



**HAL**  
open science

## Dispersion of elastic and acoustic waves in multilayer structures involving poroelastic materials

Mathieu Maréchal, Jean-Philippe Groby, Olivier Dazel, Vicent Romero-García

► **To cite this version:**

Mathieu Maréchal, Jean-Philippe Groby, Olivier Dazel, Vicent Romero-García. Dispersion of elastic and acoustic waves in multilayer structures involving poroelastic materials. Resonance -Journées Jeunes Chercheurs/Chercheuses en Acoustique Vibrations et Bruit (JJCAB), Jul 2023, Toulouse, France. hal-04178076

**HAL Id: hal-04178076**

**<https://hal.science/hal-04178076v1>**

Submitted on 7 Aug 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

# Dispersion of elastic and acoustic waves in multilayer structures involving poroelastic materials

Mathieu Maréchal

Jean-Philippe Groby, Olivier Dazel & Vicent Romero-García



LAUM Laboratoire d'Acoustique  
Le Mans Université CNRS-UMR 6613



Institut d'Acoustique  
Graduate School  
Le Mans Université

## Numerical computation of dispersion relations

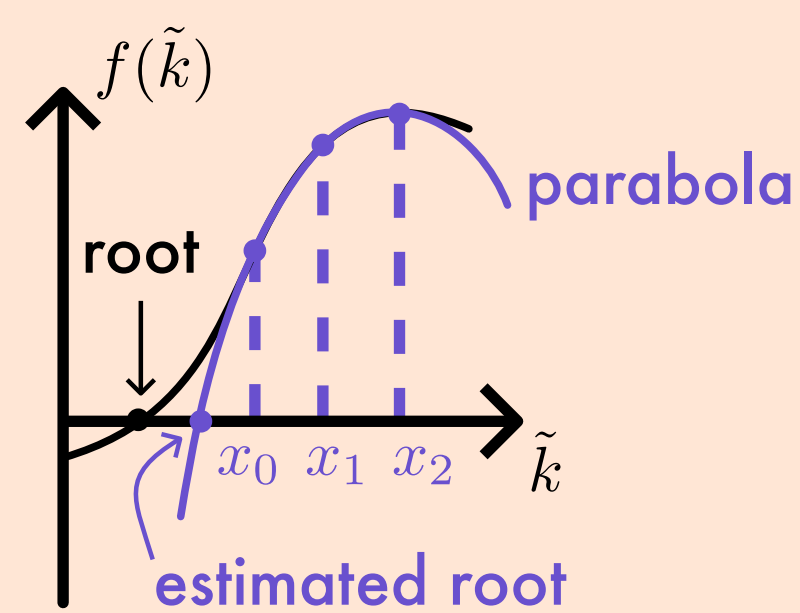
- Interest in the modal behavior of various structures: metaporous surfaces/interfaces and composite multilayer
- Influence of the losses in poroelastic materials (PEM): viscoelastic (from the elastic part), the viscothermal (from the fluid part) and leakage (in the structure and from the structure to the surrounding medium)
- Investigation of these mechanisms using  $(\tilde{k}, \omega)$  dispersion relations: complex wavenumber and real frequency
- Need of robust, versatile and easy to use numerical methods to compute the dispersion relations in these different configurations

