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MODELS AND ANALOGY: HOMOLOGIES, REPRESENTATIONS, EMBODIMENTS AND REPRODUCTIONS¹

Juan Ramon ÁLVAREZ

Models are of central importance in many scientific contexts. The centrality of models such as the billiard ball model of a gas, the Bohr atom model, the MIT bag model of the nucleon, the MIT Gaussian chain model of a polymer, the Lorenz atmosphere model, the Gaussian Lorenz atmosphere model, the Lotka-Volterra predator-prey interaction model, the DNA double helix model, agent and evolutionary models in the social sciences and general equilibrium models of markets are, in their respective domains, in their respective domains, examples of this. Scientists spend a great deal of time building, testing, comparing and revising models, and much space is devoted in journals to introducing, applying and interpreting these valuable tools. In short, models are one of the main tools of modern science.

(Frigg and Hartmann, *Models in Science*)

ABSTRACT: The term 'model' is variously used and therefore it has many meanings according to the fields where it is applied. In the present field -the theory of the scientific method construed as the theory of the subordination of the symbolic, technical, and social actions performed in the frame of the levels of resolution associated with scientific theories- models are constructs whose foundation lies in the existence of an analogy of proportion that obtains between the model and the object represented thereby. The concept of a model and its definition are supplemented in the following, through the development of the very relation of representation (a sign-object relation), with the four basic types of models: structural homologies, satisfactory representations, efficacious embodiments, and adequate reproductions.

Keywords: Analogy, expression, homology, embodiment, interpretation, model, representation, reproduction.

1. Introduction

The production of models is, together with classification, definition and inference (the *modi sciendi*), part of the basic set of theory-building procedures in the sciences. Alongside these are the procedures that satisfy the functions attributed to theories, such as describing, interpreting, understanding, establishing laws, explaining, etc., and the procedures of contrastation, such as observation, experimentation, and simulation (Cf. Álvarez, 2011).

Indeed, the content of the general methodology is constituted, as opposed to the specific techniques of the special methodologies and the more general ideas of the

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theory of scientific rationality, by these procedures. In the face of the threat, which was already warned in his day by Fernández Buey (1991), of the growing divergence in the field of methodology between scientists increasingly dedicated to the learning of highly specialized techniques and philosophers of science dedicated above all to the major conceptual frameworks of rationality, I have repeatedly insisted that there is a territory proper to the methodology of science, (Alvarez, 1988, 1999, 2011), both scientific (there is no unreflective science) and philosophical, as a theory of standardized procedures, i.e., of the reflective consideration of the subordination to norms of the symbolic, technical and social actions in scientific activity, within the framework of the levels of resolution associated with theories that link procedures to the principles of ontology semantically assumed in each science.

Principles and procedures refer correlatively to the ontology and methodology of science and, within the latter, the procedures for the construction of theories classification, definition, modeling, and inference- are established as four possible ways of linking the terms and relations of a "syntax" through the available operations.

We are indebted to Bueno (1995) for this form of presentation, which is based on a classification of the functors by Curry and which led, in the first versions of his theory of science -the theory of categorial closure- to an extensive development of the four procedures mentioned, understood as internal methods of the sciences (cf. Bueno 1987).

[T]he traditional scientific methodology identified, as early as Aristotle and the Schoolmen. three general methods which it called, respectively, definitions, methods, which were respectively called classifications/divisions, definitions, demonstrations. The gnoseology of categorial closure takes up these traditional *modi sciendi*, [38] but justifies them in another way and inscribes them in a systematic gnoseological classification (Ibid., 360-361)².

This influence having been noted, the indicated starting point of the relation between terms, relations and operations is retained here, which can be summarized as follows (Álvarez 2011). The structure of the expressions " $y = mx + b$ " and "Paul is not richer than Peter", is featured by the occurrence in them ('y', 'x', 'm', 'b' in the first and 'Paul' and 'Peter' in the second), of relations ('=' in the first and 'is richer than' in the second), and of relations ('=' in the first and 'is richer than' in the second), and operations ('+' and implicit multiplication in the first, and 'not' in the second). Let us consider the operations as procedures that lead in four different ways from one to the other of the two remaining elements. The following Table 1 shows the four types of exemplified procedures with their most characteristic form. None of the examples

² Bueno's development deserves to be studied, like his theory of science, in its entirety with close attention. A brief exposition of it can be found in a once very controversial secondary school philosophy textbook. The development does not depend so much on the starting point shared here as on the exposition of the concepts proper to Bueno's theory of science, different from those adduced here, which led him to define the model-to-modeled relationship as that existing between a formal context (which provides the assemblies) and another material one (which provides the physical referentials). The introduction of the dichotomies isological/heterological, and attributive/distributive wholes, applied to such a relation, generated a typology of models in meters, paradigms, prototypes and canons (Cf. Bueno 1987, pp. 361-365).

exhausts the category, i.e. the type, but the ones considered here are those that come from the traditional doctrine of *modi sciendi*, plus model as the fourth type.

OPERATIONS →	TERMS	RELATIONS
TERMS	Classification	Definition
RELATIONS	Model	Inference

Table 1

Classification is a procedure that leads from terms to terms, from a set to (sub)sets of the same set. Sets can be sharp or fuzzy and classifications the same (Alvarez, in preparation). Definition leads from terms to relations, from names to statements. Scientific definitions seek to be real and effective (Alvarez 1983). Inference leads from relations to relations: in its linguistic expression, from statements to statements, from premises to conclusions. Inferences can be deductive, inductive, abductive, etc.³ The construction of models leads from relations to terms. In what sense it does so is what this paper is about.

³ Herein lies my fundamental difference, despite sharing the starting point, with Bueno's development (in the unpublished project, which is where it was clearest, Bueno, 1976, v. Volume 2, pp. 2 ff.). Actually, in that place and in the later versions, rather than establishing the four *modi sciendi* in terms of transformations, they are fixed in terms of the determination of the four different relations resulting from the combinatorial subordination of the term-relationship pairs. 1) functions that from terms derive relations: models; 2) that from relations derive terms: classifications; 3) that from relations derive terms, 3) from terms draw terms: definitions; and 4) from relations draw relations: demonstrations. (Cfr. Ibid.).

