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# Characterization of condenser microphones under different environmental conditions for accurate speed of sound measurements with acoustic resonators

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Condenser microphones are more commonly used, and have been extensively modelled and characterized, in air at ambient temperature and static pressure. However, several applications of interest for metrology and physical acoustics require to use these transducers in significantly different environmental conditions. Particularly, the extremely accurate determination of the speed of sound in monoatomic gases which is pursued for a determination of the Boltzmann constant  $k$  by an acoustic method, entails the use of condenser microphones mounted within a spherical cavity, over a wide range of static pressures, at the temperature of the triple point of water (273.16 K). To further increase the accuracy achievable in this application, the microphone frequency response and its acoustic input impedance need to be precisely

determined over the same static pressure and temperature range.

Few previous works have examined the influence of static pressure, temperature and gas composition on microphone's sensitivity. In this work, the results of relative calibrations of 1/4" condenser microphones obtained using an electrostatic actuator technique are presented. The calibrations are performed in pure helium and argon gas, at temperatures near 273 K, in the pressure range between 10 kPa and 600 kPa. These experimental results are compared with the predictions of a realistic model available in the literature, finding a remarkable good agreement. The model provides an estimate of the acoustic impedance of 1/4" condenser microphones as a function of frequency and static pressure, and is used to calculate the corresponding frequency perturbations induced on the normal modes of a spherical cavity when this is filled with helium or argon gas.

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## I. INTRODUCTION

Recent advances in theoretical modelling and experimental practice of spherical and quasi-spherical acoustic resonators have led to speed of sound measurements of unprecedented accuracy [1, 2]. These results were used to obtain the currently accepted values of both the Universal Gas Constant  $R$  and the Boltzmann constant  $k$  [3], and evidenced significant differences between the thermodynamic temperature  $T$  and the International Temperature

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Scale of 1990 [4], promoting the current proposal of the future adoption of a new definition of the unit of temperature, the kelvin, which shall be based on an exact definition of the value of  $k$  [5]. In view of this significant change for metrology, an international research effort is being pursued with different experimental methods, which include the use of acoustic quasi-spherical resonant cavities [6], aiming at a further reduction of the uncertainty in the determination of  $k$ , which should be reduced to a value below 1 ppm. In order to achieve this objective, the frequency response and acoustic impedance of the microphones used to excite and detect the acoustic field inside the cavity need to be determined over the whole range of operative conditions of frequency and thermodynamic variables.

The basic relationship (whose exhaustive description and discussion are found in reference [1]) between  $k$  and the zero-static pressure speed of sound  $c_0$  in a gas of molar mass  $M$  and ideal heat capacity ratio  $\gamma_0$  at the temperature of the triple point of water  $T_w$  is

$$k = \frac{R}{N_A} = \frac{1}{N_A} \frac{c_0^2 M}{\gamma_0 T_w}, \quad (1)$$

where  $N_A$  is the Avogadro constant.

If the gas is confined within a spherical cavity of internal radius  $a$ , the speed of sound  $c_0$  is determined from the measurement of the resonance frequency  $f_{0n}$  of the  $n^{th}$  purely radial acoustic mode excited within the gas-filled cavity, leading to expressing the Boltzmann constant  $k$  as follows

$$k = \frac{M}{N_A \gamma_0 T_w} \left( \frac{f_{0n}}{z_{0n}} \right)^2 4\pi^2 a^2, \quad (2)$$

$z_{0n}$  being the eigenvalue of the  $n^{th}$  radial mode.

The frequency range of interest for an acoustic determination of  $k$  depends on the spherical cavity dimensions, the molar mass of the monoatomic gas, and the number of acoustic modes

under test (data from several resonant modes are recorded and compared to evidence possible systematic errors). As a numerical example, the data in Table I report the frequency ranges spanned by the first seven purely radial modes in helium or argon gas, for two resonators with an inner radius of 50 mm and 90 mm, respectively.

	Resonator radius 50 mm (volume 0.5 dm <sup>3</sup> )	Resonator radius 90 mm (volume 3 dm <sup>3</sup> )
Argon gas	4 kHz – 24 kHz	2 kHz – 13 kHz
Helium gas	13 kHz – 74 kHz	7 kHz – 42 kHz

Table I: Frequency ranges of interest for the determination of the Boltzmann constant.

Among several different sources of uncertainty which set the current limit of accuracy achievable in the determination of the different quantities in equation (2), one contribution is represented by perturbations to the ideal values of the resonance frequencies  $f_{0n}$  caused by a non-uniform acoustic impedance of the cavity boundary [7]. In a typical experimental configuration, one such non-uniformity is represented by the finite acoustic input impedance of the acoustic transducers used to produce and measure the acoustic field within the resonator.

To reduce the impedance mismatch with the solid metal shell comprising the resonator wall, small diameter and high input impedance transducers are thus preferred. A common choice consists in using a couple of 1/4" condenser microphones (Figure 1) flush-mounted on the wall of the resonator, one used a transmitter, and the other as a receiver.

Over the whole frequency range of interest here, shown in Table I, the acoustic input impedance of the microphone, inversely related to its sensitivity, determines the magnitude of the perturbing effect on the acoustic field within the cavity. This acoustic input impedance depends on both the mechanical parameters of the membrane and the acoustic load behind the diaphragm, represented by the fluid layer trapped between the membrane and the back-plate, connected to a small backing closed cavity (Fig. 1), which may significantly vary,

depending on the thermodynamic properties and state of the gas.

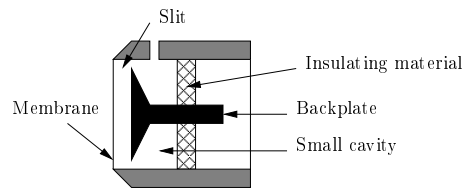


Figure 1: Condenser microphone.

On the other hand, given the extremely high quality factors of the purely radial modes of a spherical resonator, which are typically in the order of  $5 \times 10^3$  to  $2 \times 10^4$ , the flatness and smoothness of the frequency response of the microphones, even when not optimal, does not condition the achievable precision and accuracy in the determination of the modes resonance frequencies and halfwidths. However, particularly at the lower static pressures, the decreasing microphones sensitivity reduces the signal-to-noise ratio, imposing an upper operating frequency limit near the resonance frequency of the membrane. According to the manufacturers data, the frequency response of a 1/4" pressure-field microphone in air at standard reference conditions (static pressure 101.325 kPa, temperature 23°C, relative humidity rate 50 %), this operating frequency limit is at about 70 kHz.

