



Unsupervised regularized
image reconstruction
for Tomographic Diffractive Microscopy.

*Reconstruction d'image
régularisée non supervisée
en microscopie tomographique diffractive*

L. Denneulin¹, F. Momey¹, D. Brault¹, M. Debailleul², A. M. Taddese², N. Verrier², O. Haeberlé²



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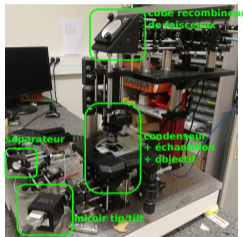
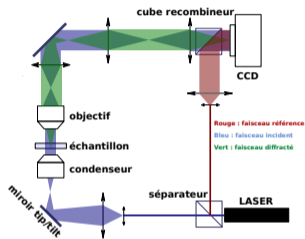
ANR HORUS (ANR-18-CE45-0010) : *High Optical Resolution for Unlabeled Samples*

Thursday December 1st, 2022

The HORUS project

Context

Observation of microscopic unlabeled samples (HIV) in Tomographic Diffractive Microscopy.



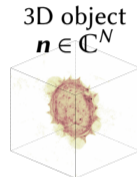
Scheme and image of the microscope.

Objectives

To obtain a precise 3D map of optical indices.

$$\text{Data } (d_\ell)_{\ell \in \{1, \dots, L\}} \in \mathbb{C}^M$$

Reconstruction \rightarrow



Example on an Jerusalem artichoke pollen
(Courtesy of IRIMAS)

Laurence's contribution :

Unsupervised regularized inverse approach.


L. Denneulin, F. Momey, D. Brault, M. Debailleul, A. M. Taddese, N. Verrier, & O. Haeberlé (2022). GSURE criterion for unsupervised regularized reconstruction in tomographic diffractive microscopy. JOSA A, 39(2), A52-A61.

Inverse approach principle


- ▶ Noisy data:

$$\mathbf{d}_\ell = \mathbf{d}_\ell^* + \eta_\ell$$

where η_ℓ is a noise vector (photon, read out, ...)



UTOPIA


$$\mathbf{d}_\ell = \mathbf{d}_\ell^* = u_\ell^*(\mathbf{n}) \Rightarrow \mathbf{n} = u_\ell^{*-1}(\mathbf{d}_\ell^*)$$

Direct or iterative inversion

Problematics :

- ▶ True u_ℓ^* unknown \Rightarrow image formation model $u_\ell(\mathbf{n})$
(**Beam Propagation Method (BPM)** [1], first Born/Rytov Approximation [2], ...)
 - ▶ Formation model are not invertible.
- \Rightarrow Inversion of $u_\ell(\mathbf{n})$ alone can propagate noise and artifacts.

This motivates the use of regularizations

Reconstruction with a regularized inverse approach

- Reconstruction by estimation of the index difference \mathbf{n} between the object and the medium:

$$\widehat{\mathbf{n}} \in \arg \min_{\mathbf{n} \in \mathcal{C}} \left\{ \underbrace{\sum_{\ell=1}^L \frac{1}{2} \|\mathbf{d}_\ell - u_\ell(\mathbf{n})\|_{\mathbb{C}}^2}_{\text{Data fidelity}} + \underbrace{f_\rho(\mathbf{n})}_{\text{Regularization(s)}} \right\}.$$

where \mathcal{C} is a sign or value separable constraint and f_ρ is composed of one or several regularization of hyperparameters ρ . Here:

- Edge preserving smoothing [3] + ℓ_1 :

$$f_\rho(\mathbf{n}) = \lambda^{\text{EP}} \sum_{k=1}^N \left(\sqrt{\|\mathbf{D}_k \mathbf{n}\|^2 + \mu^2} - \mu \right) + \lambda^{\ell_1} \sum_{k=1}^N \|\mathbf{n}_k\|_{\ell_1}$$

with $\rho = (\lambda^{\text{EP}}, \mu, \lambda^{\ell_1})$.

⇒ Resolution with VMLM-B [6].

