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# Tracking multiple targets with multiple radars using Distributed Auctions

Pierre Larrenie\*      Cédric L R Buron†      Frédéric Barbaresco‡

## Abstract

Coordination of radars can be performed in various ways. To be more resilient radar networks can be coordinated in a decentralized way. In this paper, we introduce a highly resilient algorithm for radar coordination based on decentralized and collaborative bundle auctions. We first formalize our problem as a constrained optimization problem and apply a market-based algorithm to provide an approximate solution. Our approach allows to track simultaneously multiple targets, and to use up to two radars tracking the same target to improve accuracy. We show that our approach performs sensibly as well as a centralized approach relying on a MIP solver, and depending on the situations, may outperform it or be outperformed.

**Collaborative combat, Distributed Auctions, Multi-Radar Tracker**

## 1 Introduction

The management of small and cheap sensors is a rising topic, both for civil and defence applications. In particular, recent advances have been made in the management of small radars, both in a centralized and a decentralized way. Decentralized approaches provide the system with more robust coordination, but are often expected to perform more poorly than the centralized approaches. Still, some methods have shown to be very effective, and are indeed widely used in robotics, such as auction-based ones.

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In this paper, we deal with the problem of target allocation among a set of radars with rotating antennas. In order to have a robust approach, we propose a fully decentralized approach. Our approach allows to allocate each target to one or two radars, getting lower uncertainty on the position of the target when two radars track them. Our method relies on collaborative auction. This method is among the ones that are fast enough to cope with the robotics issues [1]. To cope with the specificity of our problem –the possibility to reduce the uncertainty on the location of the target– we propose to use 2 turns of the same allocation algorithm. The first one ensures as many targets as possible are tracked, while the second one allocates remaining resources to already-tracked targets in order to reduce uncertainty.

In order to take dynamism, *i.e.* moving targets in account, the algorithm is restarted on a regular basis. It ensures that a bid made by a radar can decrease when the target moves away from it. In our applications, the targets move quickly, and their movement cannot be predicted on the long term, we therefore rely on a collaborative auctions algorithm that does not plan the following rounds. Our algorithm is however based on radars capable of anticipating the short-term positions of targets, thanks to a Kalman filter. We test our approach on multi-radar tracking scenarios where the radars, autonomous, must follow a set of targets in order to reduce the position uncertainty of the targets. Surveillance aspects are not taken in account in this paper. It is assumed that the radars can pick up targets in active tracking, with an area of uncertainty corresponding to their distance.

The paper is organized as follows: we first introduce existing works for multi-radar multi-target allocation. Section 3 presents the problem, with the radar model we use, and the mathematical optimization problem formalizing the target allocation for a set of radars. We then present our approach based on a double consensus-based bundle auction, and the simulation we have been using for the evaluation of our method. Section 5 presents the results of our method and compares it with a centralized approach relying on operational research. We summarize our work and provide possible perspectives in section 6.

## 2 Related works

Several works have focused on the use of decentralized approaches for task allocation for sensor, since the seminal work of Lesser et al. [2]. Since then, many approaches have been used among which Max-Sum based algorithms, market based algorithms, and reactive approaches [1]. Recently, for real time use cases, auction methods have gained much interest in the multi-agent community [3] for their capacity to perform good allocation in an affordable time. Many methods have been using auctions since then to allocate tasks in real-time, including to robots, sensors and radars (see for instance [4]).

One of the most successful recent algorithms is the Consensus-Based Bundle Auction algorithm (CBBA) [5], a decentralized auction-based allocation algorithm. This algorithm allows to perform the allocation in a fully decentralized way, the agents acting both as auctioneers and bidders. This algorithm has since been used several times for sensors [6, 7]. However, none of them has been taking into account the specificities of radars, *i.e.*, their collaboration through the intersection of their uncertainty ellipses. Similarly, the challenge of high dynamicity has been barely studied.