[40] 2. Representation, models and theories in Bunge

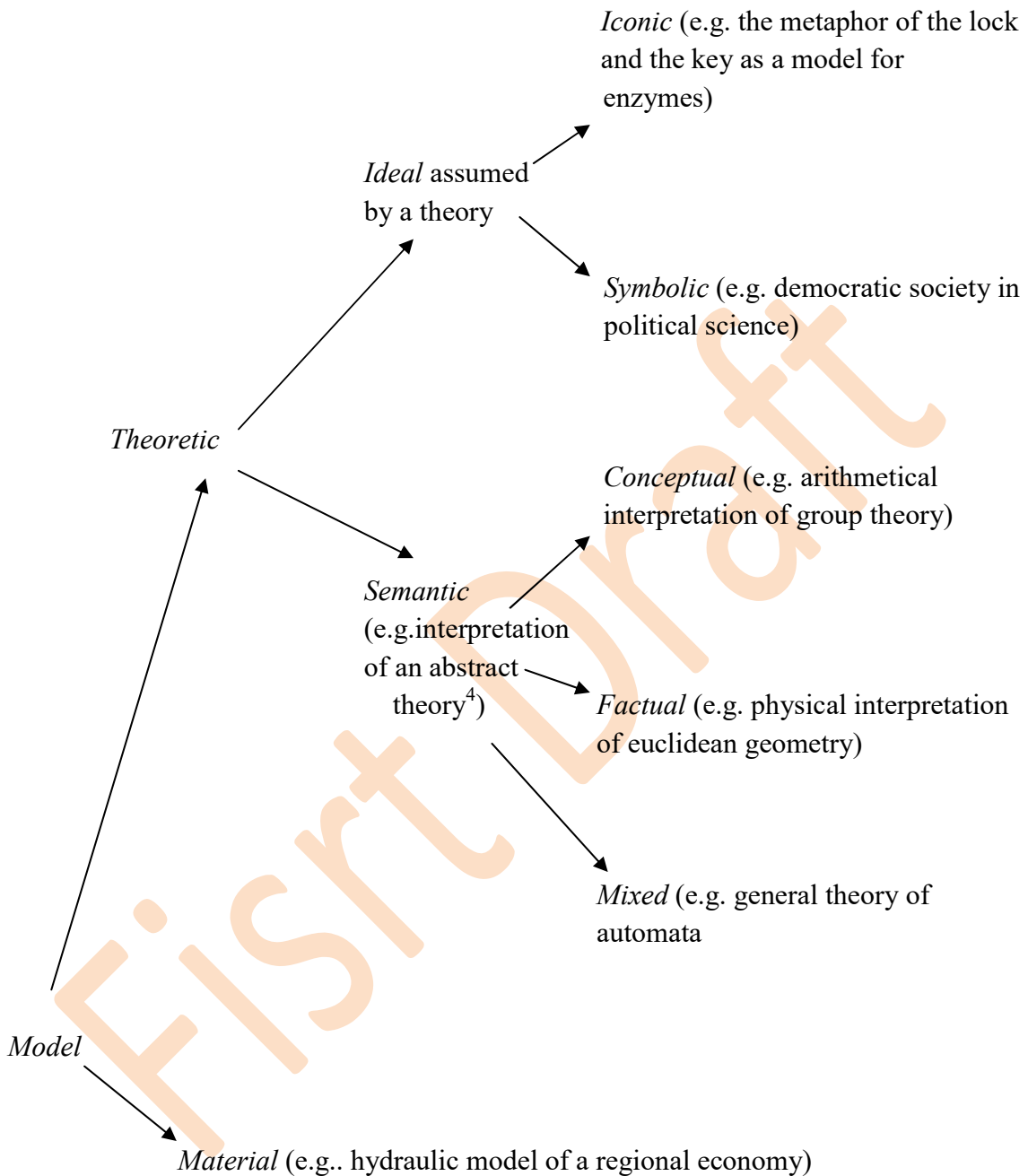


Figure 1 (Adaptaded from Bunge, 1976, p. 456)

A list of examples such as the one in the encyclopedia text at the beginning, the different meanings of the term "model" found in the most common dictionaries, or the

⁴ I introduce on the text the expression "semantic" to oppose, according to Bunge (1975), semantic model to ideal model within a theoretical model. There Bunge (p. 30) identifies semantic model with "the model as a true interpretation of an abstract theory", which on other occasions he characterizes as a satisfactory interpretation insofar as it makes the axioms of that theory true.

different typologies of models offered by the literature of the philosophy of science textbooks are a good example of the polysemy that the word carries in the most varied contexts. For the [41] extent and detail of its intent, I choose the conception elaborated in several works by Mario Bunge. Bunge, in his well-known treatise *La investigación científica* (Bunge 1976, 456), offered the above scheme of the meanings of "model" (Figure 1).

Bunge's development of the notion of a model includes in addition a fundamental distinction between three elements: 1) the *concrete object* or referent, 2) the *model object*, and 3) *the model itself*. There is no simple way of associating these three elements to the above figure, because (1) the concrete object does not appear directly in the various levels, but in the examples in parentheses: enzymes, democratic societies, the economy of a region); (2) the model object appears as the ideal type or simplified schema from which a theory starts (with two variants: iconic and symbolic); (3) but the model itself is divided into material and theoretical, and the theoretical is just the specific hypothetical-deductive theory or system that starts from the model, which is inserted in the previous one, which, on the other hand, includes semantic models as interpretations of abstract theories (with conceptual, factual and mixed variants), i.e., theories that are also specific.

Having clarified this, if one sticks to these three elements, and without prejudice to the diversity of the elements of the scheme, a simpler and no less useful approach would be the one that considers the following denominations: "concrete object", "(object) model" and "specific theory" (by development from the model object embedded in a general theory or by interpretation of an abstract theory). The model relationship might seem, in a first approximation, that between a model (object) (M) and its concrete object or referent (R), not that between M and the specific theory or theoretical model (MT).⁵ Nevertheless, Bunge's approach was not wrong in leaving outside of the models themselves (in the examples) the concrete objects, because they are not a formal part of the theory of models in science. Two different relationships are thus pointed out:⁶

(1) Relation of the models with their referents: *representation* (M, R)

(2) Relation of models to specific theories or theoretical models⁷. *embedding* (M, MT).

⁵ In the introduction to a collection of essays, Cartwright (1983, p. 4) establishes a relationship between four terms: real object, model, theory (theoretical laws), and phenomenological laws. Among them she establishes the following route. "The route from theory to reality is from theory to model, and then from model to phenomenological law. The phenomenological laws are actually true of reality -or could be. But the fundamental laws are true only of the objects in the model". I owe this indication to Oscar Esquisabel.

⁶ In chapter 3, section 3 of volume I of his *Treatise on Basic Philosophy* (Bunge, 1974) he considers four terms: *schema* (coinciding with the model object), *sketch*, *specific theory* (theoretical model) and *generic theory*, which follow the rule that the later ones integrate the earlier ones. The inclusion of the sketches does not significantly alter the scheme of the three elements (now the ideal scheme is reduced to a list, while the sketch introduces relations between the members of the list).

It is to (1) that Bunge (1975, p. 41) calls *model relation* and in which "the name of *model object* can be given to any schematic representation of an object. If the object [42] represented is concrete, then its model is an idealization thereof... [T]he representation is always partial and more or less conventional" (Ibid., p. 40).

M is characterized as a relational structure or system $\langle S, P_1, P_2, \dots, P_{n-1} \rangle$, where S is a set of elements s_i , which respond to elements r_i of R of the concrete object represented, and the P_i ($i = 1, 2, \dots, n-1$) predicates defined concerning S of which it is not known in principle whether, or to what extent, they respond to properties or relations of R , but whose function is heuristic precisely because they point to an increase in knowledge of R in terms of those properties. Having established this, Bunge formulates the relation ' M represents or models R ' as.

$$(3) \quad M \underline{\underline{=}} R.$$

That same model object M , in its structural form, is attached to a generic theory, thereby giving rise to a specific theory.

Let $M = \langle S, P_1, P_2, \dots, P_{n-1} \rangle$ be a model of a class R , that is, $M \underline{\underline{=}} R$. Suppose, furthermore, that the various coordinates with these n -tuples are logically independent of each other (i.e., not interdefinable). Then any consistent set of conditions (postulates) specifying the structure (of mathematical nature) of the n primitive concepts, as well as their factual meaning, will be a theoretical model of R . In a word, a theoretical model of R is a primitive grounded theory in $M \underline{\underline{=}} R$. (Bunge, 1975, p. 43).

If one reads carefully, the MT of R is not based on M , it includes M , but is based on $M \underline{\underline{=}} R$. M is a relational structure, to which the generic theory in which it is embedded adds the propositional conditions, the postulates, that determine MT as a theory. Thus is exposed the above relation (2).

What is characteristic of Bunge's approach, here simplified, is the identification of the model relation with a relation of *representation*. But the course he runs from a diversity of examples of models, given empirically in the disciplinary tradition, to the concept of model understood as a term of the representation relation, where the other end is an object, may be approached differently, almost in reverse. But first, it is necessary to examine comparatively the relation of representation concerning that of analogy, as conceived by Bunge, to distinguish his concept (and types) of analogy from the one that will be used in the propositive part of this study.

In "Analogy, Simulation and Representation" (Bunge, 1975, pp. 221-246), the notion of analogy as a binary relation in the universal set of objects O that obtain between elements x and y thereof, that satisfy one (or both) of the following conditions:

- (a) x and y participate in various objective properties (they are equal in some respects), and/or

b) there exists a correspondence between the parts of x or the properties of x and those of y . [43]

If x and y are analogues, we write $x \sim y$ and we say of any of the participants in the relation that it is an analogue of the other (Bunge, 1975, p. 224).

If the analogues satisfy a), they are said to be *substantially* analogous; if they satisfy b), *formally* analogous; if they satisfy a) and b) the analogy is called *homology*. "Homology implies both substantial and formal analogy and substantial analogy implies formal analogy, but not vice versa. (Ibid.) If x and y are sets, then the types or degrees of formal analogy can be characterized according to whether the relation applies to all or some of the elements of x and y .

