



HAL
open science

A Time-Stepping Volume Integral Formulation for Nonlinear Field-Circuit Coupled Problems

Mayra Hernandez Alayeto, Gérard Meunier, Loic Rondot, Olivier Chadebec,
Jean-Michel Guichon, Matthieu Favre

► **To cite this version:**

Mayra Hernandez Alayeto, Gérard Meunier, Loic Rondot, Olivier Chadebec, Jean-Michel Guichon, et al.. A Time-Stepping Volume Integral Formulation for Nonlinear Field-Circuit Coupled Problems. IEEE Transactions on Magnetics, 2024, 60 (3), 10.1109/TMAG.2023.3301566 . hal-04180908

HAL Id: hal-04180908

<https://hal.science/hal-04180908v1>

Submitted on 14 Aug 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A Time-Stepping Volume Integral Formulation for Nonlinear Field-Circuit Coupled Problems

Mayra Hernandez Alayeto^{1, 2}, Gérard Meunier¹, Loic Rondot², Olivier Chadebec¹,
Jean-Michel Guichon¹ and Matthieu Favre²

¹Univ. Grenoble Alpes, CNRS, Grenoble INP, G2ELab, Grenoble, France

²Schneider Electric, Global Technology Eybens, France

A B-conforming time-stepping Volume Integral Method (VIM) formulation for nonlinear 3D field-circuit coupled problems is presented in this paper. The advantage of the VIM with respect to the Finite Element Method (FEM) is that only the ferromagnetic regions have to be discretized, thus avoiding to mesh the air. It is an appealing approach given that it avoids numerical errors that can arise from meshing the air. A direct formulation is explained, assembling in a system the magnetic field and the circuit contributions when coils are present. An application to a current transformer is shown, comparing it to the FEM to validate its accuracy.

Index Terms—Equivalent circuit, facet-shape functions, field-circuit coupling, nonlinear, time-stepping, volume integral method.

I. INTRODUCTION

VOLUME integral method (VIM) is a powerful approach to solve magnetic problems. Its main advantage is that only ferromagnetic regions have to be discretized, thus avoiding to mesh the air as it happens in the finite element method (FEM). VIM is known since 1970s but it wasn't popular at that time because it leads to full matrices that are computationally expensive. Nevertheless, the last decade it became more accessible as a result of matrix compression algorithms such as the fast multipole method [1], enlargement of computers RAM memory and advanced integral formulations [2], [3].

The VIM performs well with a coarse mesh and it is very accurate in the computation of remote electromagnetic interactions, for instance, when the magnetic flux through external coils is needed [4]. This flux calculation is central in the framework of coupled field-circuit approaches where a distinction between electrically conductive and non-conductive media is made. Thus, it seems natural to implement the field-circuit coupling with a VIM method to treat magnetic phenomena and a classical circuit method to treat coils electrically coupled with current sources, voltage sources or more complex lumped electric elements.

A first thought to develop the circuit-field coupling is to use the harmonic regime [5], nevertheless it is only valid for linear materials and sources of sinusoidal waves, which limits the applications. A more general method would be a time-stepping scheme, in which a resolution of the system defining the problem is done every time step, which allows to deal with nonlinear materials and non sinusoidal sources. To our knowledge, this has not been developed before using the VIM, however, there are examples of time-stepping FEM formulations using a predictor-corrector method [6], the Newmark-beta scheme [7] or formulations developed to deal with eddy currents [8], [9].

Regarding the field-circuit formulations there are two types: indirect, in which the magnetic field equations and the circuit ones are treated as separate systems at each time step; and direct ones, in which the field and circuit equations are written together to form a system, being solved simultaneously.

This paper presents an original extension of the magneto-static formulation [10] to a direct time-stepping formulation for nonlinear magnetic problems coupled with circuit equations.

II. TIME-STEPPING FIELD-CIRCUIT COUPLING FORMULATION

Let us consider a problem composed of a non-conductive and nonlinear magnetic region Ω_m surrounded by an air region which includes circuit coils Ω_b (Fig. 1). The circuit coils can be coupled with lumped elements such as current sources, voltage sources or resistive circuit elements.

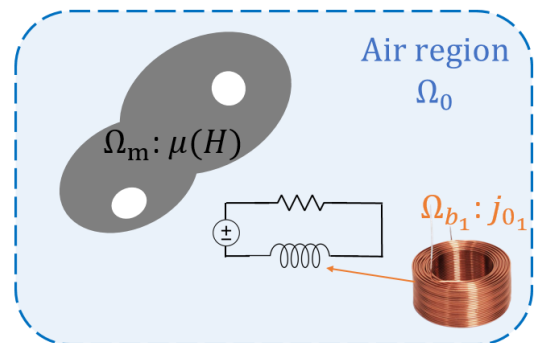


Fig. 1: General problem representation.