A few previous experimental and theoretical studies have examined the influence of static pressure, temperature and gas composition on the microphone's frequency response [8, 9]; however, the wide range of thermodynamic conditions of interest for the determination of the Boltzmann constant is outside the bounds of these investigations. For this application, the frequency perturbation due to condenser microphones on the wall of the resonator had been previously calculated [1] under the hypothesis that the motion of the membrane was limited only by its stiffness, and its acoustic input impedance was modelled as a simple compliance, estimated from the equivalent volume declared by the manufacturer in standard reference

conditions.

In this work, the acoustic input impedance, the sensitivity and the frequency response of the microphones, leading to the frequency perturbation they induce on the normal modes of a spherical cavity, are calculated using a model [10], which consistently takes into account the coupling between the movement of the membrane, the fluid air film trapped between the membrane, the backplate, and the fluid enclosed in the backing cavity, depending on the thermodynamic gas conditions. This model leads to the frequency perturbation induced on the normal modes of a spherical cavity by these microphones.

As a validation of this model, its predictions are compared to the results of an experimental characterization of 1/4" pressure-field condenser microphones, by a relative calibration performed with an electrostatic actuator method in the thermodynamic conditions of interest for the determination of  $k$ , namely, in helium and argon gas, near the temperature  $T_w = 273.16$  K, for static pressures ranging from 10 kPa to 600 kPa.

## II. ACOUSTIC MODELLING OF CONDENSER MICROPHONES ADAPTED TO SPECIFIC THERMODYNAMIC GAS CONDITIONS

The model of an electrostatic microphone, which is extensively discussed in reference [10], provides a description of the acoustic and mechanical system as a whole, taking into account the strong coupling effects between the membrane and the fluid.

The acoustic modelling of the microphone is carried out by considering the motion of the membrane in the presence of the restoring forces due to its tension and inertia, and the behaviour of the air gap, expressed by: *i*) the Poiseuille's law, which takes into account the effects of inertia, of the pressure gradient due to internal forces, of shear viscosity - whose influence is important inside the viscous boundary layer at the membrane and backing

electrode interfaces - and of the external forces due to the movement of the membrane; *ii*) the mass conservation law; *iii*) the heat conduction equation in the thermal boundary layers near the membrane and the back electrode. The boundary conditions of the equations include the effect of the backing cavity, non slip conditions and isothermal conditions on the membrane and back plate.

The solution of the problem is expressed as an expansion on the eigenmodes of both the membrane and the air gap, these solutions accounting for the strong coupling between each part of the system.

Then, assuming a time dependence of the form  $e^{j\omega t}$ , where  $\omega$  is the angular frequency, the complete open-circuit (assuming infinite input electrical impedance of the preamplifier) acoustic input admittance  $Y_a$  of the membrane (radius denoted  $R_M$  below) loaded by small acoustic elements (thin air film of thickness  $h$  between the diaphragm and the backplate, small backing closed cavity, both linked together though a slit at the periphery of the backing electrode for the 1/4" microphones) can be modelled by a lumped elements circuit, which represents the acousto-mechanical behaviour of the microphone. **The circuit depends on the acoustic impedances  $Z_n$  according to the following relation:**

$$Y_a = \sum_{n=1}^N \frac{1}{Z_n} , \quad (3)$$

where the acoustic impedances  $Z_n$  take the form

$$Z_n = \frac{j\rho_0 c k_n^2 B_n}{4\pi N_n} , \quad (4a)$$

the parameter  $B_n$ , which accounts for the tension  $T_M$  and the mass (factor  $T_M K_M^2$ ,

see below) per unit area of the membrane, being given by

$$\rho_0 c \omega B_n = T_M (K_M^2 - k_n^2) + \frac{\zeta}{\chi^2 - k_n^2}, \quad (4b)$$

the factor

$$N_n = 1 - \mu_n \frac{\sum_{l=1}^N \alpha_l}{1 + \sum_{l=1}^N \alpha_l \mu_l}, \quad (4c)$$

expressing the effect of the peripheral boundary conditions, through the factors

$\alpha_n$  and  $\mu_n$ , as follows

$$\alpha_n = \frac{\sqrt{2} \rho_0 \omega^2 Z}{[Z F_v \chi J_1(\chi R_M) - j \omega \rho_0 J_0(\chi R_M)]} \frac{1}{\rho_0 c \omega B_n (\chi^2 - k_n^2) h R_M}, \quad (4d)$$

$$\mu_n = -\frac{\sqrt{2} k_n^2 J_0(\chi R_M)}{\chi^2 - k_n^2}, \quad (4e)$$

where  $k_n R_M = z_n$  is the  $n^{\text{th}}$  zero of the zero order Bessel function of the first kind  $J_0$ ,  $J_1$  is the first order Bessel function of the first kind, and where

$$K_M^2 = \omega^2 \frac{\mu_M}{T_M}, \quad (5)$$

$K_M$  and  $\mu_M$  being respectively the propagation constant of flexural vibrations within the membrane, and the surface density of the membrane, which depends on its volume density and thickness. **The specific input impedance  $Z$  of the backing cavity of volume  $V$  and the complex wavenumber  $\chi$  (which includes the effect of viscosity and**

thermal conduction, see below) are respectively given by

$$Z = \frac{2\pi R h_s \rho_0 c^2}{j \omega V}, \quad (6)$$

where  $R$  is the radius of the backplate and  $h_s$  the width of the peripheral circular slit,

$$\chi^2 = \frac{\omega^2}{c^2} \gamma \frac{1 - (\gamma - 1) F_h / \gamma}{F_v}, \quad (7a)$$

with

$$\zeta = -\frac{\rho_0 \omega^2}{h F_v}, \quad (7b)$$

the coupling factor between the membrane and the air gap, and with

$$F_{h,v} = 1 - \frac{\tan(k_{h,v} h / 2)}{k_{h,v} h / 2}, \quad (7c)$$

$$k_v^2 = -\frac{j \omega}{c \ell'_v}, \quad k_h^2 = -\frac{j \omega}{c \ell_h}, \quad (7d)$$

$\ell'_v = \mu / (\rho_0 c)$  and  $\ell_h = \lambda / (\rho_0 c C_p)$  being the viscous and thermal characteristic lengths respectively,  $\mu$  the coefficient of shear viscosity,  $\lambda$  the coefficient of thermal conductivity,  $\gamma$  the specific heat ratio,  $\rho_0$  the mass density, and  $c$  the adiabatic speed of sound in the gas.

The open-circuit pressure sensitivity of the microphone, inversely proportional to the acoustic impedance  $Z_a = 1/Y_a$  of the microphone is given by

$$\sigma = -\frac{U_{r0}}{j \omega h S_M Z_a}, \quad (8)$$

where  $U_{r0}$  is the polarization DC voltage of the microphone, and  $S_M = \pi R_M^2$  is the surface

of the membrane.

