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Fractal Dimension of Cathode Spots in a High-Current Vacuum Arc Thruster

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Abstract

The fractal dimension of cathode spots occurring in the discharge of a high-current vacuum arc thruster (VAT) are determined according to the Minkowski-Bouligand method. Photographs taken during operation of a 30 W VAT allowed observation of spots distribution over the surface depending on the cathode material. A machine learning model as well as a heuristic analysis permit to outline that work function, cohesive energy and ionization energy of the cathode material are the main driver in the magnitude of fractal dimension and total surface area covered by the spots.

1. Introduction

1 The formation of ramified patterns has been observed for a long time in
2 the case of electrical breakdowns at atmospheric pressure [1]. Similar patterns
3 have also been seen on traces left by a vacuum arc on a stainless steel plate [2].
4 However, the mechanism at the origin of these patterns is very different.
5 Concerning atmospheric pressure electrical breakdowns, these patterns are
6 the pathways of charge-flow in the propagation medium of the arc. This is
7 well exemplified by the phenomenon of lightning during a storm. When it
8 comes to vacuum arcs though, those patterns are seen on the cathode surface
9 and are formed through the ignition and extinction of plasma emission sites.
10 These sites, named cathode spots, are distributed on the cathode surface in
11 accordance with random walk patterns [3, 4] leading to the formation of tree-
12 like structures. These structures can be described as objects with non-integer
13 dimension, namely fractals.

14 Although the fractal nature of cathode spots has already been the subject
15 of several studies [2, 4, 5, 6], most experimental arrangements involved low-

16 current long duration vacuum arcs. Consequently little is known today on
17 the fractal nature of high-current vacuum arcs. Recent application of vacuum
18 arcs in spacecraft propulsion tends to maximize the current while reducing
19 the discharge duration [7, 8]. By doing so, the thruster maximizes the in-
20 stantaneous thrust while avoiding thermal issues at the electrodes. These
21 systems, called Vacuum Arc Thrusters (VAT), rely on a vacuum arc between
22 two electrodes. The cathode material is ionized, accelerated and ejected at
23 velocities up to 50 km s^{-1} [9], leading to thrust generation in the opposite
24 direction. This technology presents inherent advantages regarding miniatur-
25 ization, simplicity and reliability through the use of a solid metal propellant.
26 However, at this stage, the formation of the arc, its dynamics and the subse-
27 quent plasma acceleration mechanism remain poorly understood. Probing in
28 depth the physics of VATs and gathering data is therefore needed to propose
29 more efficient solutions with longer lifetime able to meet the requirements of
30 a wide range of space missions.

31 Arcs and cathode spots certainly play a major role in thrust and spe-
32 cific impulse generation by VATs. In the present study we determine the
33 fractal dimension of the cathode spot structures for different cathode mate-
34 rials of a low-power high-current VAT. Calculations are performed using the
35 Minkowski-Bouligand method. The approach relies on photographs taken
36 during operation of the VAT, the characteristics of the latter being given
37 in [Experimental arrangement](#) section, along with a description of the setup.
38 The way the photographs are processed and how the fractal dimension is
39 retrieved are detailed in [Method and results](#) section. The fractal dimension
40 of the cathode spot structures is then studied by investigating the influence
41 of the material physical properties using statistical and machine learning ap-
42 proaches in [Influence of material properties](#) section. Eventually, [section 6](#)
43 outlines the conclusions.

44 **2. Experimental arrangement**

45 *2.1. The PJP thruster*

46 The VAT under study is the Plasma Jet Pack (PJP) developed by the
47 COMAT French company. This 30 W-class VAT generates a high current
48 vacuum arc through the cyclic discharge of a capacitor bank. The cathode
49 to anode voltage is continuously set to 250 V. A triggering system applies a
50 few kV pulse on the cathode surface lasting a few hundreds of ns. This pulse
51 allows ignition of the plasma through an explosive process occurring on a

52 surface region named cathode spot [10]. The high voltage applied during the
53 trigger phase initiates a breakdown by field emission [11] that is followed by a
54 spark and then an arc. During VAT operation, a thin metallic layer deposits
55 on the central insulator that separates the triggering system and the cathode.
56 Ignition of the discharge is then easier as the breakdown occurs through the
57 conductive layer. Note that when the deposited layer is thick it becomes more
58 difficult to vaporize and the thruster no longer operates properly [12]. After
59 the trigger is switched-off, the arc is forced to propagate to the anode through
60 the existing metal vapor. This second arc discharge allows roughly 3 to
61 4 kA to flow between the two electrodes with the present configuration. The
62 discharge current waveform obtained with a titanium cathode is exemplified
63 in Fig. 2. Overall, the energy released over 25 μ s into one single PJP discharge
64 reaches a few J. Measurements presented in this contribution have been
65 performed with a copper anode and various cathode materials, namely simple
66 metals such as titanium, aluminum, iron and copper, as well as metallic alloys
67 like brass (63 % Cu - 37 % Zn) and nichrome (80 % Ni - 20 % Cr). All cathodes
68 were new before photographs were taken.

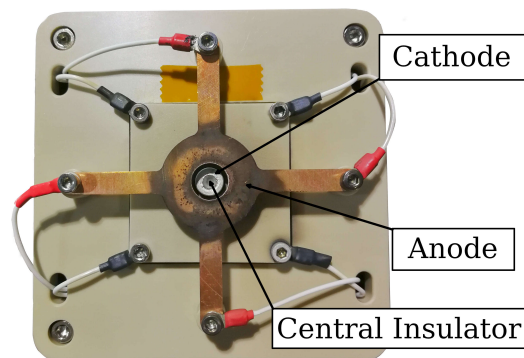


Figure 1: Plasma Jet Pack thruster: front view.

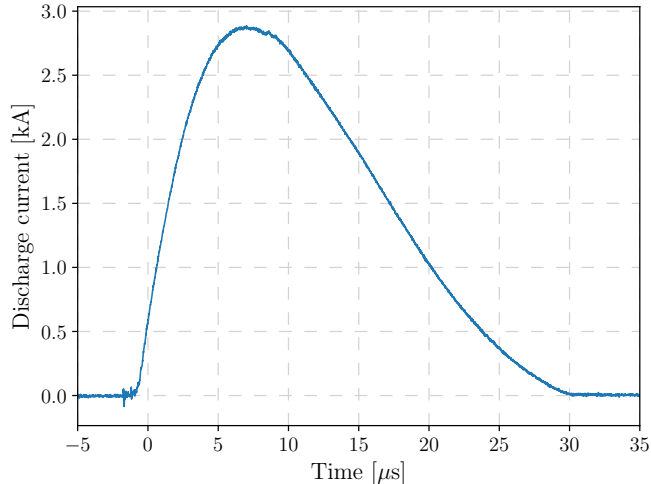


Figure 2: Typical PJP discharge current waveform during one pulse (Ti).