## 3 Problem statement

In this section, we introduce the different elements of the problem. We first describe the model that has been used for the radars. We then introduce a definition of the multi-target multi-radar allocation problem in the formalism of Constraint Optimization Problems (COP).

### 3.1 Radar model

We consider that each radar has a 2-dimensional frame in a polar coordinate system centered on itself. It is estimated that the influence of elevation is negligible, so it is not useful to use a 3-dimensional landmark.

Each target therefore has a position in the radar reference frame determined by its distance, denoted  $r$ , and its azimuth (polar angle), denoted  $\theta$ . The precision of the measurement made by the radar is noted:  $\sigma_r$  in distance and  $\sigma_\theta$  in azimuth. The resulting measurement uncertainty is represented as an ellipse. The measure itself corresponds to a centered 2-dimensional Gaussian random variable of covariance  $K = R(\theta) \begin{pmatrix} \sigma_r & 0 \\ 0 & \sigma_\theta \end{pmatrix}$ , where  $R(\theta)$  represents the rotation matrix of angle  $\theta$ . During active tracking, the aim is therefore to anticipate the next measure on the target given its past positions and its current position thanks to the use of a Kalman filter. As for the measurement, there is also a prediction uncertainty.

The signal received by the radar is assumed to be subject to Gaussian white noise. In order to reduce the size of the problem and keep it tractable, the signal to noise ratio (S/N)<sup>1</sup> is assumed to be constant. The value is set to 13, which corresponds to a common value in practice. The S/N will influence the standard deviation of the measures. The S/N corresponds to the quality of the output desired by the user. For a given S/N it is possible to choose the parameters such as the wavelength to be transmitted or the transmission power, etc.

### 3.2 Multi-sensor multi-target allocation problem

The problem of optimal multi-radar and multi-target allocation can be handled in various ways. If we want to take account of the anticipation of future actions, we can model it through Decentralized Partially Observable Markov Decision Processes. However, this problem is in the EXPTIME class (with a complexity in  $O(\exp(\exp(n)))$  where  $n$  is the number of tasks, which is clearly not applicable to common application cases, and even heuristic methods can take a very long time. Moreover, the target we consider are highly mobile and their unpredictability makes the anticipation of the future very hard to compute. We therefore propose to formalize the problem of optimal allocation on each time step, not taking the future evolution in account. This problem can be formalized as a constraint optimization problem:

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<sup>1</sup>The S/N (Signal to Noise Ratio) corresponds to the ratio between the noise power of the useful signal and the power of ambient noise.

$$\max \sum_{i,j,k} c_{ikj} \cdot w_{ikj}$$

s.t.:

$$\begin{cases} w_{ikj} = x_{Mij} \wedge x_{Okj}, \forall (i, k) \in \mathcal{I}^2, \forall j \in \mathcal{J} & (A_{ikj}) \\ \sum_i w_{ij} \leq 1, \forall j \in \mathcal{J} & (C2) \\ \sum_j \gamma_{ij} \cdot (x_{Mij} + x_{Okj} - w_{ij}) \leq L_{ti}, \forall i \in \mathcal{I} & (L) \\ (x_{Mij}, x_{Okj}) \in \{0, 1\}^2, \forall (i, k, j) \in \mathcal{I} \times \mathcal{I} \times \mathcal{J} \\ w_{ikj} \in \{0, 1\}, \forall (i, k) \in \mathcal{I}^2, \forall j \in \mathcal{J} \end{cases}$$