- Total Variation [TV: 4]:

$$f_\rho(\mathbf{n}) = \lambda^{\text{TV}} \sum_{k=1}^N \|\mathbf{D}_k \mathbf{n}\|_{\ell_{1,2}}$$

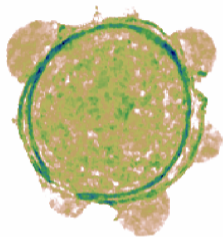
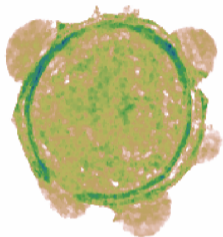
with $\rho = (\lambda^{\text{TV}})$

⇒ Resolution with Primal-Dual Condat-Vũ [7, 8].

Reconstruction with a regularized inverse approach

► Record

Example with TV



Under-regularized

← λ^{TV} →

Over-regularized

where
hyper
o Edge

on of

$$f_{\rho}(\mathbf{n}) = \lambda^{\text{EP}} \sum_{k=1}^N \left(\sqrt{\|\mathbf{D}_k \mathbf{n}\|^2 + \mu^2} - \mu \right) + \lambda^{\ell_1} \sum_{k=1}^N \|\mathbf{n}_k\|_{\ell_1}$$

$$f_{\rho}(\mathbf{n}) = \lambda^{\text{TV}} \sum_{k=1}^N \|\mathbf{D}_k \mathbf{n}\|_{\ell_{1,2}}$$

with $\rho = (\lambda^{\text{EP}}, \mu, \lambda^{\ell_1})$.

⇒ Resolution with VMLM-B [6].

with $\rho = (\lambda^{\text{TV}})$

⇒ Resolution with Primal-Dual Condat-Vũ [7, 8].

The GSURE criterion

The choice of ρ is critical to achieve a “good” reconstruction. Let $\widehat{\mathbf{n}}_\rho$ be the reconstruction obtained for a fixed ρ .

Quality criterions

estimation Mean Square Error (eMSE):

$$\text{eMSE}(\widehat{\mathbf{n}}_\rho) = \mathbb{E} \left[\|\mathbf{n}^* - \widehat{\mathbf{n}}_\rho\|_{\mathbb{C}}^2 \right]$$

prediction Mean Square Error (pMSE):

$$\text{pMSE}(\widehat{\mathbf{n}}_\rho) = \sum_{\ell=1}^L \mathbb{E} \left[\frac{1}{\sigma^2} \|\mathbf{d}_\ell^* - u_\ell(\widehat{\mathbf{n}}_\rho)\|_{\mathbb{C}}^2 \right]$$

!! \mathbf{n}^* and \mathbf{d}_\ell^* are unknown in practice !!

Generalized Stein Unbiased Risk Estimator [GSURE: 10, 11, 12]

$$\text{GSURE}(\widehat{\mathbf{n}}_\rho) = \sum_{\ell=1}^L \left[\frac{1}{\sigma^2} \|\mathbf{d}_\ell - u_\ell(\widehat{\mathbf{n}}_\rho)\|_{\mathbb{C}}^2 + 2\text{tr} \left(\frac{\partial u_\ell(\widehat{\mathbf{n}}_\rho)}{\partial \mathbf{d}_\ell} \right) - M \right].$$