In order to calculate the sensitivity of the microphone as a function of the frequency from the model described above, several different structural parameters of the transducer are needed. These could not be determined by direct, non-intrusive measurements for the specific microphone used in this work, rather, we used the typical, approximate values for the product series (here Brüel & Kjær type 4938) given by the manufacturer (Table II).

Membrane radius (mm)	2
Backplate radius (mm)	1.75
Thickness of the air film ( $\mu\text{m}$ )	18
Membrane tension (N/m)	3128
Membrane density ( $\text{kg}/\text{m}^3$ )	8300
Membrane thickness ( $\mu\text{m}$ )	6.95
Volume of the backing cavity (estimated value, $\text{mm}^3$ )	8

Table II: Typical approximate values of the structural parameters of a condenser microphone 1/4" (Brüel & Kjær , type 4938).

Using the model described above, for the two first resonant modes of the microphone [11], and the values reported in Table II, the frequency response of a 1/4" pressure microphone is calculated for frequencies from 100 Hz to 100 kHz, for several different values of the static pressure (10 kPa to 600 kPa), for two different monoatomic gases (helium and argon). The amplitude and phase of the calculated frequency responses in Figures 2 to 5 [12], do evidence some typical features of the static pressure dependence of the frequency response of a microphone, namely, *i*) at the lower pressures, an increase of the quality factor of the microphone resonance, accompanied by a resonance frequency shift towards lower frequencies, which is caused by the decrease in the gas density and an increase in the fluid layer compliance; *ii*) at the higher pressures, a similar increase of the resonance quality factor and a shift of the resonance peak towards lower frequencies as a consequence of the increasing mass-load of the membrane by the gas. This high pressure effect is more evident in the responses calculated

for argon gas, whose mass density is about ten times higher than that of helium.

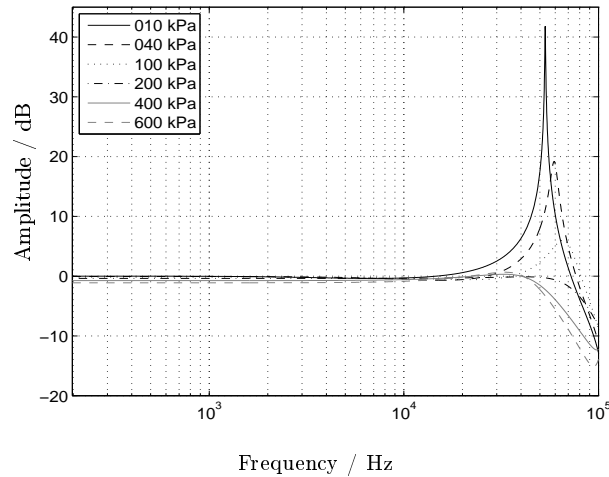


Figure 2: Amplitude of the open circuit frequency response of a 1/4" pressure microphone calculated using the model of reference [10] in helium gas at 273 K.

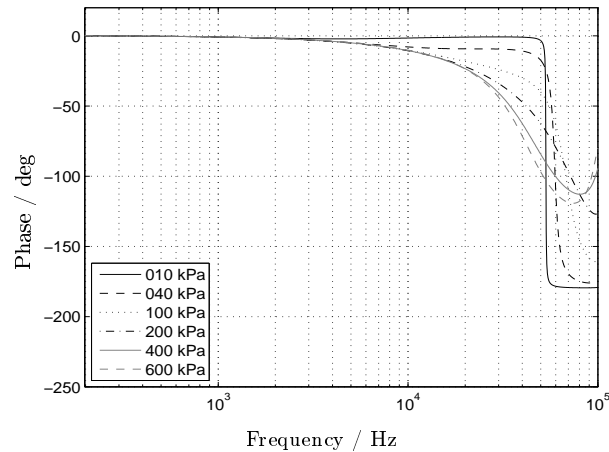


Figure 3: Phase of the open circuit frequency response of a 1/4" pressure microphone calculated using the model of reference [10] in helium gas at 273 K.

The influence on the microphone response, which is caused by the gas composition, is better evidenced by a direct comparison of the frequency responses calculated for the same microphone in helium or argon gas at the single static pressure value 600 kPa (Figures 6 and 7), the amplitudes and phases being here normalized to the value of the frequency response at 200 Hz in helium gas.

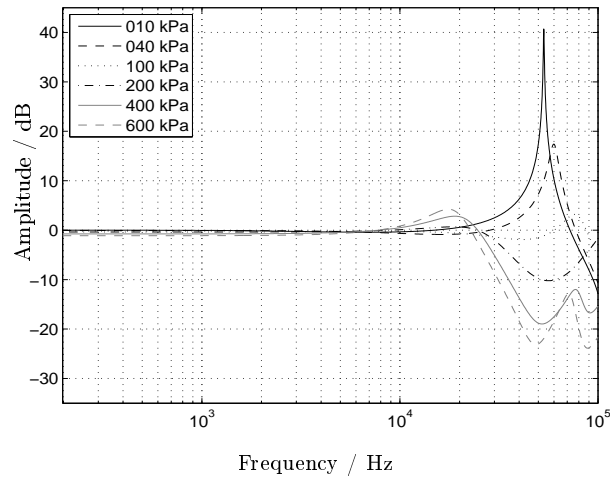


Figure 4: Amplitude of the open circuit frequency response of a 1/4" pressure microphone calculated using the model of reference [10] in argon gas at 273 K.

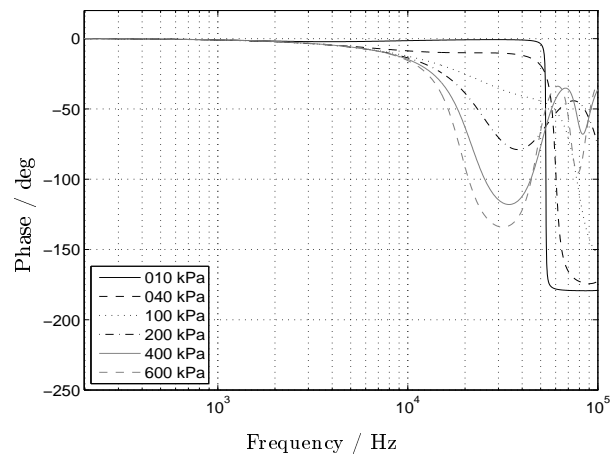


Figure 5: Phase of the open circuit frequency response of a 1/4" pressure microphone calculated using the model of reference [10] in argon gas at 273 K.