where

- $\mathcal{I}$  is the set of radars and  $\mathcal{J}$  the set of tasks. Note that we are placed here, in the framework  $\mathcal{I} \ll \mathcal{J}$ .
- $c_{ikj}$  : Corresponds to the utility that the radar  $i$  and the radar  $k$  provide to the system if the radar  $i$  handles the task  $j$  as a main radar and  $k$  as an optional radar.  $c_{ikj}$  is of the following form, with  $V(\mathcal{E}_{ij})$  (respectively  $V(\mathcal{E}_{kj})$ ) the surface of the ellipse  $\mathcal{E}_{ij}$  ( resp .  $\mathcal{E}_{kj}$ ) described by the matrix  $P_{ij}$  (resp.  $P_{kj}$ ) of the Kalman filter of the radar  $i$  (resp.  $k$ ) for the target  $j$  and  $V(\mathcal{E}_{ij} \cap \mathcal{E}_{kj})$  the intersection volume of these two ellipses.
- $x_{Mij}$  : Boolean variable,  $x_{Mij}$  equals 1 if the radar  $i$  performs the task  $j$  as the main radar, 0 otherwise.
- $x_{Oij}$  : Boolean variable,  $x_{Oij}$  equals 1 if the radar  $i$  performs the task  $j$  as an optional radar, 0 otherwise.
- $w_{ikj}$  : Boolean variable,  $w_{ikj}$  equals 1 if the radar  $i$  performs the task  $j$  as main radar and the radar  $k$  performs the task  $j$  as optional radar, 0 otherwise.

The constraints can be understood the following way:

- $(A_{ikj})$  defines  $w_{ikj}$  as  $i$  following the target  $j$  as main radar (we also write  $x_{Mij} = 1$ ), and  $k$  follows it as optional radar  $x_{Okj} = 1$ .  $w_{iik} = 1$  if there is only one radar  $i$  following the target. The operator  $\wedge$  corresponds to the logical AND operator.
- $(C2)$  lists all possible combinations of 2 sensors that track a target  $j$ . There is at most only one combination of sensors that can be chosen.
- $(L)$  models the load of the radar. If the radar is tracking the target as main or optional radar, the term between parentheses equals 1, otherwise it equals 0 and the load for the task is therefore not considered.

There is a total of  $|\mathcal{I}|^2 \cdot |\mathcal{J}| + |\mathcal{J}| + |\mathcal{I}|$  constraints.

Note that the present formulation is difficult to generalize to a set of  $n$  sensors, because it would increase the number of constraints and Boolean variables far too much. Moreover, this would make the problem insoluble<sup>2</sup> for a classical solver. In this paper, we limit ourselves to 2 radars for each target.

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<sup>2</sup>Insoluble in a reasonable time. Indeed, the presence of  $n$  Boolean variables requires performing an enumeration, ie  $2^n$  of possibilities.

## 4 Auction-based multi-radar multi-target allocation

The approach that we present in this article is based on the use of two successive CBBA algorithms: the first to make an allocation as the main radar, the second as an optional radar. In this section, we proceed in two steps. We first present the CBBA algorithm and the adaptations we have made to it so that it can consider the specificities of radars. We then present the general process allows to consider the interactions between radars and the dynamism of the mission.

### 4.1 The CBBA algorithm

The Consensus Based Bundle Algorithm [5] is an algorithm where agents bid on a sequence of tasks based on the information they have, and share information with their neighbours. The algorithm can be divided in two phases, that are repeated one after another: (1) the bidding phase during which the agent propose a bid on a sequence (or bundle) of action while trying to optimize the improvement in terms of utility in comparison with the information they have on the current situation. (2) the consensus phase, where the agent send this information, and take information from neighbors, and modify their bundle according to the newly obtained information.

The messages that agents send to each other can be represented as a set of vectors. The set of vectors that an agent sends to another one corresponds to its current knowledge of the system. It includes:

- $Y$  the winning bid utility for each target. For a radar  $i$ ,  $Y = (y_{ij})_{j \leq |T|}$
- $Z$  the identity of the winner for each target. For a radar  $i$ ,  $Z = (z_{ij})_{j \leq |T|}$ .
- $S$ , which corresponds to a “timestamp” vector, it makes it possible to manage conflicts by making it possible to keep the track of the contacts between radars. For a radar  $i$ ,  $S_i = (s_{ik})_{k \leq |A|}$ . It allows to select the most up-to-date information when there is a conflict among received information.