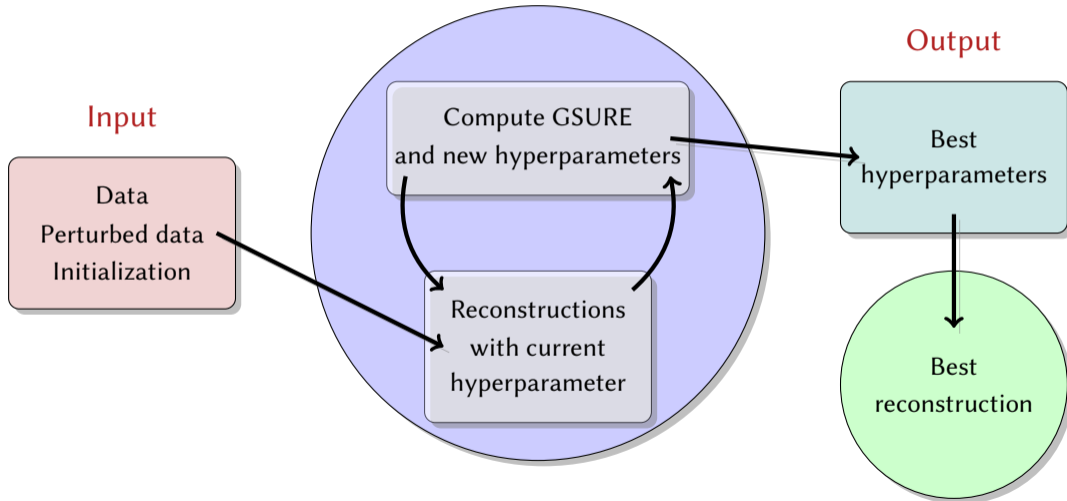

$$\mathbb{E}[\text{GSURE}(\widehat{\mathbf{n}}_\rho)] = \text{pMSE}(\widehat{\mathbf{n}}_\rho)$$

GSURE is an estimate of the pMSE and \mathbf{d}_ℓ is known $\forall \ell \in \{1, \dots, L\}$.

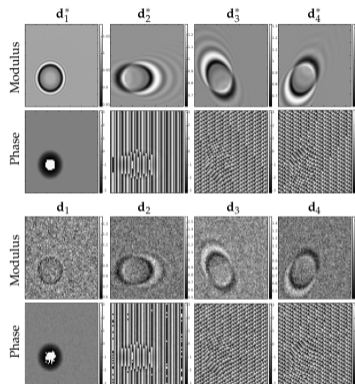
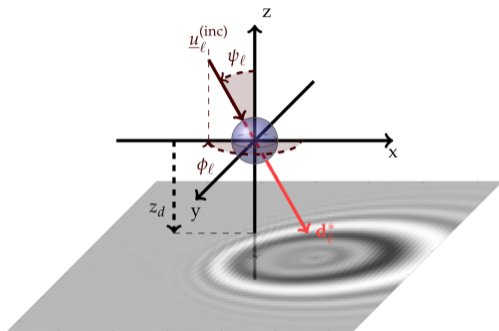
Idea: find ρ for which the reconstruction $\widehat{\mathbf{n}}_\rho$ is minimizing $\text{GSURE}(\widehat{\mathbf{n}}_\rho)$ with a Powell method [9].

Algorithmic scheme

GSURE minimization



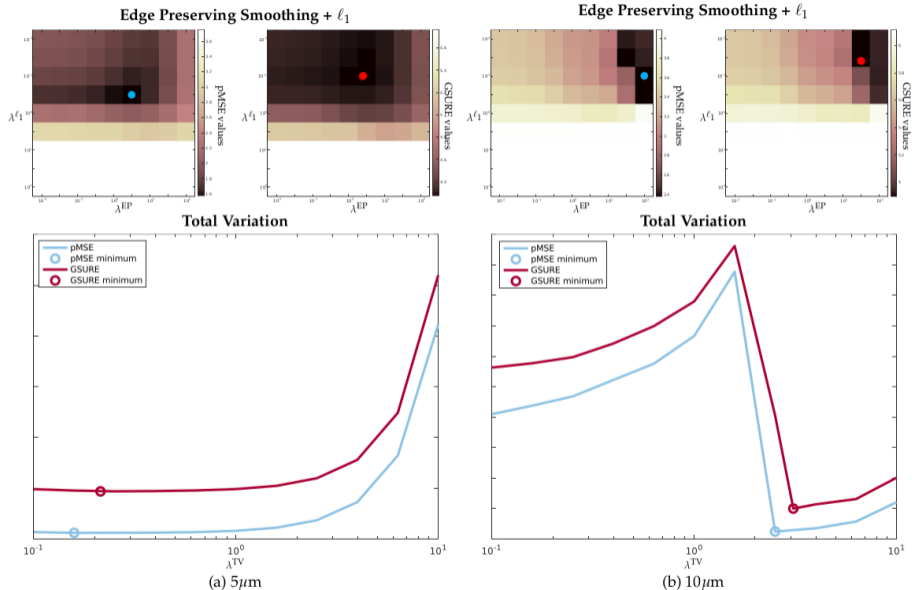
Validation on beads simulations (with the Mie scattering model)



