisfied.*

Proposition 10 ([3]). *The gradual semantics h-categorizer satisfies all the principles from* P_{CREW}*. The other principles are not satisfied.*

Note that nsa dominates M&T, i.e. it satisfies strictly more principles because Counting is satisfied by nsa but is not satisfied by M&T. It is thus the most sensible choice if one needs a gradual semantics for an application where (Strong) Self-Contradiction is satisfied. Observe that nsa and h-categorizer are incomparable in terms of principles satisfaction. Indeed, nsa represents one choice, i.e. the position to satisfy Strong Self-Contradiction and Compensation. It also satisfies all the compatible principles. h-categorizer represents another choice, namely that to satisfy Compensation, Resilience, Equivalence and Weakening Soundness. Concretely, a semantics satisfying P_{CREW} considers that a self-attacking argument is a path like the other ones. So an argument which attacks itself (and is not attacked by any other argument) can be stronger than an argument which is attacked by several arguments. On the contrary, a semantics which satisfies P_{2S2C} considers that a self-attacking argument is intrinsically flawed, without even requiring other arguments to defeat it. Note that there exist other maximal compatible sets of principles, for example the one containing Resilience and Self-Contradiction. We leave a detailed study of these maximal compatible sets of compatible principles for future work.

7 Experimental Results

We now empirically compare nsa with M&T and h-categoriser semantics. We consider a large experimental setting representing three different models used during the ICCMA competition (http://argumentationcompetition.org/) as a way to generate random argumentation graphs:

1. the Erdös-Rényi model (ER) which generates graphs by randomly selecting attacks between arguments;

Principles	M&T	h-cat	nsa
Anonymity	✓	✓	✓
Independence	✓	✓	✓
Directionality	✓	✓	✓
Neutrality	×	✓	×
Equivalence	×	✓	×
Maximality	✓	✓	✓
Weakening	✓	✓	✓
Counting	×	✓	✓
Weakening Soundness	×	✓	×
Reinforcement	×	✓	×
Resilience	×	✓	×
Cardinality Precedence	×	×	×
Quality Precedence	×	×	×
Compensation	✓	✓	✓
Self-Contradiction	✓	×	✓
Strong Self-Contradiction	✓	×	✓

Table 1: Principles satisfied by the M&T, h-categorizer and `nsa` semantics. The shaded cells contain the results already proved in the literature.

2. the Barabasi-Albert model (BA) which provides networks, called scale-free networks, with a structure in which some nodes have a huge number of links, but in which nearly all nodes are connected to only a few other nodes; and
3. the Watts-Strogatz model (WS) which produces graphs which have small-world network properties, such as high clustering and short average path lengths.

The generation of these three types of AGs was done by the AFBenchGen2 generator [17]. We generated a total of 2160 AGs evenly distributed between the three models. For each model, the number of arguments varies among $Arg \in \{5, 10, 15, 25, 50, 100, 250, 500\}$ with 90 AGs for each of these values. The parameters used to generate graphs are as follows: for ER, 10 random instances for each (Arg, probAttacks) in $Arg \times \{0.2, 0.3, \ldots, 1\}$; for BA, 9 random instances for each (Arg, probCycles) in $Arg \times \{0, 0.1, \ldots, 0.9\}$; for WS, (Arg, probCycles, β, $\mathcal{K}$) in $Arg \times \{0.25, 0.5, 0.75\} \times \{0, 0.25, 0.5, 0.75, 1\} \times \{k \in 2\mathbb{N}\ s.t.\ 2 \leq k \leq |Arg| - 1\}$. We refer the reader to [17] for the meaning of the parameters.

In order to compare the execution times of the three semantics studied in this paper, we have implemented them in C and ran the program on a cluster of identical computers with dual quad-core processors with 128 GB RAM.[9]

Figure 6 shows the average execution time obtained by each semantics for the instances classified according to the number of arguments. A first remark is that, unlike the other two semantics, the M&T semantics quickly explodes in time since it systematically reaches the timeout (900 seconds) when the number of arguments is greater than

[9] The code and benchmarks are available online at https://github.com/jeris90/nsa_code.git.

