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Intermediaries' Substitutability and Financial Network Resilience: A Hyperstructure Approach *

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Abstract

This article studies the impact of intermediaries' disappearance on firms' access to the sterling money market during the first globalization era of 1880-1914. We propose a new methodology to assess intermediaries' substitutability in financial networks featuring higher-order structures (credit intermediation chains). We represent the financial network as a *hyperstructure* and each credit intermediation chain as a *hyperedge*. This approach allows us to assess how the failure of intermediaries affects network connectivity. We apply this methodology to a unique dataset documenting the network structure of the sterling money market in the year 1906. Our results reveal that the failure of individual money market actors could only cause limited damage to the network as intermediaries were highly substitutable. These findings suggest that an international financial network without highly systemic nodes can emerge even at a time of global economic integration.

Keywords: Financial networks, systemic risk, hypergraphs, intermediation chains, bills of exchange, hyperstructures.

JEL: D85, E42, F30, G20, N20.

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1. Introduction

Financial networks are prone to systemic risk. Today’s global financial system is characterised by strong linkages between financial institutions and the presence of complex intermediation chains.¹ While financial intermediation chains play an important role in the allocation of capital between investors and borrowers, disturbances in the functioning of these chains can also have significant consequences on liquidity and credit availability across the global financial system. In particular, the failure of central intermediaries in the world’s largest money markets can lead to severe credit disruptions at the network periphery.

This article explores the impact of intermediaries’ disappearance on firms’ access to the sterling money market during the first globalization era of 1880-1914. During this period, international goods and financial markets were as integrated as in the late twentieth century (O’Rourke and Williamson, 2002). The City of London was the centre of the global financial system and its money market – the market for sterling bills of exchange – served as a global platform for short-term lending and borrowing (Accominotti and Ugolini, 2020). Firms located anywhere in the world used this market to obtain short-term loans from international lenders with the guarantee of UK-based intermediaries. As in modern money markets, intermediaries played a key role on the sterling bill market as they allowed channelling liquidity from investors to borrowers.

How damaging could the disappearance of individual intermediaries be for agents’ access to the money market? To answer this question, we exploit an original dataset of financial interlinkages between actors active on the London money market during the year 1906, which we assembled from archival sources. The dataset contains micro-level information on the 23,493 bills of exchange re-discounted by the Bank of England in 1906 and on the intermediaries involved in the underlying money market transactions (see Accominotti et al., 2021). Every bill of exchange transaction on the money market consisted of a credit chain involving three different actors: *a*. a borrowing firm located anywhere in the world (borrower); *b*. a

¹ On the concept of *intermediation chain*, see Adrian and Shin (2010), Poszar et al. (2010), Cetorelli et al. (2012) and Di Maggio and Tahbaz-Salehi (2015).

UK-based intermediary which guaranteed that firm's debt (guarantor); and ι , a bank or money market fund that lent cash to the borrowing firm (lender). Therefore, each transaction embraced both a "firm-bank" (borrower-guarantor) relationship and a "bank-bank" (guarantor-lender) relationship. Since each bill of exchange recorded the names of the borrower, guarantor and lender in the underlying transaction, we can reconstitute the precise nature of interlinkages between these actors and reconstruct the various chains of bank-bank and bank-firm relationships.

To analyse the transmission of shocks along these credit intermediation chains, we depart from the standard dyadic approach in banking network research. Network science scholars have recently shown how this approach can be misleading when transactions involve more than two actors (Bonacich et al., 2004; Estrada and Rodríguez-Velázquez, 2006; Battiston et al., 2020). In the presence of intermediation chains, any link between two nodes is conditional on the existence of other links relating each of these nodes to other ones (i.e. intermediaries). Thus, the existence of dyadic relationships between nodes depends on these nodes' belonging to a same higher-order structure (or chain). For example, on the primary corporate bond market, investors and issuing firms are connected to each other through underwriters. Therefore, a link between an investor and an issuing firm can only be formed if both are connected to a same underwriter. Similarly, modern money markets and over-the-counter securities markets feature agents that act as intermediaries between investors and ultimate borrowers. If a few of these intermediaries are central to the network and cannot be replaced – for example, because they hold asymmetric information about borrowers – their failure might deprive numerous agents from market access.

In a financial network featuring intermediation chains, a node's disappearance results in the breakdown of all the chains to which it belongs and of all dyadic relationships that compose these chains (i.e. including the links that do *not* directly involve the node). Analysing the importance of individual nodes for the network requires departing from the dyadic approach to model the financial system as a higher-order interaction network. We follow this approach here and explore interactions in the money market network using the concepts of *hypergraph* and *hyperstructure*. We represent the entire set of money market

actors as a *hyperstructure* (an association between adjacency and incidence matrices). We describe each sterling bill of exchange as a continuous intermediation chain or the *hyperedge* of a *hypergraph* that connects three different nodes, each playing one of the three roles (borrower, guarantor or lender) in the underlying credit transaction. This *hyperstructure* approach allows us to preserve each chain’s internal structure and unity (Criado et al., 2010; Lucena-Piquero et al., 2022). We propose a new method using a meso-level approach to analyse directed links between nodes within each intermediation chain. One advantage of this approach is that it allows us to consider the *gatekeeping* or *bridging* role that certain intermediaries play on the money market (Bonacich et al., 2004). Intermediaries which guaranteed borrowers’ debts in order to allow them to borrow from other financial institutions acted as gatekeepers or bridges on the London money market.

We use simple simulation techniques in order to assess the systemicness of actors on the money market and draw implications for the resilience of the system. Our main focus is on intermediaries’ *substitutability*. A given intermediary is said to be *substitutable* if its presence is non-essential to other nodes’ connection to the network (or, in economic terms, if the intermediary’s presence is non-essential to other agents’ access to the money market). We therefore consider that a given node’s degree of substitutability is inversely related to the share of actors that would remain isolated in the network in the absence of this node. A given node will therefore be considered weakly (highly) substitutable if there is a relatively large (small) number of actors whose connection to the network is strictly dependent on the node’s presence.

While most of the network literature has focused on the question of banks’ interconnectedness, regulators have recently paid increasing attention to the degree of substitutability of banks in the global financial system as an indicator of their systemicness, and they have attempted to measure their importance in the provision of financial services (Basel Committee on Banking Supervision, 2013, 2018).² The Basel Committee considers that a financial institution’s degree of substitutability is negatively related to “the

² In 2013, the Basel Committee on Banking Supervision (2013) and the Financial Stability Board (FSB) jointly published new guidelines for the assessment of banks’ systemicness. The guidelines distinguished between five dimensions of systemicness: 1. *size* (total size of the bank’s liabilities); 2. *interconnectedness* (network of contractual obligations which characterise the bank’s activities); 3. *substitutability* (the bank’s importance as a provider of client services); 4. *complexity* (business, structural, and operational complexity of the bank including its involvement in sophisticated activities such as derivatives or other off-balance-sheet exposures); and 5. *cross-jurisdictional activity* (geographical dispersion of the bank’s activities). Assessing substitutability has proved particularly difficult (Benoit et al., 2019), leading regulators to revise their guidelines on this aspect of systemicness (Basel Committee on Banking Supervision, 2018).

extent to which the bank provides financial institution infrastructure” (Bank for International Settlements, 2018, p. 6). For example, a financial institution acting as an underwriter (intermediary) for a large volume of transactions on financial markets is considered weakly substitutable because its disappearance would result in a large number of participants being deprived from access to these markets. While our definition of substitutability is narrower and more focused than the broader notion employed by the Basel Committee in its regulatory framework, it allows operationalizing this notion in the context of empirical network analysis.

