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Occam's Razor Description of Sudden Energy Transitions in a Rabi-Driven Quantum Bit Under Frequent Observation

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Abstract

The fundamental law of parsimony in science, which is colloquially referred to as Occam's razor, gives precedence to simplicity in the development of a theory. We wish to illustrate this principle by revisiting the issue related to the long-lasting continuous experimental measurement of a Rabi-driven qubit (*Z. Mineev et al. Nature 2019, 570, 200*). We propose a simple and basic explanation of the results that proceeds from the variance property of the Hamiltonian in the case of a weak Rabi drive. This variance provides a large statistical time-dependent range of available values for the qubit energy (namely, its standard deviation) that spreads out about Rabi's well-known harmonic state-flipping mean value. In the presence of a long-lasting continuous measurement process like in Mineev et al experiment, the resulting "quantum Zeno freezing" of the observed system keeps its energy frozen in its initial state as long as this remains possible within the standard deviation of the energy. Otherwise the system suddenly experiences a quantum jump between the two energy levels —dubbed a quantum Zeno jump— in order to resume its state-freezing dynamics. Some link is made with the dynamics of swapping gates in quantum information processing.

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I. INTRODUCTION

According to the quantum Zeno effect (QZE), frequent (almost continuous) measurements would inhibit the decay of unstable quantum systems [1]. It is usually explained by the property that the system evolves from the same initial state after every measurement [2]. Indeed, in agreement with the quantum measurement theory, the wave-function collapse takes place as a consequence of observation and, consequently, the density matrix of the system loses its off-diagonal components. [3]. This “state freezing effect” has been beautifully verified more than thirty years ago by use of a Rabi driven two-level (or quantum bit: “qubit”) system [4] —namely, a ground state $|G\rangle$ and an excited state $|D\rangle$ — with the addition of a 3rd “ancilla” state $|B\rangle$ that actually plays the role of the continuously-operating ground-state population measurement [5]. Specifically, this latter state $|B\rangle$ is connected by a strongly allowed transition to level $|G\rangle$ and it can decay only to $|G\rangle$. The continuous state measurement is carried out by resonantly (Rabi) driving the $G \rightarrow B$ transition with an appropriately designed optical pulse. This measurement causes a collapse of the wave function. If the system is projected into the ground-state level $|G\rangle$ at the beginning of the pulse, it cycles between $|G\rangle$ and $|B\rangle$ and emits a series of photons —hence the label $|B\rangle$ for “bright”— until the pulse is turned off. If it is projected into the excited level $|D\rangle$ (for “dark”), it scatters no photons. Therefore the wave-function collapse is due to a null measurement [6] [7]. That is, the *absence* of scattered photons when the optical pulse is applied is enough to cause a collapse of the wave function to level $|D\rangle$ [4].

A recent remarkable experiment has reproduced this experimental set-up while using it for a much longer observation time than that of [4]. The authors observed a series of sharp quantum jumps between both qubit energy levels and provide a theoretical explanation based on quantum trajectories [8]. Here we wish to show that these sharp quantum jumps —which we dub Quantum Zeno Jumps (QZJs) for reason that are explained below— actually proceed from three basic quantum properties:

- i) The commonly used rotating wave approximation (RWA) is discarded: the high-frequency terms in the transition dynamics become essential.
 - ii) These high-frequency terms define the variance property of the qubit’s Hamiltonian.
 - iii) This variance property defines the energy range in which QZE state freezing occurs.
- Therefore we claim that the QZJs are generic and we illustrate this assertion by the example of a local energy fluctuation that is a quantum-drift alternative to the standard model of a quantum jump in quantum information processing (QIP) [9] [10].