69 Figure 1 shows the front view of a laboratory version of the PJP. The 4-
70 legs electrode is the annular copper anode. The cathode corresponds to the
71 grey center-mounted hollow cylinder. The latter is screwed onto a copper
72 plate directly wired to the capacitor bank. The gap in the axial direction
73 between the cathode and the anode is approximately 1 cm. The trigger elec-
74 trode is a tiny metal tip placed exactly on the thruster axis and separated
75 from the cathode with the central insulator. The PJP target performance
76 and characteristics are given in Table 1 for the inclined reader.

Power	0-30	W
Thrust to power	10	$\mu\text{N W}^{-1}$
Average thrust at 30 W	300	μN
Specific impulse	2500	s
Total impulse	400	N s
Overall mass	1	kg
Overall volume	1	U

Table 1: PJP thruster target performance and characteristics.

77 *2.2. Experimental arrangement*

78 Experiments have been performed in the EPIC-2 vacuum chamber of the
79 laboratory. This chamber is a stainless steel cylinder 54 cm in radius and

80 104 cm in length, for an approximate capacity of 240 L. Two 2200 L s⁻¹ (N₂)
 81 magnetically levitated STP-iS2207 turbomolecular pumps from Edwards are
 82 placed on top of the chamber. Those pumps, evacuated through a 110 m³ h⁻¹
 83 dry primary pump (Edwards GV110), permit to keep a background pressure
 84 of 10⁻⁶ mbar during thruster operation. The pressure is monitored with a
 85 Pfeiffer PBR260 Pirani/Bayard-Alpert pressure gauge.

86 EPIC-2 is also equipped with several electrical feedthroughs, for both in-
 87 situ diagnostics and thruster power supply. Those feedthroughs are mostly
 88 composed of BNC or SubD connectors. Two Kodial (Borosilicate 7056) glass
 89 windows allow visual inspection of the thruster.

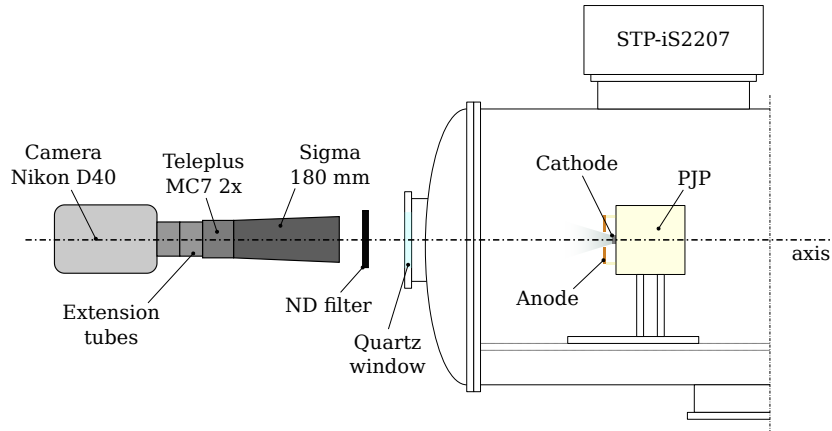


Figure 3: Schematic of the experimental setup.

90 The experimental setup is depicted in Fig. 3. In EPIC-2, the PJP is
 91 placed on a horizontal plate screwed on a rail. This mounting allows ad-
 92 justment of the distance to the front window, depending on the experiment
 93 to be performed. In this case the distance is set to have the cathode sur-
 94 face filling the whole camera field of view. In doing so, the accuracy of the
 95 Minkowski-Bouligand algorithm is increased, see section 3.2. Moreover one
 96 gets a complete view of all the cathode spots that can appear during a dis-
 97 charge. As the cathode surface is small, the PJP has been placed as closed
 98 as possible to the window. The distance was then fixed to 30 cm for this
 99 experimental campaign.

100 The camera was a Nikon D40 with a 6 Mpx CMOS sensor, which has
 101 been equipped with a Sigma 180 mm F2.8 macro lens. In order to increase

102 the magnification ratio, a Kenko TELEPLUS MC7 AF 2x lens as well as two
103 extension tubes have been added as shown in Fig. 3. The exposure time has
104 been set to 1 s so that all the light emitted during the discharge is received
105 by the sensor. In this way, one can extract the properties of all the cathode
106 spot traces over the entire surface during a PJP pulse. As spots emit a
107 large amount of light, the camera ISO has been set to the lowest option
108 available, i.e. 200, thus limiting the sensor light sensitivity. To achieve an
109 appropriate exposure, a neutral density filter with an attenuation factor of
110 10 has been mounted ahead of the lenses, as illustrated in Fig 3. In order
111 to avoid variations in the camera alignment while manually operating the
112 shutter, the sensor has been triggered by means of an infrared remote control
113 synchronized with the PJP duty cycle. The same setup and procedure apply
114 for all cathode materials. After changing of the cathode, the thruster and
115 the optical system were realigned to warranty sharp high-resolution images.

116 In total, 410 photographs have been used in this work. For each cathode
117 material, the dataset includes about 50 pictures, a large enough number to
118 ensure correct outcomes of a statistical analysis.

119 **3. Fractal dimension**

120 *3.1. Image preprocessing*

121 Figure 4 shows a photograph of PJP discharge triggered with a titanium
122 cathode along with physical boundaries of both the cathode and the anode.
123 The dendritic nature of the plasma emission sites and the arc overall shape are
124 obvious by the naked eye in Fig. 4. The luminous light blue structures extend
125 from the inner edge of the cathode to the outer edge. One can note dendrites
126 are more sparse and less luminous near the outer edge of the cathode. The
127 cathode resembles a hollow disk when seen from the front. The center of this
128 disk corresponds to the insulator, where a reddish glow is observed as can be
129 seen in the image of Fig. 4. In this photograph, parts of the internal edge of
130 the anode are illuminated. A large bright circle can be seen in Fig. 4. Since
131 these regions are facing the cathode arcs, light may originate from reflection.
132 It is for instance the case for the left region. The reddish glow at the bottom
133 right part certainly corresponds to a so-called anode mode [13].