The CBBA algorithm has a 50% performance guarantee, *i.e.*, in the worst case, the global solution obtained is greater than half of the optimal solution. This algorithm is mainly aimed at cases where planning makes sense, for instance when agents are mobile robots and where they must plan a route. But in our case, we must adapt this algorithm in order to make bids on a set of targets, not taking into account the order of the sequence.

### 4.2 Adaptation of CBBA to radars

In our case, *for the allocation as main radar*, an additional vector will also be sent, the vector  $E = (e_{ij})_{j \leq |T|}$  that groups the ellipses leading to the winning bids for each target. This notably makes it possible to calculate intersections with the latter.

In order to respect the Disminishing Marginal Gain (DMG) constraint that is required by CBBA, it is necessary to lower the utility of any new target being tracked. To take account of this constraint, we introduced the following bias:

$$c_{ij}^{\text{CBBA}} = \frac{c_{ij}}{|b_i|}$$

The algorithm operates in a closed loop and is executed at each time step; the agent makes an allocation as the main radar, then it makes the allocation as an optional radar if it has remaining budget. Each allocation is made through a CBBA algorithm, and therefore includes the two phases of the algorithm (auction and consensus) explained above. It therefore receives and sends information on its allocation as main and optional radar at each time step. A radar does not take into consideration the targets which it follows as main radar in the list of targets which it can take as optional radar.

To implement the interaction between radars, the ellipses sent by the radars are taken into account by the other radars to perform the utility calculation for the allocation as an optional radar. In order to follow as many targets as possible, when the agent computes its allocation as main radar, it considers its budget as all of its remaining budget plus the budget allocated as optional radar. If a new allocation as main radar is possible, it deallocates the tasks as optional radar with the lowest utility, and performs a reset as described in the previous section.

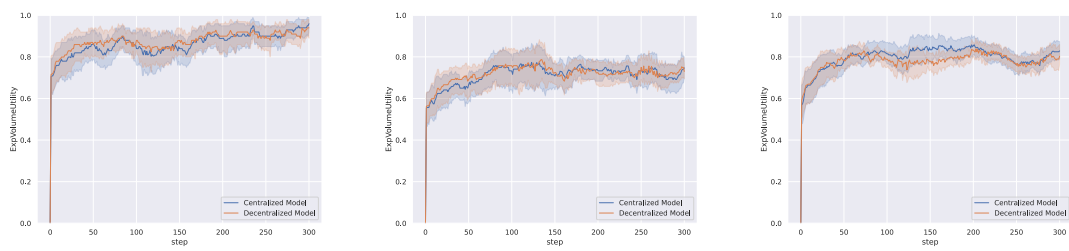
In the static case, when all the radars have the same beliefs on the allocation, we say that the consensus is established. That is to say that a distributed allocation conflict-free could be found; this therefore constitutes the end of the algorithm.

To take account of the dynamic aspect of our problem, the auctions never stop, and keep running until the end of the simulation.

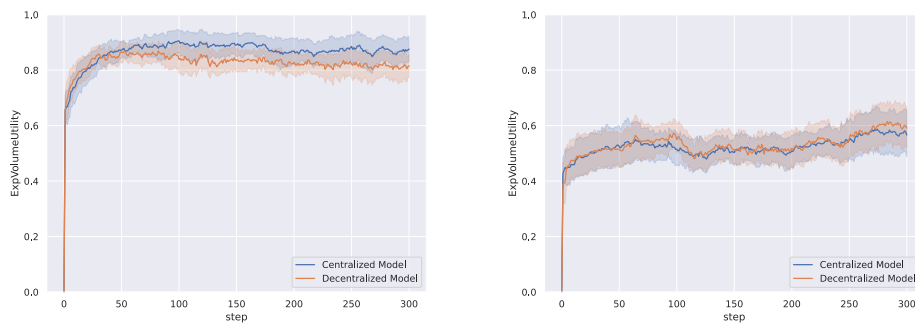
1. The radar makes the bidding phase as the main radar. To do so, it computes the uncertainty ellipses for each target, and its utility function; it also applies the Kalman filters of all the targets it is already tracking.
2. If it has remaining radar time budget, the radar proceeds to the bidding phase as an optional radar on all the targets that are not already tracked as the main radar (with its remaining budget).
3. The radar proceeds to the consensus phase as the primary radar. The vectors  $Y$ ,  $Z$ ,  $E$  and  $S$  as main radars are updated; vectors are sent to neighbors.
4. The radar then proceeds to the consensus phase as an optional radar. Vectors  $Y$ ,  $Z$  and  $S$  as optional radars are updated; vectors are sent to neighbors.
5. The radar tracks the targets it has selected, possibly by applying its Kalman filter.