II. VARIANCE OF A QUANTUM SYSTEM

Let us first review some variance properties of the Hamiltonian \mathbf{H} in a two-state quantum system defined by its time-dependent normalized spinor wavefunction $\Psi(t) = \{\psi_a(t), \psi_b(t)\}$. One simple heuristic way to proceed is to consider the Hermitian variance operator [11]:

$$\mathbf{V} = [\mathbf{H} - \langle \mathbf{H} \rangle \mathbf{I}]^2, \quad (1)$$

that is directly extrapolated from standard statistical physics (\mathbf{I} is the identity matrix). Since \mathbf{V} commutes with \mathbf{H} , it contains nothing that cannot be already obtained from measuring the energy in the sense of reconstructing the statistics of the observable. Therefore it is a mainly pragmatic quantum tool that shows how energy is spread out across its expectation

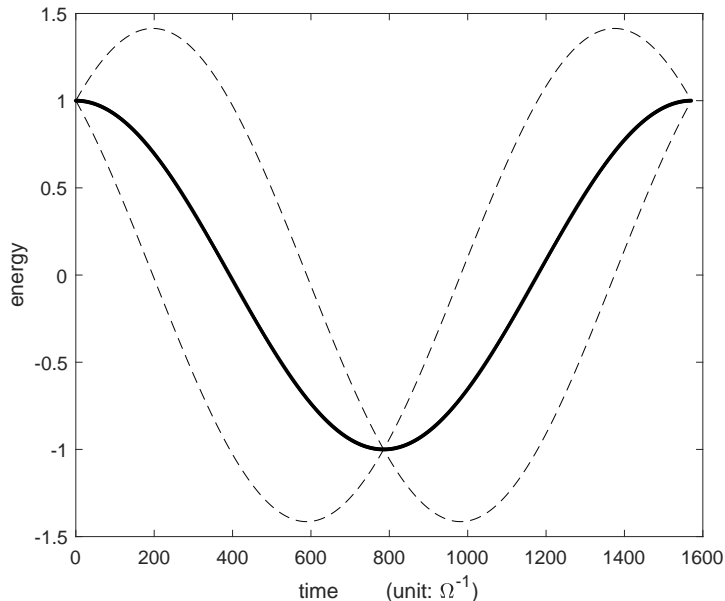


FIG. 1: The quantum standard deviations (4) —dotted profiles— about the harmonic Rabi flipping defined by the time-dependent energy expectation value (continuous profile) in the case of the weak drive $\mathcal{E}(\tau) = A \sin \tau$ with $A = 0.008$.

value $\langle \mathbf{H} \rangle = \langle \Psi | \mathbf{H} | \Psi \rangle$. The standard deviation $\mathbf{d} = \sqrt{\langle \mathbf{V} \rangle}$ is directly deduced from the variance expectation value $\langle \mathbf{V} \rangle = \langle \Psi | \mathbf{V} | \Psi \rangle$.

We define the Hamiltonian of the system (with its two respective Pauli matrices $\mathcal{E}\sigma_z$ for the drive and $K\sigma_x$ with K constant in time for the free Hamiltonian) as:

$$\mathbf{H}(t) = \begin{pmatrix} \mathcal{E}(t) & K \\ K & -\mathcal{E}(t) \end{pmatrix}. \quad (2)$$

We use the dimensionless time $\tau = \Omega t$ where $\Omega = 2K/\hbar$. This means that we assume in the present work:

$$K = 1 \quad \hbar = 2 \quad \Omega = 1. \quad (3)$$

Ω is the Larmor frequency of a spin $\frac{1}{2}$ with a time-dependent magnetic drive along the z -axis and spiraling about the x -axis that is parallel with the constant magnetic field $-K/\mu$ in model (2), the spin magnetic moment being $\mu = e\hbar/mc$. Figure 1 illustrates the “dressing” of the well-known harmonic Rabi oscillation $\langle \mathbf{H}(\tau) \rangle$ (continuous profile) by its

$$\pm \mathbf{d}(\tau) = \pm \sqrt{\langle \mathbf{V} \rangle} = \pm \sqrt{\mathcal{E}^2 - \langle \mathbf{H} \rangle^2 + 1} \sim \pm \sqrt{1 - \langle \mathbf{H} \rangle^2}, \quad (4)$$

standard deviation [11] in the case of the weak resonant drive $\mathcal{E}(\tau) = A \sin \tau$ with $A = 0.008$ (dotted profiles). It defines a wide statistical time-dependent range of available energy values about $\langle \mathbf{H}(\tau) \rangle$. Hence the following natural question: since the energy values can be scattered so widely over the whole region statistically bounded by the dashed profiles in Fig. 1, why haven’t they been observed? The purpose of the present paper is to show that they *have been* actually observed in [8].