Our concept of *substitutability* can also be related to the notion of *criticality*, which has been used in various strands of the literature in dyadic network analysis to assess the impact a node’s failure on other nodes’ connection to the network. Although different definitions of *criticality* have been proposed in the literature (Jafino et al., 2020), they all revolve around the notion of *betweenness centrality*.³ However, the concepts of *criticality* and *betweenness centrality* cannot be directly operationalized to assess nodes’ systemicness in a higher-order network (for a discussion, see Appendix A1).

Our results provide an upper-bound estimate of network fragility, as our methodology rests on the restrictive assumption that no other financial relationship between actors can be formed besides those actually observed. In other words, we assume that existing financial relationships cannot be replaced, so that an agent loses market access when the intermediaries to whom she is currently connected disappear. When applying similar node-removal methodologies to modern interbank markets, one typically finds that a few nodes are non-substitutable as removing them results in a complete breakdown of connectivity in the network (Pröpper et al., 2008).

Our main finding is that systemic risk in the sterling money market was remarkably low at the beginning of the twentieth century. We find that the money market network was resilient even to the removal of central nodes. Although our assessment of intermediaries’ systemicness constitutes an upper-bound estimate, we find that no single intermediary on the money market was highly systemic. Any node

³ For example, Goyal (2018, p. 698): “A node is said to be critical if it lies on all paths between S [a source node] and D [a destination node]”.

removal could only generate limited damage to the network. The network's various subsections were also all robust to the removal of individual nodes, as very few agents were strictly dependent on individual nodes for their money market access. Our data also allow documenting the location of money market borrowers at the city level. We study the network's geographical systemicness and find that very few cities across the world would have been cut off from the market had individual nodes been removed. Therefore, in contrast to findings obtained on modern banking networks, our analysis of the historical sterling bill market reveals that an international financial network featuring low systemicness could emerge even during a period of high global economic and financial integration.

Our article contributes to a large literature on financial network structure, going back to Allen and Gale (2000) and Freixas et al. (2000). Over the last decades, regulatory authorities have evolved from a microprudential approach focusing on the position of individual intermediaries to a macroprudential one where more attention is being paid to relational structures between banks (Basel Committee on Banking Supervision, 2013). Numerous studies have explored how links between banks are being formed and have analysed the importance of network structure for the resilience of financial systems (Allen and Babus, 2009; Glasserman and Young, 2016; Battiston and Martinez-Jaramillo, 2018; Caccioli et al., 2018; Iori and Mantegna, 2018). Most of these papers are based on data documenting bilateral relationships between financial institutions on individual countries' interbank markets but a few studies have also analysed the network structure of international financial connections (Espinosa-Vega and Solé, 2010; Minoiu and Reyes, 2013; Chinazzi et al., 2013; Minoiu et al., 2015; Hale et al., 2016; Cai et al., 2018). Similarly, our article explores the resilience of a major historical, global financial network.

Empirical studies have generally found that financial networks exhibit a strong *core-periphery* structure (sometimes referred to as a *scale-free* structure)⁴ with a small group of highly-connected actors centralizing flows and playing the role of hubs (Craig and Von Peter, 2014). This structure, which can be identified by

⁴ In theory, *core-periphery* and *scale-free* structures are not exactly equivalent. It is however difficult to distinguish between the two types of structures in empirical investigations (Iori and Mantegna, 2018, p. 645).

looking at several indicators such as the network's *degree distribution*,⁵ has important implications for the vulnerability of financial systems. While core-periphery structures are generally robust to shocks on random individual actors, they become very fragile when these shocks affect actors who are playing the role of hub (Albert et al., 2000; Newman, 2003). This characteristic is known as the *robust-yet-fragile* property of financial networks (Gai and Kapadia, 2010). The presence of a hierarchical structure appears to be a common feature of all modern interbank networks and has been recently described as “a new ‘stylized fact’” (Fricke and Lux, 2015a, p. 391).⁶ Our article however provides evidence that alternative network structures can also emerge. We describe how a major historical financial network was characterised by the absence of highly systemic hubs. While the money market network we study exhibits a structure that in certain respects resembles that of scale-free networks, it also displays much stronger resilience to shocks than these typical networks.

Our article also makes an important methodological contribution. While most studies of financial networks have focused on interactions between financial intermediaries (bank-bank networks), a handful of papers have also analysed bank-firm links alongside bank-bank links (De Masi et al., 2011; De Masi and Gallegati, 2012; Lux, 2016; Silva et al., 2018). These studies have however represented bank-bank and bank-firm relationships as different types of dyadic links within a multilayer network and have therefore treated the two types of relationships as different (albeit interconnected) networks. By contrast, our approach, which consists in analysing interactions within chains of actors, allows us to model bank-bank-firm relationships as part of a single, higher-order network. While hypergraphs have been used before in network science, our article is the first to our knowledge to apply this approach to financial networks. It is also the first to our knowledge to model the direction of links between nodes in a hyperstructure. This

⁵ The distribution of the nodes' degrees in a network defines its *hierarchy*. A network is hierarchical if a small number of nodes have a significantly higher degree than most other nodes in the network.

⁶ The core-periphery structure has been identified empirically in the case of various countries' domestic interbank networks such as Austria (Boss et al., 2004), Belgium (Degryse and Gregory, 2004), Germany (Upper and Worms, 2004), Switzerland (Sheldon and Maurer, 1998; Müller, 2006), the United Kingdom (Wetherilt et al., 2010), or the United States (Soramäki et al., 2007), as well as for country-to-country networks (Minoiu and Reyes, 2013; Chinazzi et al., 2013). Only a minority of empirical studies have found evidence of less hierarchical structures with a weaker core-periphery structure. This is the case of the domestic interbank networks of Italy (Iori et al., 2008; Fricke and Lux, 2015a, 2015b), the Netherlands (Blasques et al., 2015), and Mexico (Martinez-Jaramillo et al., 2014).

approach allows us to preserve the chains' internal configuration and unity and explore how the failure of various types of intermediaries could impact actors' access to the money market. Recently, scholars have documented and proposed explanations for the increasing length of credit intermediation chains in the modern financial system (Adrian and Shin, 2009, 2010; Poszar et al., 2010; Cetorelli et al., 2012; Colliard and Demange, 2014; Di Maggio and Tahbaz-Salehi, 2015; Glode and Opp, 2016; Shen et al., 2021; He and Li, 2021; Aymanns et al., 2023). The methodology we develop here could be adapted to study the resilience of intermediaries' networks underpinning the present-day, market-based financial system. It could also be used to study the resilience of other non-financial directed networks characterised by the presence of chains of actors such as, for example, global supply chains (Lucena-Piquero et al., 2022).

The remainder of the article is organised as follows. Section 2 describes our data and details our empirical strategy. Section 3 presents descriptive statistics on the structure of the sterling money market network. Section 4 presents our main results on intermediaries' substitutability as well as several robustness checks. Section 5 concludes.