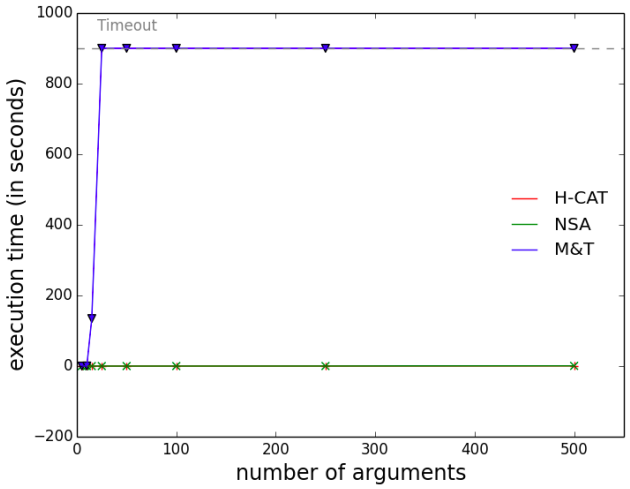


Fig. 6: Execution speed for the `nsa` (symbol green $\times$), the M&T (symbol blue $\blacktriangledown$) and the h-categorizer (symbol red $+$) semantics. x-axis shows the number of arguments of the instances ($Arg \in \{5, 10, 15, 25, 50, 100, 250, 500\}$). y-axis shows the execution time in seconds (with a timeout of 900 seconds). Note that the curves of the `nsa` and h-categorizer semantics are very close here; that is why we do not distinctly see the red curve because it overlaps with the green one. Figure 7 allows the reader a zoomed-in version of this part of the graph.

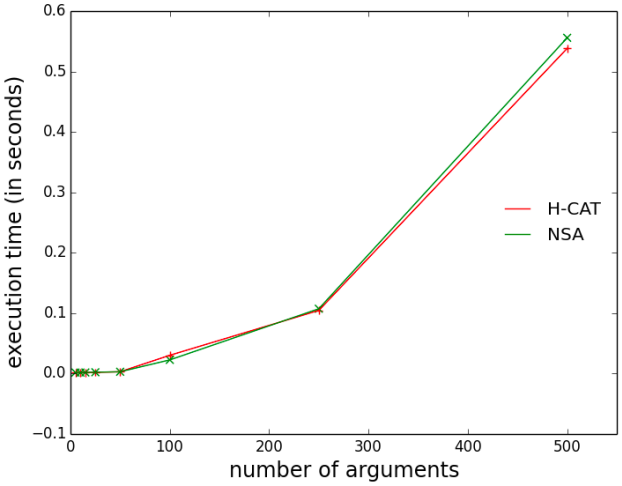


Fig. 7: A zoomed-in version of the graph from Figure 6 to better see the difference between the execution speed for the `nsa` semantics (symbol green $\times$) and the h-categorizer (symbol red $+$) semantics.

15. A second remark is that, unsurprisingly, the nsa and h-categorizer semantics have very similar execution times for each of the instances. Figure 7 shows the difference between nsa and h-categorizer semantics more precisely. The two semantics are, by definition, extremely close. The only algorithmic differences are the initialization to 0 for self-attacking arguments and keeping this score throughout the process (which in practice amounts to adding a simple condition before calculating the argument's score just as it is done for non-attacked arguments which keep the score of 1). On one hand, nsa might be more efficient than h-cat, namely when the number of self-attacking arguments is large, because a score of 0 is directly assigned to them, unlike for h-cat which calculates the score of that argument according to the score of those direct attackers. On the other hand, h-cat might be more efficient when there are not many self-attacking arguments, since no time is lost on the check whether an argument attacks itself. Moreover, a final remark is that these two semantics allow us to quickly compute (with an average smaller than one second) the degree of acceptability of each argument even for large AGs. Only a few very dense instances (i.e. those with a high probability of cycles) require between 1 and 2 seconds when $Arg = 500$.

8 Conclusion and Perspectives

We studied the question of the treatment of self-attacks by gradual semantics following a principle-based approach. We first showed links and incompatibilities between existing principles before identified two maximal compatible sets of principles (P_{CREW} which includes Equivalence and P_{2S2C} which includes Strong Self-Contradiction). Then, we defined a new semantics called no self-attack h-categorizer semantics and proved that it dominates the only existing semantics satisfying the Self-Contradiction principle. Moreover, we showed that our semantics satisfies a maximal possible amount of principles (i.e. no semantics satisfying Self-Contradiction can satisfy more principles) and is usable in practice as it returns results very quickly (on average less than 1 second) even on large and dense argumentation graphs.

