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Fast load-flow calculations based on spanning trees.

Pierrick Guichard, Nicolas Retière, Didier Mayou

***Abstract*—In this article a method is implemented for computing load flows that does not rely on the full resolution of a linear system. Instead, we solve very quickly many local load flows corresponding to dipoles (the electrical power is injected at two nodes) with an algorithm based on a Hamiltonian formalism, as it is usually done in quantum physics. The computational burden is quadratic in the size of the network when the dipoles are defined by the lines of a spanning tree of the network. This leads to a considerable speedup when compared to traditional methods relying on the full system inversion as well as methods involving other subproblems.**

***Index Terms*—Load flow, Local approach, Lanczos algorithm, Hamiltonian formalism, Spanning trees.**

I. Introduction

Efficiency is at the heart of many developed methods for estimating the state of a large electrical system, a crucial step in assessing its security [2] [3]. The complete resolution of the load flow is a time demanding operation. That is why approximate methods as the quasi-decoupled load-flow method [16] have been used. This algorithm has been extended for the specific case of distribution networks [17] and adapted for the ranking of line contingencies [18]. Despite these various advantages and its wide range of application, this method still involves solving two linear systems, which is a time consuming operation even for sparse matrices. Indeed, the usual matrix inversion methods turn out to be cubic in the size of the system N. Some algorithms lower the complexity of the inversion to $O(N^\alpha)$ arithmetic operations, with $2 < \alpha < 3$ [20]. However, these methods may not be numerically stable. Also they may even perform worse than asymptotic cubic algorithms when applied to small or middle sized problems [21]. That is why we will not use in this paper techniques relying on the full inversion of matrices (such as the nodal laplacian matrix of the network), nor spectral based methods [8]. Rather, we extend here a method presented in [1] that doesn't rely on the full resolution of a linear system and presents a substantial gain, as the number of arithmetic operations is only $O(N)$. It is based on the computation of load flows relative to specific power injections, that are dipoles.

Focusing on the DC load flow equations [5], [8], we have proposed an approach avoiding the full resolution of linear systems, by taking the physical properties of the load flow to its advantage [1]. The DC load flow equations are rewritten in the Hamiltonian formalism well known from quantum physicists. This new formulation has allowed a powerful method from that field (the Lanczos algorithm) to be successfully applied to power systems. The Lanczos method is by design well adapted for solving local problems on large systems, locality meaning that under certain circumstances, an event has little (and often, negligible) impact on distant parts of the network. The method was therefore used for the computation of the redistribution of power flows in the network after the outage of one line. This comes back to the calculation of the electrical state induced by an opposite power injection (a so-called dipole) at both endpoints of the failed line. The short size of the dipole, limited by the distance between the two nodes previously interconnected by the failed line, ensures that the power redistributions are local and can therefore be solved rapidly with the Lanczos algorithm.

Yet, in the general case the power injections are distributed throughout the network and are therefore likely to trigger a system-wide response. This mandates an adaptation of our previously proposed methodology for taking into account any possible power source belonging to the vector space $\mathcal{P}$ of zero-sum vectors (the power injections being obviously balanced between sources and loads). For that purpose, we proceed by searching a basis (p_i) of $\mathcal{P}$ such that **(1)** any vector of $\mathcal{P}$ can be decomposed easily on the basis (p_i) and **(2)** the solution corresponding to each element of the basis (p_i) can be computed rapidly. We show that a basis of dipoles that correspond to the lines of a spanning tree of the network fulfill both requirements. In particular, the short size of these dipoles guarantees that the condition **(2)** is well verified when the Lanczos algorithm is used, and a considerable speedup may be observed when compared to a situation where longer dipoles are involved.

This paper is divided into three sections after the present introduction. The first two sections briefly recall the main results stated in [1]. Section (II) specifically details the electrical model of the power system and the equivalence between the usual Laplacian formalism of the DC load flow equations and the new Hamiltonian formalism introduced in [1]. Section (III) recalls the Lanczos algorithm and how a local resolution is by design provided by this method. Then section (IV) is dedicated to the construction of the basis (p_i) defined on a spanning tree of the vector space $\mathcal{P}$. It also contains the results of tests performed on various IEEE test networks [9] [15] and on an open source model of the European network [10] [11].