III. CLASSICAL CANONICAL TRANSFORMATION OF A DRIVEN QUBIT

The mathematics have already been described in [12] [13]. They consist in a mere canonical transformation from the Hamiltonian Schrödinger description (2) to that classical Hamiltonian dynamical system (HDS) which is defined by the following Hamilton equations:

$$\frac{d}{d\tau}\alpha = -\frac{\partial\mathcal{H}}{\partial\delta} = \sqrt{1-\alpha^2}\sin\delta \quad ; \quad \frac{d}{d\tau}\delta = \frac{\partial\mathcal{H}}{\partial\alpha} = -\frac{\alpha}{\sqrt{1-\alpha^2}}\cos\delta + \mathcal{E}(\tau). \quad (5)$$

$$\mathcal{H} = \sqrt{1-\alpha^2}\cos\delta + \alpha\mathcal{E}(\tau). \quad (6)$$

The two conjugate canonical variables $\delta(\tau)$ and $\alpha(\tau)$ are defined by the spinor components:

$$\psi_a(\tau) = \sqrt{\frac{1+\alpha(\tau)}{2}}e^{i\Theta(\tau)} \quad ; \quad \psi_b(\tau) = \sqrt{\frac{1-\alpha(\tau)}{2}}e^{i[\Theta(\tau)+\delta(\tau)]}. \quad (7)$$

Therefore $\alpha(\tau) = 2|\psi_a(\tau)|^2 - 1 = 1 - 2|\psi_b(\tau)|^2 = |\psi_a(\tau)|^2 - |\psi_b(\tau)|^2$ is a direct measure of the qubit's normalized state probabilities. Hamiltonian (6) yields:

$$\mathcal{H}(\tau) = \langle \mathbf{H}(\tau) \rangle = \langle \Psi | \mathbf{H} | \Psi \rangle, \quad (8)$$

with $\Psi(t) = \{\psi_a(t), \psi_b(t)\}$ is defined by Eqs (7) while $\mathbf{H}(\tau)$ is given by Eqs (2-3). It provides the necessary physical link between both Hamiltonian descriptions; namely the quantum Schrödinger one and the classical HDS one. The dynamics of the time-dependent global —or Berry [14]— phase $\Theta(\tau)$ in (7) is defined by:

$$\frac{d}{d\tau}\Theta = -\frac{1}{2}\left[\sqrt{\frac{1-\alpha}{1+\alpha}}\cos\delta + \mathcal{E}(\tau)\right]. \quad (9)$$

Note: it *cannot* be discarded in the above canonical transformation (5-6). Indeed, if one wishes to do so —i.e. to assume $\Theta \equiv \text{constant}$ in (7)—, the vanishing r.h.s. of (9) would create the arbitrary and clearly non-physical constraint $\mathcal{H}(\tau) + \mathcal{E}(\tau) = 0$. Consequently Rabi's smooth harmonic RWA state flipping (continuous oscillation in Fig. 1) would simply disappear! Moreover, the existence of the global phase dynamics (9) in Eq. (7) yields a real physical observable effect: it defines the famous 4π symmetry of spinor wave functions (i.e. the sign reversal of the wave function under a 2π rotation [12]) which has been experimentally observed in both division-of-amplitude [15][16] and division-of-wave-front [17] neutron interferometry experiments (see also [18]).

The above HDS classical Hamiltonian description of the driven qubit's dynamics is a mere mathematical protocol aimed to stress the role of the very-high-frequency (VHF) components $E_{a,b}(\tau)$ present in the energy variance of the system. Indeed these latter are defined by [12]:

$$\langle \mathbf{H}(\tau) \rangle = \mathcal{H}(\tau) = |\psi_a(\tau)|^2 E_a(\tau) + |\psi_b(\tau)|^2 E_b(\tau), \quad (10)$$

where by use of Eqs (5), (7) and (9):

$$E_a = \frac{\mathcal{H} + \mathcal{E}}{1 + \alpha} = -2\frac{d}{d\tau}\Theta \quad ; \quad E_b = \frac{\mathcal{H} - \mathcal{E}}{1 - \alpha} = -2\frac{d}{d\tau}(\Theta + \delta). \quad (11)$$