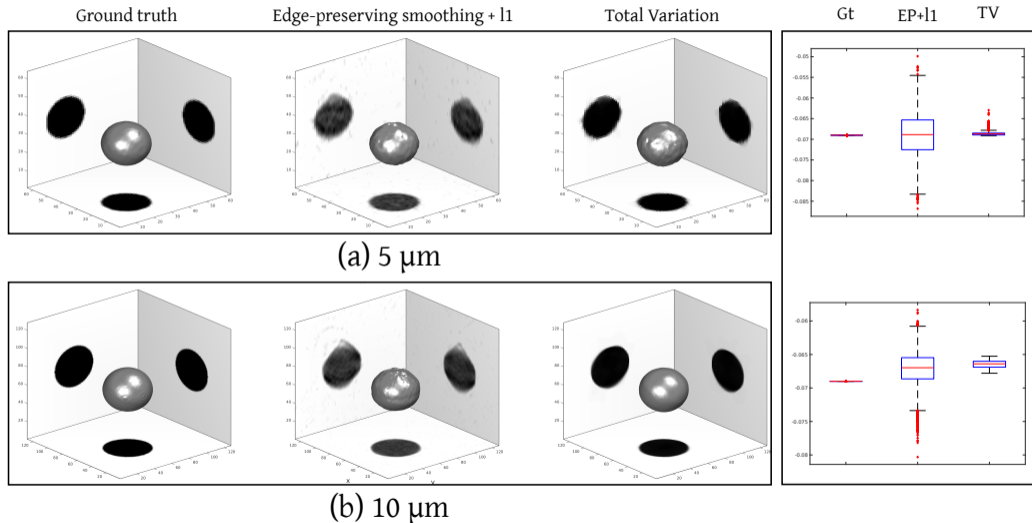
(a) $\Delta = 5\mu\text{m}$

case	Δ	$(x_{\text{bead}}, y_{\text{bead}}, z_{\text{bead}})$	ψ_L	L	M	N
(a)	$5\mu\text{m}$	$(-2.3467\mu\text{m}, -2.3467\mu\text{m}, 0\mu)$	45°	4	64×64	$64 \times 64 \times 64$
(b)	$10\mu\text{m}$	$(-2.3467\mu\text{m}, -2.3467\mu\text{m}, 0\mu)$	45°	4	128×128	$128 \times 128 \times 128$

Validation on beads simulations (with the Mie scattering model)

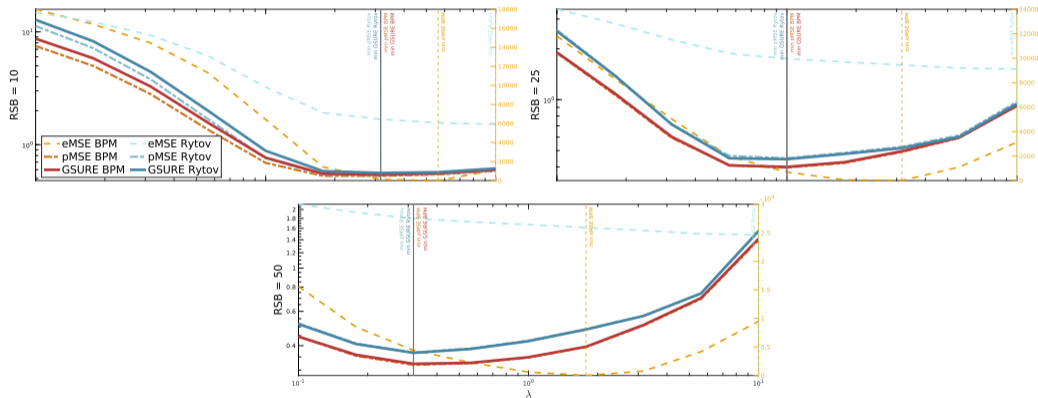


Validation on beads simulations (with the Mie scattering model)



Validation on beads simulations (with the Mie scattering model) : influence of the image formation model

► [Denneulin et al., GRETSI 2022] We studied the influence of a wrong estimation of σ ...



► Depending on the SNR, the GSURE and pSME criterions does not seem to be influenced by the image formation model but departs from the eMSE measure.