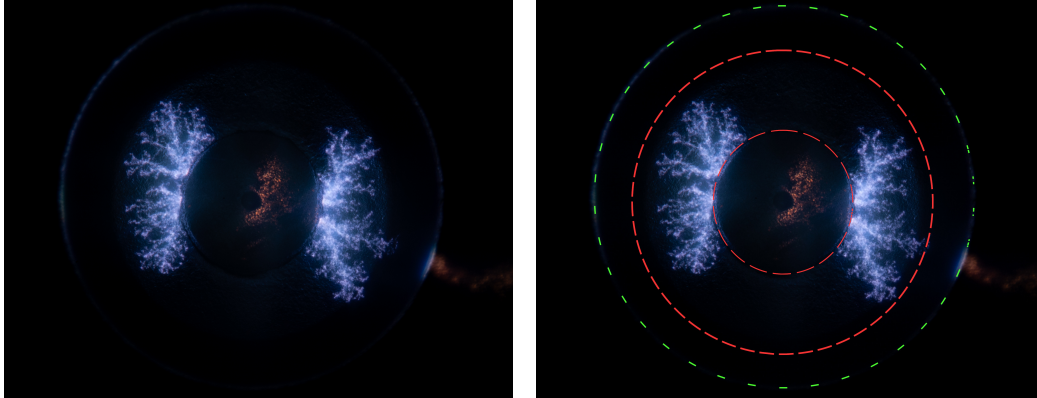


Figure 4: Raw photograph of a vacuum arc on a titanium cathode (left). Red densely-dashed and green loosely-dashed lines indicate cathode and anode boundaries respectively (right).

134 An image processing algorithm must be able to identify the structure of
 135 interest prior to analysis. Here, the structure is the cathode spot traces,
 136 i.e. points with high light intensity. However, other bright zones appear
 137 in the pictures as previously discussed. It is then crucial to remove those
 138 parasitic elements before image analysis as they can perturb data treatment
 139 and lead to erroneous results and conclusions. This situation is exemplified
 140 in Fig. 5. First, for most VAT discharges anode modes are observed [13].
 141 In short, this leads to a glow in the anode region. The anodic arc area is
 142 circled in yellow in Fig. 5. Second, a glow is visible on the center-mounted
 143 insulator due to a plasma discharge on its surface. The metal deposition that
 144 accumulates on the insulator during PJP operation is ionized at the early
 145 stage of the discharge when the high voltage is applied. This area, which is
 146 circled in red in Fig. 5, is not considered as part of the arc discharge as ionized
 147 particles do not originate from the cathode material in that case [14]. Last,
 148 the cathode spots illuminate the outer edge of the insulator, green rectangle
 149 and enlarged image in Fig. 5. This zone could be mistaken with the cathode
 150 spots by the algorithm and must be removed. The three described areas have
 151 been manually covered with black pixels by means of an image manipulation
 152 program in this work. Note the inner edge of the anode, see Fig 5, is also
 153 removed.

154 The light halo generated by the arc, see Fig. 5, must also be treated before

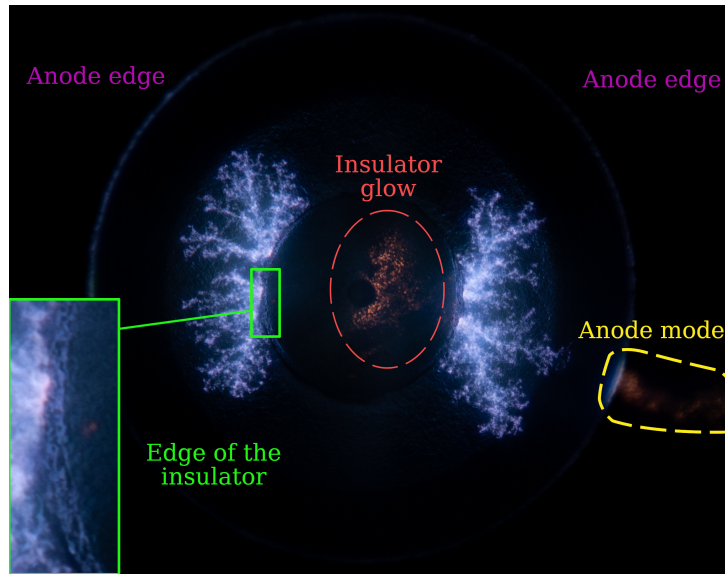


Figure 5: Unwanted zones in a photograph of a vacuum arc. Image in Fig.4 is used as a reference.

155 image processing. This halo is the diffuse light seen around the cathode
 156 spots. It results from the intense light emission of the arc. Misalignment of
 157 the camera and optics also favors its appearance. During image processing
 158 the picture is transformed into a binary matrix, indicating whether the pixel
 159 belongs to the set of cathode spots or not. This halo being especially intense
 160 in the space between the branches of the structure, the fractal nature of the
 161 arcs is lost if the halo is kept. In other words, keeping the halo would distort
 162 the calculation of the fractal dimension.

163 To decrease the impact of the luminous halo and extract the cathode
 164 spot traces, the solution is to fine tune white balance and tonal curves of
 165 each photograph. This operation permits to sharpen the color difference
 166 between the trace and the halo. The color components of the halo are strongly
 167 desaturated with regard to the ones of the spots, reducing its brightness.
 168 Afterwards conversion to gray scale is performed. It results in a gray-scale
 169 picture of the arc discharge in which the spot traces are sharper than in
 170 the original picture, without any additional numerical filter. Outcome of
 171 preprocessing is illustrated in Fig.6 for the vacuum arc image presented in
 172 Fig.4.