The VHF energy components $E_{a,b}$ are the qubit's *state energies* as clearly suggested by Eq. (10). Their respective variances are:

$$\sigma_{a,b}^2(\tau) = |\psi_{a,b}(\tau)|^2 [E_{a,b}(\tau) - \mathcal{H}(\tau)]^2. \quad (12)$$

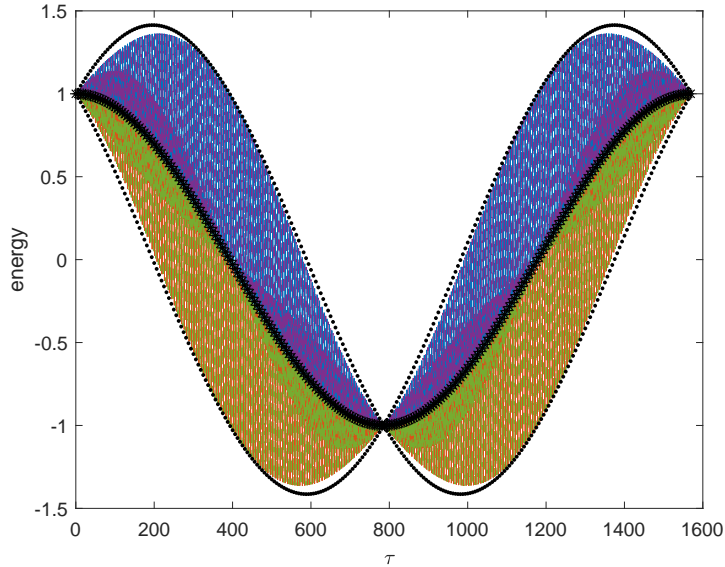


FIG. 2: $A = 8 \cdot 10^{-3}$. The colored energy ranges display the VHF standard deviation $\pm\sigma_{a,b}(\tau)$ defined by Eq. (12) about one Rabi period $4\pi/A = 1571$ (thick black plot). Dotted black plot: the quantum standard deviations $\pm\mathbf{d}(\tau)$ reproduced from Fig. 1.

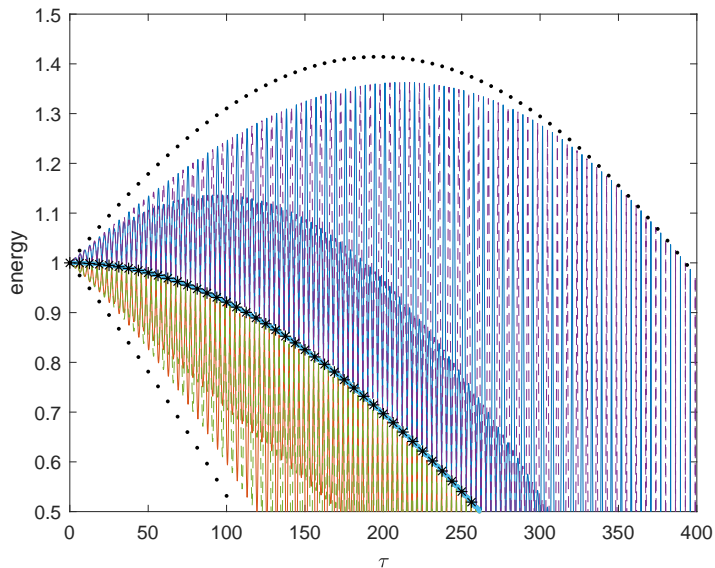


FIG. 3: Fig. 2 zoomed about the onset of the Rabi flipping in order to detail the VHF standard-deviation components $\sigma_a(\tau)$ (continuous) and $\sigma_b(\tau)$ (dotted).