II. Derivation of the Hamiltonian formalism

A power grid can be modeled by a graph with N_{nodes} nodes and N_{lines} lines. Every line l has a start node i_l and an end node j_l and every pair of adjacent nodes i and j defines a line $l(i,j)$. Every line l is modeled by a susceptance B_l. At every node $n = 1 \dots N_{\text{nodes}}$, a power load or source P_n is

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connected ($P_n = 0$ when no source nor load is connected). Under DC assumptions [4], the steady state of the system can be described at every node by the following equations, involving the Laplacian matrix $\mathbb{L}$

$$P = \mathbb{L}\theta, \quad \text{with} \begin{cases} \mathbb{L}_{ij} = -B_{l(i,j)} \text{ if } i \neq j \\ \mathbb{L}_{ii} = \sum_{j \neq i} B_{l(i,j)} \end{cases} \tag{1}$$

where θ is the vector storing the phase angles at every node. Rather than solving this Laplacian system for θ, an effective strategy has been to define the new variables (ψ_l^{L}) on the lines of the network (such as done in figure (1))

$$\forall\, l = 1 \dots N_{\text{lines}}, \quad \psi_l^{\mathrm{L}} := \sqrt{B_l}(\theta_{i_l} - \theta_{j_l}) \tag{2}$$

The Hamiltonian formalism of the DC load flow is obtained by rewriting Eq (1) in terms of the new variables (ψ_l^{L}), introducing the vectors ψ and $\mathbb{P}$ of size $N_{\text{nodes}} + N_{\text{lines}}$ and the Hamiltonian matrix $\mathbb{H}$ as

$$\underbrace{\left[\begin{array}{c|c} (0) & \mathbb{H}_{\mathrm{LN}}^{\dagger} \\ \hline \mathbb{H}_{\mathrm{LN}} & (0) \end{array}\right]}_{\mathbb{H}} \underbrace{\left[\begin{array}{c} (0) \\ \hline \mathbb{H}_{\mathrm{LN}}\theta \end{array}\right]}_{\psi := \left[\begin{array}{c} (0) \\ \hline \psi^{\mathrm{L}} \end{array}\right]} = \underbrace{\left[\begin{array}{c} P \\ \hline (0) \end{array}\right]}_{\mathbb{P}} \tag{3}$$

$\mathbb{H}_{\mathrm{LN}}$ of size $N_{\text{lines}} \times N_{\text{nodes}}$ is related to the Laplacian matrix from Eq. (1) by $\mathbb{L} = \mathbb{H}_{\mathrm{LN}}^{\dagger} \cdot \mathbb{H}_{\mathrm{LN}}$ and has null coefficients except for

$$\forall l = 1 \dots N_{\text{lines}}, \begin{cases} [\mathbb{H}_{\mathrm{LN}}]_{l,i_l} = \sqrt{B_l} \\ [\mathbb{H}_{\mathrm{LN}}]_{l,j_l} = -\sqrt{B_l} \end{cases} \tag{4}$$

Eq (3) models steady-state operations of power systems. It can however be extended for dynamical applications at low frequency, when the behaviour is mainly driven by the generator's swing equations [12] [13] [14]. In that case, the Hamiltonian formalism yields the following Schrödinger-like equation [1]

$$i\frac{\partial \psi}{\partial t} = \mathbb{H}\psi - \mathbb{P}. \tag{5}$$

It establishes an analogy with quantum physics, where equation (5) describes the injection of a quantum particle on a grid.