Note that since the radar initiates the tracks at each execution of the two phases of CBBA, which means that the algorithm does not have time to converge. In practice, this situation can generate conflicts, for instance in the case where a target on the edge between two radars, and is increasingly threatening. In this case, each radar takes its decision on a previous value of the utility of the others, and its own current value, it considers that its bid wins the bet. Similarly, targets getting less threatening (*i.e.*, with decreasing utility) is potentially tracked by none of the radars, each radar considering that another one has a better bid than itself.

Our approach makes it possible to solve the task allocation problem in an advantageous way: in the case where the radars remain in contact, even if the communication graph evolves during the mission, the algorithm keeps working as far as graph of connection of the radars remains path-connected. Finally, by carrying out two sequential auction runs, our algorithm also takes into account the possible overlapping of uncertainty ellipses, and thus makes it



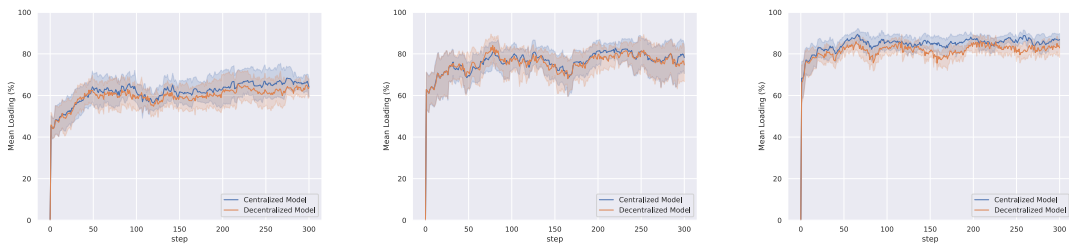
(a) Non saturated well positioned radars      (b) Few saturated well positioned radars      (c) Several saturated well positioned radars



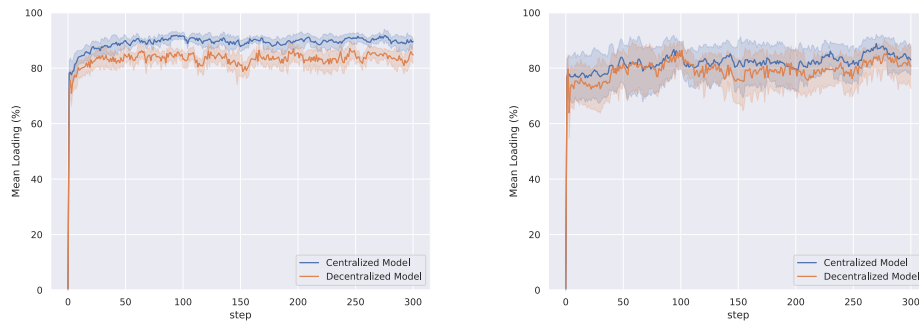
(d) Many saturated well positioned radars      (e) Saturated ill-positioned radars

Figure 1: Utility on the reference scenarios





(a) Non saturated well positioned radars      (b) Few saturated well positioned radars      (c) Several saturated well positioned radars



(d) Many saturated well positioned radars      (e) Saturated ill-positioned radars

Figure 2: Load on the reference scenarios

possible to generate an allocation favoring more precise tracking of targets when possible, while trying to track as many targets as possible (depending on radar capability).

