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Dispersion of elastic and acoustic waves in multilayer structures involving poroelastic materials

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Numerical computation of dispersion relations

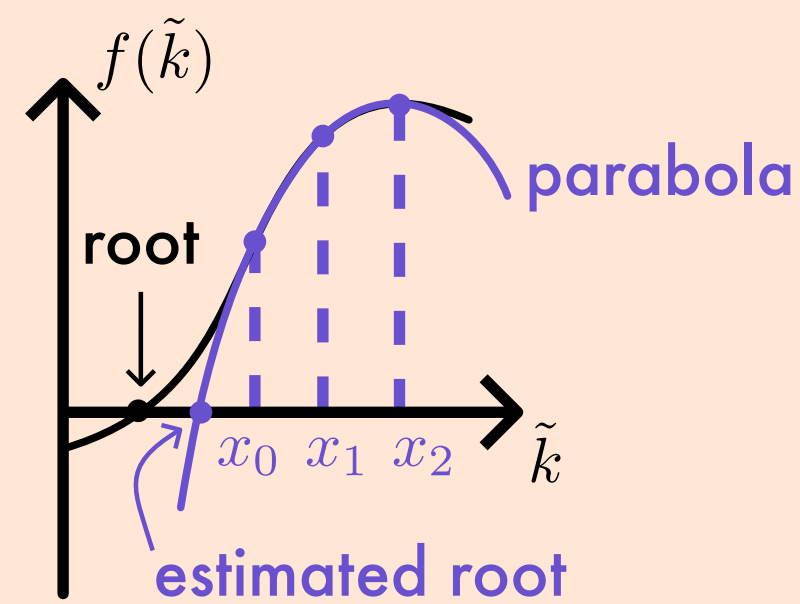
- Interest in the modal behavior of various structures: metaporous surfaces/interfaces and composite multilayer
- Influence of the losses in poroelastic materials (PEM): viscoelastic (from the elastic part), the viscothermal (from the fluid part) and leakage (in the structure and from the structure to the surrounding medium)
- Investigation of these mechanisms using (\tilde{k}, ω) dispersion relations: complex wavenumber and real frequency
- Need of robust, versatile and easy to use numerical methods to compute the dispersion relations in these different configurations

Root-finding with Müller method

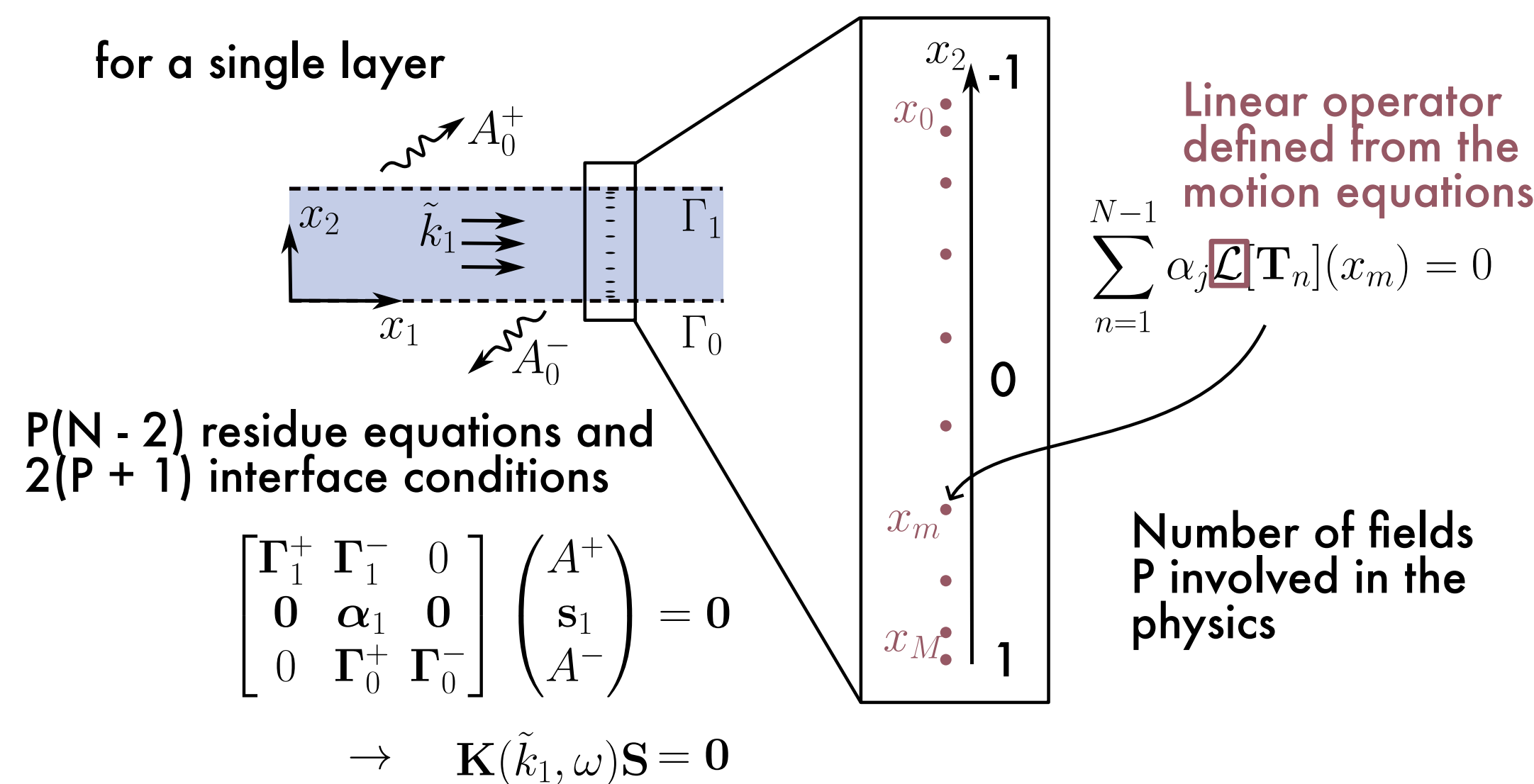
- Secant-based method with quadratic approximation
- Search of roots from an analytical description of the system