We conclude by noting several considerations for future work on this topic.

Extend the methodology to other gradual semantics. It would be interesting to extend (if possible) the approach we used for the h-categorizer semantics (i.e. force self-attacking arguments to have a minimum acceptability degree) to other existing gradual semantics.

Identify all maximal sets of compatible principles. A second line of research would be to identify all maximal sets of consistent principles from the set of principles defined in Section 3. Indeed, we have chosen to include Compensation in P_{CREW} and P_{2S2C} but it would be interesting to look at and study the maximum sets which include Cardinality Precedence or Quality Precedence.

This set of principles is yet to be augmented. Another research direction concerns the principles dealing with self-attacking arguments. Indeed, the principle of Strong Self-Contradiction can be seen as a rather strong principle in that it expresses both necessary

and sufficient conditions for an argument to have minimal degree (i.e. 0 in our case). It would be interesting to investigate weakened versions like for instance a principle that only expresses that self-contradiction is a sufficient condition for minimal degree.

Principle 17 (Weak Self-Contradiction) *A gradual semantics S satisfies Weak Self-Contradiction iff, for any AG* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ *with* $a \in \mathcal{A}$*, if* $(a, a) \in \mathcal{R}$ *then* $Deg^S_{\mathcal{F}}(a) = 0$.

In this case, some incompatibilities remain unchanged (e.g. with Resilience or with Weakening Soundness), whereas whether the same is still the case for all of them remains to be investigated. We could imagine that this way a new set of coherent principles appears (where Weak Self-Contradiction replaces Strong Self-Contradiction). However, it should be checked whether there is at least one gradual semantics that satisfies all these principles.

Towards an application-oriented axiomatic analysis. Concerning the principles, let us recall that we do not claim that all of the principles presented in Section 3 are required. However, at this level of abstraction, they allow us to compare and better understand the gradual semantics. In line with the work initiated in [29], it would be interesting to target the mandatory principles for some practical aspects of argumentation (persuasion, negotiation, online debate, etc.).

Self-attacking arguments and gradual semantics in practical applications. There have already been discussions about applications where gradual (or ranking-based) semantics can be used [24,19,1]. One such application is online debates, for example, where participants propose, in the most basic form, arguments for or against a given topic or other arguments. As the arguments are given in textual format and the relationships between them are, in the vast majority of cases, given by the participants themselves, the arguments may not be correct and/or the set of attacks may not be complete. For example, some fallacious arguments (e.g. informal fallacies) may be put forward (this is sometimes the case in social networks or in fake news). These fallacious arguments could for example be spotted via argument mining methods [23] and considered, for some of them, as self-attacking arguments because of the false reasoning (e.g. sophism[10]). It is therefore necessary to be able to have reasoning tools that can deal with them in order to correctly analyse a given debate.

9 Acknowledgements

We thank the reviewers for their useful comments on the previous version of the paper. Vivien Beuselinck was supported by the ANR-3IA Artificial and Natural Intelligence Toulouse Institute. Srdjan Vesic was supported by the AI Chair project Responsible AI[11] (ANR-19-CHIA-0008) from the French National Agency of Research (ANR).

[10] A sophism is a confusing or slightly incorrect argument used for deceiving someone. For example, the following argument is a sophism : “Everything that is rare is expensive. A cheap horse is rare. So a cheap horse is expensive.”

[11] https://ia-responsable.eu/

The Version of Record of this manuscript has been published and is available in Journal of Logic and Computation, 2023, https://doi.org/10.1093/logcom/exac093.

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A Proofs of the propositions from Section 6

Proposition 8. *The gradual semantics* `nsa` *satisfies all the principles from* P_{2S2C}*. The other principles are not satisfied.*

Proof.

Anonymity. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ be two AG. Let γ be an isomorphism from $\mathcal{F}$ to $\mathcal{F}'$. Recall the iterative version of $f_{\mathtt{nsa}}^{\mathcal{F},i}$ from Definition 6. Let us prove Anonymity by induction on i, where i is the step of the iterative algorithm. The inductive hypothesis is: for every $a \in \mathcal{A}$, $f_{\mathtt{nsa}}^{\mathcal{F},i}(a) = f_{\mathtt{nsa}}^{\mathcal{F},i}(\gamma(a))$.