III. Lanczos method and locality principle in the Hamiltonian formulation

Numerically, solving equation (3) is time demanding because it involves solving a linear system which is a cubic operation in function of its size. As it is usually done in physics, an alternative is to project the system onto a subspace of dimension $\bar{N}$ (the Krylov space) of the space of solutions. This space possesses by definition properties that accelerate the computation of the solutions when dealing with local phenomena. Indeed, it is defined as being the space spanned by the dot products between a starting state $q_1 = \frac{\mathbb{P}}{||\mathbb{P}||}$ and the first powers of the Hamiltonian matrix $\mathbb{H}$. Since applying $\mathbb{H}$ to any state comes back to transport that state to its node or line neighbours, the spanning elements of the Krylov space

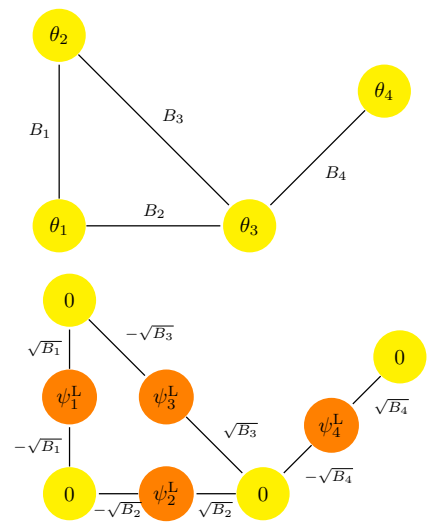

Fig. 1. Laplacian formalism (top panel) and equivalent Hamiltonian formalism (bottom panel). The arbitrary line ordering we adopted here is: $l(1,2) = 1,\ l(1,3) = 2\ , l(2,3) = 3,\ l(3,4) = 4$.

spread out concentrically in the network from the starting state. Therefore solutions that are located in the vicinity of q_1 can be computed with an acceptable accuracy well before the complete space of the solutions is spanned.

There is a unique way of defining an orthonormal basis of the Krylov space that contains q_1. This basis is computed with the Lanczos algorithm [6], [7]. At the step i, q_i is calculated by applying $\mathbb{H}$ to $q_{i-1}$and then orthonormalizing the result against q_{i-1} and q_{i-2}. The Lanczos basis (also called the recursion chain) is plotted in the figure (2). It shows the basis $(q_1 \dots q_{\bar{N}})$ as well as the action of the Hamiltonian $\mathbb{H}$ on each element of the basis. By construction, each element of the basis is only linked to its two neighbours and to itself. (α_i) define the self intercations and (β_i) give the mutual interactions between two neighbouring elements of the basis. They are respectively the diagonal elements and the extradiagonal elements of $\mathbb{H}$ in the Lanczos basis. Moreover, since $\mathbb{H}$ has two null diagonal blocks, the node states are only coupled to the line states (and reciprocally), hence all the self-interactions are equal to zero. Therefore, in the Lanczos basis, the Hamiltonian matrix has only null coefficients, except on the first superior and inferior diagonals.

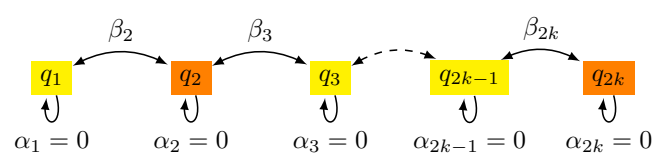

Fig. 2. Representation of the recursion chain (or Lanczos basis) with $\bar{N} = 2k$ (see [1]). The application of the Hamiltonian to a vector of the basis yields a linear combination of the adjacent vectors linked by the arrows. The node states (yellow) and the line states (orange) alternate.

The recursion chain can be used for computing the solution ψ of equation (3). Usually, the recursion is stopped before the complete space of the solutions is spanned and we compute an

approximation $\tilde{\psi}$ of the solution ψ on the recursion chain. The error can be decreased (and eventually, cancelled) by adding states to the chain. On a chain of length $\bar{N}$, $\tilde{\psi}$ is given by

$$\tilde{\psi} = \sum_{i=1}^{k} \kappa_{2i} q_{2i} \tag{6}$$

with $k := \lfloor \frac{\bar{N}}{2} \rfloor$, and where the coefficients (κ_{2i}) are given by

$$\begin{cases} \kappa_2 \beta_2 = ||\mathbb{P}|| \\ \kappa_{2i+2} = -\kappa_{2i} \frac{\beta_{2i+1}}{\beta_{2i+2}}, \quad \forall i = 1 \dots k-1 \end{cases} \tag{7}$$