Figures 2 and 3 display the corresponding standard deviations $\sigma_{a,b}(\tau)$ by superimposing them on the quantum standard deviations $\pm\mathbf{d}(\tau)$ shown by Fig. 1 (dotted black points). The quite dense colored patterns are built from the VHF oscillatory standard deviations (12) whose corresponding HDS orbit period $\sim 2\pi \ll 1571$ is extremely small at the scale of the Rabi period $4\pi/A = 1571$ [12]. The agreement with Fig. 1 is spectacular. It numerically

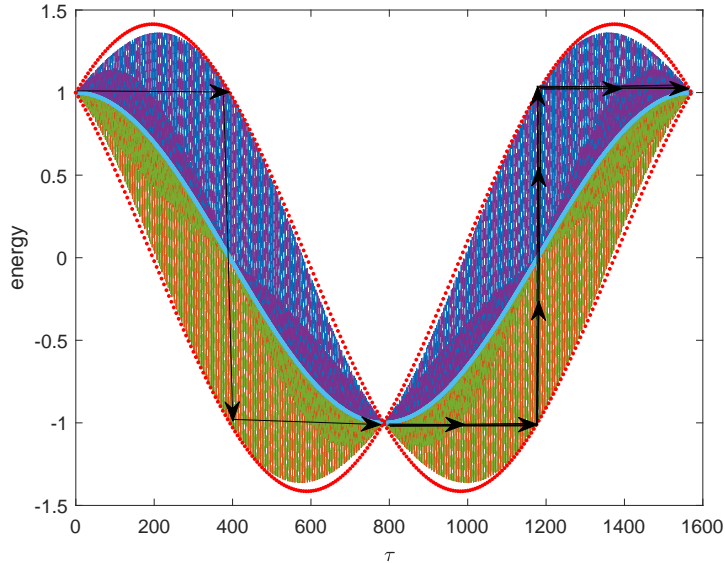


FIG. 4: The *QZE-vs-Rabi* competition sketched inside the energy patterns reproduced from Figs 1-3 for $A = 8 \cdot 10^{-3}$. The step-like energy transitions due to the QZE repetitive energy measurements are illustrated by arrows. The thick arrow path displays the abrupt energy jump occurring at $\tau = 1183$ (at $\mathcal{H} = 0$) from the ground state $E = -1$ to the excited level $E = +1$. The thinner one displays the reverse de-excitation process.

demonstrates that the quantum standard deviation (4) is quite close to $\pm\sigma_{a,b}$ (12) and that it is actually due to the VHF state energy components (10-11).

IV. QUBIT VARIANCE STATE FREEZING

This property allows us to determine the ad-hoc experimental conditions required to observe the effect of these VHF energy components in the energy variance range of a driven qubit. It has been shown in [12] that the mean value of $E_{a,b}(\tau)$ over one HDS orbit period $\sim 2\pi$ is equal to $\langle \mathbf{H}(\tau) \rangle$. This property yields the 4π symmetry related to the global phase $\Theta(\tau)$ of the spinor wavefunction by use of Eqs (11) [12]. Therefore any measurement of the qubit energy that lasts more than 2π (in our reduced units (3)) will restore the well-known energy expectation value $\langle \mathbf{H}(\tau) \rangle$: nothing will be changed with respect to the well-known low-frequency RWA description of the Rabi harmonic flipping. To the contrary, any high-frequency sequence of energy measurements (e.g. a quasi-continuous series of measurements like in experiment [8]) *must* be sensitive to the standard-deviation dispersion of the energy values about $\langle \mathbf{H}(\tau) \rangle$. This is precisely what happened in [8]: its continuous energy measurement process forces the system to explore through QZE state freezing the variance regions displayed by Figs 1-3. Once their boundaries is reached, the system burns out all of a sudden that very Rabi-flipping energy amount which has been stored during its ground state QZE frozen stage. It jumps to the next excited energy level, thus resuming its QZE state-freezing trajectory on this new level as shown by Fig. 4 [11].

Actually, the above sketch is oversimplified. Indeed the standard deviation boundary (say, at $\tau = 1183 \sim 1200$ in Fig. 4) is only statistical: it has no precise definite value.