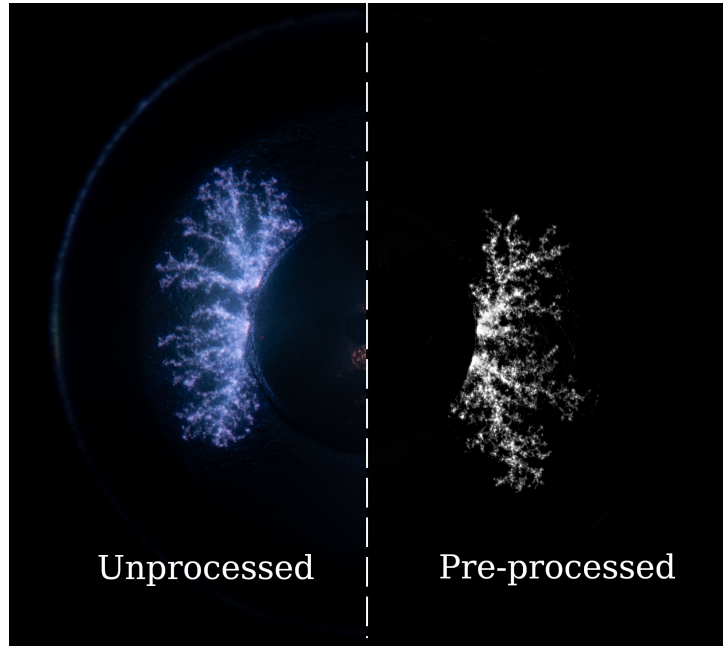


Figure 6: Vacuum arc image before (left) and after (right) preprocessing.

173 *3.2. Minkowski-Bouligand method*

174 Before determining the fractal dimension, the image must be turned into a
 175 binary matrix. In our study each pixel of a preprocessed image is associated
 176 with a value that varies between 0 and 1 according to a 8-bit gray scale.
 177 This numerical value is given by the normalized brightness of the pixel. A
 178 brightness threshold is then defined for each material. Above the threshold,
 179 the pixel is considered as part of the spot pattern and the value of 1 is
 180 assigned to the pixel. Below the threshold the value 0 is assigned to the
 181 pixel as it does not belong to the pattern. The result is a binary matrix that
 182 is used for the calculation of the fractal dimension D of the spot structure.
 183 Threshold is given in Tab. 2 for the different cathode materials. In order to
 184 assess the threshold, preprocessed images in gray scale are compared with
 185 binary images. The threshold is defined as the value that gives a binary
 186 image representative of the real image. When the threshold is low, the overall
 187 aspect of the image is changed. When it is high the image is smoother and
 188 the dendritic geometry tends to vanish. A precise value is hard to define for
 189 the threshold, fortunately it has a moderate impact on the fractal dimension.
 190 In the case of Ti for instance, the change in D is only 0.02 when considering

191 the lowest and the highest value of the threshold, which is below the standard
 192 deviation for D , see Sec. 3.3.

Material	Threshold
Ti	0.30
Al	0.25
Ni-Cr	0.40
Fe	0.35
Brass	0.48
Cu	0.40

Table 2: Threshold for different cathode materials.

193 To recover the fractal dimension of the cathode spot luminous traces, the
 194 Minkowski-Bouligand method, also known as box counting method, has been
 195 used. According to this method a fractal S is covered with an evenly spaced
 196 grid and the number of squares required to cover S is counted. The dimension
 197 is calculated by evaluating how this number changes as the grid is made finer.
 198 The Minkowski-Bouligand dimension, or box-counting dimension, is defined
 199 as:

$$\dim_{box}(S) = \lim_{\epsilon \rightarrow 0} \frac{\log N(\epsilon)}{-\log \epsilon} \quad (1)$$

200 where $N(\epsilon)$ is the number of squares of side length ϵ required to cover the
 201 structure [15, 16]. The box counting process is interrupted when ϵ equals
 202 the size of the camera pixel. However, as the camera sensor contains 6×10^6
 203 pixels, multiple iterations can still be made before reaching this limit. As
 204 the mesh is refined, N is calculated for each step. Eventually the $\log(N)$
 205 versus $-\log(\epsilon)$ curve is plotted and a line is fitted to the dataset. The slope
 206 of the linear fit gives the fractal dimension. Details about the computation
 207 methodology can be found in reference [17] for the inclined reader. Figure 7
 208 shows the curve obtained for the image in Fig. 4 along with the associated
 209 linear fit. As can be seen points are well aligned. The R^2 value is 0.996.
 210 On this example the fractal dimension is $D = 1.448$, a value close to the
 211 dimension of the Viscek snowflake and the quadratic Von Koch curve type
 212 1.

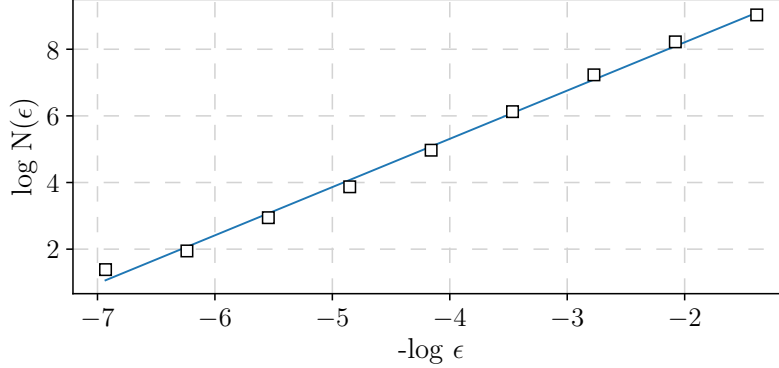


Figure 7: Plot for fractal dimension of vacuum arc image shown in Fig. 4 obtained with the box-counting method: $D = 1.448$.

213 *3.3. Fractal dimension*

214 The mean fractal dimension determined with the box-counting algorithm
 215 for each VAT cathode material along with the associated standard deviation
 216 are presented in Table 3, sorted in ascending order. As can be seen, the
 217 standard deviation remains low for all materials with a relatively constant
 218 magnitude. Moreover, except for Al and NiCr datasets, each element has a
 219 D value outside the $D \pm \sigma_D$ interval of the other elements. It means that
 220 there is a significant difference between the fractal structures of all studied
 221 materials. Titanium has the highest D whereas copper exhibits the lowest.

Material	D	σ_D
Ti	1.450	0.028
Al	1.329	0.038
Ni-Cr	1.326	0.034
Fe	1.288	0.019
Brass	1.198	0.032
Cu	1.145	0.021

Table 3: Mean fractal dimension and standard deviation for different cathode materials.