GSURE computation issues

- ▶ Let's get back to the GSURE formula :

$$\text{GSURE}(\widehat{\mathbf{n}}_\rho) = \sum_{\ell=1}^L \left[\frac{1}{\sigma^2} \left\| \mathbf{d}_\ell - u_\ell(\widehat{\mathbf{n}}_\rho) \right\|_{\mathbb{C}}^2 + 2\text{tr} \left(\frac{\partial u_\ell(\widehat{\mathbf{n}}_\rho)}{\partial \mathbf{d}_\ell} \right) - M \right].$$

- ▶ The computation of the « trace term » is performed this way :

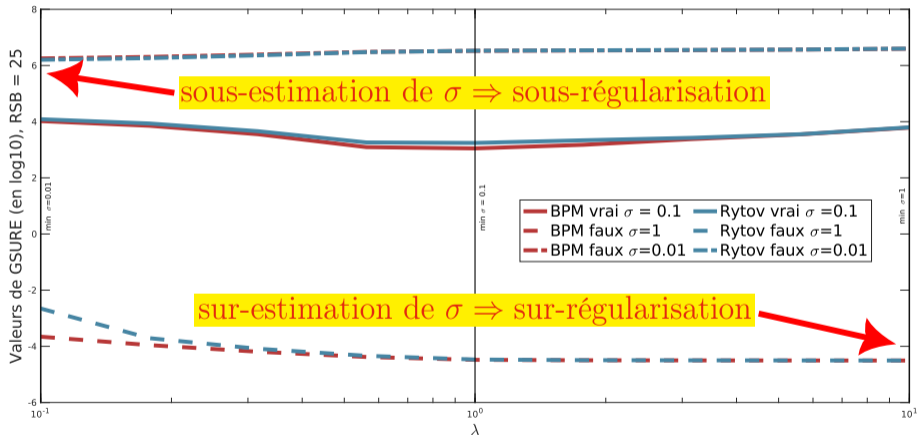
$$\text{tr} \left(\frac{\partial u_\ell(\widehat{\mathbf{n}}_\rho)}{\partial \mathbf{d}_\ell} \right) = \frac{\langle \mathbf{d}_\ell^\delta - \mathbf{d}_\ell, u_\ell(\widehat{\mathbf{n}}_\rho^\delta) - u_\ell(\widehat{\mathbf{n}}_\rho) \rangle}{\varepsilon^2}.$$

where \mathbf{d}_ℓ^δ is a perturbed dataset with an i.i.d Gaussian noise $\mathcal{N}(0, \varepsilon)$ and ε is chosen as 0.1σ .

- ▶ The computation of GSURE requires 2 reconstructions from \mathbf{d}_ℓ and \mathbf{d}_ℓ^δ , i.e. **2 reconstructions per iteration** of the unsupervised process.
- ▶ An **estimation** of the **data noise standard deviation σ** is required.

GSURE computation issues : influence of σ

- [Denneulin et al., GRETSI 2022] We compared the use of Rytov and BPM models on GSURE reconstructions for $5\mu\text{m}$ bead case (a) ...



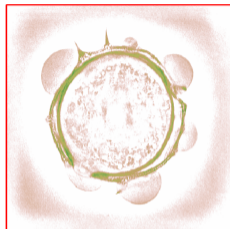
- **The estimation of σ is critical.**

Results on an artichoke pollen

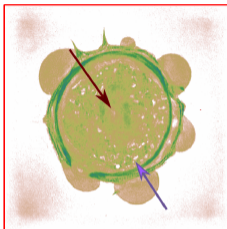
GP (L=600)	GP (L=20)	TV (L=20)

Results on an artichoke pollen

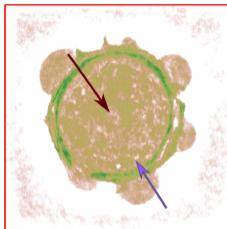
Rytov (L=600)



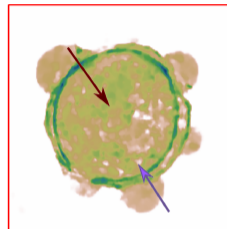
GP (L=600)