Base: Let $i = 0$. From the formal definition of `nsa` we have that for each $a \in \mathcal{A}$, $f_{\mathtt{nsa}}^{\mathcal{F},0}(a) = 0$ if and only if a is self-attacking in $\mathcal{F}$ and that $f_{\mathtt{nsa}}^{\mathcal{F},0}(a) = 1$, otherwise. Likewise, $f_{\mathtt{nsa}}^{\mathcal{F}',0}(a') = 0$ if and only if a' is self-attacking in $\mathcal{F}'$ and that $f_{\mathtt{nsa}}^{\mathcal{F}',0}(a') = 1$, otherwise.

Step: Let us suppose that the inductive hypothesis is true for every $k \in \{0, 1, \dots i\}$ and let us show that it is true for $i + 1$. Let $a \in \mathcal{A}$ and let $a' \in \mathcal{A}$ such that $a' = \gamma(a)$. Let $\mathtt{Att}_{\mathcal{F}}(a) = \{b_1, \dots, b_n\}$. From the inductive hypothesis, for each $j \in \{1, \dots, n\}$, $f_{\mathtt{nsa}}^{\mathcal{F},i}(b_j) = f_{\mathtt{nsa}}^{\mathcal{F},i}(\gamma(b_j))$. Hence, $f_{\mathtt{nsa}}^{\mathcal{F},i+1}(a) = f_{\mathtt{nsa}}^{\mathcal{F},i+1}(\gamma(a))$.

By induction, we conclude that for every i, for every $a \in \mathcal{A}$, $f_{\mathtt{nsa}}^{\mathcal{F},i}(a) = f_{\mathtt{nsa}}^{\mathcal{F},i}(\gamma(a))$. Hence, for every $a \in \mathcal{A}$, $Deg_{\mathcal{F}}^{\mathtt{nsa}}(a) = Deg_{\mathcal{F}'}^{\mathtt{nsa}}(\gamma(a))$.

Independence. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$.
Let us recall that $\forall a \in \mathcal{A}$,

$$Deg_{\mathcal{F}}^{\mathtt{nsa}}(a) = \begin{cases} 0 & \text{if } (a,a) \in \mathcal{R} \\ \dfrac{1}{1 + \sum_{b \in \mathtt{Att}_{\mathcal{F}}(a)} Deg_{\mathcal{F}}^{\mathtt{nsa}}(b)} & \text{otherwise} \end{cases} \tag{9}$$

Let $\mathtt{X} \subseteq \mathcal{A}$ be a set of arguments. Let us define $\mathtt{Att}_{\mathcal{F}}^{0}(\mathtt{X}) = \bigcup_{x \in \mathtt{X}} \mathtt{Att}_{\mathcal{F}}(x)$ as the union of the set of direct attackers of each $x \in \mathtt{X}$ and $\mathtt{Att}_{\mathcal{F}}^{i+1}(\mathtt{X}) = \mathtt{Att}_{\mathcal{F}}(\mathtt{Att}_{\mathcal{F}}^{i}(\mathtt{X}))$. Let $\mathtt{Att}_{\mathcal{F}}^{\star}(\mathtt{X}) = \bigcup_{i \geq 0} \mathtt{Att}_{\mathcal{F}}^{i}(\mathtt{X})$. Since $\mathcal{A} \cap \mathcal{A}' = \emptyset$ and $\mathtt{Att}_{\mathcal{F}}^{\star}(\{a\}) \subseteq \mathcal{A}$, since the definition of $\mathrm{Deg}_{\mathcal{F}}^{\mathtt{nsa}}(a)$ depends only on attackers of a and, in view of the recursion, on $\mathtt{Att}_{\mathcal{F}}^{\star}(\{a\})$, we have $\mathrm{Deg}_{\mathcal{F}}^{\mathtt{nsa}}(a) = \mathrm{Deg}_{\mathcal{F} \oplus \mathcal{F}'}^{\mathtt{nsa}}(a)$.

Directionality. Trivial

Maximality. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AG and $a \in \mathcal{A}$ such that $\mathtt{Att}_{\mathcal{F}}(a) = \emptyset$. By definition, if $\mathtt{Att}_{\mathcal{F}}(a) = \emptyset$ then we have $\sum_{b \in \mathtt{Att}_{\mathcal{F}}(a)} Deg_{\mathcal{F}}^{\mathtt{nsa}}(b) = 0$. Consequently, $Deg_{\mathcal{F}}^{\mathtt{nsa}}(a) = 1$.