This method has proven to be particularly efficient when the power injections defining the starting state $q_1 = \frac{\mathbb{P}}{||\mathbb{P}||}$ from which the recursion is started are located on nodes that are at a small distance from each other in the network [1]. As mentioned in the introduction, this is typically the case when a local perturbation occur, e.g a line failure. In that case, $\mathbb{P}$ is a dipole located at the termination nodes of the failed line and it therefore fulfills completely our locality requirement. In that case, the numerical cost of the resolution is linear in the number of nodes of the network, as only a bounded number of Lanczos steps are needed, every step involving a sparse matrix-vector dot product that has a linear cost in the number of nodes.

IV. Calculating the load flow resulting from system-wide power injections

When the power is injected at various nodes of the network, the solution of Eq (3) is likely to present a network-wide extension. So the locality principle is verified in a weaker sense. A natural way of restoring strong locality is to decompose the space $\mathcal{P}$ on a basis of short dipoles. Even if there are $N_{\text{nodes}}-1$ solutions (one for each element of the basis) to compute, this method turns out to be quadratic in the number of nodes of the network, whereas strategies involving the resolution of a linear system remain cubic in the number of nodes of the network. That is why we will in this section focus only on dipole-based strategies to evaluate their benefits for the computation of load-flows.

A. Decomposition of any power injections on a dipole basis

The method consists in selecting a basis (p_i) of the space $\mathcal{P}$ of admissible power injections (which is the $(N_{\text{nodes}}-1)$-dimensional space of vectors that have a null sum). Then, any power injection $\mathbb{P}$ can be decomposed on this basis such as $\mathbb{P} = \sum_{i=1}^{N_{\text{nodes}}-1} \lambda_i p_i$, hence the corresponding solution ψ of Eq (3) can be written

$$\psi = \sum_{i=1}^{N_{\text{nodes}}-1} \lambda_i \phi_i \tag{8}$$

where (ϕ_i) are the solutions of equation (3) relatively to the power injections (p_i). So the problem goes back to finding a set of well-adapted power injections (p_i) and a strategy for computing the related (λ_i) and (ϕ_i). This method is beneficial if **(1)** the coefficients (λ_i) are easily obtainable in function of $\mathbb{P}$ and **(2)** the solutions (ϕ_i) can be computed rapidly with the locality principle. Inspired by our previous work, we focus on power injections (p_i) that are dipoles.

B. Two different methods for defining the basis (p_i)

They are many possible choices for defining a basis (p_i) of $\mathcal{P}$. Below we present two choices that are found to either slow down convergence or speed it up, respectively.

- **A strategy based on a common origin (long dipoles):** First, it is possible to define a basis of $\mathcal{P}$ without trying to minimize the size of the dipoles, meaning the distance between the two nodes of the dipole. For instance we consider that the dipole p_i injects powers of opposite value $(-1, +1)$ at the nodes 1 (called the "common origin") and $i+1$, respectively. The results for this strategy are presented in table (I) under the denomination "Long dipoles".
- **A strategy based on a spanning tree (short dipoles):** We can also use a strategy that relies on the minimization of the size of the dipoles and that fulfills both requirements **(1)** and **(2)**. An oriented spanning tree of the network is constructed and we define the basis (p_i) as being the basis of dipoles associated to the lines of that tree (the powers of opposite value $(-1, +1)$ are injected at the start and end nodes of each line of the tree, respectively). A simple 9-node case example [19] is presented in figure (3).

