



HAL
open science

IMC for Compensating Periodic Disturbance of a Time-Delay System

Can Kutlu Yuksel

► **To cite this version:**

Can Kutlu Yuksel. IMC for Compensating Periodic Disturbance of a Time-Delay System. Process Control 2023, Jun 2023, Štrbské Pleso, Slovakia. hal-04176001

HAL Id: hal-04176001

<https://hal.science/hal-04176001>

Submitted on 2 Aug 2023

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

IMC for Compensating Periodic Disturbance of a Time-Delay System

1. Introduction and Motivation

Despite their known negative impacts on system stability, delays have surprising benefits when it comes to improving the control performance of a close-loop control system.

By treating delays from the very beginning, this poster presents a straightforward controller design to make systems suffering from delays or approximated by simple time-delay models track/reject periodic signals.

For a control system to track or reject a periodic signal expressible by a finite Fourier Series as in

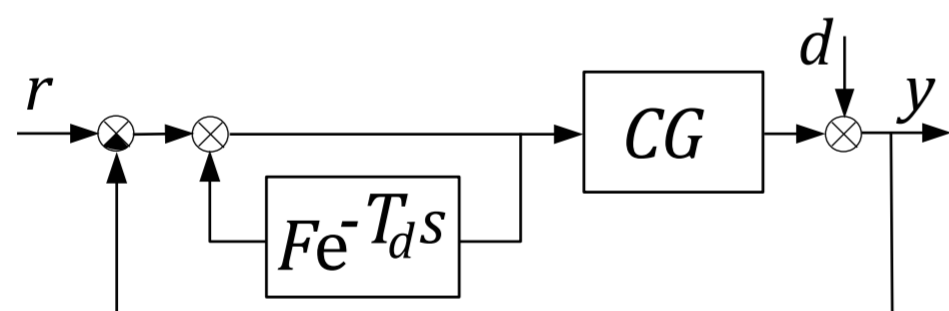
$$v(t) = v_0 + \sum_{i=-M_d}^{M_d} v_i e^{j\Omega_i t}, \quad (1)$$

its close-loop sensitivity $S(s)$ needs to be stable and must have zeros on the imaginary axis at targeted frequencies:

$$S(0) = S(\pm j\Omega_i) = 0 \quad (2)$$

2. Existing Solution: Repetitive Control

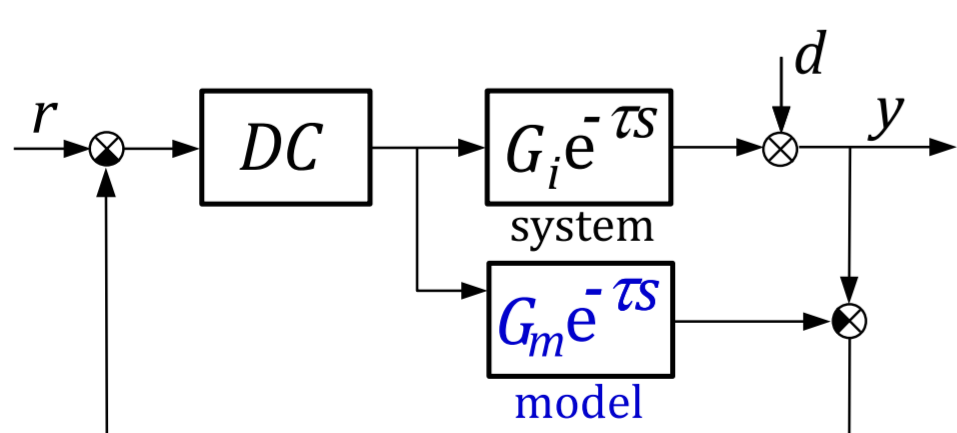
The periodic control is ideally achieved by employing a time delay model of the signal in the control loop. However, the ideal concept is applicable to only biproper systems and, in general, the low-order filter F that is associated with the delay e^{-sT_d} to relax the conditions on stabilization prevents zeros from being at their ideal positions on the imaginary axis.



3. Alternative Solution: Internal Model Control (IMC)

In contrast to Repetitive Control, we propose an approach based on IMC arrangement, which employs a model of the process in the loop instead of the signal model.

The advantage of IMC scheme is that in the nominal case i.e. when the plant and the model perfectly match, a stable IMC controller implies a stable close-loop. In addition to this feature, the close-loop reduces to a finite-dimensional system despite the occurrence of delays.



We consider a plant of the form

$$G(s) = \frac{y(s)}{u(s)} = G_i(s)e^{-s\tau} \quad (3)$$

where τ is the input delay to be compensated and $G_i(s)$ is a proper and invertible transfer function. We assume that $G(s)$ can be approximated by a first-order model,

$$G_m(s) = \frac{K}{Ts + 1} \quad (4)$$

with an input delay $e^{-s\tau_m}$.