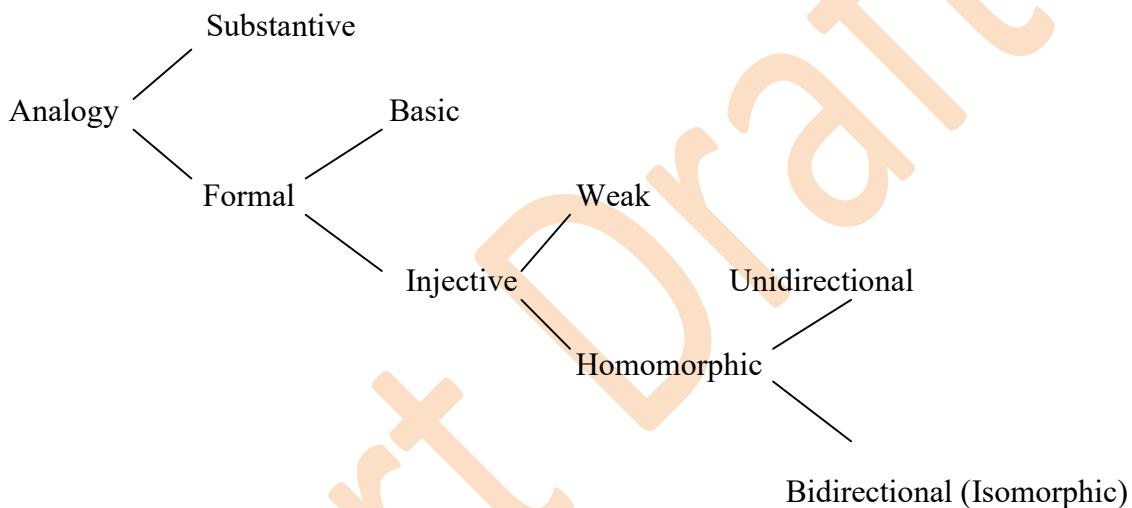


Figure 2 (Adapted from Bunge, 1973, p. 116)

Basic (plain) analogy: some elements of x correspond to some elements of y .

Injective analogy: to all elements of x corresponds an element of y . There are two forms: the weak and the homomorphic ("an all-some analogy that preserves the structure <of x >") (Ibid., p. 225).

Bijjective analogy: to all elements of x corresponds one and only one of y and reciprocally (the inverse analogy is also injective). He only considers this case within the homomorphic analogy. The resulting division conforms to the schema of Figure 2 (Bunge, 1973, p. 116, with the addition of the homomorphic analogy).⁸

[44]

⁸ The scheme is still incomplete, since it only considers the bijjective analogy for the homomorphic and not the weak bijjective. Bijjective is opposed to injective, and isomorphic to homomorphic.

3. Concept and types of model

I develop in this part, first of all, a concept of model based on a notion of analogy different from the one put forward by Bunge and, in general, by many scholars of the subject. Bunge's comparison between representation and analogy is based on the common character of both relations as binary relations. I argue, on the other hand, that following an idea of relation with four arguments, such as that which Leibniz called *expression*, and which embodies Bunge's idea of analogy, it is possible to lay the foundations of the model relation, as distinct from that of representation understood as binary. Secondly, having established the model relation as a proportional analogy, I will turn to the internal development of the semantic relation of representation (a relation of the sign-object type), as I have proposed in different places in the framework of a semiotic approach, called context *C*, where representation coexists with embodiment (an object-sign type relation). as one (as opposed to the other) of two semantic relations. This development yields a basic typology of models that should recover, to a large extent, the plurality of examples that overwhelm us with their casuistry.

3.1 Model relation: Analogy and expression

However, despite the importance of the notion of expression in Leibniz's work, it has received surprisingly little attention from commentators.

(Chris Swoyer, Leibnizian Expression)⁹.

Although the term "model" and the relation that includes it has taken hold over the past 20th century in the most prominent approaches to the philosophy of science, in which we here wish to remain without extending the use of the word to other scientific, philosophical, and even everyday contexts, the idea is not as recent as it may seem. The idea of expression, as defined by Leibniz in his 1678 opuscle *Quid sit idea* (What is an idea?) has a history that does not need to be recounted here (Cf. Soto Bruna, 1994 in an exposition where "expression" is much more extensive than the one discussed herein), and in different variants in Leibniz's writings (Cf. Debuiche, 2009) offers us a good handle to face the question of what kind of relation is that in which something is a model of a thing, and in what does this representative character consist. Leibniz' text written in 1678, as translated by Leroy E. Loemker, reads thus:

That is said to express a thing in which there are relations [*habitudines*] which correspond to the relations of the thing expressed. But there are various kinds of expression; for example [45], the model of a machine expresses the machine itself, the projective delineation on a plane expresses the solid, speech expresses thoughts and truths, characters express numbers, and an algebraic equation expresses a circle or some other figure. What is common to all these expressions is that we can pass from a consideration of the relations in the expression to knowledge of the corresponding propertie

⁹ This quotation corresponds to the beginning of Swoyer (1995). Twenty-two years later, the opposite can be affirmed. I bring it up just to mark this change.

of the thing expressed. Hence it is clearly not necessary for that which expresses to be similar to the thing expressed, if only a certain analogy is maintained between the relations. (Leibniz, 1969, 207-208).¹⁰

Although the idea of expression has in Leibniz's philosophy not only a formal but also a metaphysical, epistemological, etc. meaning¹¹, this does not preclude us from taking the above text as the definition of a relation between four terms: the expressive thing, the thing expressed, the relations in the expressive thing and the thing expressed, where the relation of expression between A and B (expressed thing) is made to depend on the relation [of analogy?: "provided that a certain analogy is preserved for the relations"¹²] between $R(A)$ (relations in A) and $R(B)$ (relations in B). Thus understood, the relation of expression welcomes a relation common to the similes and arithmetical proportions. Let the Homeric simile be the modular piece of an oral tradition:

He said, and, greatly encouraged, he hastened on his way to the city; like the victorious steed in the chariot race trots swiftly through the field, so lightly did Achilles move feet and knees. (*Iliad*, Book XXII)

and the proportion 2: 4:: 8: 16, where there are four numerals. In the simile there are also four terms: Achilles: victorious steed :: movement of feet and knees: swift trot across the field. Undoubtedly, it is convenient to order Leibniz's quotation and to specify its parts and its structure.

[46]

1. The relation of expression is established between A (expressive thing) and B (expressed thing). It is, then, a binary relation, but that binary relation obtains if

¹⁰ The latin text in the referenced edition reads thus:

Exprimere aliquam rem dicitur illud, in quo habentur habitudines, quae habitudinibus rei exprimendae respondent. Sed eae expressiones variae sunt; exempli causa, modulus Machinae exprimit machinam ipsam, scenographica rei in plano delineatio exprimit, solidum, oratio exprimit cogitationes et veritates, characteres exprimunt. numeros, aequatio Algebraica exprimit circulum aliamve figuram: et quod expressionibus istis commune est, ex sola contemplatione habitudinum exprimentis possumus venire in cognitionem proprietatum respondentium rei exprimendae. Unde patet non esse necessarium, ut id quod exprimit simile sit rei expressae, modo habitudinum quaedam analogia servetur (Leibniz, 1678, 263-264)

¹¹ "Expression is a notion which, in Leibniz, is used as frequently (in almost all fields of his thought, mathematical, logical, gnoseological, linguistic, physical, metaphysical) as it is lacking in explicitness. The diversity and quantity of its appearances are thus paradoxically linked to a certain conceptual imprecision, or at least to a deceptive simplicity that tends to define it as an analogy between relations. However, it seems that despite this almost universal applicability, the notion of expression maintains a privileged relationship with mathematics" (Debuiche, 2009, p. 1).

¹² This paper is not a scholarly study of the notion of expression in Leibniz, about which much has been well and abundantly written. Apart from Debuiche (2009), Soto Bruna (1994) and the authors cited by them, in Spanish one can consult Esquisabel (2008, 2016, 2016, unpublished) and Herrera Castillo (2012). More in line with this formulation is the study of Swoyer (1995) with his idea of the correlation relation, which serves as a basis for the analogical reasoning that he calls subrogatory reasoning. There (Ibid., p. 81) he argues that the condition of isomorphism is excessively strong to characterize the concept of expression: "such complete similarity of structure is an excessive requirement for expression". Less demanding than a relation *unto* is a relation *from to*. In his view, it is not necessary that what he calls "correlation" be biunivocal. It is enough that it is given in only one direction: "we can think of this relation as running in one direction or the other" (Ibid. italics added): he does not call it either way, although it is strictly a homomorphism.