## Root-finding with Müller method

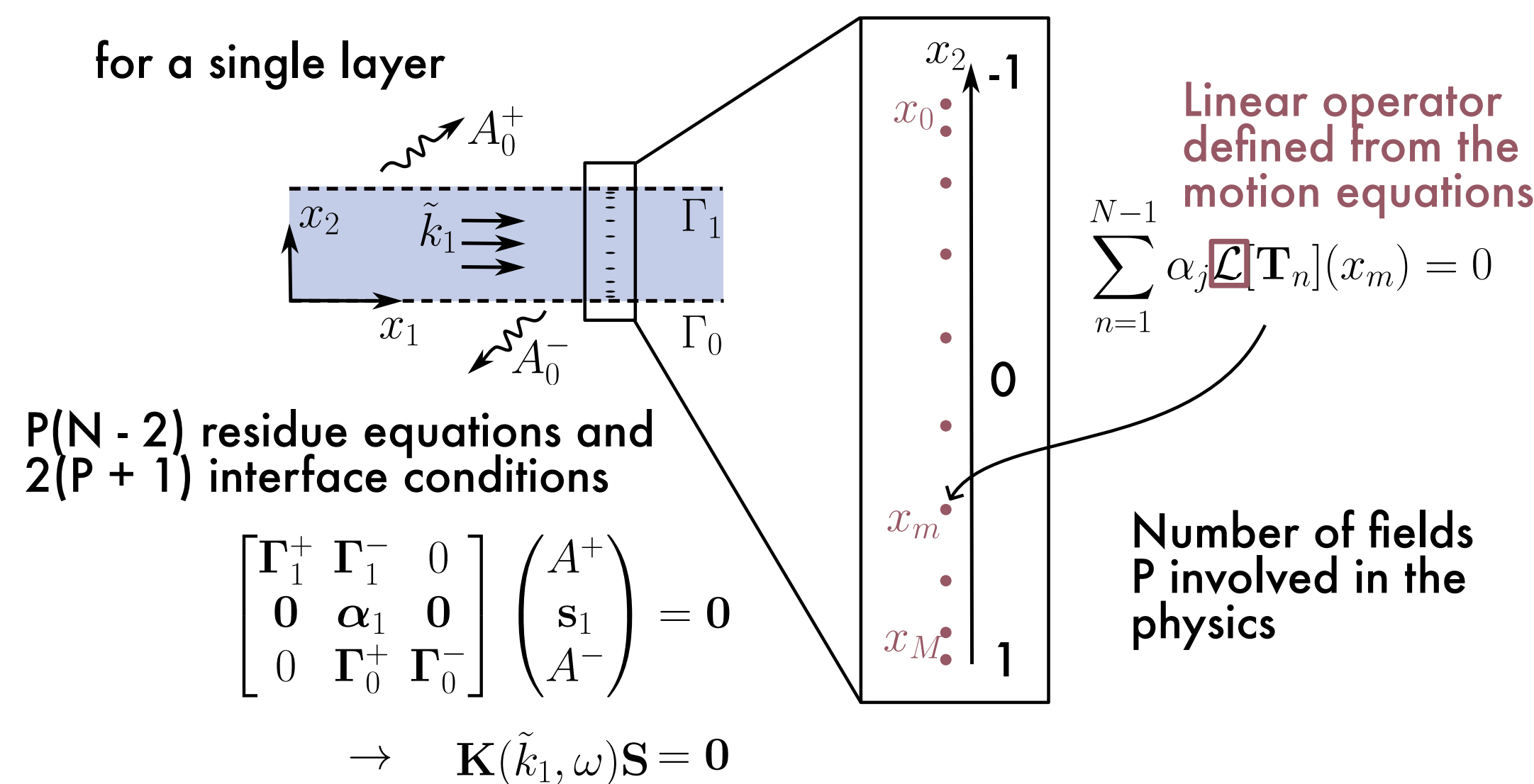
- Secant-based method with quadratic approximation
- Search of roots from an analytical description of the system

$$\det(M(\tilde{k}, \omega)) = 0$$

- Sensitivity to initial values and conditioning of  $M$
- Slow and computationally expensive method
- No certainty of finding all roots at a given  $\omega$