$$\det(M(\tilde{k}, \omega)) = 0$$

- Sensitivity to initial values and conditioning of M
- Slow and computationally expensive method
- No certainty of finding all roots at a given ω



1 - Discretization across the layers of the structure



3 - Solution of the problem

$$(k_1^2 K_2 + k_1 K_1 + K_0 + ik_2^{(f)} K_1') S = 0$$

$$S = (A_{0-}, s_0, \dots, s_n, \dots, s_N, A_{0+})$$

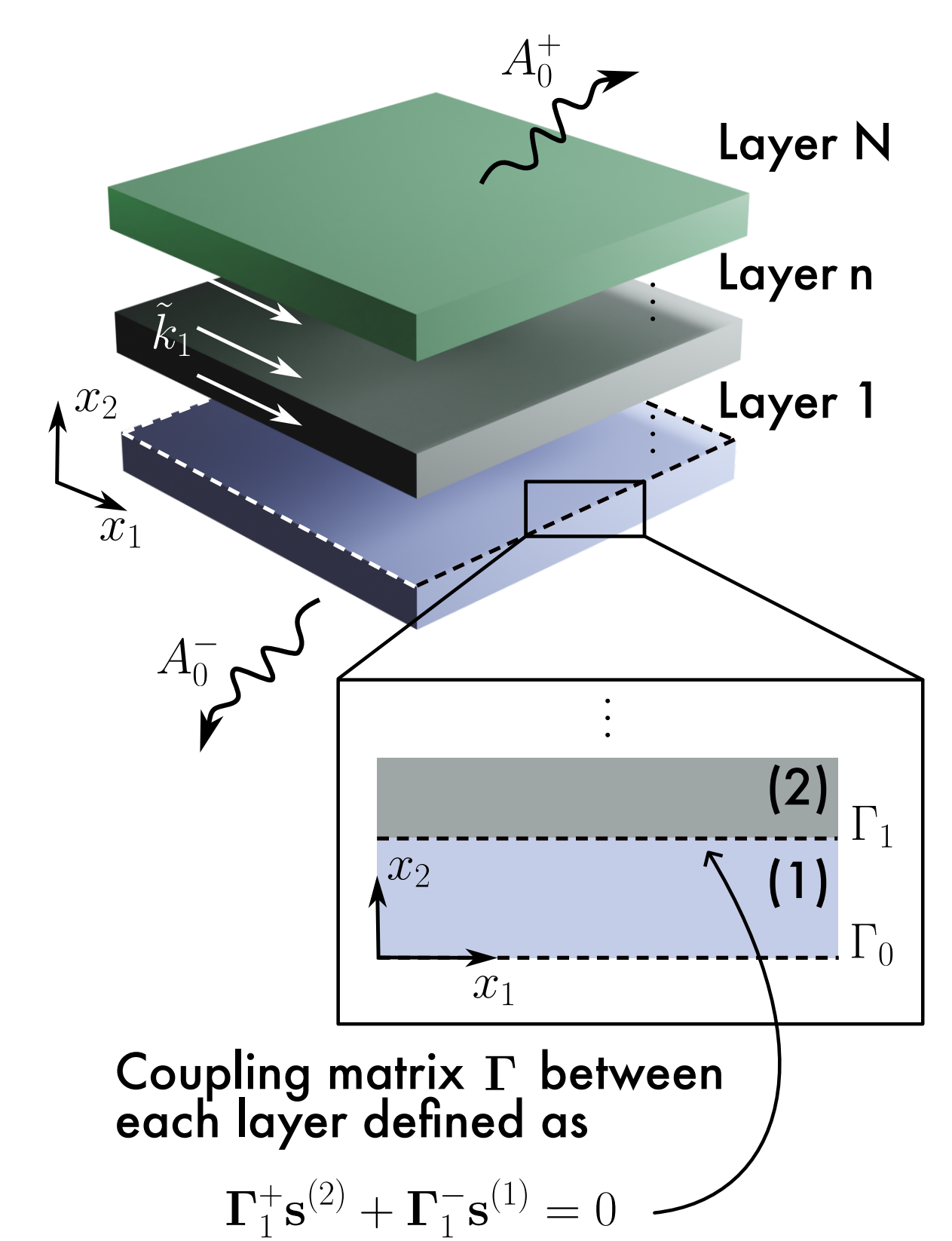
Algebraic manipulations to linearize the problem