Weakening. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AG and let argument $a \in \mathcal{A}$ such that $\exists b \in \mathtt{Att}_{\mathcal{F}}(a), Deg_{\mathcal{F}}^{\mathtt{nsa}}(b) > 0$. Clearly, argument b cannot be a self-attacking argument because $Deg_{\mathcal{F}}^{\mathtt{nsa}}(b) > 0$.
We have two possibilities for a:

- If a is a self-attacking argument then, by definition, we have $Deg_{\mathcal{F}}^{\mathtt{nsa}}(a) = 0 < 1$ which satisfies the principle.

– If a is not a self-attacking argument then we have

$$\sum_{b' \in \mathtt{Att}_{\mathcal{F}(a)}} Deg^{\mathtt{nsa}}_{\mathcal{F}}(b') > 0$$
$$1 + \sum_{b' \in \mathtt{Att}_{\mathcal{F}(a)}} Deg^{\mathtt{nsa}}_{\mathcal{F}}(b') > 1$$
$$\frac{1}{1 + \sum_{b' \in \mathtt{Att}_{\mathcal{F}(a)}} Deg^{\mathtt{nsa}}_{\mathcal{F}}(b')} < 1$$
$$Deg^{\mathtt{nsa}}_{\mathcal{F}}(a) < 1$$

showing that the principle is satisfied.

Counting. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AG and $a, b \in \mathcal{A}$ such that i) $Deg^{\mathtt{nsa}}_{\mathcal{F}}(a) > 0$ and ii) $\mathtt{Att}_{\mathcal{F}}(b) = \mathtt{Att}_{\mathcal{F}}(a) \cup \{y\}$ with $y \in \mathcal{A} \backslash \mathtt{Att}_{\mathcal{F}}(a)$ and $Deg^{\mathtt{nsa}}_{\mathcal{F}}(y) > 0$.
Clearly, a cannot be a self-attacking argument because $Deg^{\mathtt{nsa}}_{\mathcal{F}}(a) > 0$.
In addition, if b is a self-attacking argument then $Deg^{\mathtt{nsa}}_{\mathcal{F}}(b) = 0 < Deg^{\mathtt{nsa}}_{\mathcal{F}}(a)$ which satisfies the principle.
So, if a and b are not self-attacking arguments, by definition, we have:

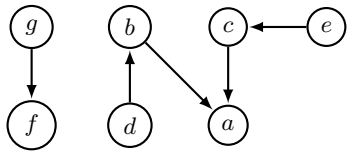
$$\sum_{b' \in \mathtt{Att}_{\mathcal{F}}(b) \backslash \{y\}} Deg^{\mathtt{nsa}}_{\mathcal{F}}(b') = \sum_{a' \in \mathtt{Att}_{\mathcal{F}}(a)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(a')$$
$$\sum_{b' \in \mathtt{Att}_{\mathcal{F}}(b)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(b') = \sum_{a' \in \mathtt{Att}_{\mathcal{F}}(a)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(a') + Deg^{\mathtt{nsa}}_{\mathcal{F}}(y)$$

Since $Deg^{\mathtt{nsa}}_{\mathcal{F}}(y) > 0$, we have:

$$\sum_{b' \in \mathtt{Att}_{\mathcal{F}}(b)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(b') > \sum_{a' \in \mathtt{Att}_{\mathcal{F}}(a)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(a')$$
$$1 + \sum_{b' \in \mathtt{Att}_{\mathcal{F}}(b)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(b') > 1 + \sum_{a' \in \mathtt{Att}_{\mathcal{F}}(a)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(a')$$
$$\frac{1}{1 + \sum_{b' \in \mathtt{Att}_{\mathcal{F}}(b)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(b')} < \frac{1}{1 + \sum_{a' \in \mathtt{Att}_{\mathcal{F}}(a)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(a')}$$
$$Deg^{\mathtt{nsa}}_{\mathcal{F}}(b) < Deg^{\mathtt{nsa}}_{\mathcal{F}}(a)$$