2. Data and methodology

2.1. Data

Our empirical analysis is based on an original dataset of international financial interlinkages during the first globalization (1880-1914). At that time, London was the unrivalled global financial centre and the sterling-denominated *bill of exchange* was the staple international money market instrument (Accominotti and Ugolini, 2020). Our dataset was hand-collected from one archival source (the Bank of England's *Discount Ledgers*) and includes information on 23,493 bills of exchange issued on the sterling money market and discounted by the Bank of England during the calendar year 1906. A detailed discussion of the nature and representativeness of these data can be found in our historical companion paper (Accominotti et al., 2021).⁷

⁷ The sterling bill market was an over-the-counter market and no systematic information was collected on transactions taking place on that market. However, archival records allow reconstituting all relationships between actors involved in the origination

Figure 1 here

The market for sterling bills of exchange was the world's dominant money market in the early twentieth century. As illustrated in Figure 1, any sterling bill transaction involved three actors: a borrower (called the *drawer*), a guarantor (called the *acceptor*), and a lender (called the *discountor*). Borrowing firms located anywhere in the world and willing to obtain short-term sterling funds could draw a bill on a UK-based agent (an *acceptor*) with whom they had a relationship. The agent accepted the bill by putting its signature on it and, in so doing, agreed to repay the bill at maturity (typically, after three months) in the expectation that she would herself have received payment from the borrower in the meantime. After obtaining the signature of a guarantor (*acceptor*), the borrower (*drawer*) could discount the bill to another UK financial institution willing to lend funds on the money market (the *discountor*). For each sterling bill originated, our archival source provides information on the identity of the borrower, guarantor, and lender. We are therefore able to document all borrower-guarantor ("firm-bank") and guarantor-lender ("bank-bank") relationships and reconstruct the complete network of interlinkages between agents operating on the money market. Our static network for the year 1906 contains 4,970 nodes, of which only roughly one third were located in the UK. The other nodes consisted of borrowing firms spread across the entire world.

Given the still opaque nature of most interbank connections today, empirical financial network research is often based on estimated rather than observed data. Relationships between banks are typically inferred from balance sheet or payments data (Furfine, 1999; Upper and Worms, 2004; Allen and Babus, 2009; Upper, 2011).⁸ One advantage of our historical database is that it is solely based on observed and systematically recorded links between money market participants. Our approach therefore does not require making any assumption to reconstruct interactions between money market actors. One limitation of our

and distribution of bills purchased (re-discounted) by the Bank of England. Bills rediscounted by the Bank of England represented only a small minority of all sterling bills issued. However, Accominotti et al. (2021) perform a series of cross-checks and do not identify any serious bias in this sample.

⁸ The Italian interbank network is one of the only networks for which complete transaction data have been available. See Iori et al. (2008), Fricke and Lux (2015b), Iori et al. (2015), and Temizsoy et al. (2015). Although researchers have produced interesting results based on estimated data, several scholars have also questioned the reliability of this method (Upper, 2011; Mistrulli, 2011; Anand et al., 2018).

archival source however is that it does not allow measuring the extent of cross-balance-sheet exposure between actors. Nevertheless, balance sheet data are not strictly necessary for our analysis. Indeed, our approach consists in assessing a given intermediary's substitutability by looking at the number of nodes which are strictly dependent on that intermediary for their money market access. When a given intermediary disappears from the network, we consider that all agents that are strictly dependent on this intermediary will lose market access regardless of the size of their financial exposure.

2.2. Hyperstructures

Financial transactions often take the form of an intermediation chain involving three or more actors. For example, in the modern financial system, credit is often channelled from investors to ultimate borrowers through a chain of intermediaries involving commercial banks, asset-backed security issuers, and money market funds (Adrian and Shin, 2010). Similarly, on securities markets, individual investors are related to issuers through one or multiple underwriters and dealers (Glode and Opp, 2016; Shen et al. 2021). Financial intermediation implies that the different bilateral relationships between actors involved in a transaction cannot exist independently of the transaction itself. This was the case for sterling bill transactions where the borrower-guarantor and guarantor-lender relationships were the indissociable components of one same intermediation chain, which was designed as one single contract – i.e., as an organic whole.

In a network featuring such intermediation chains, the disappearance of a given node results in the breakdown of all chains to which the node belongs and of all dyadic links that compose these chains. For example, in our network, a borrower and lender can only be linked together through a guarantor. Thus, unless these actors are linked to each other through another chain in the network, the disappearance of one of them results in the collapse of all dyadic links between them. For example, the disappearance of a lender in the borrower-guarantor-lender chain results in the breakdown not only of the guarantor-lender but also of the borrower-guarantor relationship featured in the chain. This means that, even if the guarantor remains in the network, the borrower will lose market access (unless she has access to a different lender

through another intermediation chain). At the same time, within each chain, the dyadic links between a borrower and a guarantor (on the one hand) and between a guarantor and a lender (on the other hand) are not of the same economic nature, and intermediaries playing various roles might therefore have different degrees of systemicness in the network. In order to assess the systemicness of various types of intermediaries, it is necessary to adopt an empirical methodology that allows taking account of *a.* the fact that dyadic links between nodes are embedded within higher-order structures (chains), and *b.* the fact that although in any given chain each node plays one given role (borrower, guarantor, or lender), the role played by a given node is not necessarily the same in all the chains it participates to.

The traditional dyadic approach is not suitable to analyse intermediation chains, because it does not allow explicitly modelling the fact that dyadic interactions between nodes take place at a higher-order level (i.e. within chains) while preserving the internal structure of the chains in which dyadic relationships are embedded. Since this approach does not account for the fact that the existence of dyadic links between nodes is conditional on these nodes' belonging to a same higher-order structure, it does not allow simulating the implications of a chain's breakdown on the network and measuring the comprehensive effect of nodes' disappearance on network connectivity. In order to account for this, we rely on the concept of *hyperstructure*.

Following Criado et al. (2010), we define a hyperstructure as a combination of an *adjacency matrix* (a matrix recording the presence or absence of a dyadic link between each pair of nodes) and an *incidence matrix* (a matrix recording the hyperedges to which each node belongs). To present their intuition, Criado et al. (2010) give the example of a subway network. Such a network is composed of a set of subway stations (the nodes) and a set of trunks connecting pairs of stations (the edges). Stations and trunks are grouped into subway lines (the hyperedges). A passenger travelling between two stations separated by the same number of trunks will face substantially different situations if these two stations are located on the same line or if they are on two different lines. For instance, if each subway ticket is valid on one line only, interchange will not be an option, and a passenger holding a ticket will only be able to reach stations located on the corresponding line. Similarly, a money market borrower might only be able to reach investors via

certain credit intermediation chains. Our analysis builds on Criado et al. (2010)'s intuition and extends it to analyse non-symmetrical (directed) relationships within hyperstructures.

More formally, let us represent the money market network as a finite set of individuals $V = \{i_1, i_2, i_3 \dots i_n\}$. Each bill can be represented as a chain $C_k \in \{C_1, C_2, C_3 \dots C_m\}$, defined as a non-empty set $\{a, b, c\} \in V$ in which there exist both a borrower-guarantor relationship (aTb) and a guarantor-lender relationship (bUc) so that $C_k = (aTbUc) \forall \{a, b, c\} \in V \wedge \{T, U\} \in R$, where: a , b , and c indicate the roles of (respectively) borrower, guarantor, and lender; T and U indicate (respectively) the borrower-guarantor and guarantor-lender relationship; and R is the full set of relationships in the network. For any node i , we indicate the chains to which i belongs as $C_k^i = (aTbUc) \forall \{a, b, c\} \in V \wedge i \in \{a, b, c\}$.⁹ We represent the network of chains as a hypergraph $H = (V, E): \forall C_k \exists E_k$, where $E_k \in \{E_1, E_2, E_3 \dots E_m\}$ is a set of hyperedges. Our hyperstructure S will therefore be composed of the chains C_k (each of which associates three linked nodes (a, b, c)) and of the hyperedges E_k that integrate them. Representing the entire set of chains as a hyperstructure allows preserving these chains' unity (captured by their affiliation to a given hyperedge) and the specific ordering of nodes within each of them (captured by the dyadic links between nodes). In addition, the concept of hyperstructure provides a flexible analytical framework and allows characterising networks' structural properties through simple social network measures such as, for example, degree centrality measures.