## 1 - Discretization across the layers of the structure



## 3 - Solution of the problem

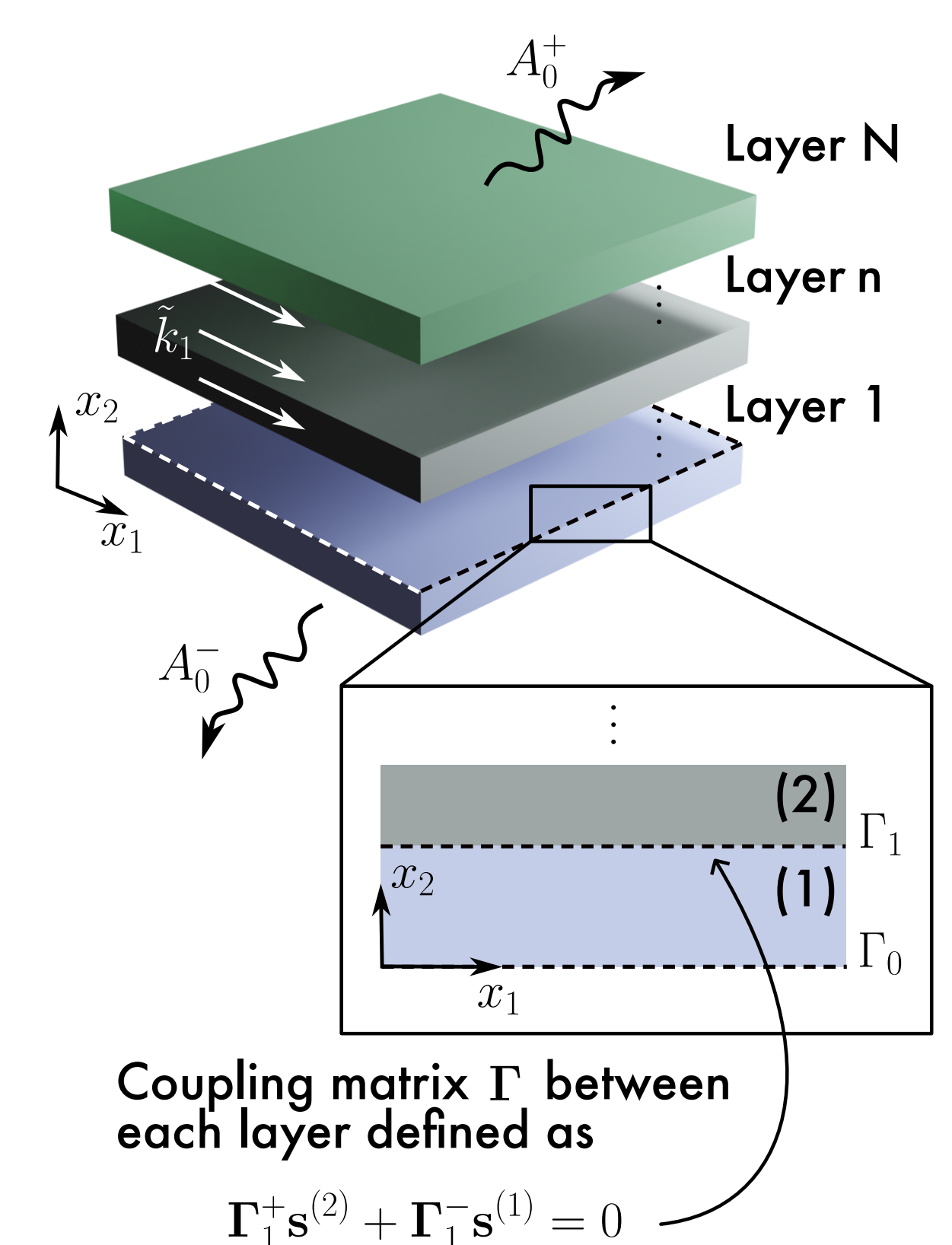
$$(k_1^2 \mathbf{K}_2 + k_1 \mathbf{K}_1 + \mathbf{K}_0 + ik_2^{(f)} \mathbf{K}_1') \mathbf{S} = 0$$

$$\mathbf{S} = (A_{0-}, s_0, \dots, s_n, \dots, s_N, A_{0+})$$

Algebraic manipulations to linearize the problem

eigenvalues: wavenumbers  $\gamma^3 \mathbf{S}$   
eigenvectors: mode shapes  $(\mathbf{A} - \gamma \mathbf{B}) \mathbf{S} = 0$   
 $\begin{pmatrix} \gamma^3 \mathbf{S} \\ \gamma^2 \mathbf{S} \\ \gamma \mathbf{S} \\ \mathbf{S} \end{pmatrix}$