2. There exist in A relations $R(A)$ and in B relations $R(B)$, such that $R(A)$ respond to $R(B)$, and if
3. A certain analogy of relations $R(A)$ and $R(B)$ is saved or preserved (*servetur*).
4. In sum. It suffices that a certain analogy of $R(A)$ with $R(B)$ is given for A to express B . The analogy (a binary relation between $R(A)$ and $R(B)$) is a sufficient condition of the expression (binary relation between A and B).

Having clarified this, I take the text as incitement and not as piece of doctrine, except for the troubling simplicity to which Debuiche hints. Up to here the incitement, the proposal is the following.

5. A is said to express B if and only if there exists a one-to-one structure-preserving correspondence from $R(A)$ to $R(B)$.

These are the differences between the inducement and the proposal. Leibniz's "responds" is strengthened by "corresponds univocally" and the "some analogy is preserved" by "the structure is preserved" (in passing from $R(A)$ to $R(B)$). The sufficient condition (if...) is completed by the necessary condition (only if...) in the definition's biconditional "if and only if". And the analogy of the second relation is transferred to the relation of the relations between A and B and $R(A)$ and $R(B)$: that is to say

$$A : B :: R(A) : R(B).$$

But thereby what has just been formulated is an analogy of proportionality¹³, where the relations (*rationes*) of the proportion (analogy) are the expression and the Leibnizian "analogy" between $R(A)$ and $R(B)$. It is also possible that whoever reads this will transfer these words to others that are better known in algebra textbooks, evoked by proposition 5).

By "one-to-one correspondence" is meant the function that assigns to each element of A one and only one element of B . $R(A)$ and $R(B)$ are the structure of A and B expressed relationally. For minimal structures of sets, they can be expressed by an operation or law of internal composition, for example, $*$ for A and \perp for B . Then the form of the analogy can be formulated as a homomorphic function or homomorphism:

Given two nonempty sets A (of generic element a) and B (of generic element b) endowed respectively with operations or laws of internal composition $*$ and \perp , and a function f from A to B :

¹³ The Aristotelian tradition distinguished two kinds of analogy. The analogy of attribution (*analogia attributionis*), such as that which calls a walk healthy because it contributes to the good health of the walker, and the analogy of proportionality in the form of a *quaternio terminorum*, in which the first term is to the second as the third is to the fourth.

$$f: \langle A, * \rangle \longrightarrow \langle B, \perp \rangle$$

[47] the following definitions can be given:

6. f is a homomorphism if and only if

$$\forall a_i, a_j, \in A, \quad [f(a_i * a_j) = f(a_i) \perp f(a_j)], \quad (i, j=1, 2, \dots, n)$$

i.e., if and only if the image of the composites of elements of A is equal to the composite of the images of the elements A by the operation of B .

7. f is an isomorphism if and only if, being a homomorphism, its inverse f^{-1} exists and is also a homomorphism, that is, if and only if

$$\forall a_i, a_j, \in A \quad [f(a_i * a_j) = f(a_i) \perp f(a_j)] \wedge \forall b_i, b_j, \in B \quad [f^{-1}(b_i \perp b_j) = f^{-1}(b_i) * f^{-1}(b_j)]$$

($i, j=1, 2, \dots, n$).

8. If $A = B$, f is an automorphism.

Therefore, A expresses or is a model of B , if and only if the correspondence of $R(A)$ with $R(B)$ is a homomorphism. This is the basic form of the model relation. But the characterization of the model relation employing the notion of homomorphism (or even of isomorphism) is merely formal or syntactic (an any-object—any-object relation¹⁴; sign-sign is already an interpretation). It expresses the formal conditions of modeling, namely, for the correctness of modeling the condition of homomorphism must be satisfied.¹⁵

Now, this presupposed, whatever the models are, they all have to satisfy that the model relation in which they stand is homomorphic. But the question of what are the fundamental types of models is developed from the semantic relation of representation, which is now supported by that of expression in its elementary form of homomorphism.

I will call homomorphisms, and other morphisms, *structural homologies*.

3.2 The basic types of model: internal development of the representational relation

In the last one hundred years -to approximate a period -semiotics as a general theory of signs has been associated with the American philosophers Charles Sanders Peirce and

¹⁴ Gonthier (1936) termed logic as "the physics of any object" (*physique de l'objet quelconque*) and in combinatorial logic, Curry and Feys (1967) established relations between *obs* (undetermined objects). Yet in logic, the last level of analysis before abandoning the formalized language in which it unfolds remains unattained.

¹⁵ Like other theory-building procedures, model building has to satisfy formal or correctness requirements, before material or adequacy requirements. The classification attempts to go from set-theoretic requirements (precise or fuzzy) to naturality (Álvarez, in preparation), in definitions those of coextension and non-circularity prior to reality and effectiveness conditions (Álvarez, 1983).

Charles W. Morris. The former, dispersed and multifaceted in his vast and varied production, has given rise to the widest variety of affections, accessions, revisions and adaptations. Peirce's greatness is beyond doubt, but the choice of Morris as the spokesman for the search for such a theory [48] of signs or Semiotics is also a permanent acquisition of our cultural tradition. Since the publication in 1938 of his paper "Foundations of the Theory of Signs" (Morris, 1938), it has been the standard choice. In that textual locus (Morris, 1938) Semiotics was typified as a general theory of signs with three fundamental dimensions of analysis: syntax, semantics, and pragmatics, dealing, respectively, with the relations of signs to each other, "the relation of signs to their *designata*" (Morris, 1938) and, therefore, to the objects that they may denote" (Ibid. p. 21), and of the relations of signs with the subjects that use them.

Three disciplines which deal with semiotic relations properly speaking, because they include signs, but leave out of consideration others that also include them, such as the inverse of the sign-object and sign-subject relations to which Morris reduced, respectively, the semantic and pragmatic approaches. Since Álvarez (1984, 1988, et al.) I have argued that semantic relations are both the one pointed out by Morris, and its inverse, the object-sign relation to which the material conditions of symbolization must be associated, symbolic *embodiment*: signs in their supports, mainly bodies, such as those which materialize in rulers, clocks, discs, nervous systems, organic apparatuses, faces, limbs, etc. For this reason, the inverse of the object-sign relation, the material conditions of symbolization, the symbolic embodiment, is to be associated with the signs on their supports, mainly bodies, such as those materialized in rulers, clocks, discs, nervous systems, organic apparatuses, as a second class of semantic relations¹⁶.

But the sign-object relation is not unique, it is rather a class of different relations, the object-sign relation. That class constitutes the realm of *assignation*. In this respect, we find that we are dealing with various relations, such as, for example, designation, denotation, reference, representation and proxying (cf. Bunge 1974, pp. 91 ff.), among which representation is one among others and, therefore, the semantics of models, if they are understood as representations, is one of them among others and, therefore, the semantics of models does not exhaust semantics as such. Even so, what is true for the class is true for the members thereof and the representation relation will be (re)presented by the class to which it belongs, the sign-object relation, hereafter (σ , O) and its inverse (O , σ). The development of the direct semantic relation is construed as the Cartesian product of the set $\{\sigma, O\}$ with itself, whose elements are σ and O , i.e. $\{\sigma, O\}^2$ in the following way:

¹⁶ It is a difference between kinds of relations: the relation of designation is one among other relations of signs to objects. The relations of the inverse class are different relations of objects to signs, among them the relation of embodiment. The same is true of the sign-subject relations, which Morris identified without further ado with pragmatic relations, to which the must be added the inverse subject-sign relations, which in the same pages I have called symbolic relations. But these are not indispensable here.

$\{\sigma, O\}^2$	Sign	Object
Sign	(σ, σ) I. Structural homologies	(σ, O) II. Satisfactory Representations
Object	(O, σ) III. Efficacious embodiments	(O, O) IV. Adequate reproductions

Table 2

[49]

3.2.1 Types I and II

In this semiotic approach, we are already *within* a universe in which the formal relations are already syntactic, whereas, in the previous consideration of the relation of expression, the relations were considered to be between indeterminate objects. Structural homologies are now homomorphic relations between sign systems. Objects are not simple indeterminate *obs.*, but objects in general (existing in space and time and subject to causal actions). *Ob* gain here also the status of signs. The relations of signs to objects are not only representational (cf. *supra*, also designative, referential, etc), but among them are those that are of interest to this typology of models.