An alternative to the methodology we develop here would have consisted in transposing the supra-dyadic structures featured in our observed higher-order network into dyadic ones and analyse them through a traditional dyadic approach. However, as shown by Faccin (2022), in the case of directed hypergraphs such transpositions (generally referred to as *graph expansions of hypergraphs*) cannot be performed without a substantial loss of information about the network's properties. For example, one possible transposition is the so-called *star expansion* of a hypergraph. While this approach would have allowed

⁹ Note that nodes' specialization was not absolute. Each actor could play different roles in the various transactions in which it was involved. In our data, however, such nodes playing more than one role are relatively rare (see Table 1 below).

simulating the impact of an individual node’s disappearance on overall network connectivity, it would not have allowed distinguishing between the substitutability of intermediaries playing various roles, as any transposition of higher-order interactions into dyadic ones results in losing information about the roles played by each node in the different chains to which it belongs. We discuss in more detail the differences between our methodology and the star expansion approach in Appendix A2.

2.3. Shock simulations

In order to measure the resilience of the money market network to shocks, we perform a simple node removal simulation analysis.¹⁰ We adopt a straightforward approach for our shock simulations, which consists in measuring to what extent the removal of individual nodes causes damage to network connectivity.¹¹ This approach allows us to assess the degree of *substitutability* of the various intermediaries in the network. According to our definition above, an intermediary is considered substitutable if other nodes are not strictly dependent on it for their access to the money market (or connection to the network). More precisely, one given node A is said to be substitutable with respect to another given node B if A does not appear in all chains to which B belongs. In that case, A’s removal from the network does not leave B isolated, as node B can still connect to the network through other chains that do not involve A. One given actor’s degree of substitutability is therefore inversely proportional to the share of nodes that would remain isolated if the actor was removed from the network.

¹⁰ Following Allen and Gale (2000), shock simulations have become a standard method to assess the fragility of financial networks (Gai and Kapadia, 2010).

¹¹ The literature has generally estimated financial network resilience by simulating default cascades on the basis of interbank exposures. In that case, each node receives a shock from its incoming links and spreads it to its outgoing links. In these types of simulations, the effect of one bank’s default on another bank is generally assumed to be proportional to the bilateral exposure between the two banks (Eisenberg and Noe, 2001; Müller, 2006; Battiston et al., 2012a, 2012b; Acemoglu et al., 2015; Glasserman and Young, 2015). This approach requires obtaining information on the magnitude of bilateral interbank exposures, or reconstructing this information based on partial data. Our data do not allow constructing weighted links between nodes in the network. An alternative approach to measuring resilience consists of estimating the damage to network connectivity generated by the removal of individual nodes. This method has been used extensively in network analysis (Albert et al., 2000; Newman, 2003; Cohen and Havlin, 2010; Li et al., 2015; Jafino et al., 2020) and has also been applied to financial networks (Pröpper et al., 2008). One advantage of this method with respect to the *fictitious default algorithm* initially developed by Eisenberg and Noe (2001) is that it does not require making any assumption about how shocks propagate from bank to bank (Allen and Babus, 2009; Upper, 2011).

In any chain of our network, there is one node situated in position 1 (borrower), one node situated in position 2 (guarantor) and one node situated in position 3 (lender). We simulate the removal of individual nodes situated in position 2 or 3 of each chain (and thus playing the role of either guarantor or lender) and assess how many actors would lose market access in case of their removal. A given node i 's structural relevance is therefore measured through the number of money market actors that are strictly dependent on i for their market access. A node is considered independent from i if it is connected to other nodes that can grant it market access; or, in other words, if it has access to other paths allowing it to participate into a full borrower-guarantor-lender chain. The number of nodes losing market access when i is removed is an indicator of i 's degree of substitutability.

Figure 2 here

The hypothetical example presented in Figure 2 illustrates our methodology. The figure represents four different chains corresponding to groups of agents involved in four different bills of exchange: (A,B,C), (D,B,C), (D,B,F), and (D,E,F). Each chain involves a borrower (in position 1), a guarantor (in position 2) and a lender (in position 3). For example, in the chain (A,B,C), A plays the role of borrower, B the role of guarantor and C the role of lender. Each combination of two same-coloured arrows constitutes a hyperedge that integrates the three nodes and their links included in the corresponding chain.

Let us now suppose that node C (a lender) is removed from the network. This would result in the suppression of chains (A, B, C) and (D, B, C) and of all dyadic links that compose these chains. As a result, node A (a borrower) would remain isolated and be cut off the money market as the only path through which that node can access a lender is path (A, B, C). C's disappearance results in the breakdown of not only the B-C but also the A-B dyadic link, and A's connexion to the network is therefore strictly dependent on the existence of C. By contrast, node D (another borrower) would still be able to borrow from the money market even if lender C disappeared, as there exist two alternative paths (D, B, F) and (D, E, F) connecting that borrower to another lender (node F). This example illustrates the importance of preserving

the integrity of observed chains when performing our shock simulation analysis. Ignoring the compound nature of relationships in the network would lead us to erroneous conclusions about the impact of individual node removals. For example, a standard dyadic approach would have led us to conclude that the suppression of node C has no impact on node A's connectivity. This would have been inaccurate, as there exists in reality no chain (A, B, F) that could connect borrower A to the other lender (node F) in the network.

When simulating the impact of a given node i 's removal, we first identify all chains in which i plays the role of guarantor or lender and single out all nodes involved in these chains (the reference set). We then check whether these individual nodes are involved in other chains in the network. If an individual node j is present in at least one other chain that does *not* involve i , it means that an alternative path exists allowing j to access the money market even in the absence of i ; in other words, j is not strictly dependent on i for its market access. By contrast, if all chains to which node j belongs also include node i , then j loses market access and becomes isolated when i is removed. Formally, consider a node $i \in (bUc)$ of the chain $C_k^i = (aTbUc)$. Any node $j \in C_k^i \wedge j \neq i$ has an alternative access to the money market if $\exists C_k^j: i \notin C_k^j \wedge j \in C_k^j$.

Our methodology assumes that an actor remaining isolated as a consequence of another node's removal cannot build new, alternative paths to access the market. This assumption leads us to bias our results towards finding higher systemicness for lenders and guarantors, and against our hypothesis that the sterling money market network featured few highly systemic intermediaries.

3. Descriptive statistics

3.1. Network demography

In this section, we provide descriptive statistics to characterise the money market network's topology. We start with its demography. On the sterling bill market, borrowers (bill drawers) could be located

anywhere in the world, while it was a legal requirement for guarantors (bill acceptors) and lenders (bill discounters) to reside in the UK.

Table 1 here

Intermediaries were most often specialized in one of these three roles, but a few agents were *hybrid*, i.e., they played different roles in the different bills in which they were involved. Table 1 shows the distribution of money market actors according to their role and location. Borrowers were by far the most numerous group, followed by guarantors and lenders.¹² While a large majority of borrowers (61.71%) appear in one single chain of the network, this is only the case of 40.47% of the guarantors and of 17.93% of the lenders. This distribution resembles a funnel-shaped structure in which the number of potential individuals playing a given role is reduced at every stage of the transaction.