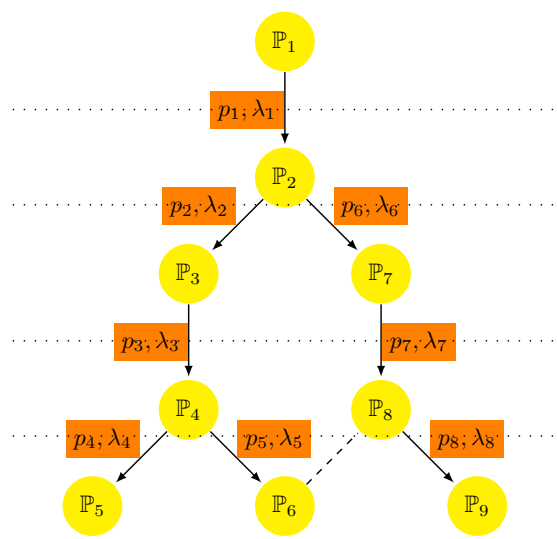

Fig. 3. Example of a spanning tree of a 9-node graph and its related basis of dipoles $(p_i)_{1 \leq i \leq 8}$. The dashed line figures a line of the network that was not selected in the spanning tree.

The main interest in using a tree for the short dipole strategy is that the decomposition of a vector $\mathbb{P}$ in this basis can be obtained easily. Let's look for the decomposition of a zero sum vector $\mathbb{P}$ in the basis $p_1 \dots p_8$, for the example introduced in Figure (3). This decomposition is given by

$$\mathbb{P} = \sum_{i=1}^{8} \lambda_i p_i \tag{9}$$

The scalar coefficients (λ_i) related to the 9-node example presented in figure (3) are equal to

$$\begin{aligned}
&\lambda_1 = \mathbb{P}_2 + \mathbb{P}_3 + \mathbb{P}_4 + \mathbb{P}_5 + \mathbb{P}_6 + \mathbb{P}_7 + \mathbb{P}_8 + \mathbb{P}_9 \\
&\lambda_2 = \mathbb{P}_3 + \mathbb{P}_4 + \mathbb{P}_5 + \mathbb{P}_6 \qquad \lambda_6 = \mathbb{P}_7 + \mathbb{P}_8 + \mathbb{P}_9 \\
&\lambda_3 = \mathbb{P}_4 + \mathbb{P}_5 + \mathbb{P}_6 \qquad \lambda_7 = \mathbb{P}_8 + \mathbb{P}_9 \\
&\lambda_4 = \mathbb{P}_5 \qquad \lambda_5 = \mathbb{P}_6 \qquad \lambda_8 = \mathbb{P}_9
\end{aligned}$$

So each scalar coefficient λ_i is equal to the sum of the components of $\mathbb{P}$ located below the line defining the corresponding dipole p_i (see figure (3)). This result, that can be generalised beyond this simple example, shows that zero sum vectors can be decomposed easily on the short dipole basis defined by spanning trees and that the obtention of the related coefficients (λ_i) is straightforward. It is worth mentioning that this method is the same as the backward sweep of the backward/forward sweep algorithms used for load flow calculations on radial distribution networks [22].

C. Fast computation of the basis of solutions (ϕ_i) using the Lanczos algorithm

Besides the possibility of decomposing quickly any source vector in the dipole basis, using a tree-defined dipole basis also allows a quick resolution employing the Lanczos algorithm. Indeed, this method has proven to be well adapted for calculating the solution related to local power injections.

Many different spanning trees can be constructed for a given network. Every tree can be characterized by its weight, defined as being the sum of the susceptances of all the lines $\mathcal{L}(T)$ of the tree T divided by the number of dipoles

$$\text{Weight}(T) = \frac{1}{N_{\text{nodes}} - 1} \sum_{l \in \mathcal{L}(T)} B_l \tag{10}$$

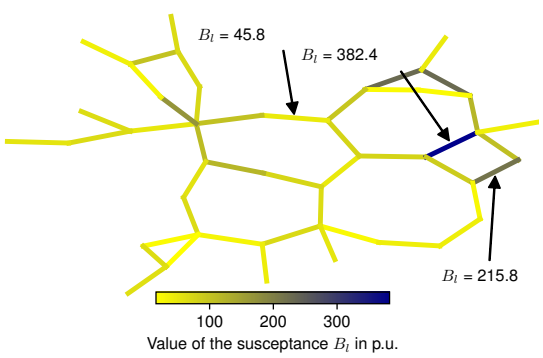

Fig. 4. Susceptances of the case 39 IEEE network.