Assuming that the form of the idea of representation is already assumed in the relation of expression, when it is considered (interpreted) that representation is the homomorphic relation between (systems of) signs and (structures of) objects, we find ourselves in the second position in the first row of the table where the expressive thing is the system of signs and the expressed thing is the structure of objects represented by them. This representational condition is one that enables knowledge by analogy -by subrogatory reasoning (Swoyer, 1991, 1995; Contessa, 2007) -as Leibniz himself was already concerned to point out:

[...] it is only by contemplation of the relations of that which expresses it that we can arrive at the knowledge of properties that correspond to the thing that is to be expressed (Leibniz, 1678, p. 264).¹⁷

¹⁷ Although the subject of subrogation inference is not the object of this section, I offer an accurate characterization thereof, provided by Esquisabel (unpublished), whom I thank for sending it to me.

"As such, the subrogatory inference contains at least three fundamental steps: first, there must be a translation or transfer, so to speak, of structural aspects of the represented object to the system of representation; second, a series of operations and transformations are performed within the representational system; and third, these results are transferred back to the object itself. The efficiency of the representation system consists in the fact that, through the operations with and on it, it is possible to obtain as final result properties or behaviors of the object that are not part of the initially represented properties".

Here the rule is what Esquisabel (2008, p. 14) has very aptly called the *transference principle*, "by means of which one can validly pass from true propositions about the one to the corresponding propositions of the other without loss of the rigor of the substantiation". Now, since this principle governs subrogatory reasoning, which, as such, produces new knowledge from other existing knowledge, they must be dealt with in another study on models, theory and explanations, which will follow the present one.

Moreover, as I wish to point out here, representation is the expression, *satisfactorily* interpreted, in a given field. It is customary to refer to semantic models as satisfactory interpretations of abstract theories, i.e., interpretations that make the axioms of such theories true. (Cf., for example, Bunge, 1976, pp. 448 et seq., where a micro-theory of a partial order is formulated in three axioms concerning reflexivity, antisymmetry and transitivity, and eight interpretations that make them true, in as many fields). Thus, the homomorphism condition is satisfied by the [50] interpretation which assigns numbers to the elements of A and B , operation multiplication (\times) to A and addition ($+$) to B and to f the logarithmic function:

$$9) \quad \log_b : \langle A, \times \rangle \longrightarrow \langle B, + \rangle ,$$

such that it is true that

$$10) \quad \forall a_i, a_j, \in A [\log_b (a_i \times a_j) = \log_b (a_i) + \log_b (a_j)], \quad (i=1,2,\dots,n)$$

Formal examples (mathematical, logical, etc.) are always very useful, although reiterative or informatively almost null. In turn, the notion of satisfactoriness is plural and is not only commensurable with truth but also with the stability of systems (as will be seen in the fourth type of models, the adequate reproductions, in the part-whole relation in connection with organs and parts as "satisfactory models", appropriately called there adequate). That is to say, satisfactoriness is always concerning some clearly determined criterion. Restricting ourselves to the context assignation, a representation is satisfactory concerning a given criterion, such as truth, verisimilitude, certain principles, etc.

The well-known representation of natural selection by artificial selection in *The Origin of Species* (Darwin, 1876) has been studied in abundance. Artificial selection is a satisfactory representation of natural selection concerning the principles relating to the underlying mechanisms, which are the very same. Darwin puts it this way in Chapter XV:

[...] In the domestic state we see much variability produced, or at least stimulated, by the change of conditions of life; but often in so obscure a manner that we are tempted to regard the variations as spontaneous. Variability is governed by many complex laws [...] Variability is not really produced by man; man merely exposes, and unintentionally, organic beings to new conditions of life, and then nature works upon the organization and makes it vary. But man can and does select the variations presented to him by nature, and thus accumulates them in the desired manner. Thus man adapts animals and plants to his own benefit or taste. He may do this methodically, or he may do it unconsciously, retaining the individuals that are most useful or agreeable to him, with no intention

of modifying the castes. [...] This unconscious process of selection has been the principal agent in the formation of the most distinct and useful domestic races.

There is no reason why the principles which have worked so effectually in the domestic state, should not have worked also in the state of nature. (Darwin, 1676, pp. 410-411. Italics added).¹⁸

Between the concise formulation of the logarithmic representation and the prolixity of the selectionist analogy, which considers the *endomorph*ic representation of natural selection by artificial selection (of the whole by the part) as satisfactory, there is not a difference in genre, but in style. They are of the same nature, if we take "nature" in the exact sense in which there is an intrinsic connection between the model and what is represented. In the case of selection, the intrinsic connection consists of the identity of the underlying principles (mechanisms). The connection (I use here the same word as Leibniz in the text quoted below) can be the transformability of the represented into the [51] strongest natural expressions In (Leibniz 1678, p. 264) he distinguishes between expressions based on convention (*arbitrium*) and nature.

It is clear that some expressions have their foundation in nature, but others are founded, at least in part, on a convention (*in arbitrium*), such as expressions made using words or characters. Those that are founded in nature postulate either a similarity, such as that between a large circle and a small one, or between a region and the map of the region, or at least some connection such as that which exists between the circle and the ellipse that represents it optically, since any point of the ellipse responds to some point of the circle according to a *certain law*. Actually, *in such a case the circle would be misrepresented by a more similar figure* (Italics added)¹⁹.

Quotations cannot be arguments of authority, but they can be incitements to reflection, such as the text on the expression discussed in section 3.1. This one, belonging to the same writing, provides for the issue of representation an appreciable contribution to the free commentary.

There are natural or necessary expressions -and, therefore, representations- and conventional or arbitrary ones. The latter are those proper to oral or written language marked by discrete signs; from the examples, the latter seems to be like the spatial figures (representatives or represented) rather continuous. Of these natural representations, some are based on resemblances and others on legal connections, the latter being more satisfactory than those based on resemblances, although -may I use a pun- in the latter the model and the object are more "similar". Perhaps what Leibniz wished to communicate would become clearer with a distinction often attributed to Poincaré: "'roundel' is a general concept, "circle" is a scientific concept".²⁰ In the case

¹⁸ Cfr. Álvarez (2000, 2010)

¹⁹ *Patet etiam expressiones alias fundamentum habere in natura, alias vero saltem ex parte fundari in arbitrio ut sunt expressiones quae fiunt per voces aut characteres. Quae in natura fundantur, eae vel similitudine aliquam postulant, qualis est inter circulum magnum et parvum, vel inter regionem et regionis tabulam geographicam; vel certe connexio qualis est inter circulum et ellipsin quae eum optice repraesentat, quodlibet enim punctum ellipseos secundum certam quandam legem alicui puncto circuli respondet. Imo circulus per aliam figuram similiores in tali casu male repraesentaretur.* (Leibniz, 1678, p. 264).

²⁰ It may happen with this as with Lord Acton's famous alleged quotation on power and corruption that the quotation is repeated and its source is not identified. In any case, it may be helpful.

of similarity, there is no particular law according to which a roundel drawn to represent a circle is a scientific concept, although it may *resemble* it more than an ellipse. But the circle, an ellipse of equal axes, shares the nature thereof as its particular case. *Resemblance is extrinsic, connection is intrinsic.* In outline (Figure 3):

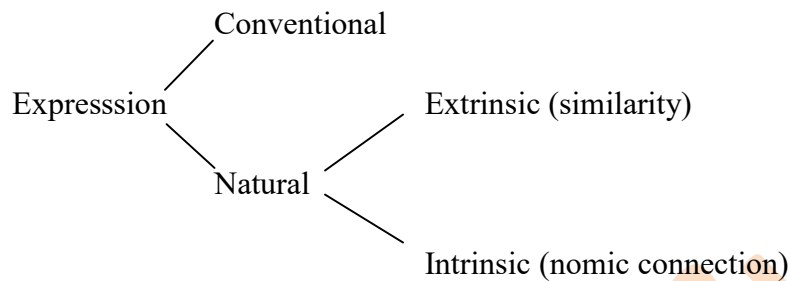


Figure 3

[52]

The second type of model, *satisfactory representations*, has at least the diversity shown in the scheme of Figure 3. The diversity of satisfactoriness now abounds in an orderly fashion in the casuistry with which its consideration began. It is possible to go further in this development, and it will surely be interesting to do so, but now it is necessary to attend to types III and IV