It is apparent, from the observation of these frequency responses, and a comparison with the frequency ranges which are of interest for an acoustic determination of  $k$  reported in Table I, that, at this moderately high static pressure, particularly for the case of a small volume acoustic resonator ( $0.5 \text{ dm}^3$ ), many normal modes of the resonator might fall close to, or even at higher frequency than the first resonance of the microphone. In practice, this situation might correspond to a significant decrease in both the precision and accuracy of a

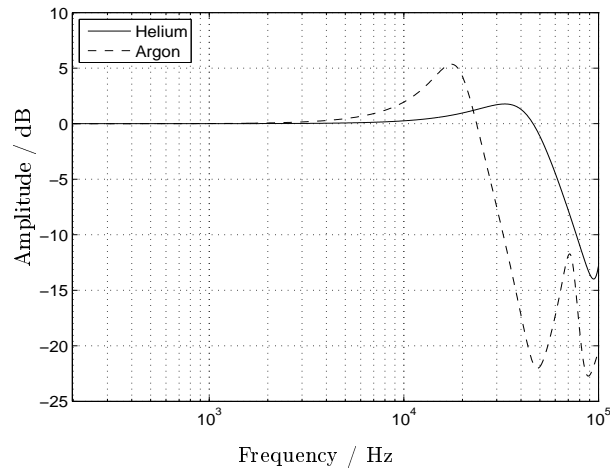


Figure 6: Amplitude of the open circuit frequency response of a 1/4" pressure microphone calculated using the model of reference [10] in helium and argon gas at 273 K and 600 kPa.

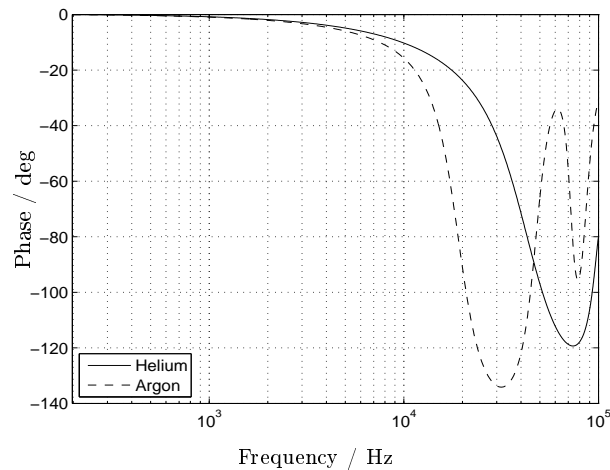


Figure 7: Phase of the open circuit frequency response of a 1/4" pressure microphone calculated using the model of reference [10] in helium and argon gas at 273 K and 600 kPa.

measurement of the speed of sound, the precision being limited by the significant decrease in the microphone sensitivity which takes place at frequencies higher than the first microphone resonance, the accuracy being affected by an increasing frequency perturbation and energy loss for those modes of the cavity which happen to fall close to the microphone resonance frequency itself.

### III. CALIBRATION METHODS FOR THE CHARACTERIZATION OF CONDENSER MICROPHONES

For the particular microphone application which is of interest in this work, i.e. the determination of the frequencies and halfwidths of a series of radial acoustic modes in a gas-filled cavity, only the pressure-field sensitivity, directly linked to the acoustic impedance of the microphone (eq. 8), needs to be determined.

#### A. Absolute calibration of microphones

The absolute method of pressure reciprocity calibration, which makes use of cylindrical cavities (called plane wave couplers) is extensively described in international standards [13] and widely used for 1" and 1/2" Laboratory Standard Microphones. The use of this method for the calibration of 1/4" microphones would require the use of specific adaptors for appropriate mounting of 1/4" cartridges in 1/2" plane wave couplers. The impact of such custom modifications to the dimensions and the geometry of the couplers on the reliability of the calibration is hard to predict, as the accuracy of this method is significantly affected by the capability to model the acoustic field in the cylindrical coupler cavity [14]. In the current standard, the analytical model of the acoustic field in the cavity assumes a plane wave solution. However, as it is discussed in reference [15], the radial wave-motion, which is mainly influenced by the velocity profile of the microphones, should also be taken into account, especially in the highest frequency bandwidth.

Mainly for these reasons, in addition to the practical and technical difficulties to implement the experimental scheme associated with this method at static pressures and temperatures different from ambient, the possibility to use the reciprocity calibration method to

determine the sensitivity of 1/4" microphones over the whole frequency range of interest with the required uncertainty, is discarded.

### B. Relative pressure calibration of microphones using an electrostatic actuator

The electrostatic actuator method is a well established technique for the relative calibration of condenser microphones [16], and is particularly suited for being performed over a wide range of environmental conditions [9]. This method consists in applying on the membrane of the microphone an equivalent acoustic pressure generated by an electrostatic force

$$\begin{aligned} F(t) = F_0 + f(t) &= \frac{\epsilon S_a}{2D(t)^2} U(t)^2 \\ &= \frac{\epsilon S_a}{2D(t)^2} (U_0 + u(t))^2, \end{aligned} \quad (9)$$

which is provided by a circular metallic grid (actuator) of equivalent surface area  $S_a$  placed in front of the microphone membrane at a distance  $D(t)$ ,  $\epsilon$  being the dielectric constant of the gas.

In response to the electrostatic excitation, the displacement of the microphone membrane generates an output voltage  $U_r(t) = U_{r0} + u_r(t)$  at the microphone electric terminals (Figure 8).

The electrical current through the microphone (of capacitance  $c$  "at rest") is  $I(t) = I_0 + i(t)$ . The distance between the diaphragm (surface  $S_M$ , vibrating with the mean velocity  $v$ ), and the backplate is  $d(t) = h + \xi(t)$ , and the distance between the actuator and the diaphragm is  $D(t) = D_0 - \xi(t)$ . The mechanical impedance of the microphone is denoted  $Z_M = Z_a S_M^2$  ( $Z_a = 1/Y_a$  being given by eq. 3), and the acoustic load in front of the diaphragm is represented by its acoustic impedance  $Z_L$ .

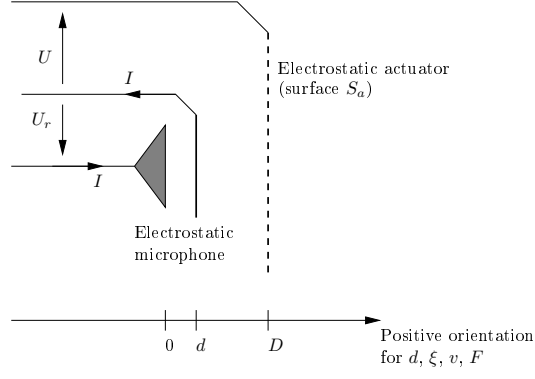


Figure 8: Scheme of the the relative calibration of microphones using an electrostatic actuator technique.