3.2. Network topology: method

To describe the network's topology, we follow the standard approach consisting in measuring its *node degree distribution* and in comparing it with that of *null* models, expressly simulated to display specific properties (Craig and Von Peter, 2014; Martinez-Jaramillo et al., 2014). We compare the node degree distribution of the observed network to that of 250 simulated *random* (Erdős-Renyi) networks and 250 simulated *scale-free* networks, each displaying the same number of nodes and hyperedges (and, thus, the same number of agents and chains) as the observed network. To ensure comparability, we constrain our simulations so that each individual borrower in simulated networks appears in the same number of chains as in the observed network. We also ensure that the simulated and observed networks feature the same number of borrowers, guarantors, and lenders.

¹² Note that the lenders (or discounters) in our dataset were mainly financial institutions (commercial banks, investment banks, money market funds) which purchased bills of exchange and re-sold them to other investors. They were therefore wholesale lenders and played an intermediary role on the money market (Accominotti et al., 2021).

In a random network, every group of three nodes has the exact same probability of being connected through a hyperedge. Comparing the observed network’s topology with that of simulated random networks allows assessing whether link creation in the observed network is guided by any kind of non-random relational dynamics (Iori et al., 2015; Chinazzi et al., 2013). In a scale-free network, by contrast, link creation is governed by a specific process known as *preferential attachment dynamics*. When simulating scale-free networks, we assume that individual actors have a greater tendency to establish links with well-connected nodes than with weakly-connected ones. This relational dynamic is conducive to the core-periphery network structure characteristic of most modern interbank systems (Martinez-Jaramillo et al., 2014; Iori and Mantegna, 2018). Applied to our network, this structure would involve that a large number of borrowers (forming the network’s periphery) are connected to a small number of lenders (forming the network’s core) through the intermediation of guarantors. Simulated scale-free networks provide a useful baseline to assess whether the sterling money market exhibited such a topology.

3.3. Network topology: metrics

We compare nodes’ degree centralities in the observed and simulated networks. In a hypergraph, a node will be considered central if it has many hyperedges (i.e. belongs to a large number of chains) and/or if it is connected to many other actors through its hyperedges (Kapoor et al., 2013; Battiston et al., 2020). Any hyperedge (or chain) to which a given node belongs is said to be *incident* to that node. All nodes sharing a same hyperedge (belonging to a same chain) are said to be *hyperedge-adjacent*. Hence, we define a node i ’s degree centrality in terms of:

- 1) its *in-degree* Id_i : the number of nodes to which it is connected through an input-arc (i.e., an incoming link):¹³

$$Id_i = \sum_{j=i_1}^{i_n} |\{(j, i) \in R\} \cap \{i, j\} \in V|$$

¹³ By construction, a node playing the role of borrower in all its chains is always situated at the beginning of these chains and has an in-degree of zero.

- 2) its *hyperedge degree* Hd_i : the number of its incident hyperedges (i.e., the number of chains to which it belongs):

$$Hd_i = |\{C_k^i \in E : i \in C_k^i\}| \forall C_k^i \in E$$

- 3) and its *hyperedge-adjacent nodes degree* $HAND_i$: the number of its hyperedge-adjacent nodes (i.e., the number of actors to which node i is linked through a hyperedge):

$$HAND_i = |\{\{j \in E_k^i : j \in V\} \setminus \{i\}\}| \forall E_k^i : i \in E_k^i$$

The hypothetical example presented in Figure 2 can be used to illustrate the three network degree centrality metrics. In the case of node B in Figure 2 (a guarantor), $Id_B = 2$ (B is connected with two different nodes, A and D, by incoming links), $Hd_B = 3$ (B has three incident hyperedges or, in other words, belongs to three different chains), and $HAND_B = 4$ (B is connected to four different actors through its hyperedges). For the six nodes represented in Figure 2, Table 2 reports the value of each of the three network degree centrality metrics.

Table 2 here

3.4. Network topology: evidence

Using the three above-defined degree metrics, we now compare the topology of the observed network to that of simulated benchmark networks. For each of the three metrics, Figure 3 shows the degree distribution of the observed network versus that of the 250 simulated random networks and of the 250 simulated scale-free networks. The observed network's degree distributions significantly differ from those of simulated random networks. Therefore, we can rule out that link creation between actors on the money market followed a random or near-random process. At the same time, the observed network's nodes degree

distributions look much closer to those of simulated scale-free networks.¹⁴ While the large majority of nodes only appear in a small number of chains and are connected to a few actors only, the network also contains a very small number of intermediaries (lenders and guarantors) that appear in a very large number of chains and are connected to many other nodes through these chains.

Figure 3 and Table 3 here

Despite apparent similarities in degree distributions, the observed network differs from the scale-free model in that its degree distribution is lighter-tailed. Table 3 shows that the most central node exhibits lower degree centrality in the observed network than in simulated, scale-free networks. For each of the three degree metrics, the table reports descriptive statistics on the maximum degree values in the 250 simulated random networks and 250 scale-free networks, and compares them with the maximum degree value in the observed network. The results indicate that simulated random networks did not feature any node that was as central as the observed network's most central node. However, in all but one of the 250 simulated scale-free networks, the maximum values of Id_i , Hd_i and $HAND_i$ are greater than the corresponding maximum values in the observed network. Therefore, the observed money market network does not feature as highly central nodes (so-called mega-hubs) as typical scale-free networks.

¹⁴ Note that the distribution of $HAND_i$ is significantly different in the observed and simulated networks. This is because, by construction, in simulated networks, $HAND_i$ tends to mostly take even values. In random networks, the probability that a link is formed is equal for each pair of nodes. Hence, a given actor is unlikely to be connected to the same node through more than one chain. As a result, actors will tend to be connected to twice as many nodes as the number of chains to which they belong. For example, any actor belonging to one single chain will be connected to exactly two nodes while most actors belonging to two different chains will be connected to exactly four other nodes. In scale-free networks, by construction, a large majority of nodes have a very low degree value and those actors are unlikely to belong to two different chains featuring the same nodes. Only the small minority of actors exhibiting high degree values are likely to be connected to a given node through more than one chain and to exhibit an odd $HAND_i$ value.

4. Results

4.1. Absolute and local systemicness

We now explore the resilience of the sterling money market network through a more detailed analysis of actors' systemicness. In order to assess the *absolute systemicness* of money market intermediaries (i.e., nodes playing the role of guarantor or lender), we perform shock simulations and remove them one by one from the network. We then identify the chains impacted and count how many nodes remain isolated from the network when a given intermediary is removed. We define a node i 's *absolute systemicness* AS_i as the percentage of the total number of nodes that lose market access when i is removed. We also compare each node's AS_i to its *market share* MS_i , defined as the percentage of nodes in the network which belong to a hyperedge in which i is present (i.e., the percentage of nodes which are hyperedge-adjacent to i). The formal definitions of AS_i and MS_i are provided in Algorithm 1.

Algorithm 1 here and Table 4 here

In Table 4, we first compare the maximum values of absolute systemicness ($\max(AS_i)$) and market share ($\max(MS_i)$) in the observed network and in the simulated random and scale-free networks. For each of the two variables, the maximum value is higher in the observed network than in any of the 250 simulated random networks but lower than in all but one of the 250 simulated scale-free networks. This indicates that all simulated scale-free networks but one featured at least one node that was more systemic than the observed network's most systemic node. Interestingly, the ratio between the median $\max(AS_i)$ and median $\max(MS_i)$ appears to be higher for simulated scale-free networks than for the observed one. This invites a more detailed analysis of the actual distribution of AS_i and MS_i in the different networks.