3.2.2 Types III and IV

Types I and II are indicated in the development of the idea of representation by those relations of $\{\sigma, O\}^2$ which carry the signs in determinative position (in the reading order, to the left), i.e., where the other term of the relation is subordinate to the first. In the syntactic relations (homogeneous relations, of equal terms) that are at the base of the structural homologies and in the semantic ones (heterogeneous, sign-object), of which we are mainly interested in the representative ones, the symbolic term dominates. On the other hand, in the second row of Table 2, the relations in the boxes are determined by objects versus signs in the efficacious embodiments (heterogeneous relations) and objects versus other (different or not) objects (homogeneous relations). If the relations of the first row refer mainly to contexts of semiotic predominance, those of this second row refer mainly to biological and technological contexts. The following sections have the particular difficulty that the basic concepts do not enjoy an analytical tradition similar to that granted to type I and II models by linguistics, mathematics and philosophy of science. For this very reason, the need to coin them with an appeal to disciplines that develop in biological and technological contexts, such as genetics, morphology, ethology, cognitive sciences, archaeology, computer science, and communication theory, implies a new effort that can only be glimpsed here.

3.2.2.1 Embodiment versus assignation

If type II is configured in what can be called *representational* semantics, type III is configured in what can be called *embodimental* semantics. Bueno (Cfr. García Sierra, 2000, p. 95) characterized materialism by a restrictive condition, namely, the *inexistence of incorporeal living beings*. This same condition can be applied [52] to signs in the sense that every sign is embodied: although it is not reduced to a body, it is in a body. Said in the negative, *there are no incorporeal signs*; it is another thing what consistency or form of existence the meanings have. Clock dials on which figures appear, rulers of various materials with stripes and numbers, but also networks of knotted strings or computers whose switch-structured circuits embody the machine language that *run* sets of instructions that, in turn, have served to analyze networks of knotted strings, all these products of technique (artisanal or scientific-industrial) are corporeal systems that we call *efficacious embodiments* and that constitute type III of models.

[53]

We cannot *inscribe* stable marks in a puddle of water; we can, on the other hand, inscribe them in snow or a sand bank for as long as the stability of these bodies is maintained (as long as the snow does not melt or the sand does not collapse). In the center of the Sun or the pure spirit, there is no semiosis, because in the processes of nuclear fusion of the former or the supposed immateriality of the latter there are no bodies sufficiently stable for it to take place. A certain stability of bodies is the first necessary condition for the embodiment of signs: writing requires a certain density of ink and absorptive capacity of paper or solidity of stone or clay, etc. The footprint of a footstep is a mark on the ground that is produced under certain conditions of stability, solidity and other conditions of the ground so that it can be a sign for "subjects" of various species. The mark left by the footprint of an animal is the *effect* of the heavy limb of the walker on a ground of little resistance (on rock, concrete or cold asphalt it would not occur).

There are, therefore, structural and causal conditions for the production of signs which, like the mark of a footprint, the smoke of a fire, or the fever in a disease, have been called, respectively, traces, indications and symptoms, which fall in the tradition in use under the denomination "natural signs". Stability and causality are requisites of *natural signs* which would not be so without their reception and interpretation by animals for whom they have adaptive value within the framework of causal knowledge.

[Causal knowledge evolved in response to the demands of a changing and increasingly variable environment and [...] rested on three cognitive foundations: an innate tendency, driven by curiosity, to recognize and categorize natural signs and causally connect them to covariant categories drawn from memory; an aptitude to simultaneously represent in [...] incipient working memory perceived signs and remembered categories, and to infer a causal connection between them; and an ability to

embody casual relationships in cognitive maps, as a way of facilitating rapid access during the subconscious selection of behavior (Stuart-Fox, 2015, p. 250; italics added).

That the effects become natural signs depends on other effects occurring in other bodies, those of hominins (which would also have a fever and would perceive it in their conspecifics). These would perceive, then, the presence of smoke, fever, of footprints, but also in other animals' colorations, figures, gestures, actions, etc., that is, all the perceptual content characteristic of what Portmann (Cfr. Alvarez, 185, pp. 124-128) called the exhibition or self-presentation (*Selbstdarstellung*) of some animals before others. The animals themselves, in their morphology and behavior, are symbolic bodies for other animals and, especially, for the descendants of the hominins who today dedicate themselves to analyses such as this one. The Darwinian tradition, crossed today not only by genetics but also by the cognitive sciences (as the above quotation from Stuart-Fox shows) and others such as Biosemiotics, which began with Uexkull's theory of the ambient world (*Umwelt*) and continued with Sebeok's zoosemiotics, all insist on this symbolic world based on bodies of all kinds. Indeed, symbolic embodiments are as much physical and biological as they are technological, the biological ones being the most ancient. Natural signs are not signs save for animals that react according to their constitution by behaving in certain ways, i.e., with preordained or acquired patterns of action.

[54]

Marks on physical supports, variable traits in animals, behaviors that affect conspecifics or alter-specifics, advantageous or detrimental reactions to what is perceived, all this proves that there are bodies that carry, receive, emit and execute the information embodied in them. The embodiment of signs takes place in various ways, the detail of which deserves a detailed study that does not fit here, but which should be noted. In this note, suffice it to point out that just as in representational semantics the relation of representation was one among other relations that, in general, can be called *assignments* (those that rely on the generic sign-object form), here variants of embodiment that occur both in nature and in technology -natural signs and artificial signs- rely on the object-sign form, are the object of *embodimental* semantics. In it, both biological and technological processes of sign embodiment have to be taken into consideration.

Recently Poznic (2016), in a study about models in science and engineering, has confronted the relation of representation with the relation (or, better, procedure) of *design*. In one direction (a *direction of fit*, an expression he takes from Searle), the direction from the vehicle to the target, the representational relation is established; in the opposite direction, from the target to the vehicle, the (relation or procedure) of design. Poznic's approach leaves aside, however, the biological embodiments which, if integrated into it, would again leave us affected by the old *problem of design or purpose* in biology, which is not a question of addressing either by principle or opportunity in this study. Just to point out, for a development elsewhere, that in the realm of

embodiment -biological and technological embodiments taken into account- a distinction can be made, respectively, between adaptation (mainly in the ateleological sense of *exaptation*) and design (always teleological): between what it *serves for* and what it *is made for*.

Certain products of technology constitute information systems in which physical structures embody symbolic structures by an expressive relation (in the sense of section 3.1). Digital computers are technological systems in which the physical structure of switch circuits embody a Boolean algebra that operates the machine by following the "instructions" of a binary language (cf. Kaye, 1970): *hardware embodies software*. As machines that transform energy into work, computers conform to *efficiency*; as machines that operate according to algorithms, they are defined by the mathematical notion of *effectiveness*. To conflate efficiency and effectiveness I make use of the term "efficacy," and I qualify models of this type as *efficacious embodiments*.

An efficacious embodiment is a corporeal system whose structure embodies at least one data structure. Beynon-Davies, (2007) has applied contemporary concepts such as information system, techniques or information technologies to the results of archaeological studies that reveal the existence of pre-writing information systems in Ancient Sumer (the "Paleolithic informatics": 8000 to 3000 BC) and in the Inca civilization (C. 1200-1475). The Incas, who had neither writing nor the wheel, managed to build an information system whose elements were (1) an extensive and efficient transportation network, (2) specialized personnel, messengers who ran relay races on the network's routes and those in charge [55] of encoding and decoding the messages contained in the *quipu*. (3) The *quipu* was a network or assembly of strings, made of cotton or wool, colored and knotted, which served to record the data of a complex message (Figure 4). Apparently the color, connection, location, shape of the knots and their relative position were the basis of the coding performed by the *quipu* keepers. The complete assembly of strings in a *quipu* embodies a data structure.²¹

²¹ I will not dwell further on the *quipu*, an object studied extensively and repeatedly, and interpreted in various ways, as it remains to be studied in more detail and the digital computer as effective embodiments (Alvarez, in progress).