The formulation is developed in two parts, first the magnetic field contribution is addressed and then the circuit equations are added to build a system.

A. Magnetic field contribution

For the magnetic field contribution, which is based on the magnetostatic formulation [10], let us consider the magnetostatic Maxwell equations in differential form:

$$\begin{cases} \text{rot}\mathbf{H} = \mathbf{J} \\ \text{div}\mathbf{B} = 0 \end{cases} \quad (1)$$

where \mathbf{H} is the magnetic field, \mathbf{J} is the electric current density and \mathbf{B} is the magnetic flux density. The first of them will be strongly imposed whereas the second one will be automatically verified with an equivalent circuit approach. The constitutive law allows to write a relation between the magnetic field \mathbf{H} and the magnetic induction \mathbf{B} as:

$$\mathbf{H}(\mathbf{B}) = \nu(\mathbf{B})\mathbf{B} + \mathbf{H}_c \quad (2)$$

where ν is the reluctivity of the material and \mathbf{H}_c is the coercive field of the magnetic materials.

Maxwell-Ampere equation is used to decompose \mathbf{H} following:

$$\mathbf{H} = -\text{grad}\varphi_r + \mathbf{H}_b \quad (3)$$

where φ_r is the reduced magnetic scalar potential and \mathbf{H}_b is the magnetic field created by the circuit coils, which is expressed as:

$$\mathbf{H}_b = \sum_k \mathbf{h}_{b,k} I_{b,k} \quad (4)$$

where $\mathbf{h}_{b,k}$ is the magnetic field created by coil k with current 1 A and $I_{b,k}$ is the current of coil k .

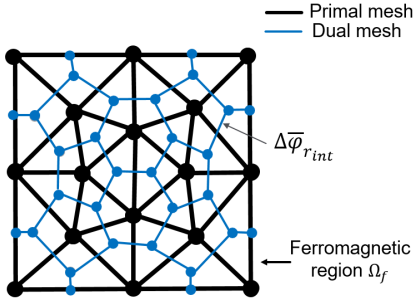


Fig. 2: Primal and dual mesh of a problem.

A mesh of the domain is considered with N_f facet elements (Fig. 2), and facet-shape functions \mathbf{w}_j (Whitney 2-form [11]) associated to that mesh are taken. Induction \mathbf{B} is discretized with these functions such that:

$$\mathbf{B} = \sum_{j=1}^{N_f} \mathbf{w}_j \Phi_j \quad (5)$$

where Φ_j is the magnetic flux flowing across facet j , which corresponds to the branches of the dual mesh, Fig. 2. Now, projecting eq. (3) with the same facet-shape function \mathbf{w}_i (Galerkin projection) [10], the following system arises:

$$\mathbf{R}_m \Phi + \mathbf{L}_{mb} \mathbf{I}_b = \Delta \bar{\varphi}_r^{\text{int}} + \mathbf{U}^{\mathbf{H}_c} \quad (6)$$

where \mathbf{R}_m is a finite element matrix:

$$R_{m,i,j} = \int_{\Omega_m} \mathbf{w}_i \cdot \nu(\mathbf{B}) \mathbf{w}_j d\Omega_m \quad (7)$$

$i = 1, 2, \dots, N_f$, $j = 1, 2, \dots, N_f$, $\Delta \bar{\varphi}_r^{\text{int}}$ is the average potential difference between two elements that are part of the magnetic region, $\mathbf{U}^{\mathbf{H}_c}$ is a term coming from the coercive field

$$\mathbf{U}_i^{\mathbf{H}_c} = - \int_{\Omega_m} \mathbf{w}_i \cdot \mathbf{H}_c d\Omega_m \quad (8)$$

and \mathbf{L}_{mb} is the contribution of the external coils:

$$\mathbf{L}_{mb_{i,k}} = - \int_{\Omega_m} \mathbf{w}_i \cdot \mathbf{h}_{b,k} d\Omega_m. \quad (9)$$

System (6) describes the magnetic field contribution of the internal branches, meaning the branches inside the magnetic region. Now, to take into account the contribution of the air, a common convention with integral methods is to define an external node called infinity node to which the branches of the boundary of the magnetic region are linked, Fig. 3. We call the latter external branches.

