

# Möbius strip microlasers: Quantum chaos on curved surfaces

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**Abstract:** Möbius strip microlasers are fabricated by direct laser writing. Experiments and 3D FDTD numerical simulations evidence that laser modes propagate along periodic geodesics, a geodesic being the shortest path between two points on a surface. © 2022 The Author(s)

The research field of quantum chaos is dedicated to semiclassical physics [1], i.e., studying the relationship between a quantum or wave-dynamical system and its classical counterpart. Experimentally these relations can be investigated in a plethora of different wave system, from quantum dots to acoustic resonators, microwave cavities as well as optical systems. In particular microlasers are a very useful platform for investigating quantum chaos in open systems [2].

One of the major hypotheses in the field is the localization of eigenmodes on classical periodic trajectories [3], which was demonstrated with quasi two-dimensional microlasers [4–6]. Recently it became possible to fabricate three-dimensional (3D) microlasers with high optical quality by direct laser writing [see Fig. 1(a)], in particular microlasers on curved surfaces. We investigate Möbius strip microlasers and demonstrate by experiments and FDTD simulations that their modes are located on periodic geodesics [7]. Geodesics are the generalization of straight lines on curved surfaces and are the shortest path between two points on a surface. A typical wave function and its corresponding periodic geodesic are presented in Fig. 1(c), which evidences the connections between waves and rays in a non-Euclidean photonic device.

Non-Euclidean photonic devices can exhibit properties impossible for flat, two-dimensional cavities. Hence, such devices will allow an improved control over light on the microscale to create for example complex polarization states and engineer the spectral properties of microlasers using geodesics and Berry phases.

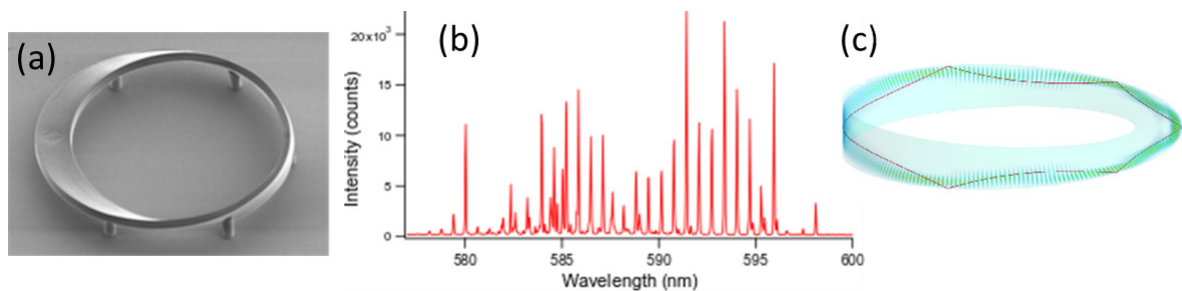


Fig. 1. (a) Scanning Electronic Microscope (SEM) image of a Möbius strip microlaser with diameter 100 μm. (b) Typical experimental lasing spectrum from a Möbius strip microlaser. (c) High- $Q$  wave function of a Möbius strip resonator calculated by 3D FDTD simulation, which is located on the periodic geodesic indicated by the red line.

## References

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