222 It is known, referenced and usually admitted that the cathode spot pat-
 223 terns depends on the surface condition, i.e. the degree of contamination by
 224 oxides [18, 19, 20] and the roughness [4, 2, 5]. In order to verify if the frac-
 225 tal dimension is impacted by the cathode surface properties under our arc

226 condition, experiments have been performed with a brand new Ti cathode
 227 and a worn-out cathode exposed to more than 10^4 discharges. A set of 100
 228 photographs has been collected and analyzed for the two cathodes. The
 229 fractal dimension is given in Table 4. It results that there is no significant
 230 differences between the two values as they overlap when accounting for the
 231 standard deviation. Two explanations can be proposed then. Either the
 232 surface roughness and state of the two cathodes are comparable in spite of
 233 a very different exposure to the high current plasma discharge, or there is
 234 indeed no influence of the surface roughness on D . The second explanation
 235 is more likely as the erosion of the “old” cathode was clearly noticeable with
 236 a change in the geometry. The fractal dimension of the vacuum arc cathode
 237 spots pattern would therefore mostly depends on the material properties
 238 and contamination by oxides. There is unfortunately no way to quantify
 239 the degree of contamination by oxygen with the present experimental setup.
 240 We ensured that the operating frequency and the background pressure were
 241 the same for all cathodes, respectively 1 Hz and 10^{-6} mbar. Moreover, the
 242 thruster is fired during about 2 minutes (~ 100 pulses) after installation in
 243 the vacuum chamber to clean the cathode and anode surface.

Cathode	D	σ_D	S [mm ²]	σ_S
Ti new	1.45	0.03	4.2	0.4
Ti old	1.42	0.03	4.6	0.5

Table 4: Fractal dimension and covered area for a fresh and a worn titanium cathode.

244 3.4. Covered surface area

245 The cathode material has a strong impact on D as previously demon-
 246 strated. It is also obvious from the pictures that the area covered by the
 247 spots depends upon the material. Figure 8 shows photographs of the vacuum
 248 arc cathode spots for the six different cathode materials. While for titanium
 249 the spots show large branches that separate into small dendrites, for copper
 250 and brass the sub-structures are much less developed and they form small
 251 and compact islands, especially for copper. A common point between all
 252 images is that the spots seem to develop and propagate mainly in the radial
 253 direction with the external edge of the central insulator as origin. It should
 254 be noted that when more than one spot is active, which is the case here, the
 255 direction of new ignitions is influenced by the magnetic field of the current
 256 paths (active spots) [21, 22].

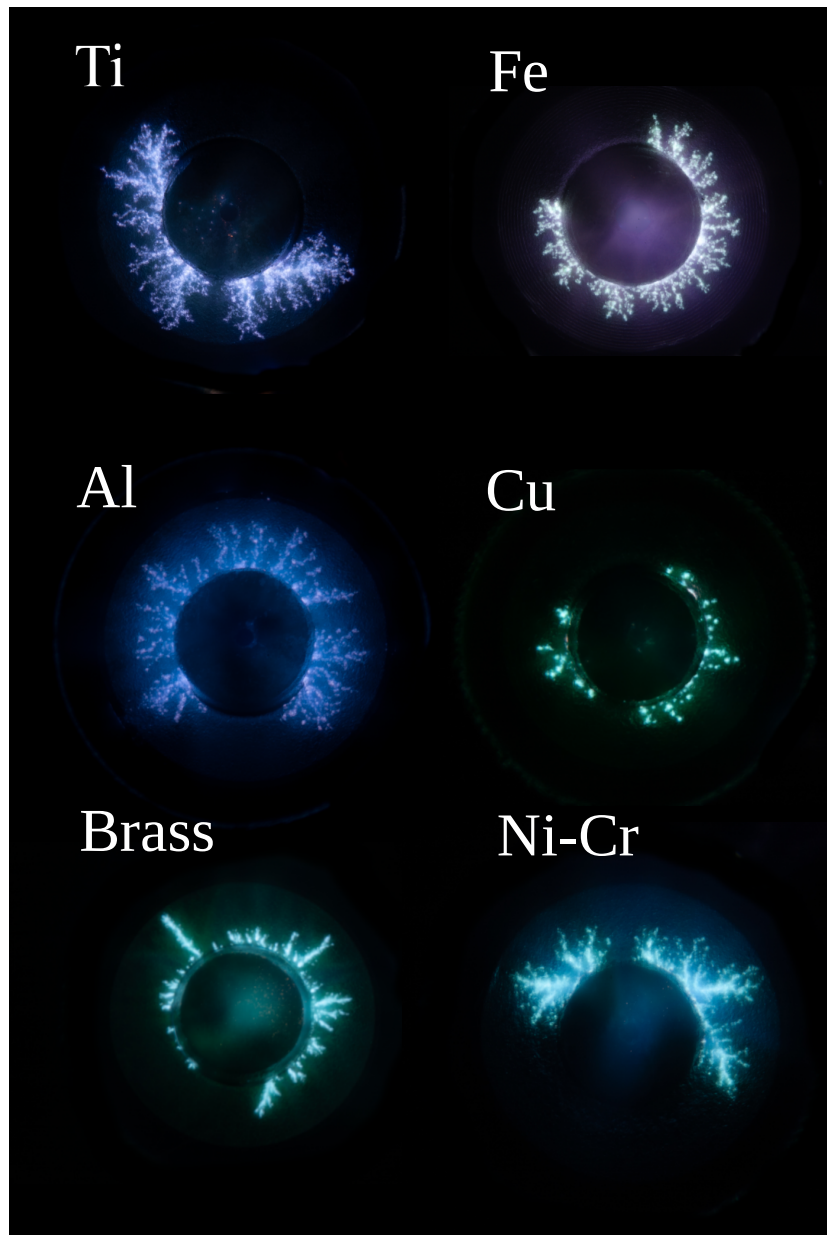


Figure 8: Raw photograph of cathodic arcs using different materials.

257 From the binary matrix of an arc, one can easily determine the surface
258 area covered by the spots. Scaling the picture allows to determine the exact
259 size of one pixel. Then, the total area of cathode spots is given by the

260 number of pixels associated with value 1 in the binary matrix times the area
 261 that corresponds to one single pixel. Results are presented in Table 5, with
 262 area sorted in descending order.

Material	S (mm ²)	σ_S	% of the cathode area
Ti	4.2	0.4	11.1
Ni-Cr	3.2	0.3	8.5
Al	2.6	0.4	6.9
Fe	2.4	0.3	6.3
Brass	1.4	0.1	3.7
Cu	0.9	0.09	2.3

Table 5: Mean surface area covered by spots for different cathode materials.