Compensation. Figure 8 is an example showing that there exists an AG such that i) $Deg^{\mathtt{nsa}}_{\mathcal{F}}(a) > 0$; ii) $|\mathtt{Att}_{\mathcal{F}}(a)| = |\{b, c\}| = 2 > 1 = |\{g\}| = |\mathtt{Att}_{\mathcal{F}}(f)|$; iii) $Deg^{\mathtt{nsa}}_{\mathcal{F}}(g) > Deg^{\mathtt{nsa}}_{\mathcal{F}}(b)$ and $Deg^{\mathtt{nsa}}_{\mathcal{F}}(g) > Deg^{\mathtt{nsa}}_{\mathcal{F}}(c)$ and $Deg^{\mathtt{nsa}}_{\mathcal{F}}(a) = Deg^{\mathtt{nsa}}_{\mathcal{F}}(f)$.

Strong Self-Contradiction. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an AG and $a \in \mathcal{A}$.
($\Leftarrow$) Let us suppose that $(a, a) \in \mathcal{R}$. By definition of $\mathtt{nsa}$, $Deg^{\mathtt{nsa}}_{\mathcal{F}}(a) = 0$.
($\Rightarrow$) Let us suppose that $Deg^{\mathtt{nsa}}_{\mathcal{F}}(a) = 0$. Again, by definition, for any AG and any non-self-attacking argument a, we have $\dfrac{1}{1 + \sum_{b \in \mathtt{Att}_{\mathcal{F}}(a)} Deg^{\mathtt{nsa}}_{\mathcal{F}}(b)} > 0$. Consequently, the only way to obtain $Deg^{\mathtt{nsa}}_{\mathcal{F}}(a) = 0$ is when $(a, a) \in \mathcal{R}$.



$Deg^{\texttt{nsa}}_{\mathcal{F}}(d) = Deg^{\texttt{nsa}}_{\mathcal{F}}(e) = Deg^{\texttt{nsa}}_{\mathcal{F}}(g) = 1$
$Deg^{\texttt{nsa}}_{\mathcal{F}}(b) = Deg^{\texttt{nsa}}_{\mathcal{F}}(c) = 0.5$
$Deg^{\texttt{nsa}}_{\mathcal{F}}(f) = 0.5$
$Deg^{\texttt{nsa}}_{\mathcal{F}}(a) = 0.5$

Fig. 8: `nsa` satisfies Compensation

Self-Contradiction. Implied by Strong Self-Contradiction which is satisfied by `nsa`.

The other principles are not satisfied because of incompatibilities :

- Equivalence and Self-Contradiction are incompatible (see Proposition 3).
- Neutrality and Strong Self-Contradiction are incompatible (see Proposition 7).
- Reinforcement and Strong Self-Contradiction are incompatible (see Proposition 6).
- Weakening Soundness and Strong Self-Contradiction are incompatible (see Proposition 5).
- Cardinality Precedence and Compensation are incompatible (see [4]).
- Quality Precedence and Compensation are incompatible (see [4]).
- Resilience and Strong Self-Contradiction are incompatible (see Proposition 4).

Proposition 9. *The gradual semantics M&T satisfies Anonymity, Independence, Directionality, Maximality, Weakening, Compensation, Self-Contradiction and Strong Self-Contradiction. The other principles are not satisfied.*

Proof.

Satisfied principles

Anonymity. See [14].

Independence. See proof of Proposition 9 in [25].

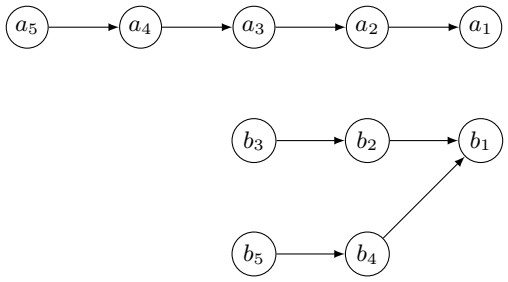
Directionality. The sets of strategies of the players are the same in the (G, x) and (G', x) games because the set of arguments remains unchanged. This implies that there is no impact on the payoff matrix. Therefore, the acceptability of a given argument only depends on arguments which have a path to this argument.

Maximality. See proof of Proposition 4 in [25].