Radars must also be able to differentiate between first-round allocation messages – one that tracks as many targets as possible – and one that improves accuracy by generating an intersection of uncertainty ellipses. Each radar  $i$  proceeds by trying to maximize  $\sum_j x_{ij} \cdot c_{ij}$  for the main allocation (resp.  $\sum_{j,k} w_{ikj} \cdot c_{ikj}$  for the optional allocation), that is to say the sum of the utilities corresponding to the targets that it tracks, while taking into account the information received from the other radars, in particular the bids made by the latter.

The implementation must include an additional target disambiguation mechanism, making it possible to identify the targets present at several radars, and in particular a plot merging algorithm, making it possible to match the targets of the different radars. This induces the sending of additional information enabling this operation to be carried out, such as the estimated speed and position of the targets.

## 5 Results

The implementation of our model has been performed on the MESA framework [8], along with the Kalman filter package [9], on the same simulator as the one used in [10]. As in this article, we rely on 5 kinds of scenarios, similar but not identical to those of [10]; results are averaged on 10 such scenarios for each graph:

- Non saturated well positioned radars (5 radars, 10 targets)
- Few saturated well positioned radars (3 radars, 12 targets)
- Several saturated well positioned radars (5 radars, 20 targets)
- Many saturated well positioned radars (8 radars, 30 targets)
- Saturated ill-positioned radars (4 radars, 20 targets)

In order to evaluate our work, we compare it to an centralized allocation, performed with the Coin-OR Branch and Cut tool (CBC) [11]. Results are represented on figs. 1 and 2 and are averaged on 10 similar scenarios. The blue (resp. orange) curves correspond to the decentralized (resp. centralized) approach. The standard deviation is represented in light blue/light orange.

The utility of the system on the scenarios is represented on fig. 1. The decentralized approach is pretty close to the results of the centralized approach, in particular for situations where the radars are not saturated. This is pretty interesting because for similar utility, the decentralized approach gets lower load. This can be explained by the fact that the optimization is performed only on the current step and the centralized approach computes a new allocation through optimization algorithm each step, while the CBBA only makes changes if it has remaining budget, which makes it more “stable”, and therefore improves the result of the Kalman filters. It is also the case for ill-positioned radars, where the “jumps” between radars do not have big influence on the utility, as the improvement is equivalent to the loss in terms of the Kalman filter. The case with few radars is almost similar for the centralized and decentralized approaches, the gaps and loss in optimality balancing each other.

In cases where the radars are saturated and numerous, on the contrary the radars jump from target to target for both approaches but centralized approach makes a more complete

exploration of the possible allocations. This explains the higher utility for the centralized approach and also the higher load. The coverage (proportion of tracked targets, not presented for space reasons) is also higher for the centralized approach, which confirms our hypothesis.

Overall, our approach performs almost as well as the centralized approach. When the radars are not saturated, it even performs better. In cases where the radars are saturated and numerous, a less complete research leads to targets not being tracked, and the centralized approach is better.

## 6 Conclusion

In this paper, we presented a novel approach for allocating target to a team of radars in a totally decentralized way. This approach is based on a fully decentralized auction algorithm, CBBA. We showed that, when taking into account the intersection of uncertainty ellipses, the results of this algorithm is comparable to the centralized allocation, better in cases where the radars are not too numerous and not saturated, and a bit worse when the radars are numerous and saturated.

Future works include the design of a more generic approach handle an arbitrary number of radars following the same target. We also would like to make our approach more dynamic, for instance by including replanning approaches that have been proposed to improve CBBA [12] and evaluate this approach in the setting imposed by our use-case. Finally, including a possible variation of the SNR would allow a finer allocation and therefore improve our approach.

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