Figure 4 here

Figure 4 presents a scatter plot of the two variables for all actors playing the role of guarantor and/or lender on the money market. We report the comparison for nodes in the observed network as well as in one representative simulated random network and one representative simulated scale-free network.¹⁵ Two main features emerge from the figure. First, AS_i is low for all individual actors i in the observed network. Out of the 1,535 actors playing the role of guarantor or lender in the observed network, there are only two whose removal impacts more than 4% of market participants.¹⁶ At the other end of the spectrum, 597 actors have no impact at all. Second, AS_i rises less than proportionately with MS_i . Actors are situated on the 45-degree line on the figure when their removal impacts 100% of their hyperedge-adjacent nodes. Thus, the further to the right intermediaries are from the 45-degree line, the less dependent other nodes are on them for their market access. The figure reveals that the most central nodes in the observed network are situated well below the 45-degree line. This means that even the most highly systemic nodes in the observed network were relatively substitutable as few actors depended exclusively on them for accessing money market facilities. By contrast, in the simulated scale-free network, nodes with a high AS are also situated closer to the 45-degree line, indicating that they are much less substitutable. Overall, these results indicate that the observed money market network featured less systemic actors than corresponding scale-free networks with the same demography.

Figure 5 here

¹⁵ These two representative simulated networks have been generated using the same procedure as for the previous 250 random and 250 scale-free networks. In the representative random network, $\max(Id_i)=81$, $\max(Hd_i)=105$, and $\max(HAND_i)=196$; in the representative scale-free network, $\max(Id_i)=1,667$, $\max(Hd_i)=3,334$, and $\max(HAND_i)=2,200$: compare this with mean and median values in Table 3. In the representative random network, $\max(AS_i)=0.543$ and $\max(MS_i)=3.219$; in the representative scale-free network, $\max(AS_i)=26.685$ and $\max(MS_i)=43.514$: compare this with mean and median values in Table 4.

¹⁶ These two actors are Union Discount Company (7.83%), a large money market fund of the City of London at the beginning of the twentieth century, and Anglo-Foreign Banking Corporation (5.41%), a commercial bank specialised in foreign lending through its activities as guarantor and lender for overseas firms and banks.

For each node i , we also measure its *local systemicness* $LS_i = AS_i/MS_i$, i.e., the share of i 's hyperedge-adjacent nodes which lose market access when i is removed (see Algorithm 1). Figure 5 plots the frequency distribution of LS_i across all nodes playing the role of guarantor and all nodes playing the role of lender in the observed network. A large number of guarantors exhibit intermediate levels of local systemicness. For 48.64% of guarantors, LS_i is situated between 20% and 50%. By contrast, only 36.55% of lenders display such intermediate levels of local systemicness and 17.24% of them exhibit a LS_i higher than 50%.¹⁷ Overall, these results shed light on the differences between various types of intermediaries on the money market. While lenders displayed on average higher absolute systemicness than guarantors, they exhibited relatively lower levels of local systemicness.

Algorithm 2 here

While the removal of individual nodes could not cause significant damage to the network, the cumulative disappearance of the most systemic intermediaries might nonetheless have led to its rapid breakdown. Removing any actor from the network results in the disappearance of all chains in which it is involved. To estimate the damage potentially caused by cumulative shocks, we present a measure of *spare-chain connectivity* obtained by sequentially removing nodes from the network in the order of their absolute systemicness (AS_i). Figure 6 reports the percentage of chains C_k of the original observed network that are preserved following the sequential removal of various numbers of nodes. Algorithm 2 formally details this procedure. We also compare how the sequential removal of systemic nodes affects the number of chains in the observed network versus in one simulated random network and one simulated scale-free network (selected as in Figure 4).

Figure 6 here

¹⁷ Note that these locally-systemic lenders were small on average. The mean and median hyperedge degrees Hd_i are equal to 6.36 and 2.00, respectively, for the 25 lenders whose local systemicness LS_i is greater than 50%.

Removing the four most systemic nodes altogether from the observed network results in the disappearance of 26.7% of its chains. The observed network still conserves 49.6% of its original chains after the ten most systemic actors are removed and 6.9% after the removal of the fifty most systemic intermediaries. By contrast, the removal of the four most systemic nodes from the simulated scale-free network leads to the disappearance of as many as 82.9% of its chains and this network breaks down completely after the seven most systemic actors are removed. The random network is (unsurprisingly) much more resilient to the removal of its most systemic nodes than both the observed and scale-free networks. However, while the random network collapses completely following the removal of its 158 most systemic nodes, it takes the removal of 1,421 nodes for the observed network's chains to all disappear. This pattern indicates the presence on the money market of a significant number of intermediation chains featuring agents weakly connected to the rest of the network.

Overall, these results indicate that the sterling money market network was much more resilient to shocks than typical scale-free networks. It is also worth noting that, as stated above, our shock simulation methodology assumes that money market actors cannot build alternative paths to access the money market when the chains in which they are involved disappear. This assumption leads us to overestimate individual nodes' systemicness. Despite this upwards bias however, we find that the money market network was not subject to the *robust-yet-fragile* feature characteristic of most modern interbank networks (Gai and Kapadia, 2010). In the remainder of this section, we perform a number of robustness checks to corroborate this finding.

4.2. Sampling bias

The entire set of links within a financial network is rarely observable in full. Our historical dataset records information on a large sample of sterling money market transactions for the year 1906 and documents all links between actors involved in these transactions. Yet, given the over-the-counter nature of money market dealings, not all transactions were being recorded and links between money market actors

are therefore not all observable. Using incomplete data to infer the structure of a true network can result in sampling biases as recently emphasized by the literature on ecosystems (Fründ et al., 2016; Henriksen et al., 2018). How do these sampling effects affect our conclusions about systemic risk on the sterling money market?

In the absence of complete data, empirical network analysis is often based on a random sample of nodes drawn from the true network. This method can however lead to underestimate the true network's resilience, as highly systemic nodes are generally very few and the likelihood of randomly selecting them is therefore low (Stumpf et al., 2005). By contrast, our observed money market network was built from a sample of bills of exchange (money market transactions) each involving three different nodes. In other words, the network was constructed from the sampling of chains rather than nodes. Since highly systemic nodes are by definition present in a large number of chains, the likelihood of selecting those nodes is much higher when the network is constructed from a sample of chains than from a sample of nodes. Hence, it is plausible that our sampling method leads us to overestimate rather than underestimate the true money market network's resilience to shocks.

Figure 7 here

In order to verify this intuition, we perform two types of checks. We first assess how maximum absolute systemicness in the observed network evolves with sample size. We randomly select subsections of our observed network of various sizes and compute for each of them $\max (AS_i)$ – i.e., the absolute systemicness of the most systemic node in the network. We start with a sample that consists of 1,000 chains randomly selected from the observed network. We then add 1,000 additional (randomly-selected) chains to the previous sample and repeat this procedure until all chains are included in the sampled network. This leads us to generate eight sampled networks, whose size varies from 1,000 to 8,000 chains. For each sampled network, we compute $\max (AS_i)$. The procedure is then reproduced 100 times. Figure 7 reports $\max (AS_i)$ for each of the eight sampled networks included in each of the 100 simulations. Each

black line corresponds to a set of eight sampled networks of increasing size. For any given sample size, the red line reports the mean value of $\max (AS_i)$ observed across all 100 sampled networks.