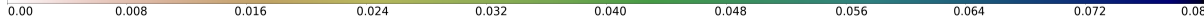
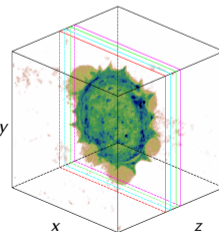
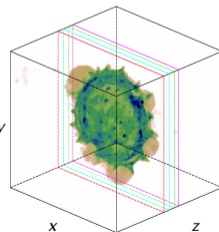
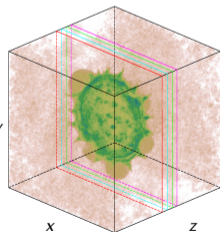
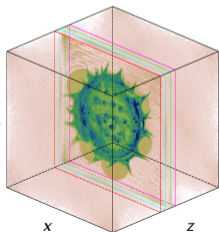
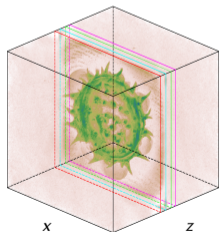
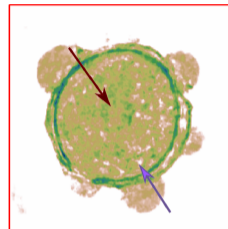
GP (L=20)



EP+ ℓ_1 (L=20)

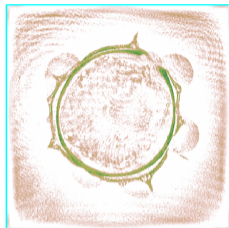


TV (L=20)

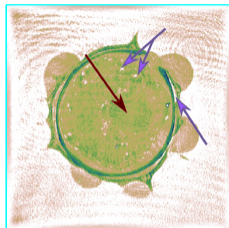


Results on an artichoke pollen

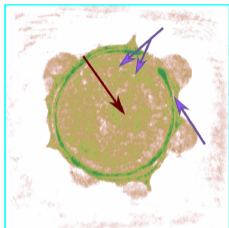
Rytov (L=600)



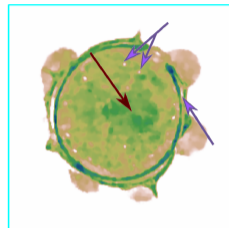
GP (L=600)



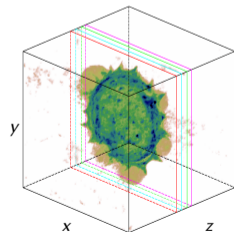
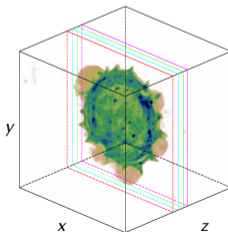
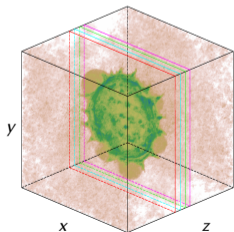
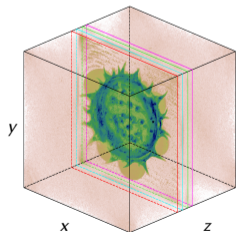
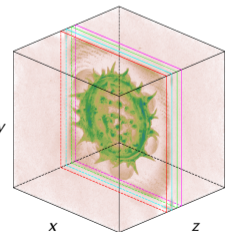
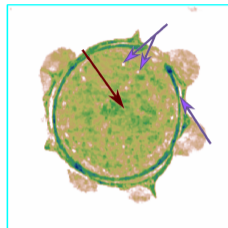
GP (L=20)



EP+l1 (L=20)

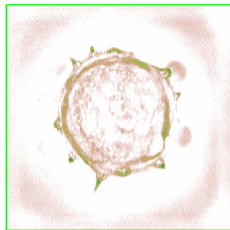


TV (L=20)

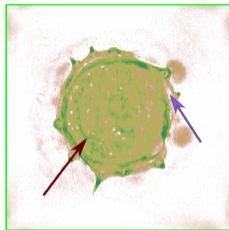


Results on an artichoke pollen

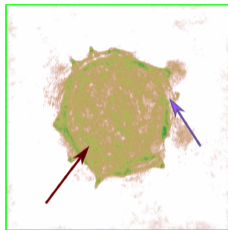
Rytov (L=600)



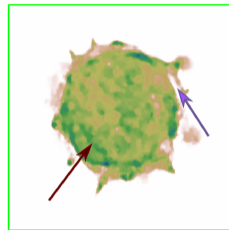
GP (L=600)