eigenvalues: wavenumbers $\gamma^3 S$

eigenvectors: mode shapes $(A - \gamma B) S = 0$

$$\begin{pmatrix} \gamma^3 S \\ \gamma^2 S \\ \gamma S \\ S \end{pmatrix}$$

2 - Construction of the multilayer numerical scheme



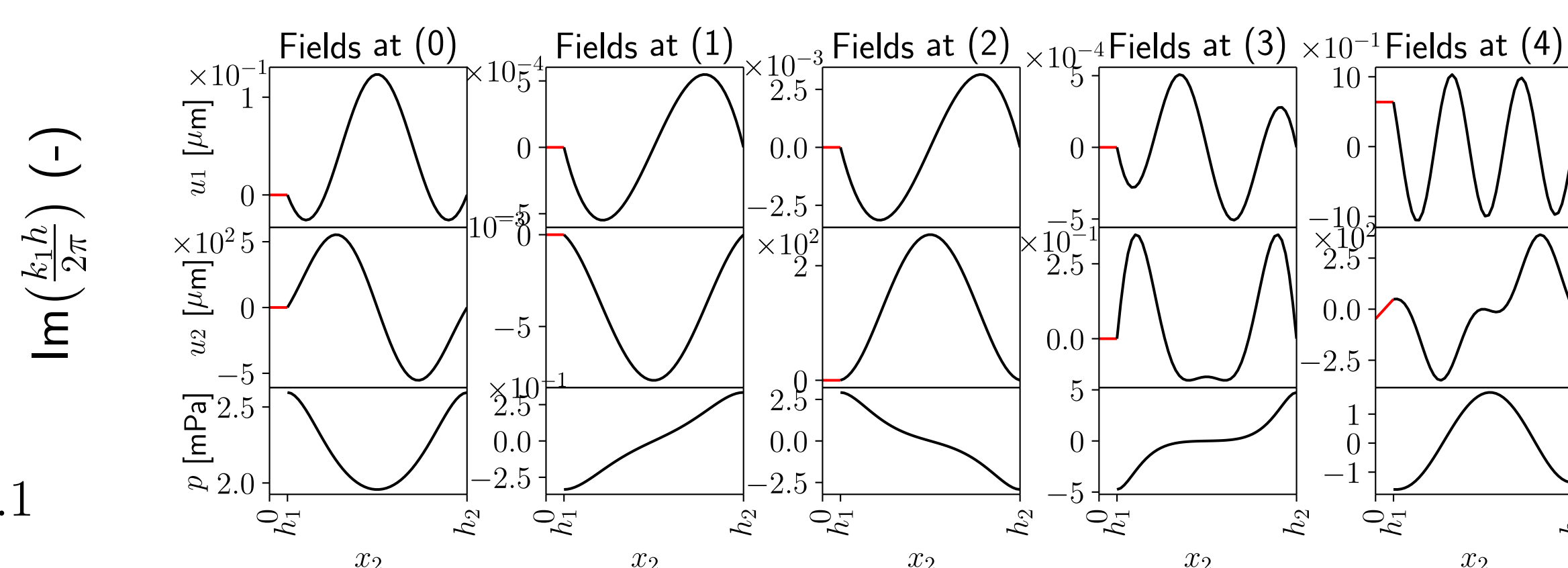
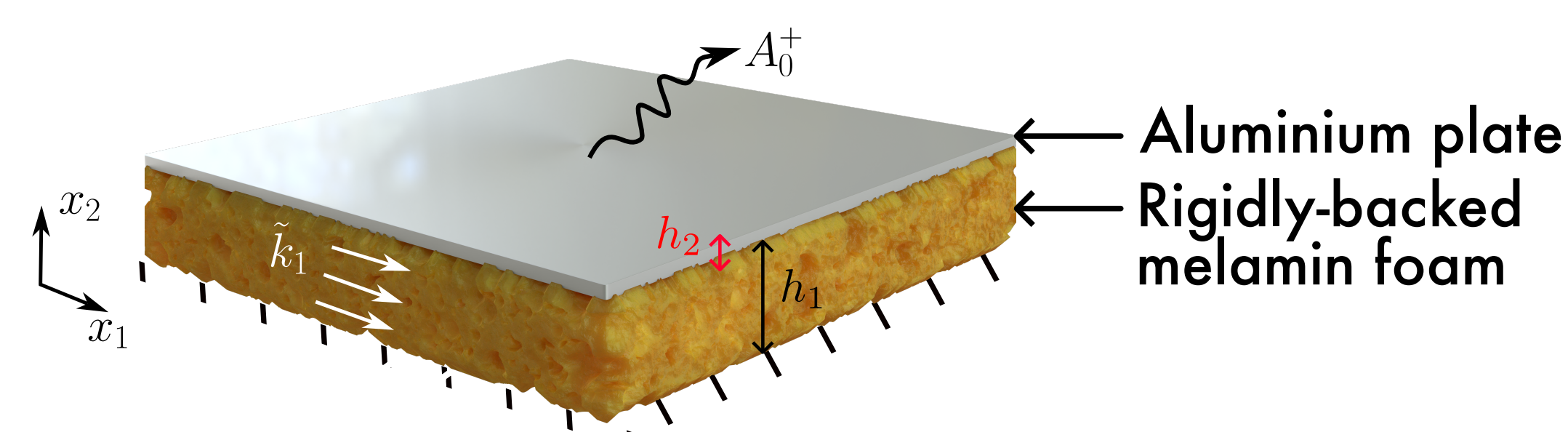
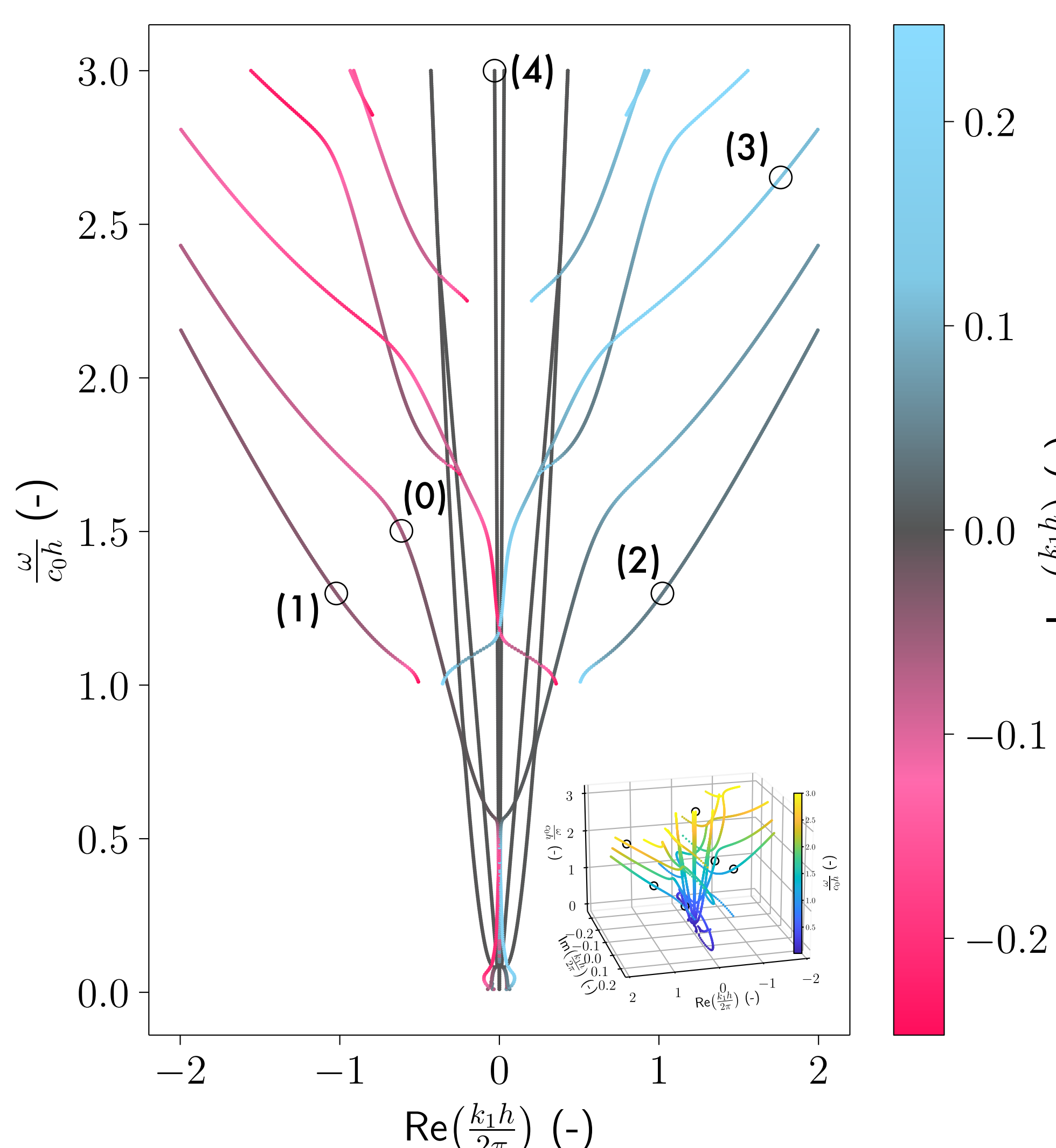
Solution of the eigenvalue problem $(A - \gamma B) S' = 0$

- Linearization: change of variable for k_1 and k_2 to go from a nonlinear problem to a polynomial problem
- Recovery of the physical fields mode shapes at no additional computational cost.

Poroelastic materials (PEM)

- Biphasic material: fluid and solid phases
- Full Biot model accounting for the skeleton motion
- Formulation (u^s, p) used to express the motion equations, involving 3 physical fields in the PEM

Dispersion of waves in a bilayer aluminium-melamin structure



Overview of the SCM

- Numerical computation of dispersion relations for general multilayer structures
- Computationally very efficient and guarantees to yield all solutions of the system
- Further developments: experimental validation and study of similar structures with embedded periodic inclusions

Results

- Aluminium plate: $h_2 = 1 \text{ mm}$, $M_2 = 5$
- Melamine foam: $h_1 = 5 \text{ cm}$, $M_1 = 17$

- Symmetry in the results, with $\tilde{k}_1 = -\tilde{k}_1^*$
- A0 mode with a cutoff frequency from the rigid-backing (0)
- Large attenuation for most branches: fluid-coupling and losses in the material
- Aluminium plate modes not visible in this frequency range: large thickness difference

References

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