263 The largest spot area is obtained with Ti whereas the smallest area is
 264 obtained with Cu. In fact the fractal dimension D and the surface area S are
 265 ordered in the same way keeping in mind that D is very similar for Al and
 266 Ni-Cr. Said differently our study reveals a strong link between the fractal
 267 dimension of the vacuum arc cathode spots and the covered area: a large D
 268 corresponds to a large S . This was expected in fact as the fractal dimension
 269 characterizes the space-filling capacity of a pattern.

270 Results are similar when comparing a fresh and a worn cathode: the
 271 covered area does not depend on the wear degree. Table 4 shows that the
 272 value of S is about 4.4 mm² in the case of a titanium cathode whatever the
 273 cathode surface state. Figure 8 shows in addition an interesting character-
 274 istic of the arc discharge. There is always a sector of the cathode with no
 275 arcs and spots. The sector location and size change from one pulse to the
 276 other. Empty sectors have been observed for all photographs for the six
 277 tested materials. The existence of such sectors and their dynamics remain
 278 unexplained. They, however, participate in the fact that the area covered by
 279 the arcs is a small fraction of the total cathode area as illustrated in Tab. 5.

280 3.5. Discussion

281 The dendritic aspect of the traces left on cathode surfaces has been con-
 282 sistently observed either on metal plates [2] or on films [5]. Note that for
 283 studies described in references [2, 3] vacuum arcs were operating at 50 A, a
 284 current magnitude much below the one of the PJP thruster. Striking sim-
 285 ilarities were found between the cathode spots patterns and Lichtenberg

286 figures, or even structures obtained by diffusion-limited aggregation [23]. In
 287 our work, despite the fact that the discharge current I_d is almost 100 times
 288 higher, fractal patterns are still identified, especially when taking a close view
 289 to the dendrites. Similarities come from the fact that a spot cell can sustain
 290 a limited current density j [24]. When j becomes too high, another spot is
 291 initiated elsewhere and j decreases as the current flows through two different
 292 spot cells. Hence, the number of spots increases linearly with I_d [25]. On
 293 the other hand, cathode spots have a limited lifetime which is proportional
 294 to j^2 . At the end of its lifetime, the spot extinguishes and another one ig-
 295 nites elsewhere. This sequence of ignitions and extinctions spreads over the
 296 cathode surface in an random walk process [3, 26]. Note that the random
 297 walk in the case of vacuum arcs is not necessary a self avoiding walk, which
 298 means that a previously active spot may reignite later. Photographs taken
 299 in this study certainly contain spots that have been ignited several times. As
 300 spot and vacuum arc dynamics is governed by the current density, it is not
 301 surprising that high-current short duration arcs show structural similarities
 302 with low-current long duration arcs when the photograph exposure time is
 303 longer than the discharge duration in such a way all spots have been cap-
 304 tured. The main difference between low- and high-current arcs is just the
 305 number of simultaneously active spots [21].

306 The size of a group of spots, generally speaking, is closely related to the
 307 pre-explosion specific action of the cathode material [10, 27]. This parameter
 308 h is written in wire explosion theory as :

$$h = \int_{t_0}^{t_{br}} j^2 dt \quad (2)$$

309 where t_0 is the time at which the energy is released in the circuit, t_{br} the
 310 instant the explosion occurs and j is the current density. This quantity
 311 represents the “thermal toughness” of the material heated with a current
 312 pulse [28, 29]. This parameter h has been linked to the ecton formation
 313 criterion in several studies [24, 30]. The ecton can be seen in this case as
 314 a potential candidate to the physical spatial cutoff of the fractal structure.
 315 It should be emphasized that in the present contribution, the spatial cutoff
 316 is the pixel size. It has been shown in [10] that h strongly depends on the
 317 cathode material, in such a way that it can be considered a function of the
 318 cathode material only, regardless of the energy input in the discharge or
 319 the cathode temperature. Unfortunately, values of h are not available in

320 the literature neither for pure metals nor for alloys. Therefore h could not
321 be accounted for the fractal dimension analysis. However, it confirms that
322 the structure of the plasma emission sites strongly depends on the cathode
323 material for a similar geometry. It is therefore of interest to examine the
324 physical properties of the different cathode materials. The objective is here
325 twofold, namely: (i) to find which material properties influence the most the
326 cathode spot structure during a plasma discharge and (ii) to reveal a possible
327 relation between the fractal dimension of the vacuum arc cathode spots and
328 cathode material characteristics.

329 4. Influence of material properties

330 To the best of the authors knowledge, there is currently no equation ruling
331 the evolution of the fractal dimension with the cathode material properties.
332 This work shows, however, that when applying the same voltage and releas-
333 ing the same energy in the discharge for the same cathode geometry, the
334 fractal dimension and the covered area are material-dependent. **This idea is**
335 **supported by the fact that the ordering in D and S follows the ordering of the**
336 **mean diameter of macropots in [21], for titanium, aluminium and copper,**
337 **despite different experimental conditions.** Two different yet complementary
338 approaches, namely Machine Learning and Heuristic method are proposed
339 here to investigate the impact of the cathode material on the arc pattern.

340 The physical properties considered in these two approaches are the fol-
341 lowing: thermal conductivity λ [W/(m · K)], electrical conductivity σ [S/m],
342 density ρ [kg/m³], vaporization energy E_{vap} [kJ/mol], cohesive energy, E_{coh}
343 [kJ/mol], ionization energy E_{ion} [kJ/mol] and work function W_f [kJ/mol].
344 Values have been taken from the handbook of chemistry and physics [31] as
345 well as from [32] and are presented in Tab. 6. Exact values of energies for
346 alloys could not be found from the literature. Therefore the weighted average
347 with respect to the composition of the alloy was used. According to several
348 works [33, 34, 35, 36, 37] such an approximation is reasonable.

349 4.1. Machine learning

350 In order to determine a relation between the material properties and the
351 fractal dimension D , an approach based on machine learning (ML) has first
352 been considered as it allows to quantify each term contribution to the predic-
353 tion function. Such insight is useful to capture the most contributing physical
354 properties to the change in fractal dimension. The numerical model consists

Material	λ [W/(m · K)]	$\sigma \times 10^6$ [S/m]	$\rho \times 10^3$ [kg/m ³]	E_{coh} [kJ/mol]	E_{vap} [kJ/mol]	E_{ion} [kJ/mol]	W_f [kJ/mol]
Ti	22	2.34	4.51	468	421	659	418
Al	237	37.7	2.69	327	293	577	402
Ni-Cr	11	0.93	8.55	421*	365*	721*	488*
Fe	80	9.93	7.87	413	350	762	457
Brass	125	15.6	8.45	259*	231*	803*	419*
Cu	401	59.6	8.96	336	300	746	459

* Average between the properties of the materials composing the alloy

Table 6: Physical properties of cathode materials considered in this study.