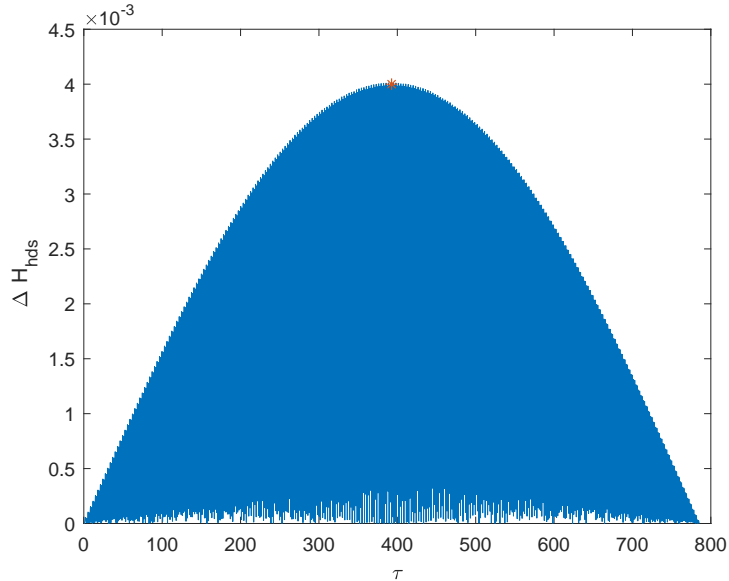


FIG. 5: The quasi-stochastic energy input $\Delta\mathcal{H} = \alpha\mathcal{E}$ in Eq. (6) over half a Rabi period $2\pi/A$ when the origin of time is now taken at $\tau = 785$ in Fig. 4. Its seemingly two-dimensional dense pattern results from its extremely fast oscillations at the scale of the Rabi period and will be regarded as the uncertainty in the energy input at a given time τ . The star indicates the maximum at $\tau = 393$ (or $\mathcal{H} = 0$ in Fig. 4) that scales this uncertainty in Eq. (13).

Therefore the above-described quantum Zeno jump (QZJ) may actually occur at any time within an interval $\Delta\tau$ about this boundary (e.g. about $\tau = 1183$). When trying to evaluate $\Delta\tau$, one should regard the driving term $\Delta\mathcal{H} = \alpha\mathcal{E}$ in Hamiltonian (6) as a quasi-stochastic (in fact quasi-uniformly dense) energy input pattern as shown by Fig. 5. Therefore it yields in our units (3) an estimate of the uncertainty $\Delta\tau$:

$$\Delta\mathcal{H}\Delta\tau \geq \frac{\hbar}{2} = 1, \quad (13)$$

during which any step-like QZJ illustrated by Fig. 4 can statistically occur. We recover Bohr’s undeterministic picture of an instantaneous quantum transition in agreement with the Heisenberg uncertainty principle. Since this QZJ formerly appears at half the gap defined by $\mathcal{H} = 0$, we take the corresponding value $\Delta\mathcal{H} = \Delta\mathcal{H}_{max} = 4 \cdot 10^{-3}$ in (13): see the star in Fig. 5. Therefore $\Delta\tau_{min} \sim 1/\Delta\mathcal{H}_{max} \sim 250$. This time interval is pictured by the two vertical bars about $\mathcal{H} = 0$ in Fig. 6 where the experimental data given in [8] are reproduced by use of red circles, using for the normalization of our τ -axis the experimental Rabi period $T_{Rabi} = 50 \mu s$ given there (the $\sim 13\%$ deviation from the excited eigenvalue $+1$ is due, the authors say, to imperfections, mostly excitations to higher levels). We see that $\Delta\tau_{min} \sim 250 \sim 2\Delta_{mid}$ agrees fairly well with the so-called “time-of-flight” value $\sim 2\Delta_{mid}$ of experiment [8]. Moreover, $\Delta\tau_{min}$ is also in good agreement with HDS action quantization when the system crosses the separatrix of the system at $\mathcal{H} = 0$ [19]. Recall that Heisenberg’s l.h.s. of inequality (13) has indeed the dimension of an action.