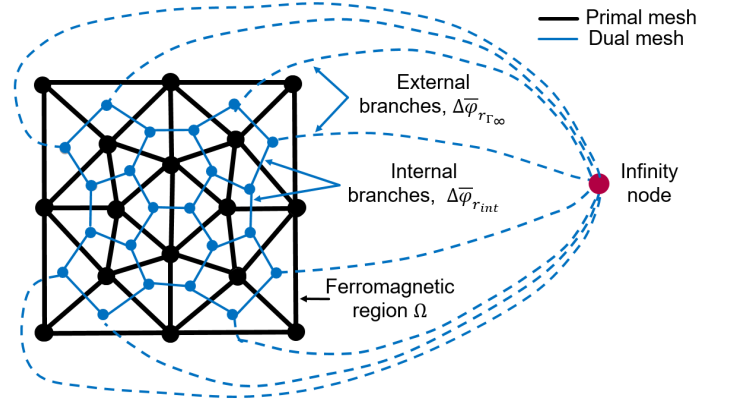


Fig. 3: Discretization of the domain with infinity point.

The external branches contribution is taken into account writing the reduced magnetic potential φ_r in terms of the magnetization \mathbf{M} :

$$\varphi_r = \frac{1}{4\pi} \int_{\Omega_m} \mathbf{M} \cdot \text{grad}\left(\frac{1}{r}\right) d\Omega_m \quad (10)$$

where r is the distance between the evaluation point of φ_r and the integration point in Ω_m . Assuming a constant reluctivity per mesh element (which is already true for simplexes elements) to use the divergence theorem, expressing the magnetization as $\mathbf{M} = (\nu_0 - \nu)\mathbf{B} + \mathbf{H}_c$ and using the computation of the flux through a facet j : $\Phi = \mathbf{B} \cdot \mathbf{n} S_j$ with \mathbf{n} the normal vector of facet j , φ_r can be written:

$$\varphi_r \approx \frac{1}{4\pi} \sum_j \int_{\Gamma_j} \frac{1}{S_j} \frac{\delta \nu_j}{r} d\Gamma \Phi_j + \frac{1}{4\pi} \int_{\Omega_m} \mathbf{H}_c \cdot \text{grad}\left(\frac{1}{r}\right) d\Omega. \quad (11)$$

where $\delta \nu_j$ is the reluctivity difference of the two elements that share facet j or $(\nu_j - \nu_0)$ if facet j belongs to the border of the ferromagnetic region. Assuming null potential at the infinity node, the average potential of the external facets that are going to the infinity node is computed by:

$$\bar{\varphi}_r^{\text{ext}} = \frac{1}{S_i} \int_{\Gamma_{\text{ext}}} \varphi_r d\Gamma \quad (12)$$

where the over line means average and Γ_{ext} refers to the external branches (Fig. 3). The potential difference between the average potential of the external facets reads:

$$\Delta\bar{\varphi}_r^{\text{ext}} = \mathbf{P}_m\Phi + \mathbf{U}^I \quad (13)$$

where

$$\mathbf{P}_{m_{ij}} = \frac{1}{4\pi} \int_{\Gamma_{\text{ext}_i}} \frac{1}{S_i} \left(\int_{\Gamma_j} \frac{1}{S_j} \frac{\delta\nu_j}{\mathbf{r}} d\Gamma \right) d\Gamma, \quad (14)$$

$$\mathbf{U}_i^I = \frac{1}{4\pi} \int_{\Gamma_{\text{ext}_i}} \frac{1}{S_i} \left(\int_{\Omega_m} \mathbf{H}_c \cdot \mathbf{grad}\left(\frac{1}{\mathbf{r}}\right) d\Omega \right) d\Gamma. \quad (15)$$

Then, assembling the internal and external branches, the system taking into account the magnetic field contribution is obtained:

$$\mathbf{R}_m\Phi + \mathbf{P}_m\Phi + \mathbf{L}_{mb}\mathbf{I}_b = \Delta\bar{\varphi}_r + \mathbf{U}^m \quad (16)$$

where $\Delta\bar{\varphi}_r = \Delta\bar{\varphi}_r^{\text{int}} + \Delta\bar{\varphi}_r^{\text{ext}}$ and $\mathbf{U}^m = \mathbf{U}^{\mathbf{H}_c} + \mathbf{U}^I$.