355 in a multivariate polynomial regression (MPR). A correlation between the
356 fractal dimension and relevant material properties is sought in the form

$$D = \sum_k \sum_i \alpha_{i,k} x_i^k \quad \text{with} \quad \alpha_{i,k} \geq 0 \quad (3)$$

357 where D is the predicted fractal dimension, $\alpha_{i,k}$ are linear coefficients and
358 x_i are physical constants relative to the cathode material, here extracted
359 from Tab. 6. Each x_i appears in the expression with exponent k . As for the
360 polynomial order, since most scaling laws in physics feature exponents ± 0.5 ,
361 ± 1 and/or ± 2 , it is chosen to limit k to these values. No intercept is allowed
362 in the model shown in Eq. 3, which means that a null fractal dimension is
363 assumed when all physical properties are equal to zero. Physically, it means
364 that no discharge can occur without a propagation medium.

365 The analysis consists in determining the value of each $\alpha_{i,k}$ that outputs
366 the best fit to the experimentally observed fractal dimension. For this pur-
367 pose, a ML program was developed in Python. Having selected 7 physical
368 properties and 6 exponents, 42 coefficients are to be determined. In practi-
369 cal terms, the algorithm targets at minimizing the residual sum of squares
370 between the experimentally observed fractal dimension and the one retrieved
371 by the polynomial approximation. The dataset on which the analysis is per-
372 formed comprises 6 materials and 411 pictures. This is randomly split into
373 a training set (85 %) and a test set (15 %). The training and test sets are
374 depicted in Fig. 9 along with the fractal dimension predicted by the ML pro-
375 gram. The quality of the model is quantitatively assessed on the test set
376 through the MAE and R^2 metrics, which read 2.15 % and 0.94, respectively.
377 These parameters outline that the model does satisfactorily predict the ex-
378 perimental findings.

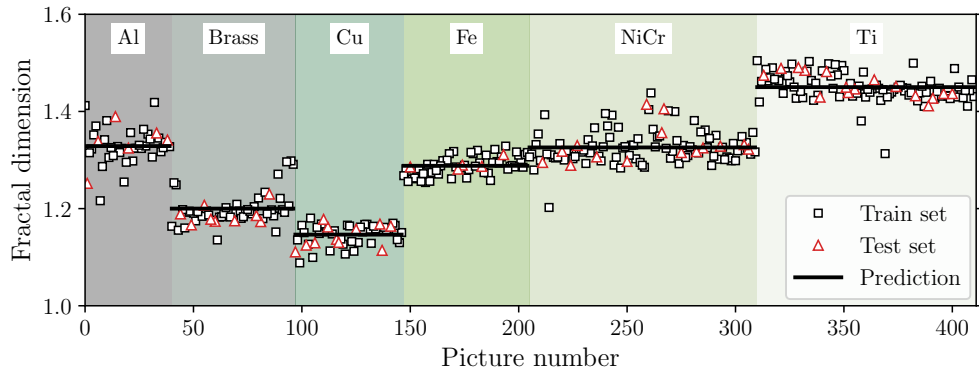


Figure 9: Scatter plot of train set, test set and predicted value of fractal dimension.

379 Figure 10 shows the normalized values of every $\alpha_{i,k}x_i^k$ for the six tested
 380 materials, that means their respective contribution to the fractal dimension
 381 prediction function. In all cases, the reciprocal of the work function term
 382 contribute more than 55 % to the fractal dimension. The sum of the squared
 383 ionization energy and squared cohesive energy accounts for more than 35 %,
 384 with varying proportions. The remaining quantities visible at the bottom
 385 of Figure 10 appear with variable weights depending on how the dataset
 386 is randomly divided into train and test sets. This effect possibly results
 387 from the relatively limited amount of data available. Therefore the precise
 388 influence of ρ , λ and σ is difficult to quantify with this approach, however it
 389 certainly remains secondary. Note that the contribution of the vaporization
 390 energy is null in each case.

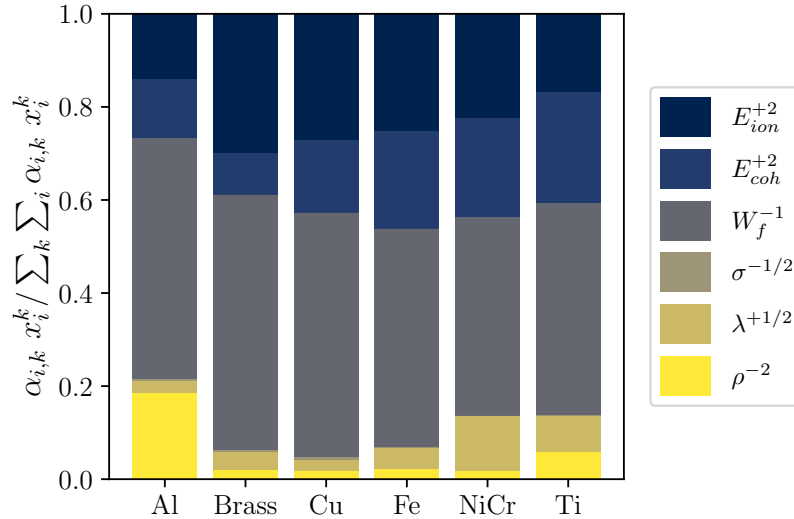


Figure 10: Normalized cumulative bar plot of the absolute value of each $\alpha_{i,k}x_i^k$. Omitted units are consistent with Table 6.