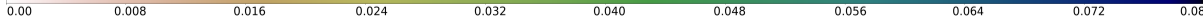
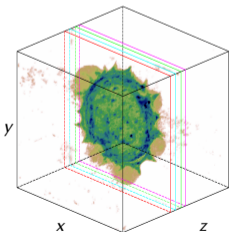
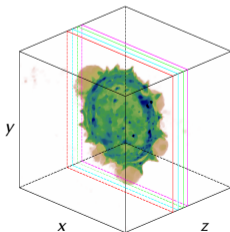
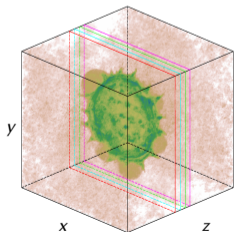
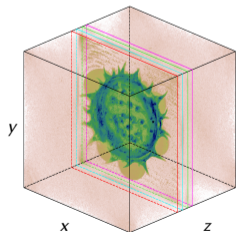
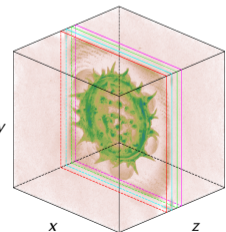
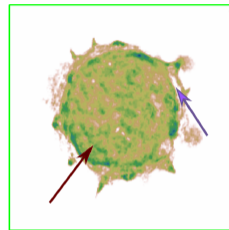
GP (L=20)



EP+ ℓ_1 (L=20)

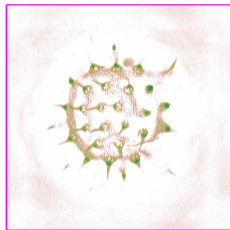


TV (L=20)

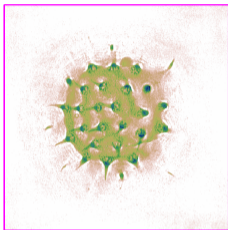


Results on an artichoke pollen

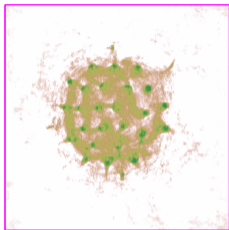
Rytov (L=600)



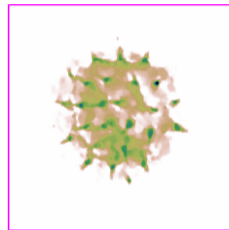
GP (L=600)



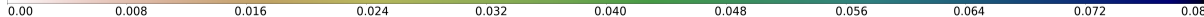
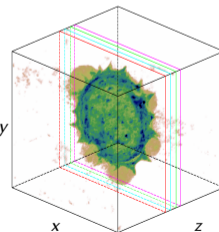
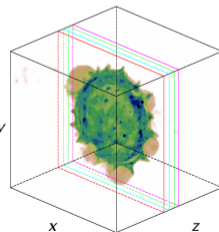
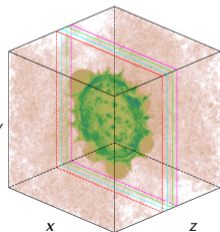
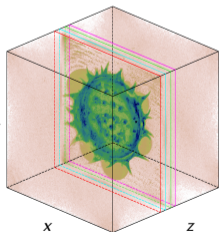
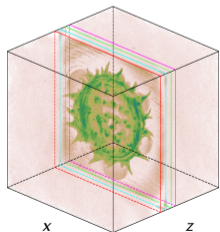
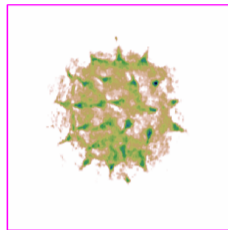
GP (L=20)



EP+l1 (L=20)



TV (L=20)