The coupled transduction equations of the microphone [17] are, for the harmonic motion of angular frequency  $\omega$  considered here:

$$u_r = \frac{1}{j\omega c} i + \frac{U_{r0}}{j\omega h} v, \quad (10a)$$

$$\frac{f}{S_M} - Z_L S_M v = -p = \frac{U_{r0}}{j\omega h S_M} i + \frac{Z_M}{S_M^2} S_M v. \quad (10b)$$

Assuming that the polarization voltage  $U_0$  is much higher than the amplitude of superimposed sinusoidal AC voltage  $u$ , and the distance  $D_0$  much larger than displacement  $\xi$ , the total electrostatic force  $F$  can be written as

$$F = F_0 + f \approx -\frac{\epsilon S_a}{2D_0^2} (U_0^2 + 2U_0 u),$$

which corresponds to a variation of the electrostatic force

$$f \approx -\frac{\epsilon S_a U_0}{D_0^2} u. \quad (11)$$

The output voltage  $u_r$  of the microphone, used as a receiver, is

$$u_r = -Z_e i, \quad (12)$$

where  $Z_e$  is the electrical input impedance of the preamplifier connected to the microphone.

Combining equation (12) and the first transduction equation (10a) gives straightforwardly

$$v = \frac{j\omega d_0}{U_{r0}} \left( 1 + \frac{1}{j\omega c Z_e} \right) u_r, \quad (13a)$$

and then, by substitution of equations (11) and (13a) in the second transduction equation (10b), leads to

$$\frac{u}{u_r} = -\frac{D_0^2}{\epsilon\alpha U_0} \left[ -\frac{U_{r0}}{j\omega h S_M Z_e} + \frac{j\omega h S_M Z_a}{U_{r0}} \left( 1 + \frac{Z_L}{Z_a} \right) \left( 1 + \frac{1}{j\omega c Z_e} \right) \right], \quad (13b)$$

$\alpha$  being the ratio  $S_a/S_M$ .

The open-circuit transfer function  $(u_r/u)_{i=0}$  (obtained if  $Z_e$  is an infinite impedance) is then written as

$$\left( \frac{u_r}{u} \right)_{i=0} = -\frac{\epsilon\alpha U_0}{D_0^2} \sigma \frac{Z_a}{Z_a + Z_L}, \quad (14)$$

where  $\sigma$  is the open-circuit pressure sensitivity of the microphone, as previously defined in equation (8).

Assuming that the acoustic impedance  $Z_a$  of the microphone is much higher than the load impedance  $Z_L$ , the transfer function  $(u_r/u)_{i=0}$  is then proportional to the sensitivity of the microphone.

Strictly, the open-circuit tension  $(u_r)_{i=0}$  should be measured making use of the insert tension technique [13]. However, the typical electrical input impedance of 1/4" Brüel &

Kjær preamplifiers is great enough to justify the assumption that  $(u_r)_{i=0} \approx u_r$ .

In order to avoid the possible cross-talk between the membrane of the microphone and the actuator, and between the different cable connections, we chose to set the DC voltage  $U_0$  on the actuator to zero. In this case, the electrostatic force applied on the membrane of the microphone is

$$F = f = -\frac{\epsilon S_a}{2D_0^2} u^2.$$

Making explicit the time-dependence of the voltage  $u$ , in the form  $|u| e^{j\omega t}$ , leads then to

$$f = -\frac{\epsilon S_a |u|^2}{2D_0^2} e^{2j\omega t}. \quad (15)$$

Thus, the angular frequency of the electrostatic force and of the equivalent acoustic pressure applied on the microphone is  $2\omega$  while the angular frequency of the electric input signal is  $\omega$ , minimizing crosstalk between the electrostatic source and the microphone.

The choice of setting the polarization voltage  $U_0$  to zero, eliminating the static force produced by the actuator, which increases the mechanical tension of the membrane, eliminates a possible calibration error.

A drawback of this choice is the strong decrease of the output microphone voltage. In the present work, this decrease is well balanced by using a sensitive lock-in detection technique which provides sufficient signal-to-noise ratio, over the wide range of different environmental conditions examined.

#### IV. EXPERIMENTAL DETERMINATION OF THE FREQUENCY RESPONSE OF CONDENSER MICROPHONES: INFLUENCE OF GAS CONDITIONS

##### A. Experimental set-up

A schematic diagram of the experimental set-up is shown in Figure 9. A 1/2" electrostatic actuator is fitted with a 1/4" adaptor onto the microphone cartridge to be calibrated, following the indications of the current IEC standard [16]. The cartridge and a 1/4" preamplifier are clamped and held in vertical position using a massive aluminium optical mount, within a vacuum and pressure-tight stainless steel cylindrical vessel, which is suspended and immersed in a stirred liquid bath thermostat filled with ethanol.

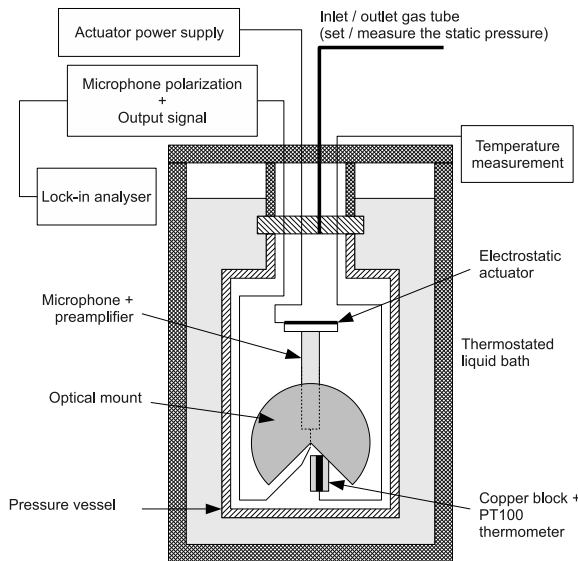


Figure 9: Experimental set-up for the environmental characterization of 1/4" condenser microphones with the electrostatic actuator technique.

The vessel could be alternatively evacuated by means of a system composed by a dry scroll and a turbo-molecular pump, or filled with extremely pure (99.999 %) samples of helium or argon gas, up to a maximum pressure of 800 kPa.

The static pressure is measured with a relative uncertainty of 0.05% using an absolute capacitive transducer, which has been calibrated by comparison with a primary standard (piston gauge pressure balance).