Weakening. This result is a direct consequence of the result from Proposition 5.b in [25] stating that if there exist n attacks against an argument x, then $Deg^{\mathtt{MT}}_{\mathcal{F}}(x) < 1 - \frac{1}{2}f(n)$ where $f(n) = \frac{n}{n+1}$. Indeed, one can easily deduce from this formula that whatever the number of attackers of x and regardless of their degree of acceptability, the degree of x will always be strictly less than 1 because $\forall n > 0, Deg^{\mathtt{MT}}_{\mathcal{F}}(x) < 1 - \frac{1}{2}f(n) < 1$, in accordance with the Weakening principle.

Compensation. Figure 9 is an example showing that there exists an AG such that :

i) $Deg^{\mathtt{MT}}_{\mathcal{F}}(b_1) > 0$;
ii) $|\mathtt{Att}_{\mathcal{F}}(b_1)| = |\{b_2, b_4\}| = 2 > 1 = |\{a_2\}| = |\mathtt{Att}_{\mathcal{F}}(a_1)|$;
iii) $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_2) > Deg^{\mathtt{MT}}_{\mathcal{F}}(b_2)$ and $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_2) > Deg^{\mathtt{MT}}_{\mathcal{F}}(b_4)$ and $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_1) = Deg^{\mathtt{MT}}_{\mathcal{F}}(b_1)$.



$Deg^{\mathtt{MT}}_{\mathcal{F}}(a_5) = Deg^{\mathtt{MT}}_{\mathcal{F}}(b_3) = Deg^{\mathtt{MT}}_{\mathcal{F}}(b_5) = 1$
$Deg^{\mathtt{MT}}_{\mathcal{F}}(a_4) = Deg^{\mathtt{MT}}_{\mathcal{F}}(b_2) = Deg^{\mathtt{MT}}_{\mathcal{F}}(b_4) = 0.25$
$Deg^{\mathtt{MT}}_{\mathcal{F}}(a_3) = 0.5$
$Deg^{\mathtt{MT}}_{\mathcal{F}}(a_2) \simeq 0.386$
$Deg^{\mathtt{MT}}_{\mathcal{F}}(a_1) = 0.5$
$Deg^{\mathtt{MT}}_{\mathcal{F}}(b_1) = 0.5$

Fig. 9: The gradual semantics M&T satisfies the Compensation principle

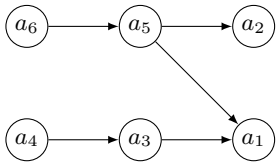
Self-Contradiction. See [14].

Strong Self-Contradiction. See proof of Proposition 3 in [25].

Unsatisfied principles

Neutrality. Incompatible with Self-Contradiction which is satisfied (see Proposition 7).

Equivalence. Incompatible with Self-Contradiction which is satisfied (see Proposition 3).



$Deg^{\mathtt{MT}}_{\mathcal{F}}(a_6) = Deg^{\mathtt{MT}}_{\mathcal{F}}(a_4) = 1$
$Deg^{\mathtt{MT}}_{\mathcal{F}}(a_5) = Deg^{\mathtt{MT}}_{\mathcal{F}}(a_3) = 0.25$
$Deg^{\mathtt{MT}}_{\mathcal{F}}(a_2) = 0.5$
$Deg^{\mathtt{MT}}_{\mathcal{F}}(a_1) = 0.5$

Fig. 10: The gradual semantics M&T falsifies the Counting principle

Counting. To show that the semantics M&T does not satisfy the Counting principle, consider the AG represented in Figure 10.

The principle says that $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_2) > Deg^{\mathtt{MT}}_{\mathcal{F}}(a_1)$ because i) $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_2) > 0$ and ii) $\mathtt{Att}_{\mathcal{F}}(a_1) = \mathtt{Att}_{\mathcal{F}}(a_2) \cup \{a_3\}$ where $a_3 \notin \mathtt{Att}_{\mathcal{F}}(a_2)$ and $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_3) > 0$. However, when the semantics is applied on $\mathcal{F}$, we have $Deg^{\mathtt{MT}}_{\mathcal{F}}(a_2) = Deg^{\mathtt{MT}}_{\mathcal{F}}(a_1)$, contradicting the principle.

Weakening Soundness. Incompatible with Strong Self-Contradiction which is satisfied (see Proposition 5).

Cardinality Precedence. Incompatible with Compensation which is satisfied.

Quality Precedence. Incompatible with Compensation which is satisfied. □