We let the IMC controller composed of two components, D and C . The subsystem D contains the controller's delay component. As demonstrated below, it leads to two different design approaches depending on whether the delay is lumped or distributed. The subsystem C is the controller that improves the stabilization and performance of the closed-loop. It has the form

$$C(s) = \frac{1}{G_m(s)}F(s) \quad (5)$$

The low-pass filter $F(s)$ with unity static gain is introduced to ensure the properness of the IMC controller when required.

As a result, the sensitivity transfer function of the scheme in the nominal case becomes

$$S(s) = 1 - D(s)F(s)e^{-s\tau_m}. \quad (6)$$

3.1 Lumped-Delay based Compensator Design [1]

This configuration is mainly suitable for signals with one harmonic component, i.e. $v(t) = v_0 + A_d \cos(\Omega_d t + \phi_d)$. The design is carried out entirely analytically.

The subsystem D corresponds to a tuneable lumped delay $e^{-s\theta}$. A filter F , in the form

$$F(s) = \frac{\alpha T_f s + 1}{T_f s + 1} \frac{\bar{\Omega}^2}{s^2 + 2\xi\bar{\Omega}s + \bar{\Omega}^2} \quad (7)$$

is analytically set to get $|F(j\Omega_d)| = 1$ with $\|F(s)\|_\infty \approx 1$. Once the filter parameters are determined, the associated delay θ is evaluated by

$$\theta = \frac{2l\pi + \arg F(j\Omega_d)}{\Omega_d} - \tau, \quad (8)$$

Based on this approach, a configuration aiming at multi-harmonic signals do exist.

3.2 Distributed-Delay based Compensator Design [2]

This configuration is suitable for multi-harmonic signals.

The subsystem D corresponds to a distributed delay with length T_D and form

$$D(s) = \frac{1}{s} \sum_{k=0}^N a_k e^{-sk\vartheta} \quad (9)$$

where $\vartheta = \frac{T_D}{N}$ and $N + 1$ is the number of coefficients a_k to be tuned. In this case, filter $F(s)$ is only there to ensure the properness of the controller.

The distributed delay is shaped by optimizing the weighted sensitivity through gains a_k subjected to a linear constraint that ensures (2):

$$\begin{aligned} \min_x & \| (1 - D(j\omega)F(j\omega)e^{-j\omega\tau_m})W(j\omega) \|_\infty \\ \text{s.t.} & Ax = B \\ & A_n x \leq B_n \end{aligned} \quad (10)$$