The temperature of the gas within the vessel is inferred from the readings of a commercial capsule-type 100  $\Omega$  platinum resistance thermometer, which has been calibrated by comparison with a secondary standard, and was inserted within a massive copper sleeve attached to the optical mount sustaining the preamplifier and the microphone. This arrangement also serves as a heat sink, to minimize the heating from the preamplifier on the microphone and the surrounding environment. The temperature within the bath and the vessel is precisely maintained close to 273.16 K, by a fine external PID control. The absolute uncertainty achieved in the determination of the gas temperature is then less than 0.02 K.

The electric connections from the voltage supplies to the preamplifier and the actuator, and the cables driving the output signal from the microphone are interrupted through the top of the vessel by vacuum and pressure-tight feedthroughs. The AC voltage fed to the actuator, is initially synthesized by a function generator at the level of 1 Vrms and successively amplified to 100 Vrms with no DC component (cf. § III B) by an electrostatic actuator supply.

The polarization voltage applied to the microphone under test is reduced from the typical 200 V value to 125 V to eliminate sporadic arcing phenomena which occurred at the lower working pressures in helium, and were revealed by the detecting electronic system as high voltage pulses.

The in-phase and quadrature components of the output microphone signal is recorded with a lock-in analyser locked on the second harmonic of the reference synthesizer signal. The frequency scan spans the range from 200 Hz to 100 kHz in 1000 discrete steps differently

distributed along several frequency sub-ranges, from a minimum of 8 Hz separation between successive points at the lower frequencies to a maximum separation of 200 Hz at the highest frequencies.

### B. Experimental frequency responses

The extremely good agreement between the predictions of the theoretical model, previously discussed in section II, of the amplitude and phase responses of a 1/4" pressure-field condenser microphone (Brüel & Kjær type 4938), for both helium and argon gas over the wide static pressure range 10 kPa to 600 kPa, is apparent by a comparison of the Figures 2 to 7 with the experimentally recorded data for both gases over the same range of thermodynamic conditions, which are illustrated below in Figures 10 to 15.

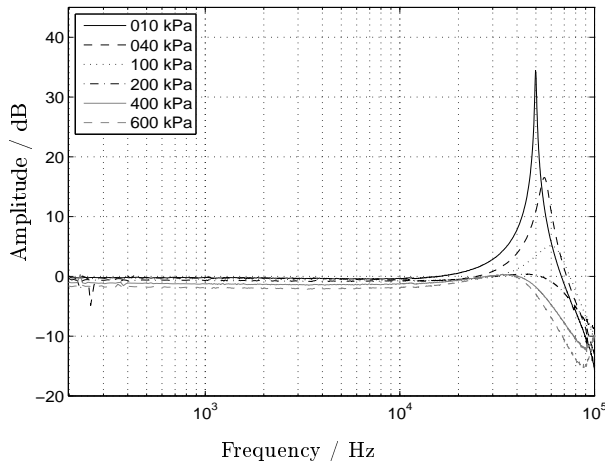


Figure 10: Amplitude of the response of a 1/4" pressure microphone to the electrostatic actuator measured in helium gas at 273 K.

Despite both quantitative and qualitative satisfactory agreement between the theoretical model and the experimental data, a few discrepancies are apparent and are worth discussion. **At the lowest static pressures, a maximum difference of 9 % is found between the model prediction and the experimentally observed microphone resonance**

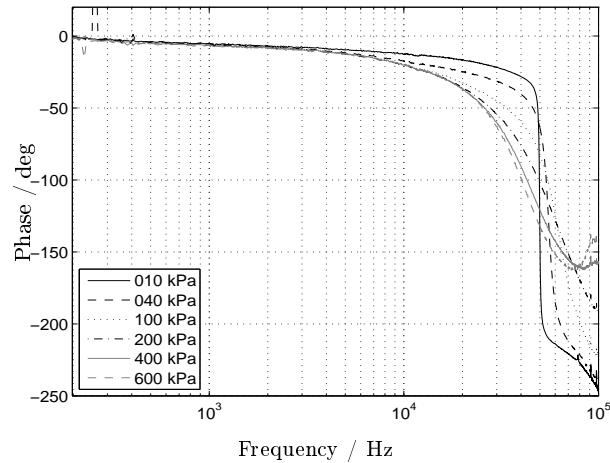


Figure 11: Phase of the response of a 1/4" pressure microphone to the electrostatic actuator measured in helium gas at 273 K.

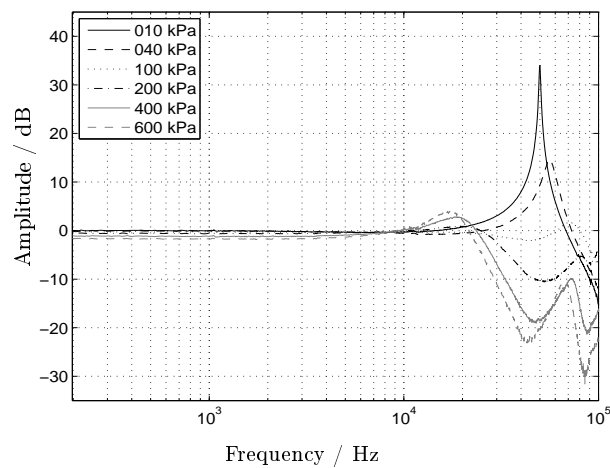


Figure 12: Amplitude of the response of a 1/4" pressure microphone to the electrostatic actuator measured in argon gas at 273 K.

frequency (Figures 16 and 17). This is likely to be mainly due to the imperfect knowledge of the intrinsic mechanical parameters of the membrane, whose approximated values are inferred from those declared by the manufacturer. **According to the analytical model describing the behaviour of the microphone, this discrepancy would be justified if the assumed estimate of the tension or the thickness of the membrane was wrong by the same relative amount. It seems likely that the reproducibility**

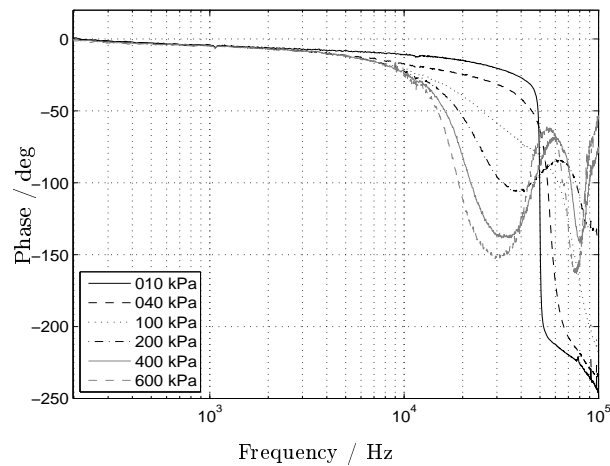


Figure 13: Phase of the response of a 1/4" pressure microphone to the electrostatic actuator measured in argon gas at 273 K.