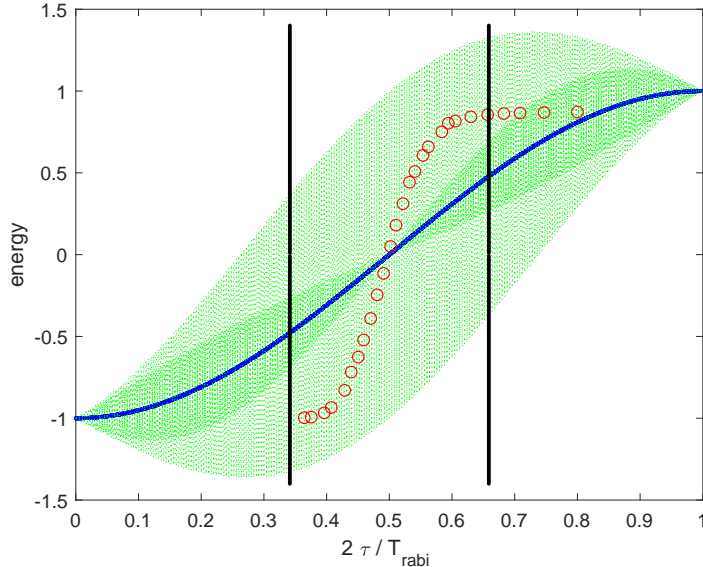


FIG. 6: Second (r.h.s.) part of Fig. 2 illustrating the actual *QZE-vs-Rabi* competition sketched in Fig. 4 by use of the experimental data provided in [8]. Blue plot: Rabi’s mean energy $\langle \mathbf{H}(\tau) \rangle = \mathcal{H}(\tau)$. Circles: the quantum jump reproduced from Fig. 3b of Ref. [8], using its $T_{Rabi} = 50 \mu s$ Rabi period for the normalization of the τ -axis. The two vertical bars display the minimum duration $\Delta\tau_{min}$ of the transition obtained as the consequence of the uncertainty principle (13) when the driving $\Delta\mathcal{H}$ illustrated by Fig. 5 is maximum (star). These bars fit quite well with the duration of the so-called “time-of-flight” value $\sim 2\Delta_{mid}$ given in [8].

V. CONCLUSION: THE GENERIC QUANTUM ZENO JUMP

We have shown that the “time-of-flight” values obtained by the continuous measurement routine in experiment [8] can be obtained by a first-principle consequence of both the energy variance shown in Figs 1-3 and the quantum Zeno effect (QZE) displayed in Fig. 4. They proceed from a generic quantum process and do not depend on the specific repetitive measurement procedure provided this latter is long-lasting enough. *Any* such measurement yields QZE freezing of the corresponding quantum states in their initial configuration [1] [20]. And *any* such QZE freezing lasts as long as the continuous measurement allows the resulting “frozen” qubit dynamics to lie inside of its energy variance range. Since energy is continuously fed into the system by the external driving, this energy excess with respect to the frozen energy state must sooner or later be burnt out. The proposal of the present paper is that this occurs at the very last time, when the variance properties of the system no longer allows it to stay within the statistical time-dependent standard deviation range of available energy about the well-kown RWA Rabi energy mean value. Then the system experiences all of a sudden a quantum jump between the two energy levels in order to resume its state freezing trajectory and stay further in the new frozen state. Due to its simplicity, this approach is in the spirit of Occam’s razor description [21]. It stresses the generic importance of the *QZE freezing vs Rabi flipping* competition resulting in the existence of quantum Zeno jumps (QZJs) in experiment [8] as shown by Fig. 6.

In quantum information processing (QIP), a phase transition can be described as a func-

tion of a given control parameter —e.g. an empirical “friction coefficient”— by a non-analytic behavior of the ground state energy in the quantum dynamics of a Rabi-oscillating system tunneling coherently between two levels [22]. This Rabi oscillatory dynamics is shown to “freeze” when the interaction with the quantum environment exceeds a certain critical strength. A resulting “frozen phase transition” appears as a consequence of a dynamical QZE where the transition is from an isolated system that is weakly perturbed to a state in which the effect of the environment is no longer perturbative. The link with the present work is illuminating. Indeed a series of repetitive measurements of the ground state energy similarly yields, above a given threshold of the measurement frequency [4], a non-analytic —since non-perturbative— QZE “freezing” quantum dynamics in the variance range of the Rabi-oscillating system. Therefore any external energy feeding will result in the same kind of *QZE freezing vs Rabi flipping* competition as in the present work.