Results on an artichoke pollen

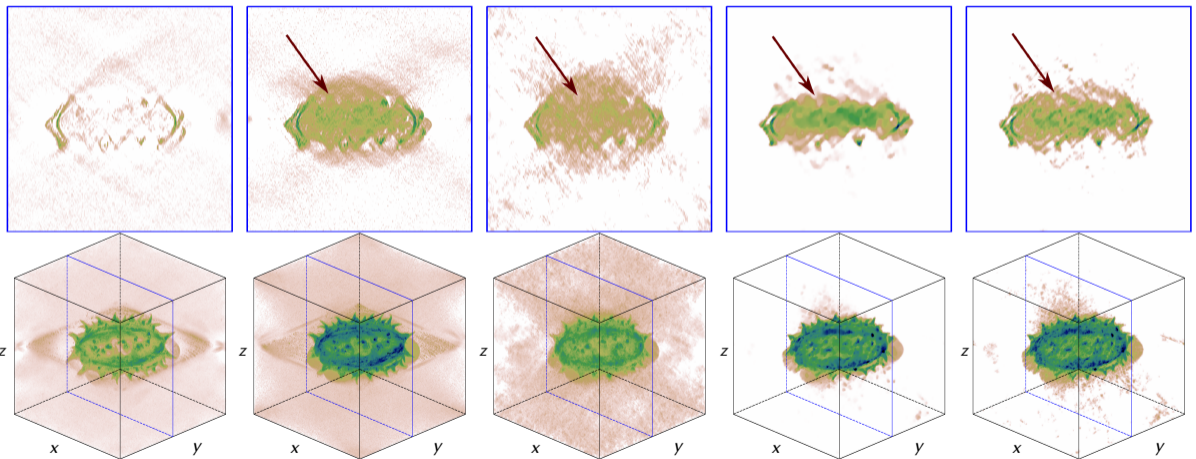
Rytov (L=600)

GP (L=600)

GP (L=20)

EP+ ℓ_1 (L=20)

TV (L=20)



Conclusion

- ▶ We proposed an unsupervised regularized inverse approach using GSURE.
- ▶ GSURE accurately estimates the pMSE in case of complex TDM data.
- ▶ Moreover, it allows a satisfying reconstruction of the object without the knowledge of the ground truth.
- ▶ GSURE is a promising tool to develop unsupervised reconstructions method but require an accurate model of the data formation model. However it is quite time consuming.

However ...

- ▶ Time consuming \Rightarrow **2 reconstructions per iteration of the unsupervised GSURE minimization process.**
- ▶ Hardly dependent on the **knowledge of the noise** on the data \Rightarrow **requires a pre-estimation of σ .**

Perspectives

- ▶ Find a way to avoid the « 2 reconstructions » step for GSURE minimization, or study « smarter » reconstruction strategies to limit the computation time.
- ▶ Apply GSURE on an end-to-end method using data without pre-processing ⇒ could help to get a better (more predictable?) estimate of the noise.

Références

- [1] U. S. Kamilov, I. N. Papadopoulos, M. H. Shoreh, A. Goy, C. Vonesch, M. Unser, and D. Psaltis : Learning approach to optical tomography. *Optica*, 2015.
- [2] A. Berdeu, F. Momey, B. Laperrousaz, T. Bordy, X. Gidrol, J.-M. Dinten, N. Picollet-D'hahan, and C. Allier : Comparative study of fully three-dimensional reconstruction algorithms for lens-free microscopy. *Applied Optics*, 2017.
- [3] P. Charbonnier, L. Blanc-Féraud, G. Aubert, and M. Barlaud : Deterministic edge-preserving regularization in computed imaging. *IEEE TIP*, 1997.
- [4] L. I. Rudin, S. Osher, and E. Fatemi : Nonlinear total variation based noise removal algorithms. *Physica D : Nonlinear Phenomena*, 1992.
- [5] F. Slimani, G. Grehan, G. Gouesbet, and D. Allano : Near-field Lorenz-Mie theory and its application to microholography. *Appl. Opt.*, 1984.
- [6] É. Thiébaud : Optimization issues in blind deconvolution algorithms, *Astronomical Data Analysis II*, 2002.
- [7] L. Condat : A Primal–Dual Splitting Method for Convex Optimization Involving Lipschitzian, Proximal and Linear Composite Terms, *Journal of Optimization Theory and Applications*, 2013
- [8] B. C. Vũ : A Splitting Algorithm for Coupled System of Primal–Dual Monotone Inclusions, *Journal of Optimization Theory and Applications*, 2015
- [9] M. J.D. Powell : The NEWUOA software for unconstrained optimization without derivatives, *Large-scale nonlinear optimization*, 2006.
- [10] C. M. Stein : Estimation of the mean of a multivariate normal distribution. *The annals of Statistics*, 1981.
- [11] Y. C. Eldar : Generalized SURE for exponential families : Applications to regularization. *IEEE TSP*, 2008.
- [12] S. Ramani, T. Blu, and M. Unser : Monte-Carlo Sure : A Black-Box Optimization of Regularization Parameters for General Denoising Algorithms. *IEEE TIP*, 2008.