B. Circuit contribution

For the circuit contribution, a circuit relation is needed for each coil k :

$$\Delta V_{b_k} = \mathbf{R}_{b_k}\mathbf{I}_{b_k} + \frac{d}{dt} \sum_l (\mathbf{L}_{b_{kl}}\mathbf{I}_{b_l}) + \frac{d\Phi_{mb_k}}{dt} \quad (17)$$

where ΔV_{b_k} is the voltage of coil k , \mathbf{R}_{b_k} is the resistance of coil k , $\mathbf{L}_{b_{kl}}$ is the self and mutual inductance between coils k and l and Φ_{mb_k} is the magnetic region contribution to the flux of coil k . The latter flux can be expressed as

$$\Phi_{mb_k} = \int_{\Omega_{b_k}} \mathbf{j}_{b_k} \cdot \mathbf{A}_m d\Omega_{b_k} \quad (18)$$

where \mathbf{A}_m is the magnetic vector potential generated by the magnetization on magnetic regions and \mathbf{j}_{b_k} is the current density imposed on coil k .

Following [12], the contribution of the magnetic region to the flux of coil k can be integrated over the magnetic region instead of the coil region as:

$$\int_{\Omega_{b_k}} \mathbf{j}_{b_k} \cdot \mathbf{A}_m d\Omega_{b_k} = \int_{\Omega_m} \mathbf{h}_{b_k} \cdot \mathbf{M} d\Omega_m. \quad (19)$$

Then, considering $\mathbf{M} = (\nu_0 - \nu)\mathbf{B} + \mathbf{H}_c$ and the discretization of \mathbf{B} , the equation (17) for a coil k can be extended to a matrix system that considers all the coils as:

$$\frac{d}{dt}(\mathbf{L}_{bm}\Phi) + \mathbf{R}_b\mathbf{I}_b + \frac{d}{dt}(\mathbf{L}_b\mathbf{I}_b) = \Delta V_b \quad (20)$$

where

$$\mathbf{L}_{bm_{kj}} = \int_{\Omega_m} \mathbf{h}_{b_k} \cdot (\nu_0 - \nu)\mathbf{w}_j d\Omega_m \quad (21)$$

and \mathbf{H}_c term vanishes given that it does not depend on time. Finally, assembling the magnetic field and circuit contributions, the formulation system defining the problem reads:

$$\begin{cases} \mathbf{R}_m\Phi + \mathbf{P}_m\Phi + \mathbf{L}_{mb}\mathbf{I}_b = \Delta\bar{\varphi}_r + \mathbf{U}^m \\ \frac{d}{dt}(\mathbf{L}_{bm}\Phi) + \mathbf{R}_b\mathbf{I}_b + \frac{d}{dt}(\mathbf{L}_b\mathbf{I}_b) = \Delta V_b. \end{cases} \quad (22)$$

III. EQUIVALENT CIRCUIT APPROACH

This formulation can be expressed in a generic circuit form, composed of branches and nodes (primal and dual mesh, Fig. 2), which reads:

$$\mathbf{R}(\mathbf{I})\mathbf{I} + \frac{d}{dt}\mathbf{L}(\mathbf{I})\mathbf{I} = \Delta\varphi + \mathbf{U}_{sc} \quad (23)$$

where \mathbf{I} is the magnetic flux of an equivalent circuit that has been discretized and the electric currents of the coils, $\Delta\varphi$ is the electric or magnetic potential difference between the branches of the electric and magnetic circuit respectively, \mathbf{U}_{sc} is a vector term that comes from the external sources and finally \mathbf{R} and \mathbf{L} are matrices that come from the magnetic circuit and they are obtained integrating on the primal mesh.

The equivalent circuit approach (23) allows to solve (22) with a circuit solver, then either Kirchoff's mesh rule or Kirchoff's node rule is applied for the resolution, which automatically ensures the free divergence of \mathbf{B} . Both methods make use of an incidence matrix that can be obtained with tree and co-tree techniques [13].

The formulation makes the coupling between magnetic regions, external circuits and coils with a topological description of the circuits, where electric connections between them are ensured thanks to common circuit nodes. Circuit components of resistance and inductance type, current sources, voltage sources and capacitors can be introduced in (22).