391 *4.2. Heuristic approach*

392 As a means to complete the ML model and examine whether a product
 393 relation between the material properties can also explain the change of fractal
 394 dimension D , a heuristic analysis has been implemented. It relies here on an
 395 expression in the form

$$D \propto \prod_i x_i^k \tag{4}$$

396 where x_i is a physical parameter relative to the cathode material and k are
 397 usual exponents in physical laws, in a similar fashion to the approach dis-
 398 cussed beforehand. Values of k are limited once again to $k = 0, \pm 0.5, \pm 1, \pm 2$.
 399 As this approach relies on an expression in a product form, a risk of overfit-
 400 ting the data can arise. One should then limit the number of x_i 's. In that
 401 respect, since the ML model brings out the main contributing properties, the
 402 method proposed here only considers parameters with a contribution above
 403 10% in the ML model i.e. E_{coh} , E_{ion} and W_f . Note however that the heuris-
 404 tic model considers a product of the various physical properties while the
 405 ML model is based on a sum. All the 343 possible combinations have been
 406 tested in an iterative way for the six materials using a custom program in

407 Python. For each combination a linear regression analysis is performed that
 408 includes the six materials. Unlike with the previous model, the intercept of
 409 the linear regression is not forced to be null. The expression giving both the
 410 lowest mean absolute error (MAE) and the highest R^2 is considered the most
 411 probable and the most appropriate relation of proportionality between the
 412 physical parameters of the material and the fractal dimension. Following this
 413 procedure, the best suited relation between D and the material properties is:

$$D \propto \frac{E_{coh}^2}{W_f E_{ion}^2} \quad (5)$$

414 The evolution of the fractal dimension D with this combination of pa-
 415 rameters is displayed in Fig. 11. With the relation given by Eq. 5, the linear
 416 regression yields a mean absolute error of 0.025 and a R^2 equal to 0.865. It
 417 is quite clear from this figure that the copper is responsible for the decrease
 418 in R^2 . The linear regression without taking Cu into account has an R^2 of
 419 0.987. This may be due to the accuracy of the box counting method with the
 420 copper dataset. The copper spots covering the smallest surface with small
 421 filaments, the box counting algorithm is less accurate as the size of the spot
 422 pattern is close to the pixel size.

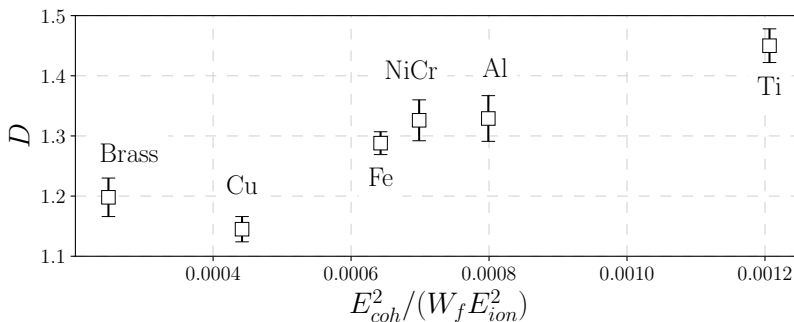


Figure 11: Evolution of the fractal dimension according to Eq. 5.

423 The five relations having the most linear behavior, as well as the coef-
 424 ficients of the regression and their respective R^2 and MAE, are given and
 425 ranked in Tab. 7. This table indicates that the cohesive energy increases
 426 the fractal dimension while the work function and the ionization energy de-
 427 creases D . Studies have shown that the noise amplitude in the vacuum arc
 428 burning voltage scales linearly with the cohesive energy of the cathode ma-
 429 terial [38, 39]. According to the theory of colored noise [40, 41], this noise

Relation	a	b	R^2	MAE
$\frac{E_{coh}^2}{W_f E_{ion}^2}$	305.7	1.08	0.865	0.025
$\frac{E_{coh}}{W_f \sqrt{E_{ion}}}$	13.4	0.86	0.841	0.030
$\frac{E_{coh}}{W_f^2 \sqrt{E_{ion}}}$	5096.8	0.91	0.813	0.030
$\frac{E_{coh}}{W_f^2 E_{ion}}$	111107.8	0.98	0.771	0.039
$\frac{\sqrt{E_{coh}}}{W_f^2}$	5616.3	0.72	0.623	0.049

Table 7: Most appropriate relations of proportionality between the physical parameters of the material and the fractal dimension.

430 in burning voltage could possibly indicate a Brownian motion of the active
431 cathode spots, often approximated with random walk models. A link could
432 therefore be expected between the spot distribution over the cathode surface
433 and the cohesive energy of the cathode material. On the other hand, the ion-
434 ization energy and the work function counteract the cohesive energy in the
435 relation, playing a crucial role in reducing the fractal dimension magnitude.

436 5. Conclusion

437 Photographs of cathode spots taken during firing of a high current vacuum
438 arc thruster reveal a fractal structure on the cathode surface. The dimension
439 of these structures was determined by the Minkowski-Bouligand method. It
440 was found that the fractal dimension varies considerably as a function of the
441 cathode material. Differences cannot be attributed to the typical dispersion
442 resulting from several separated discharges, or to surface conditions. For the
443 same energy released in the discharge, D spreads from 1.14 ± 0.02 for copper
444 to 1.45 ± 0.03 for titanium. The total surface covered by the spots during a
445 discharge shows the same variation and the same ordering.

446 Two methods have been proposed to investigate the role of the cathode
447 material physical properties in relation to both the fractal dimension and
448 the total surface. A machine learning approach as well as a heuristic analysis
449 show that the work function, the ionization energy and the cohesive energy
450 of the material play an important role in the fractal pattern. Variation of D
451 as a function of the cathode material could be further examined by extending
452 the investigation to materials with large and small E_{coh} respectively.

453 Numerous studies have been carried out in the past about cathode spots
454 of vacuum arcs fractal nature. This work however proposes a quite unique
455 arc regime in terms of current intensity and pulse duration as well as an un-
456 usual application in the field of satellite propulsion. In addition to determine
457 the arc fractal dimension for several cathode materials, we also investigate
458 the relation between D and the material physical properties and we suggest
459 an ordering. This work is not an end in itself, just a step towards a broader
460 and more meaningful conclusion as the ultimate goal is to identify a possible
461 relation between the fractal dimension and the VAT performances in terms of
462 specific impulse, impulse bit and thrust efficiency. A link between fractal di-
463 mension, cathode material properties and performances would certainly help
464 researchers and engineers in the development and optimization of thrusters
465 and offer paths for better understanding the complex physics at play. Look-
466 ing for a relationship between D and thruster performances requires thrust
467 measurements, that means the building of a dedicated thrust balance. We
468 are currently working on that specific point. Measurements performed by
469 the PJP thruster manufacturer nevertheless indicate the thrust is large with
470 a titanium cathode and much smaller when copper is used, what we would
471 expect if thrust is driven by the fractal dimension.

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