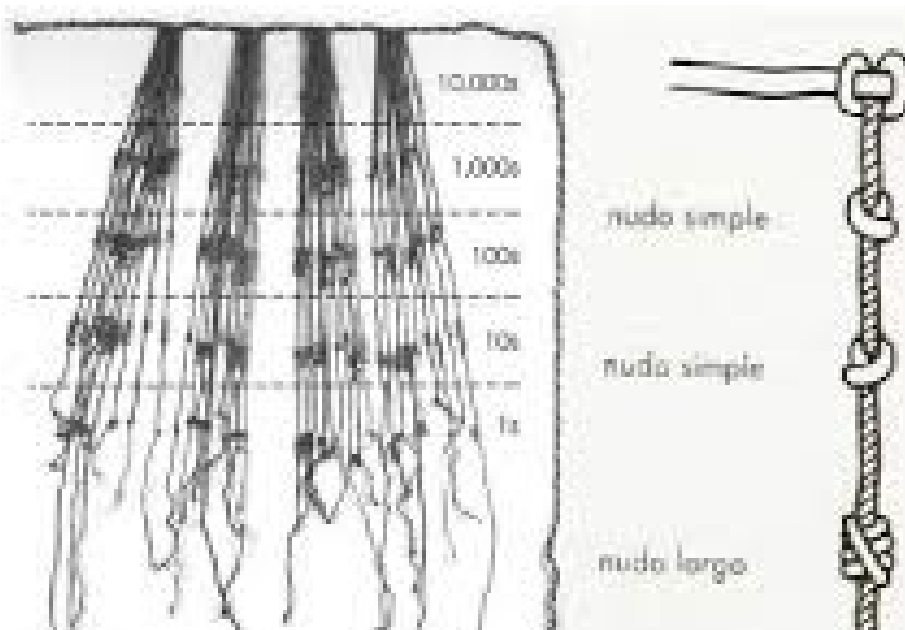


Figure 4: A quipu

The digital computer, a product of 20th-century technology, is a patent and explicit example of this conjoint relationship, but it should not be forgotten that the oldest embodiments were (and are, if they are preserved) those produced by biological evolution, in which organisms that possessed highly complex (very efficient) nervous systems developed resolute (effective) communication systems, thus acquiring the status of efficacious embodiments²². There are, therefore, natural and artificial embodiments.²³

[56]

²² Biosemioticians, especially the Copenhagen school and in particular Hoffmeyer, maintain the existence in organisms of a code duality: digital, that of the genetic code, and analogical, that of the organism. Hoffmeyer summarizes the issue as follows:

"[...] DNA does not contain the key to its interpretation [...] The interpreter of the DNA message is buried in the cytoskeleton of the fertilized egg (and of the growing embryo), which, in turn, is the product of history, that is, of the billions of molecular habits acquired through the evolution of the eukaryotic cell (Margulis 1981) [. ...] Therefore, life shows a non-trivial, that is, semiotic interaction between two states, the analogically coded state of the organism itself and its redescription in the digital code". (Quoted in Alvarez, 2007, p. 238).

Subsequently, the code duality theory has been criticized as a simplification of the theory of organic codes, developed by Barbieri, which intercalates between the genetic code and the linguistic code several levels of organic codes. (Cf. Barbieri, 2003).

²³ The natural/artificial (conventional, technological) dichotomy has here another meaning than the one derived from Leibniz's text (vid. supra) concerning nature-based representations. The circle and the ellipse share the nature of the latter, and both share the nature of the conics with the parabola and the hyperbola; the roundel and the circle, however, do not share the same nature which is the one preserved concerning certain transformations (laws, in Leibniz's terminology): no transformation turns a roundel into a circle.

3.2.2.2.2 *Reproduction vs. homology*

Type IV models are diagonally opposed (in Table 2) to type I models. Suitable reproductions are corporeal objects that have a homomorphic relationship with others. An easily evoked example is the maquette of a building, for example, of a house. Let us suppose that we commission an architect to design a house and that he does not limit himself to presenting the bound volume of documents that usually make up the project, but with a gesture of complacency, he hands us a maquette. The most popular encyclopedia today -*Wikipedia*- presents the following definition of the architectural maquette:

An **architectural maquette** is a reduced-scale physical representation of a building, urban project, or parts of it.

Our architect gives us the maquette as a foretaste of the house, so it is somewhat problematic to consider it "a physical representation" of the house, otherwise still non-existent. It would not be a *model of*, but rather a *model for*²⁴ the (construction of) the house: the corporeal form of a norm of action. Once the house is built, it could then be considered an adequate reproduction of the model, although this is not our current way of speaking. It is also convenient to specify the existence of three elements at play and to modify the terminology of, the above definition.

There are three elements: the project, the maquette and the building. Between these terms, there is a relationship from the project to the maquette and from the maquette to the house. The project is a document referring to an object in the field of assignation, namely, a representation of something that does not exist, that is to say, that only improperly is a representation²⁵. The maquette is made in accordance with the project, it is the first *embodiment* of the project; the building is also constructed in accordance with the project, it is a second *embodiment* of the project. The relationship of maquette and building does not take place, as a *reproduction*, until both are built²⁶. In scheme:

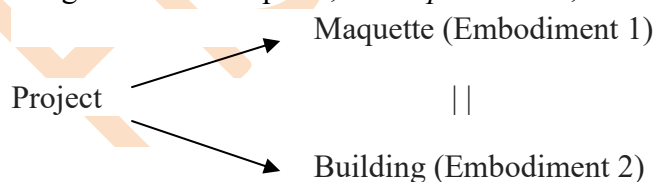


Figure 5

²⁴ I will not go into this distinction here, which has to do with two different normativities that I have dealt with elsewhere: *methodological* normativity and *technological* normativity (Cf. Alvarez, 1994).

²⁵ Oscar Esquisabel warns me that the model does not really represent in the sense of *stare pro aliquo*, and points out to me the opposition between *Vorstellung* and *Darstellung*, as well as the theatrical representation which is an execution or performance of something, not a representation of something.

²⁶ In his concept of *indirect representation* Poznic (2016) also uses a threefold scheme: model description, model system and target model. Between the first two he establishes a relationship of specification (which he ends up assimilating to design) and between the last two that of *representation*.

where " | " means "reproduces a". In this case, it is a scale reproduction, which is usually called a scale or reduced model. Not only is the size reduced, but also the amount of information [57], although this is not very accurate.²⁷ On the one hand, the relationship between the maquette and the building is a *morphological* relationship and, on the other hand, a relationship of everything to everything (whole maquette-whole building).

A part or spare part of a machine, for example, the drain pump of a washing machine or the carburetor of a lawnmower, may or may not be the original "of the model", but if it were not and could be adapted and maintain with its coupling the proper (normal) operation of the machine, it would be said to be an adequate partial reproduction of the machine (the part-machine relationship is from part to whole), but it is not (totally) morphological, but *functional*.

It follows from these considerations that there are two relevant oppositions to be taken into account. The *part/whole* opposition and the morphological/functional opposition. Unlike embodiments, where biological evolution and human history show that the oldest are the biological ones, in reproductions the oldest are also some biological ones such as specific offspring and lactation between different species, more recent the artisanal and technological ones proper, recent contemporary medical ones (organ transplants) and in the not too distant future the manufacture by three-dimensional printing of prostheses and of tissues and even organs.

The oldest biological reproductions, in the all-all relationship, are the results of the processes of reproduction of organisms, characterized by the *generatio univoca*, where adequacy is defined by *species preservation* and the genotypic and phenotypic traits of the parents.²⁸ The most recent biological reproductions, in this all-all relationship, have a good example in the cloning of the sheep Dolly (1996), an adequate reproduction until it died in 2003, whose adequacy is questioned by reduced viability. *Model organisms* are being used more recently and more and more continuously, as microbes, plants or animals allow the study of specific biological processes in other organisms where technical or ethical constraints prevent experimentation.²⁹

²⁷ It depends on the level of detail contained in the project, and the scale of the model. In the finished house the screws may be star or slotted, hex or Allen headed, etc., but it is clear that this level of detail is not embodied in the maquette.

²⁸ What manifests itself in the concept of species defined in terms of the reproductive process and which is mostly associated with Mayr: two individuals (of different sexes) belong to the same species if they leave viable and fertile offspring. This definition not only predates Mayr, but also Darwinian theory. For the concept(s) of species in Mayr, Cf. de Queiroz (2005). In order to extend this criterion to reproduction by duplication, already in the 1970s Pratt (1976) suggested defining species as groups whose members possess genotypes where homologous genes are substitutable for each other, without altering the viability of the resulting organisms: "two genotypes of the same species resemble each other to the extent that in general any gene of one could replace the corresponding gene of the other forming a genotype that would produce a viable organism" (Ibid., p. 357).