where the weighting function is

$$W(s) = \frac{\frac{1}{b_u}s + \omega_b}{s + b_l\omega_b} \quad (11)$$

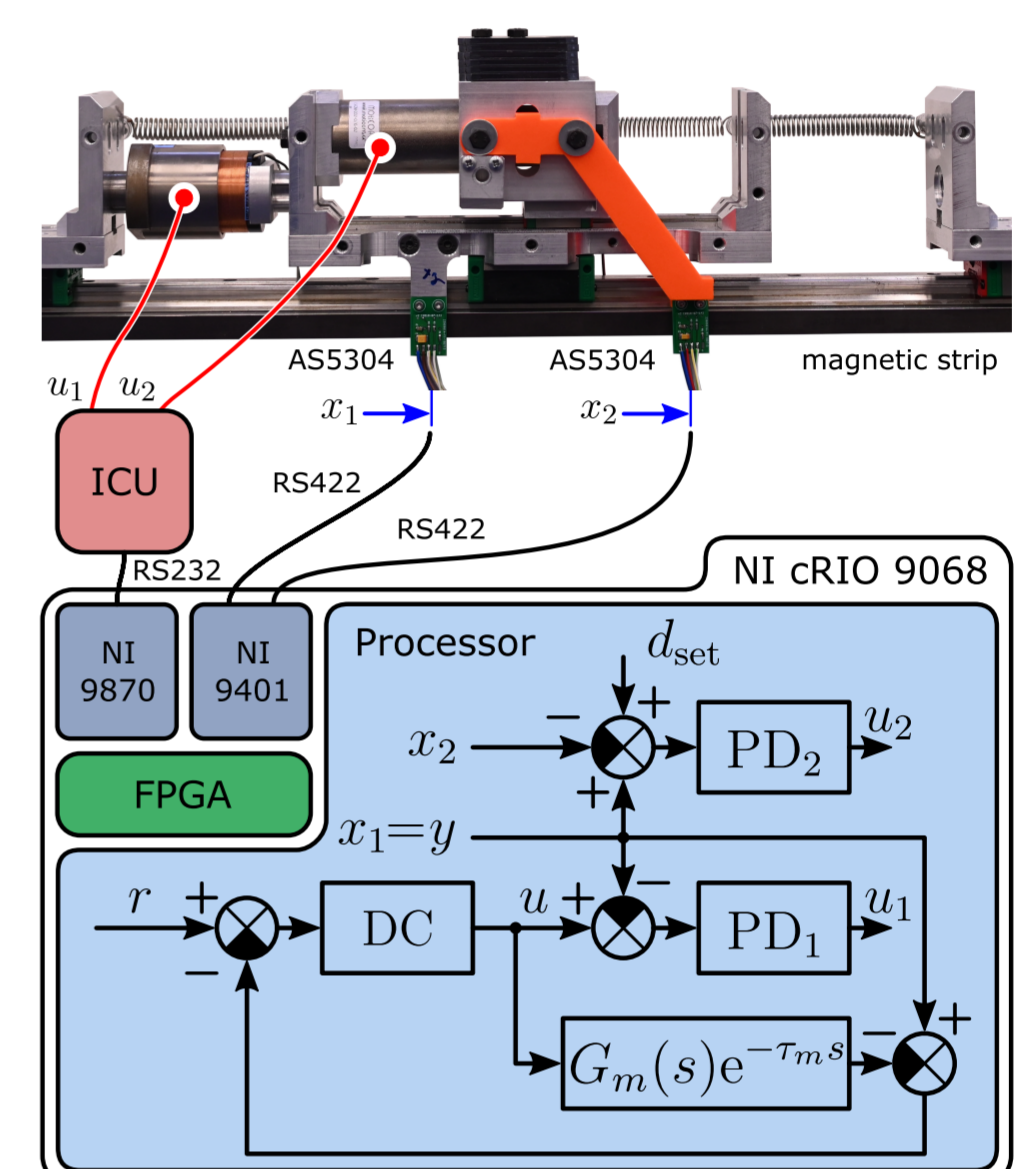
and $x = [a_0, a_1, \dots, a_N]^T$,

$$A = \begin{bmatrix} 1 & \cos(\Omega_1\vartheta) & \dots & \cos(\Omega_1 N\vartheta) \\ \vdots & \vdots & \dots & \vdots \\ 1 & \cos(\Omega_{M_d}\vartheta) & \dots & \cos(\Omega_{M_d} N\vartheta) \\ 0 & \sin(\Omega_1\vartheta) & \dots & \sin(\Omega_1 N\vartheta) \\ \vdots & \vdots & \dots & \vdots \\ 0 & \sin(\Omega_{M_d}\vartheta) & \dots & \sin(\Omega_{M_d} N\vartheta) \\ 1 & 1 & \dots & 1 \\ 0 & \vartheta & \dots & N\vartheta \end{bmatrix}, B = \begin{bmatrix} R_1 \\ \vdots \\ R_{M_d} \\ I_1 \\ \vdots \\ I_{M_d} \\ 0 \\ -1 \end{bmatrix}$$

By setting $N + 1 > M_d$, the linear constraint attains infinitely many feasible solutions.

4. Experimentation

The proposed methods are experimentally validated on a serially connected two mass-spring-damper system. The original 4th-order system with input delay is made approximable by a 1st-order time delay model using an inner PD control loop. Later on, the IMC controller forms the outer control loop to make position x_1 unaffected by the periodic disturbance given through input u_2 .



Scan the QR codes below to access the video records of the experiments based on (a) lumped-delay and (b) distributed-delay-based compensators:



(a)



(b)

5. References

- [1] Yuksel, C. K., Vyhliđal, T., Bušek, J., Anderle, M., Niculescu, S. I. (2023). Harmonic Disturbance Compensation of a System With Long Dead-Time, Design and Experimental Validation. IEEE/ASME Transactions on Mechatronics.
- [2] Yuksel, C. K., Bušek, J., Anderle, M., Vyhliđal, T., Niculescu, S. I. (2023). A distributed delay based controller for simultaneous periodic disturbance rejection and input-delay compensation. Mechanical Systems and Signal Processing, 197, 110364.

6. Acknowledgement

This work was supported by the Czech Science Foundation project No. 21-07321S - *Persistent problems of repetitive control*. The work of the first and last authors was also supported by a public grant overseen by the French National research Agency (ANR) as part of the Investissements d'Avenir program, through the "ADI 2020" project funded by the IDEX Paris-Saclay, ANR-11-IDEX-0003-02. The first author was also supported by the Student Grant Agency of the Czech Technical University in Prague, student grant No. SGS23/157/OHK2/3T/12.