IV. RESOLUTION STRATEGY

The resolution is carried out with a theta scheme time-stepping method with a nested Newton-Raphson (NR) loop to solve the nonlinear term. The jacobian matrices of the system have to be computed. The only matrices depending on the unknown are \mathbf{R}_m , \mathbf{L}_{bm} and \mathbf{P}_m . The contribution of \mathbf{R}_m matrix to the jacobian is:

$$J(\mathbf{R}_{m_{i,j}}) = \int_{\Omega_m} \mathbf{w}_i \cdot \left[\frac{\partial \mathbf{H}}{\partial \mathbf{B}} \right] \mathbf{w}_j d\Omega_m \quad (24)$$

where J refers to the jacobian. For the other two matrices, given that the reluctivity ν has significant less weight than ν_0 , $(\nu_0 - \nu)$ is approximated by ν_0 , which is constant at each NR iterations, therefore we approximate their influence to the jacobian matrices by themselves, without deriving the reluctivity tensor. This allows to accelerate the computations. Regarding the time-stepping theta scheme, we generally choose $\theta = 1$ (Euler implicit) or $\theta = 0.5$ (Crank-Nicolson) for stability and accuracy respectively.

To accelerate the simulation and avoid the computation of the integral matrix \mathbf{P}_m at every time step, since the hypothesis of constant reluctance per element is set, we can write $\mathbf{P}_{m_{i,j}} = \delta\nu_j\mathbf{Q}_{ij}$, computing matrix \mathbf{Q} just once, where

$$\mathbf{Q}_{i,j} = \frac{1}{4\pi} \int_{\Gamma_{\text{ext}_i}} \frac{1}{S_i} \left(\int_{\Gamma_j} \frac{1}{S_j} \frac{1}{\mathbf{r}} d\Gamma \right) d\Gamma \quad (25)$$

which only depends on the geometry and it can be multiplied by $\delta\nu_j$ at each NR iteration. Another strategy to speed the computations is to consider a matrix compression technique, such as fast multiple method on matrix \mathbf{P}_m . A GMRES iterative solver is then considered and it is accelerated with a LU type preconditioner on the finite element matrix \mathbf{R}_m .

V. APPLICATION

The proposed formulation has been applied to a current transformer to validate its accuracy. The reference is the solution obtained with a FEM commercial software with a mesh that has converged.

The current transformer is composed of a magnetic region, a primary coil and a secondary coil as Fig. 4 shows. The primary

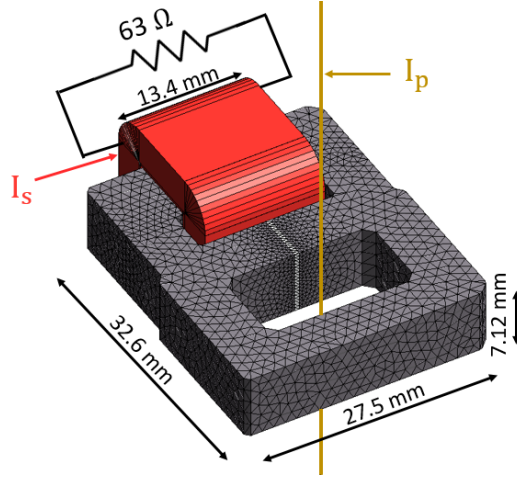


Fig. 4: Current transformer geometry.

coil has 1 turn and an imposed current of $300 \sin(2\pi ft)$ where t is the time and f is the frequency, 50 Hz. The secondary coil has 980 turns and it is connected to a resistance of 63Ω . The simulation is done for three periods, therefore $t \in [0, 0.06]$ seconds, with 50 time steps per period. The magnetic region is nonlinear following an isotropic analytic saturation of arc tangent type with two coefficients: $\mu_r = 1000$ and saturation magnetization = 1.2 T.

The converged mesh has 46,523 tetrahedral elements in the magnetic part of the current transformer for VIM. For FEM, the mesh of the magnetic part is similar and there are 261,863 additional elements in the air region.

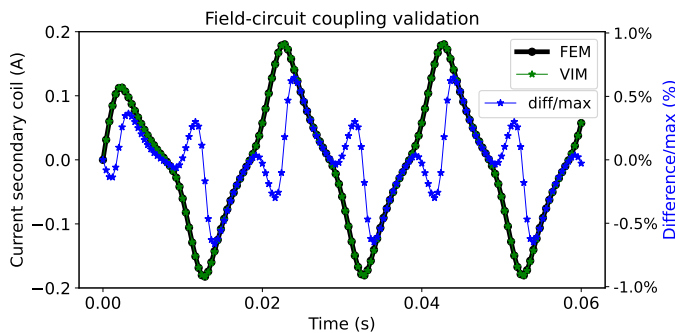


Fig. 5: Current secondary coil for VIM and FEM.