## 2 - Construction of the multilayer numerical scheme



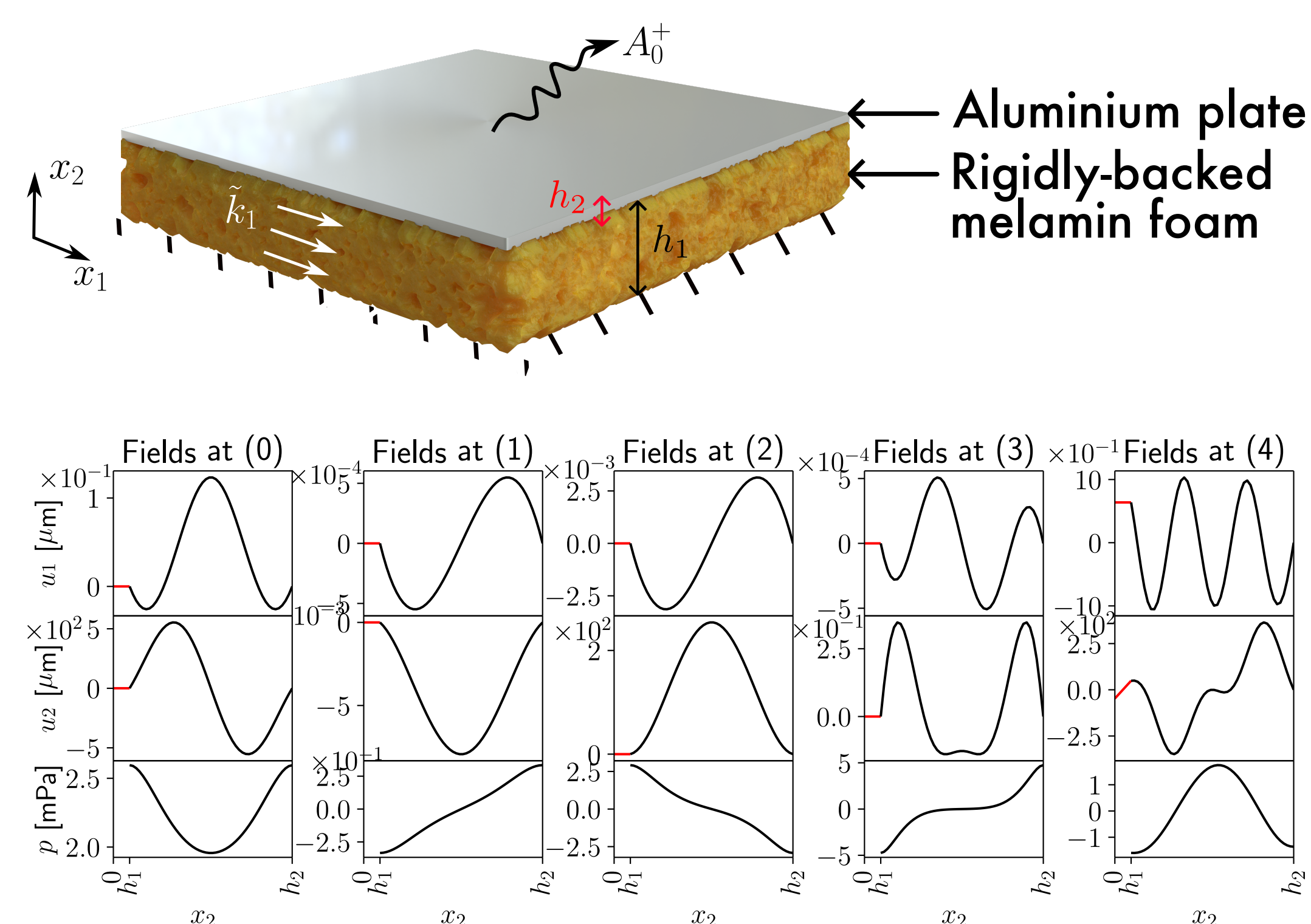
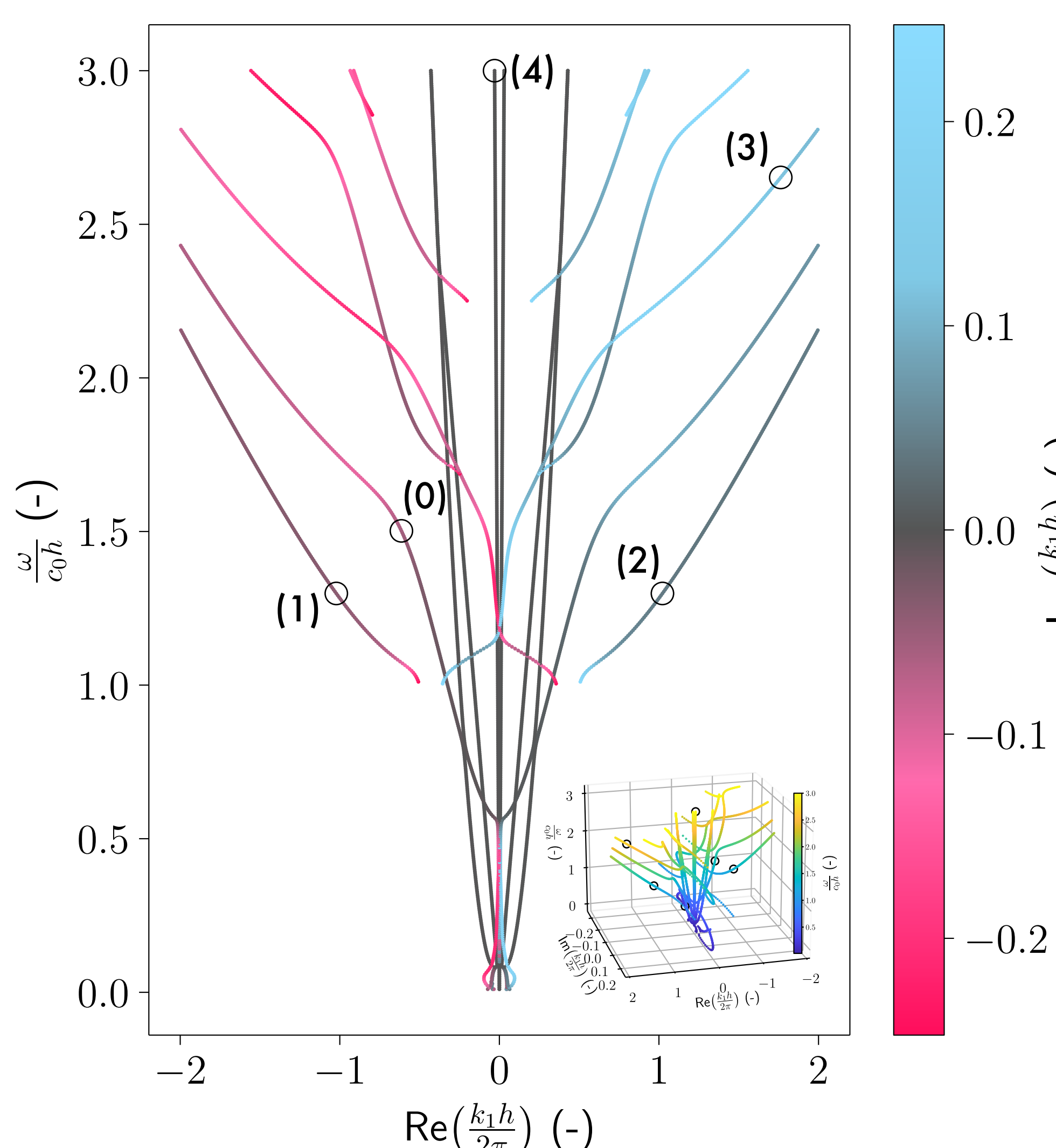
## Solution of the eigenvalue problem $(\mathbf{A} - \gamma \mathbf{B}) \mathbf{S}' = 0$