In Table (I), we present the average number of Lanczos steps needed for the computation of one element of the basis of solutions (ϕ_i) with a 5% precision. We test on various examples the long dipole method against the short dipole method for a minimum and a maximum spanning tree. For large networks, using a short dipole strategy reduces the number of Lanczos steps up to a factor of 10, compared to a long dipole strategy. Also, an additional gain of 20% steps can be made by maximizing the weight of the chosen spanning tree. Indeed, selecting high susceptance lines for constituting the spanning tree (as shown in figures (4) and (3)) turns out to reduce the number of iterations of the Lanczos algorithm. When a line has a high susceptance, then the electrical current generated by the dipole injection defined by this line is mostly located on this precise line. Therefore, the electrical current on further parts of the network is low, and may even be neglected. This enforces the locality principle and decreases even more the number of Lanczos iterations needed for the computation of the basis of solutions (ϕ_i). This shows that the cost of dipole based methods can be decreased considerably by choosing an adapted basis (p_i).

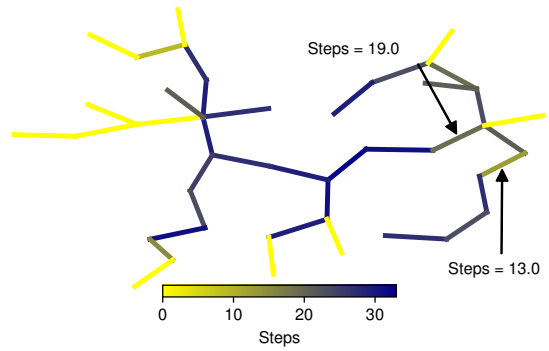

Fig. 5. Maximum spanning tree of the IEEE case 39. The colors indicate the number of Lanczos steps necessary to resolve the dipole defined by the nodes of a given line. Whereas an average of 23.8 Lanczos steps per dipole is needed for solving the maximum spanning tree, the number of steps is decreased, respectively increased when a high, respectively low susceptance line is considered.

TABLE I
AVERAGE NUMBER OF LANCZOS STEPS PER DIPOLE FOR METHODS BASED ON DIFFERENT DIPOLES: THE LONG DIPOLES, AND THE DIPOLES RELATED TO THE MINIMUM SPANNING TREE AND THE MINIMUM SPANNING TREE.

Network	Average data per dipole				
	Long dipoles	Minimum tree		Maximum tree	
	Steps	Weight	Steps	Weight	Steps
case 30	21.7	4.6	19.7	9.4	18.4
case 39	30.6	62.5	25.4	90.2	23.8
case 57	35.7	6.1	30.2	11.3	28.0
case 89	66.1	273.0	49.2	856.0	36.5
case 118	67.9	12.1	42.5	24.4	33.8
case 145	77.4	36.6	47.5	107.1	41.3
case 300	172.6	31.8	79.1	61.9	71.4
case 1354	426.3	364.2	95.8	415.2	79.8
case 1888	681.5	669.3	161.7	741.7	131.8
case 1951	702.1	655.3	165.9	725.9	135.9
pantagruel 3809	1165.8	115.9	127.5	188.6	98.7

V. CONCLUSION

The specific purpose of this article was to calculate efficiently the load flows for multiple system-wide power injections. These power injections live in the vector space $\mathcal{P}$ that can be spanned by a basis (p_i) of short dipoles. Such a basis presents locality properties and can therefore be solved in a couple of iterations by the Lanczos algorithm. This leads to a significant speedup compared to the case involving longer dipoles. Finally, we observe on several examples that the locality effect is even increased by maximising the weight of the spanning tree. It is expected that this approach could

be useful to anyone wishing to calculate many approximate load flows under challenging time constraints or computing capacities.

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VI. Biography

Pierrick Guichard Pierrick Guichard was born in Le Creusot, France, in 1997. In 2021, he received the Eng. degree from ENSTA Paris and a Master's degree from the Polytechnic Institute of Paris. He is a Ph.D. student at the CNRS in the Institut Néel at Grenoble. His principal research interest include the modeling of power systems and their analogy with physical models.

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