Fig. 5 shows the current of the secondary coil for FEM formulation (black line) and the VIM formulation developed (green line) on the left y-axis. The difference point by point of VIM minus FEM over the maximum of FEM for all the points is displayed on the right y-axis (blue line) and its absolute

value is always inferior to 0.6 % thus validating the VIM formulation. Regarding the computation time, FEM took 45 hours whereas the VIM formulation took 26 hours, using both of them a computer of CPU 2.9 GHz, 128 GB of RAM.

VI. CONCLUSION

A time-stepping volume integral formulation for nonlinear non-conductive materials has been developed. This formulation takes into account the field-circuit coupling between magnetic regions and external circuits that are linked to coils. An equivalent circuit approach has been considered, writing the field-circuit coupling system in a generic circuit form, which allows to use a circuit solver for its resolution. This resolution implements a time-stepping theta scheme and a nested Newton-Raphson loop to deal with the non linearity of the system. Furthermore, some additional numerical strategies are taken into account to decrease the computation time. Finally, the developed formulation is applied to a current transformer whose secondary coil is linked to a resistance. The result is compared to a FEM simulation, obtaining a difference inferior to 0.6 %, which validates it.

REFERENCES

- [1] L. Greengard and V. Rokhlin, "A fast algorithm for particle simulations," *J. Comput. Phys.*, vol. 73, no. 2, pp. 325–348, 1987.
- [2] A. Canova and M. Repetto, "Integral solution of nonlinear magnetostatic field problems," *IEEE Trans. Magn.*, vol. 37, no. 3, pp. 1070–1077, May 2001.
- [3] A. Carpentier, O. Chadebec, N. Galopin, G. Meunier and B. Bannwarth, "Resolution of Nonlinear Magnetostatic Problems With a Volume Integral Method Using the Magnetic Scalar Potential," *IEEE Trans. Magn.*, vol. 49, no. 5, pp. 1685–1688, May 2013.
- [4] L. Huang, G. Meunier, O. Chadebec, J-M Guichon, Y. Li and Z. He, "General Integral Formulation of Magnetic Flux Computation and its Application to Inductive Power Transfer System," *IEEE Trans. Magn.*, vol. 53, no. 6, June 2017.
- [5] A. E. Ruehli, "Inductance Calculations in a Complex Integrated Circuit Environment," *IBM Journal of Research and Development*, vol. 16, no. 5, pp. 470-481, 1972.
- [6] E. Vassent, G. Meunier, A. Foggia and G. Reyne, "Simulation of induction machine operation using a step by step finite element method coupled with circuits and mechanical equations," *IEEE Trans. Magn.*, vol 27, no. 6, pp. 5232-5234, 1991.
- [7] S. Yan and J-M Jin, "Theoretical Formulation of a Time-Domain Finite Element Method for Nonlinear Magnetic Problems in Three Dimensions" (Invited Paper), *Progress In Electromagnetics Research*, vol 153, pp. 33-55, 2015.
- [8] I. A., Tsukerman, A. Konrad and J. D. Lavers, "A method for circuit connections in time-dependent eddy current problems," *IEEE Trans. Magn.*, vol. 28, no. 2, pp. 1299-1302, 1992.
- [9] D. Shen, G. Meunier, J. Coulomb and J. Sabonnadiere, "Solution of magnetic fields and electrical circuits combined problems," *IEEE Trans. Magn.*, vol 21, no. 6, pp. 2288-2291, 1985.
- [10] V. Le-Van, G. Meunier, O. Chadebec and J-M Guichon, "A volume integral formulation based on facet elements for nonlinear magnetostatic problems," *IEEE Trans. Magn.*, vol. 51, no. 7, July 2015.
- [11] A. Bossavit, "Whitney forms: A class of finite elements for three dimensional computations in electromagnetism," *IEE Proc. A Phys. Sci., Meas. Instrum., Manage. Edu.-Rev.*, vol. 135, no. 8, pp. 493–500, Nov. 1988.
- [12] L. Huang, G. Meunier, O. Chadebec, J. M. Guichon, Y. Li, and Z. He, "General integral formulation of magnetic flux computation and its application to inductive power transfer system," *IEEE Trans. Magn.*, vol. 53, no. 6, pp. 1-4, 2017.
- [13] P. Dular, A. Nicolet, A. Genon and W. Legros, "A discrete sequence associated with mixed finite elements and its gauge condition for vector potentials," *IEEE Trans. Magn.*, vol. 31, no. 3, pp. 1356-1359, May 1995.