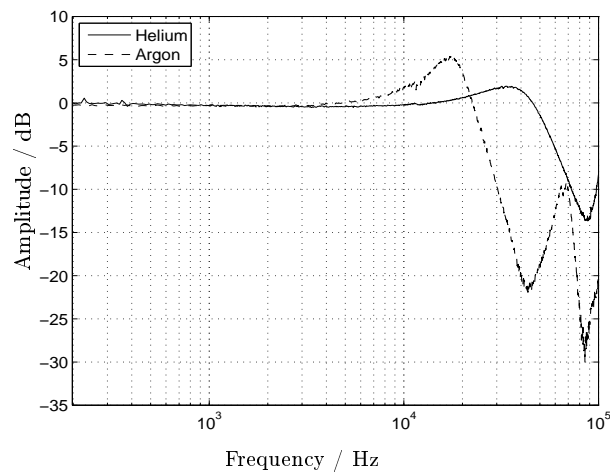


Figure 14: Amplitude of the response of a 1/4" pressure microphone to the electrostatic actuator measured in helium and argon gas at 273 K and 600 kPa.

of the manufacturing procedure and materials may vary within these bounds. Also, the ageing of the microphone may significantly vary the membrane tension. As far as we know, there is currently no non-destructive method which would provide a direct measurement of these mechanical parameters.

Also, at the lowest static pressures examined in this work, the effect of the structural damping of the membrane should become significant in comparison to the damping of the

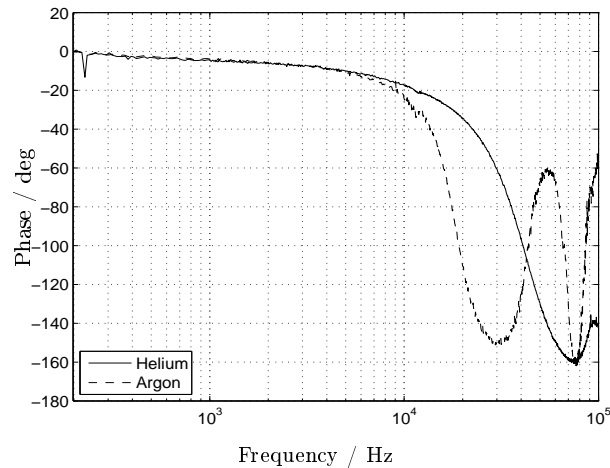


Figure 15: Phase of the response of a 1/4" pressure microphone to the electrostatic actuator measured in helium and argon gas at 273 K and 600 kPa.

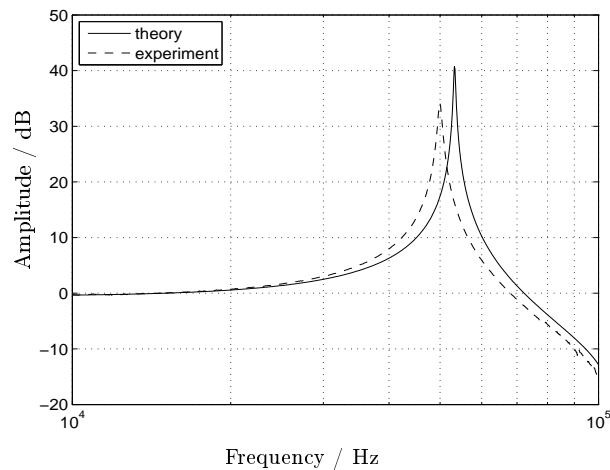


Figure 16: Amplitude of the theoretical and experimental frequency response of a 1/4" pressure microphone in argon gas at 273 K and 10 kPa.

membrane motion due to thermal and viscous dissipation mechanisms taking place in the fluid layer confined between the membrane and the backplate. However, the structural damping of the membrane is neglected and not accounted for by the model.

The smooth decreasing phase slope which is observed over the whole frequency range can not be predicted or explained by the model. Possibly, the assumptions that the electrical impedance  $Z_e$  of the complete measurement set-up

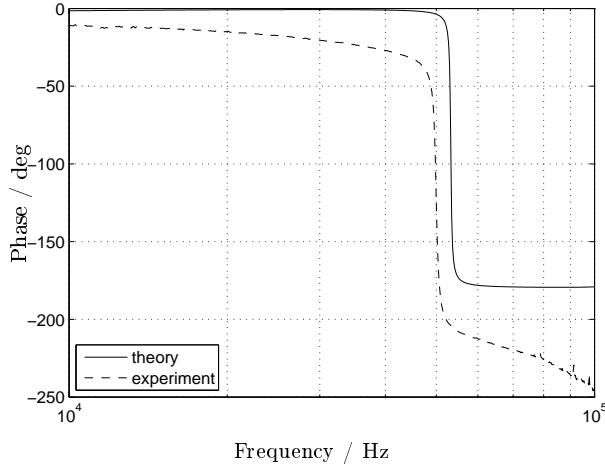


Figure 17: Phase of the theoretical and experimental frequency response of a 1/4” pressure microphone in argon gas at 273 K and 10 kPa.

(including the microphone preamplifier, the cables and measurement electronics) is infinite, and that the acoustic load impedance  $Z_L$  in front of the microphone is negligible might be incorrect and should be re-considered to include in a consistent manner the delay causing this phase slope in the complete model of the electro-mecano-acoustic transduction system.

## V. PERTURBATIONS INDUCED BY CONDENSER MICROPHONES ON THE RADIAL RESONANCE MODES OF A SPHERICAL RESONATOR

The effect of a non-uniform boundary admittance on the ideal, unperturbed eigenfunctions of a hollow resonant cavity, can be calculated using boundary perturbation theory [7]. Particularly, the shifts in the eigenfrequencies of the purely radial modes  $f_{0n}$ , do not depend on the relative angular position of the transducers, given the radial symmetry of the associated eigenfunctions. The frequency shifts  $\Delta f_{0n}$  due to the presence of a couple of acoustic transducers of total surface area  $2S_M$ , which are flush-mounted on the inner resonator wall,

are expressed as follows:

$$\frac{\Delta f_{0n}}{f_{0n}} = \operatorname{Re} \left( \frac{j \rho_0 c Y_a}{2\pi a^2 z_{0n}} \right). \quad (16)$$

By making the simplifying assumption that the acoustic input admittance of a microphone may be considered as a simple compliance  $C_M = S_M^2/8\pi T_M$ , the relative perturbation to the frequencies of the radial modes is

$$\frac{\Delta f_{0n}}{f_{0n}} = -\frac{\rho_0 c^2 \chi_M S_M}{2\pi a^3}, \quad (17)$$

where  $\chi_M = C_M/S_M$  is the compliance of the transducer per unit area.