By construction, as the number of chains increases, the maximum absolute systemicness in sampled networks converges towards its actual value in the entire observed network (7.83%). In line with the intuition presented above, we also find that $\max (AS_i)$ tends to decrease when sample size increases. This finding is due to the fact that an incomplete network constructed from randomly sampling chains will tend to disproportionately feature highly systemic actors. Hence, our sampling method leads us to overestimate rather than underestimate actors' systemicness on the money market.

Figure 8 here

Second, we check whether this finding obtained on the observed network also holds when considering larger sample sizes and alternative network structures. We simulate 10 random networks and 10 scale-free networks featuring three times as many nodes and chains as the observed network (14,910 nodes and 26,664 chains). We then construct sampled subsections of these simulated networks as described above; first, by randomly selecting 2,000 chains; and then, by successively adding 2,000 randomly-selected chains to each sampled network until all 26,664 chains of the simulated network are included in the sample. We repeat this procedure five different times for each of the 10 simulated network in order to obtain 50 simulations. At every stage, we compute $\max (AS_i)$ for each sampled network.

Figure 8 reports the results. They reveal that, for both types of simulated networks, maximum systemicness tends to decrease as sample size increases. These results confirm that sampling biases lead us to overestimate systemicness in the true network and can therefore not affect our main conclusion that systemicness was relatively low on the sterling money market at the start of the twentieth century.

4.3. Group systemicness

We then investigate whether the sterling money market was resilient to shocks affecting specific groups of money market intermediaries. For that purpose, we identify four different groups of intermediaries in our network whose role has been described by contemporaries and financial historians alike:

1. *Discount houses*: these 20 institutions were comparable to modern money market funds and specialized in investing on the money market by purchasing large amounts of sterling bills of exchange (King, 1936; Accominotti et al., 2021).
2. *Anglo-foreign banks*: these 45 institutions were UK-based multinational commercial banks that intermediated credit through their many overseas branches (Jones, 1993);
3. *Merchant banks* (or *acceptance houses*): this group is composed of the top-10, globally-renowned investment banks (merchant banks) of the City of London. These banks specialized in guaranteeing (accepting) bills of exchange on account of their domestic and overseas clients (Chapman, 1984).
4. *Clearing banks*: This group is composed of the 11 banks that dominated domestic commercial banking in the UK (Sykes, 1926).

Table 5 here

To analyse the significance of these four groups for agents' access to the sterling money market, we remove each group $G_x \in V$ from the network and compute their *absolute systemicness* AS_{G_x} and their *market share* MS_{G_x} .

The procedure is similar to that described in Algorithm 1, except that we now measure the joint absolute systemicness of all nodes that belong to a given group G_x rather than the systemicness of

individual nodes.¹⁸ The results of these computations are shown in Table 5. It is evident that discount houses (money market funds) were the overly dominant lenders on the money market with a combined market share of 65.65%. If all these intermediaries had failed at the same time, 42.1% of agents would have lost access to the sterling money market. While this would have represented significant damage for the financial network, this also suggests that there existed alternative routes that allowed accessing London's financial facilities without going through these institutions. Absolute systemicness was lower for other groups of intermediaries ranging from 5.7% for the UK clearing banks (domestic commercial banks) to 21.2% for the Anglo-foreign banks (UK-based multinational commercial banks). The relatively higher systemicness of the latter group of banks arises from their joint activity as guarantors and lenders of short-term funds for their overseas clients. Nevertheless, while the removal of each of these various groups would have caused significant disruptions to the money market, no group of intermediaries was sufficiently systemic for its removal to cause a complete collapse of the financial network and cut off all agents' access to that market.

4.4. Geographic systemicness

Borrowers on the London money market were located everywhere in the world and UK financial intermediaries often specialized along geographic lines (Accominotti et al., 2021). Therefore, it is possible that certain geographic regions were strongly dependent on specific actors for their access to short-term sterling credit facilities. For example, if all borrowers from a certain region accessed the money market through the intermediation of one single UK guarantor or lender, the failure of these intermediaries would have resulted in an entire region being cut off from the London money market.

Figure 9 here

¹⁸ Note that a given group G_x 's absolute systemicness differs from the sum of absolute systemicness values of all individual nodes that compose it. This is because all nodes of a given group are removed at the same time and these nodes often appear in the same chains.

We thus exploit our data on money market borrowers' geographic location to assess how dependent individual cities were on specific UK intermediaries. Our dataset includes 617 cities. In Figure 9, we report the frequency distribution of these cities according to the number of borrowers they comprise. Money market borrowers were scattered across the world and many of them were located in relatively small cities. Hence, 53.65% of cities in the network featured only one borrower, while only 21.1% had five or more borrowers. Table 6 reports the list of all cities in the network classified according to the number of borrowers they featured.

Table 6 here

For each node i playing the role of guarantor or/and lender in the network, and for each city c in our database, we compute a *city market loss rate* $CMLr_{ci}$ corresponding to the share of borrowers of city c that lose market access when i is removed from the network. $CMLr_{ci}$ is formally defined in Algorithm 3.

Algorithm 3 here

We consider that a money market intermediary i is geographically systemic with respect to a given city c when $CMLr_{ci}$ is equal or higher than 50% or, in other words, when more than half of that city's borrowers lose market access as a result of the intermediary's failure. Out of the 1,535 guarantors and lenders in our dataset, only 63 were geographically systemic for at least one city. Of course, cities featuring one borrower who only appears in one chain in the network are by definition fully dependent on one guarantor and one lender for their market access. For those cities therefore, the lender's and guarantor's city market loss rates are both equal to 100% by construction.

Figure 10 here

In Figure 10, we focus on cities that comprise at least two borrowers. We represent relationships between all these cities and intermediaries in a matrix form. A dot on the graph indicates that a given intermediary is geographically systemic for a given city. The dot's size varies according to the number of borrowers present in that city while its color varies according to the intermediary's level of systemicness with regards to the city (measured through its city market loss rate). For example, looking at the interaction of the first column and bottom line in the graph, we see that Boston featured 23 different borrowers on the London money market and that 61% of them were dependent on one London intermediary (the discount house Union Discount Company) for their market access.

Several UK intermediaries that specialized in intermediating credit for borrowers located in specific regions of the world were geographically systemic for specific cities in these regions. This is the case, for example, of the Canadian Bank of Commerce (a multinational bank that specialized in the financing of North American trade), on which borrowers in San Francisco, Pensacola, Toronto, Wilmington, Paris (Texas), Vancouver, and Navasota were strongly dependent. Similarly, C Murdoch & Co. (a merchant bank that specialized in guaranteeing bills for customers located in Africa) was a key intermediary for most borrowers in Casablanca, Mogador, Tangier, and Saffi. At the same time, even the largest discount houses or money market funds of the City of London (Union Discount Company, National Discount Company, Alexanders & Co.) did not exhibit much geographic systemicness with regard to any city in the world. Overall, the results reveal that, except for a few of them, cities did not depend on one single intermediary for accessing the London money market.

4.5. City vulnerability

Last, we measure to what extent each city in the network was vulnerable to shocks on UK intermediaries. A money market borrower was particularly vulnerable if its market access depended exclusively on one single guarantor or lender. For example, a borrower might have been involved in five different chains featuring five different lenders but only one guarantor. In that case, the borrower was strictly dependent on one guarantor for its market access but did not depend on any specific lender. We

consider that a given borrower is *vulnerable* when its market access depends on one single guarantor or lender. We also distinguish between *guarantor-vulnerable* borrowers (i.e., borrowers that depend on one single guarantor) and *lender-vulnerable* borrowers (i.e., borrowers that depend on one single lender).