This process can also be regarded as a model for decoherence in time-dependent transport described by a dynamical formulation of the Landauer-Büttiker equations [23]. It yields a form of wave function which undergoes a smooth stochastic drift of the phase in a local basis. This latter is nothing else but a local energy fluctuation that is a quantum-drift alternative in QIP to the standard model of a quantum jump. It resembles the QZJ process, as shown by the stroboscopic representation of the inhomogeneous system-environment interaction [9] [10]. The authors introduce a fictitiously homogenized interaction rate in the dynamics of a swapping gate, showing a quantum dynamical phase transition where the dynamical behavior changes from a swapping phase to an overdamped phase. Here again, this last regime can be associated with QZE since frequent projective measurements prevent the quantum evolution. This dynamical Zeno effect is produced by interactions with the environment and freezes the oscillations of the system like in the present work.

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- [1] B. B. Misra and E. Sudarshan, *J. Math. Phys.* **18**, 756 (1977).
 - [2] D. D. Home and M. Whitaker, *Phys. Lett. A* **173**, 327 (1993).
 - [3] J. Von Neumann, *Die mathematischen Grundlagen der Quantenmechanik* (Springer Berlin, 1932).
 - [4] W. Itano, D. Heinzen, J. Bollinger, and D. . Wineland, *Phys. Rev. A* **41**, 2295 (1990).
 - [5] R. J. Cook, *Phys. Scr.* **T21**, 49 (1988).
 - [6] M. Porrati and S. Putterman, *Phys. Rev. A* **36**, 929 (1987).
 - [7] J. M. Raimond, *La Recherche* **555**, 41 (2020).
 - [8] Z. Mineev, S. Mundhada, S. Shankar, P. Reinhold, R. Gutiérrez-Jáuregui, R. Schoelkopf, M. Mirrahimi, H. Carmichael, and M. Devoret, *Nature* **570**, 200 (2019).
 - [9] G. Álvarez, E. Danieli, P. Levstein, and H. Pastawski, *J. Chem. Phys.* **124**, 194507 (2006), <https://doi.org/10.1063/1.2193518>.
 - [10] G. Álvarez, E. Danieli, P. Levstein, and H. Pastawski, *Phys. Rev. A* **75**, 062116 (2007), doi: 10.1103/PhysRevA.75.062116.
 - [11] G. Reinisch, arXiv:2205.12763 [quant-ph] (2022), <https://doi.org/10.48550/arXiv.2205.12763>.
 - [12] G. Reinisch, *Physica D* **119**, 239 (1998).
 - [13] G. Reinisch, *Phys. Lett. A* **238**, 107 (1998).
 - [14] J. C. Solem and L. C. Biedenharn, *Foundation of Phys.* **23**, 185 (1993).
 - [15] H. H. Rauch, A. Zeilinger, G. Badurek, A. Wilting, W. Bauspiess, and U. Bonse, *Phys. Lett.*

- A **54**, 425 (1975).
- [16] S. Werner, R. Colella, A. Overhauser, and C. Eagen, Phys. Rev. Lett. **35**, 1053 (1975).
 - [17] A. Klein and G. Opat, Phys. Rev. Lett. **37**, 238 (1976).
 - [18] M. E. Stoll, E. K. Wolff, and M. Mehring, Phys. Rev. A **17**, 1561 (1978), doi.org/10.1103/PhysRevA.17.1561.
 - [19] G. Reinisch, Results in Physics **29**, 104761 (2021), doi.org/10.1016/j.rinp.2021.104761.
 - [20] W. Itano, Current Science **116**, , 201 (2019).
 - [21] I. Mazin, Nature Physics **18**, 367 (2022).
 - [22] H. Pastawski, Phys. B **398**, 278–286 (2007), arXiv:0705.0773v1 [cond-mat.other] 7 may 2007.
 - [23] J. Fernández-Alcázar and H. Pastawski, Phys. Rev. A **91**, 022117 (2015).

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Data availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflict of Interest

The authors have no conflicts to disclose.