The above simplifications were adopted to estimate the transducer corrections in the latest determination of the Boltzmann constant  $k$  [1], which was obtained by measuring the speed of sound in argon, using experimental data for the five lower order radial modes of a spherical cavity with an inner radius  $a$  of about 90 mm. In that case, the corresponding frequency shift was estimated (using equation 17 and the nominal value  $\chi_M = 1.5 \times 10^{-10}$  m/Pa), to be a linear function of the static pressure  $P_0$  within the cavity:  $\Delta f_{0n}/f_{0n} = -0.8 \times 10^{-6} P_0/\text{MPa}$ , and therefore considered to be negligible.

However, the above discussed simplifying assumptions would not longer hold if an accurate speed of sound measurement is pursued in helium gas within a cavity with a smaller volume. Actually, as it is apparent from observation of the simplified model in equation 17, the amplitude of the frequency shift is expected to be inversely proportional to the cavity volume, as is the bandwidth spanned by the first few radial modes of the cavity which are subject to measurement. Particularly for a light gas like helium, the upper limit of this bandwidth is higher than the first microphone resonance, where the assumption that the motion of the membrane is limited by stiffness is not appropriate.

Finally, while it is true that, for the sake of an acoustic determination of  $k$ , the extrapolation of experimental data to zero static pressure would reduce the systematic influence of the microphone perturbation close to zero, this is strictly true only if the perturbation is assumed to be a monotonic smooth function of the gas pressure.

A more realistic estimate of the amplitude and the static pressure dependence of the frequency shifts is obtained using equation 16, with the microphone input admittance given by equation 3 (derived from the complete model of reference [10] described in section II). The calculated relative frequency perturbations for the first seven purely radial modes excited within a spherical cavity having an inner radius of 50 mm, filled with argon or helium gas at 273.16 K, at static pressures up to 800 kPa, are shown on Figures 19 and 18.

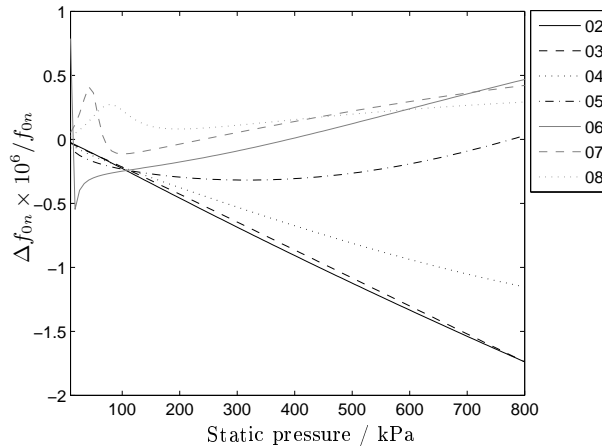


Figure 18: Frequency shifts from the resonance frequencies of (0,2) to (0,8) radial modes of the spherical resonator due to two 1/4" pressure microphones calculated using the model of reference [10] in helium gas at 273.16 K.

The calculated perturbations show a smooth monotonic linear trend over the whole pressure range for the lower order radial modes (0,2) to (0,5) when the cavity is filled with argon (Fig. 19). The corrections for the higher frequency modes (0,6) to (0,8) lose their monotonic trend in the highest static pressure range as a consequence of the significant decrease of the microphone resonance which can be observed in the sensitivity response shown in

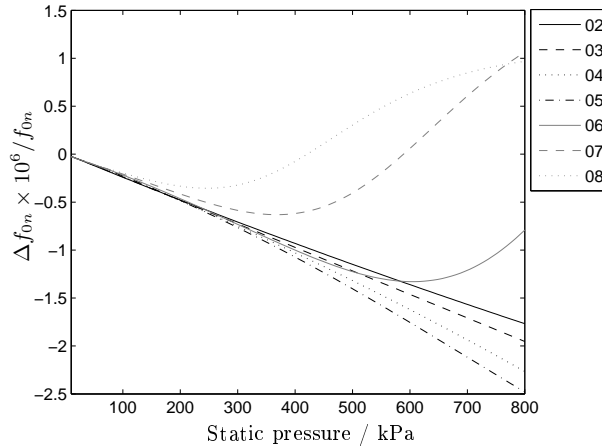


Figure 19: Frequency shifts from the resonance frequencies of (02) to (08) radial modes of the spherical resonator due to two 1/4” pressure microphones calculated using the model of reference [10] in argon gas at 273.16 K.

Figure 4.

When the cavity is filled with helium gas, the perturbations to different modes show a more complicated trend. Particularly, the curves of three highest frequency resonances show a sharp peak at the lowest static pressures, due to their proximity with the first microphone resonance, which is itself a varying function of the static pressure, as it was shown in Figure 2. It is worth noticing that, over the whole static pressure range, the slope of the calculated perturbations changes sign from negative (as in the case of argon) to positive, for those cavity radial modes whose frequencies are respectively lower or higher than the first microphone resonance.

## VI. CONCLUSION

A complete model currently available in the literature [10] is used to calculate the acoustic input impedance and the sensitivity of 1/4” condenser microphones as a function of the frequency, for two different monoatomic gases, over the wide static pressure range which is

of interest for an acoustic determination of the Boltzmann constant.

The validity of the model was confirmed by a relative calibration of the microphone with an electrostatic actuator in the same thermodynamic conditions. These results allowed to estimate the frequency perturbations to the normal modes of resonant spherical cavity induced by the impedance mismatch between the transducer and the cavity wall.

Some minor discrepancies between the model predictions and the experimental results are liable of a twofold interpretation: *i*) they indicate that the model might be improved, possibly to take into account several minor effects which are currently neglected in its formulation (namely the non-uniform motion of the membrane and its structural damping, the impedance of the acoustic load in front of the membrane, the electrical input impedance of the preamplifier), *ii*) given the sensibility of the calculated frequency responses to slight changes in the intrinsic mechanical parameters of the microphone, the discrepancies indicate that these parameters are only imperfectly known.

For the sake of improving the model, the present experimental results, which are recorded in reproducible conditions with careful experimental practice, may represent a valid aid. Alternatively, if an improved version of the model could be available in the future (possibly based on more complete model of electrostatic transducers like the one described in reference [18]), it might be used to interpolate the experimental data to achieve a more reliable estimate of the microphone intrinsic parameters. If achieved, these advances might help the study, design and realization of custom electrostatic transducers for specific applications in

environmental conditions different from ambient.

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