²⁹ If you look closely, it is the name that is recent, although the reality is ancient. The food tasters of kings and emperors were a well-known case, where the reasons that saved the "royal" recipients from risks were not ethical, but political. On the other hand, experimentation on humans has not been as scarce as desirable -except for voluntary experimentation for altruistic reasons.

In the part-whole relationship, surrogate nursing from one species to another (the she-wolf of Rome or the more humble goat's milk) is very ancient. Much more recent are [58] blood transfusions, prostheses and organ transplants (the most famous, a heart transplant by Barnard, dates from 1967; a kidney transplant from 1936). In these cases, adequacy is defined as feasibility.

Table 3 summarizes the combinatorics of these dichotomies, in which type IV is broken down into eight subtypes:³⁰

REPRODUCTIONS	TOTAL	PARTIAL	REPRODUCTIONS
MORPHOLOGICAL	Global anatomical models	Implants and corsets	BIOLOGICAL
MORPHOLOGICAL	Maquettes	Pieces, partial anatomical models	TECHNOLOGICAL
FUNCTIONAL	Clones, model organisms	Organs and prostheses	BIOLOGICAL
FUNCTIONAL	Dies, molds, 3D printers	Spare parts	TECHNOLOGICAL

Table 3

4. Conclusions

Two parts of the analysis should be synthetically reflected in these conclusions. The first comprises the exposition of the concept of the model relationship as an analogical relationship of proportionality. In this sense, it was emphasized that, although the denomination is of recent historical vintage (it reached its apogee in the last century and continues in the present), the idea has an ancient and long history that comes from Aristotle, is augmented in the Scholastic theory of analogy and manifests itself in the form of the Leibnizian theory of expression. As numerous excellent studies have pointed out, the latter has in the philosopher of Hannover a general character since it crosses all the dimensions of his philosophy, not limiting itself to formal aspects -of course, in Leibniz this distinction is artificial and false.

The Leibnizian idea of expression, freely interpreted as an analogy of proportionality, allows us to formulate the model relation as a relation between four terms where something is a model of a thing (expressed) if and only if it exists in that something (expressive thing) relations that correspond univocally, preserving the structure thereof, to the relations existing in the expressed thing. In more common terminology one would say that the relation of expression is at least that of homomorphism. The revision of the model relation led to this point.

[59]

³⁰ The usefulness of a classification is always relative to a scale of analysis and its detail should not be extended beyond what is reasonable. The classification of a set A of elements a_i into unit subsets $\{a_i\}$, based on exhaustive information about a_i , is correct, but informatively useless. This will be discussed shortly in a forthcoming paper.

Once this characterization was achieved, having also taken into account the relation between expression and representation, it was emphasized that there is a rather generalized tendency to understand the model relation as a relation of representation. Having taken this assimilation seriously, we proceeded to develop internally, within a semiotic framework, the very idea of representation. This led to distinguishing, first of all, in the semantic relations between the realm of *assignation* (determined by sign-object relations, among which is that of representation to which that of the model is most often assimilated) and that of *embodiment* (determined by object-sign relations, insofar as every sign is realized materially and above all corporeally), an equally semantic realm. From such a development of the relation of representation, we obtain a quadripartite typology of models.

Two types are mainly linked to the whole of the semiotic sciences and the corresponding philosophies. The first, structural homologies, establishes the formal conditions of any model relation in line with the idea of analogy developed in the first part. The second one, satisfactory representations, now take up the notion of representation bounded in the field of interpretation procedures and in relation to truth. The third and fourth types, in contrast to the two previous ones, are more characteristic of biological and technological contexts than of logical and epistemological ones.

The third type, efficacious embodiments, has to do with the various ways in which signs are realized in organisms and artifacts, thus providing a second semantic perspective without which the first -the representational- would lack an appropriate material basis. In the realm of embodiment, one shifts from the natural history to the cultural (technological) history of semantics. The ascription of the predicate "efficacious" to models of this type (embodiments) combines their condition of efficient systems (transformers of energy into work) with the effectiveness of resolute systems (even algorithmic in some cases) of materially realized signs. Finally, the fourth type, the adequate reproductions, introduces a perspective diagonally opposite to that of the first type, in which there is the distinction between being a model of and model for - respectively, methodological normativity and technological normativity- is given according to biological and technological entities of different history and complexity. This approach culminates with the presentation of eight subtypes of the adequate reproductions, according to three oppositions: partial/total, morphological/functional and biological/technological.

[60]

Appendix:

The Latin text of Leibniz's opusculum *Quid sit idea*, as it appears in Gerhardt's edition, volume VII (Cf. Bibliography), with page numbers included, is reproduced below.

III

Quid sit Idea.

[263] Ante omnia (autem) Ideae nomine intelligimus aliquid, quod in mente nostra est; vestigia ergo impressa cerebro non sunt ideae, pro certo enim sumo Mentem aliud esse quam cerebrum aut subtiliorem substantiae cerebri partem.

Multa autem sunt in mente nostra, exempli causa cogitationes, perceptiones, affectus, quae agnoscimus non esse ideas, etsi sine ideis non fiant. Idea enim nobis non in quodam cogitandi actu, sed facultate consistit, et ideam rei habere dicimur, etsi de ea non cogitemus, modo data occasione de ea cogitare possimus.

Est tamen et in hoc difficultas quaedam, habemus enim facultatem remotam cogitandi de omnibus, etiam quorum ideas forte non habemus, quia facultatem habemus eas recipiendi; idea ergo postulat propinquam quandam cogitandi de re facultatem sive facilitatem.

Sed ne hoc quidem sufficit, nam qui methodum habet quam si sequatur ad rem pervenire possit, non ideo habet ejus ideam. Ut si ordine enumerem Coni sectiones, certum est me venturum in cognitionem Hyperbolarum oppositarum, quamvis nondum earum ideam habeam. Necesse est ergo esse aliquid in me, quod non tantum ad rem ducat, sed etiam eam exprimat.

Exprimere aliquam rem dicitur illud, in quo habentur habitudines, quae habitudinibus rei exprimendae respondent. Sed eae expressiones variae sunt; exempli causa, modulus Machinae exprimit machinam ipsam, scenographica rei in plano delineatio exprimit, solidum, oratio exprimit cogitationes et veritates, characteres exprimunt. numeros, aequatio Algebraica exprimit circulum aliamve figuram: et quod expressionibus istis commune [264] est, ex sola contemplatione habitudinum experimentis possumus venire in cognitionem proprietatum respondentium rei exprimendae. Unde patet non esse necessarium, ut id quod exprimit simile sit rei expressae, modo habitudinum quaedam analogia servetur.

Patet etiam expressiones alias fundamentum habere in natura, alias vero saltem ex parte fundari in arbitrio ut sunt expressiones quae fiunt per voces aut characteres. Quae in natura fundantur, eae vel similitudine aliquam postulant, qualis est inter circulum magnum et parvum, vel inter regionem et regionis tabulam geographicam; vel certe connexio qualis est inter circulum et ellipsin quae eum optice repraesentat, quodlibet enim punctum ellipseos secundum certam quandam legem alicui puncto circuli respondet. Imo circulus per aliam figuram similiorem in tali casu male repraesentaretur. Similiter omnis effectus integer repraesental causam plenam, possum enim semper ex cognitione talis effectus devenire in cognitionem suae causae. Ita facta cujusque repraesentant ejus animum, et Mundus ipse quodammodo repraesentat DEUM. Fieri etiam potest ut ea sese mutuo exprimant quae oriuntur ab eadem causa, exempli

causa gestus et sermo. Ita surdi quidam loquentes non ex sono, sed ex motu oris intelligunt.

Ideam itaque rerum in nobis esse, nihil aliud est, quam DEUM, autorem pariter et rerum et mentis, eam menti facultatem cogitandi impressisse, ut ex suis operationibus ea ducere possit, quae perfecte respondeant his quae sequuntur ex rebus. Etsi itaque idea circuli non sit circulo simili tamen ex ea veritates duci possunt, quas in vero circulo experientia haud dubie esset confirmatura.

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