- Linearization: change of variable for  $k_1$  and  $k_2$  to go from a nonlinear problem to a polynomial problem
- Recovery of the physical fields mode shapes at no additional computational cost.

## Poroelastic materials (PEM)

- Biphase material: fluid and solid phases
- Full Biot model accounting for the skeleton motion
- Formulation  $(u^s, p)$  used to express the motion equations, involving 3 physical fields in the PEM

## Dispersion of waves in a bilayer aluminium-melamin structure



## Results

- Aluminium plate:  $h_2 = 1 \text{ mm}$ ,  $M_2 = 5$
- Melamine foam:  $h_1 = 5 \text{ cm}$ ,  $M_1 = 17$

- Symmetry in the results, with  $\tilde{k}_1 = -\tilde{k}_1^*$
- A0 mode with a cutoff frequency from the rigid-backing (0)
- Large attenuation for most branches: fluid-coupling and losses in the material
- Aluminium plate modes not visible in this frequency range: large thickness difference

## References

- D. Kiefer et al., J. Acoust. Soc. Am., 145, 3341 (2019)
- L. Trefethen, Spectral methods in MATLAB, SIAM (2000)
- N. Atalla et al., J. Acoust. Soc. Am. 104.3 (1998)
- A. Hood, Localizing the eigenvalues of matrix-valued functions, Cornell University (2017)

## Overview of the SCM

- Numerical computation of dispersion relations for general multilayer structures
- Computationally very efficient and guarantees to yield all solutions of the system
- Further developments: experimental validation and study of similar structures with embedded periodic inclusions

Contact:  
mathieu.marechal@univ-lemans.fr

Download the poster!