For each city c , we compute both a *guarantor-vulnerability rate* VUL_{Gc} (defined as the share of borrowers of that city that depend on one single guarantor) and a *lender-vulnerability rate* VUL_{Lc} (defined as the share of borrowers of the city that depend on one single lender). The formal definitions of VUL_{Gc} and VUL_{Lc} are provided in Algorithm 4.¹⁹

Algorithm 4 here

Figure 11, Panel A, presents a scatter plot of cities' VUL_{Gc} against their overall number of borrowers. Then, 40.0% of the cities appearing in at least two chains exhibit a guarantor-vulnerability rate of 100%, indicating that each of their borrowers is dependent on one single guarantor. All of these highly guarantor-vulnerable locations were however small cities comprising 10 borrowers or less. At the same time, not all small cities exhibited a high VUL_{Gc} . Out of the 296 cities that featured 10 borrowers or less, 16.9% had no guarantor-vulnerable borrower and 29.7% had less than half of their borrowers that were guarantor-vulnerable. At the other end of the spectrum, no single city with more than 10 borrowers exhibited a guarantor-vulnerability rate of 100%.

Figure 11 here

Figure 11, Panel B, scatters cities' VUL_{Lc} against their overall number of borrowers. On average, cities appear to have been less exposed to the removal of lenders than guarantors. The share of cities with a

¹⁹ Cities that appear in one single bill of the network have, by construction, a vulnerability score of 100%. Hence, we restrict our sample to the 360 cities that appear in at least two bills.

vulnerability rate of 100% is lower for lenders (31.9%) than for guarantors (40.0%).²⁰ In addition, while 16.7% of cities in the network feature no lender-vulnerable borrowers, only 13.9% of cities have no guarantor-vulnerable borrower.

Overall, these findings indicate that lenders were on average more substitutable money market intermediaries than guarantors. Borrowers obtained funds from a relatively diverse pool of lenders but had their debts guaranteed by a more limited number of guarantors. At the same time, the 20 biggest lenders on the money market had a significantly higher market share (85.35%) than the 20 biggest guarantors (37.52%). Therefore, despite the greater market concentration in lending than in guaranteeing, intermediaries playing the role of guarantors were less substitutable. This apparent paradox can be explained by the specific nature of these two types of activities, which were performed by different types of money market intermediaries (Accominotti et al., 2021). One implication of this market structure is that the sterling money market did not feature very large and systemic hubs, and was less prone to the *robust-yet-fragile* property characteristic of most present-day interbank networks.

5. Conclusions

We rely on a new dataset assembled from archival records in order to reconstruct the network of financial interlinkages in the dominant global money market of the first globalization era. This dataset covers both “bank-bank” (lender-guarantor) and “bank-firm” (guarantor-borrower) relationships. We represent the network of borrower-guarantor-lender intermediation chains as a hyperstructure and assess financial network resilience through an original methodology that allows preserving the unity of these higher-order structures. We apply simple shock simulation techniques and measure the effect of removing individual nodes (or intermediaries) on the overall network. This allows us to measure to what extent money market intermediaries were substitutable and to what extent borrowers across the world were dependent on a few UK agents for their access to the dominant global money market.

²⁰ It is worth noting however that two out of the 115 cities (Port Said and Amsterdam) that exhibit a vulnerability rate of 100% with respect to lenders are middle-sized cities, that featured 15 and 20 borrowers respectively.

In modern interbank networks, shock simulations involving the removal of central nodes generally result in a complete breakdown of network connectivity. This is because present-day networks are generally characterised by the presence of a few systemic and non-substitutable actors. Our findings however indicate that the sterling money market of the first globalization era did not feature any highly-systemic intermediaries, whose failure could have caused major damage to the network. These findings indicate that a global financial network with a low level of actors' systemicness can and did actually exist even at a time of high international financial integration.

Our article makes a methodological contribution. To the best of our knowledge, we are the first to apply the hyperstructure approach to the study of financial networks. The methodology we develop here could be applied to study the resilience of any directed network in which certain nodes are non-substitutable and can potentially cause severe damage to connectivity. This is the case, for example, of global supply chains or transportation networks (Lucena-Piquero et al., 2022). One limitation of our analysis is that it is based on data representing a snapshot in time. Extending the hyperstructure approach we develop here to study a dynamic financial network is beyond the scope of this article, but would constitute an avenue for future research.

Our results also have implications for financial regulators. The low systemicness of intermediaries on the sterling money market at the beginning of the twentieth century arose from the specific characteristics of the financial instruments (bills of exchange) used for money market transactions. These instruments created incentives for money market agents to produce information on borrowers and discouraged the emergence of too large intermediaries (Accominotti et al., 2021). Nineteenth-century regulators were adamant about the superiority of the bill of exchange from a supervisory viewpoint (Ugolini, 2017). This suggests that supervisors aiming to improve the robustness of financial networks should pay close attention to the microstructure of financial markets and encourage the use of instruments whose design provides disincentives to concentration.

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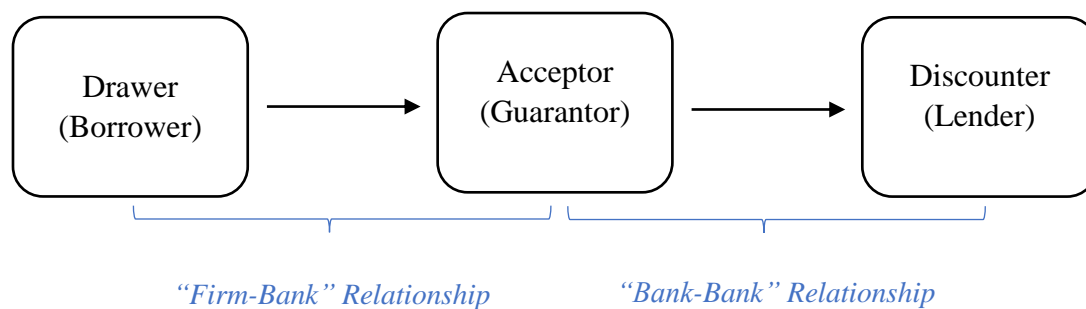
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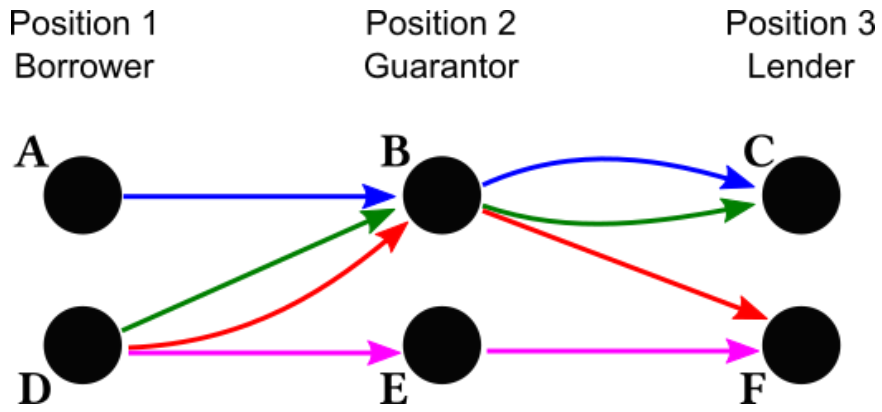
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Figure 1. Links encompassed in a bill of exchange



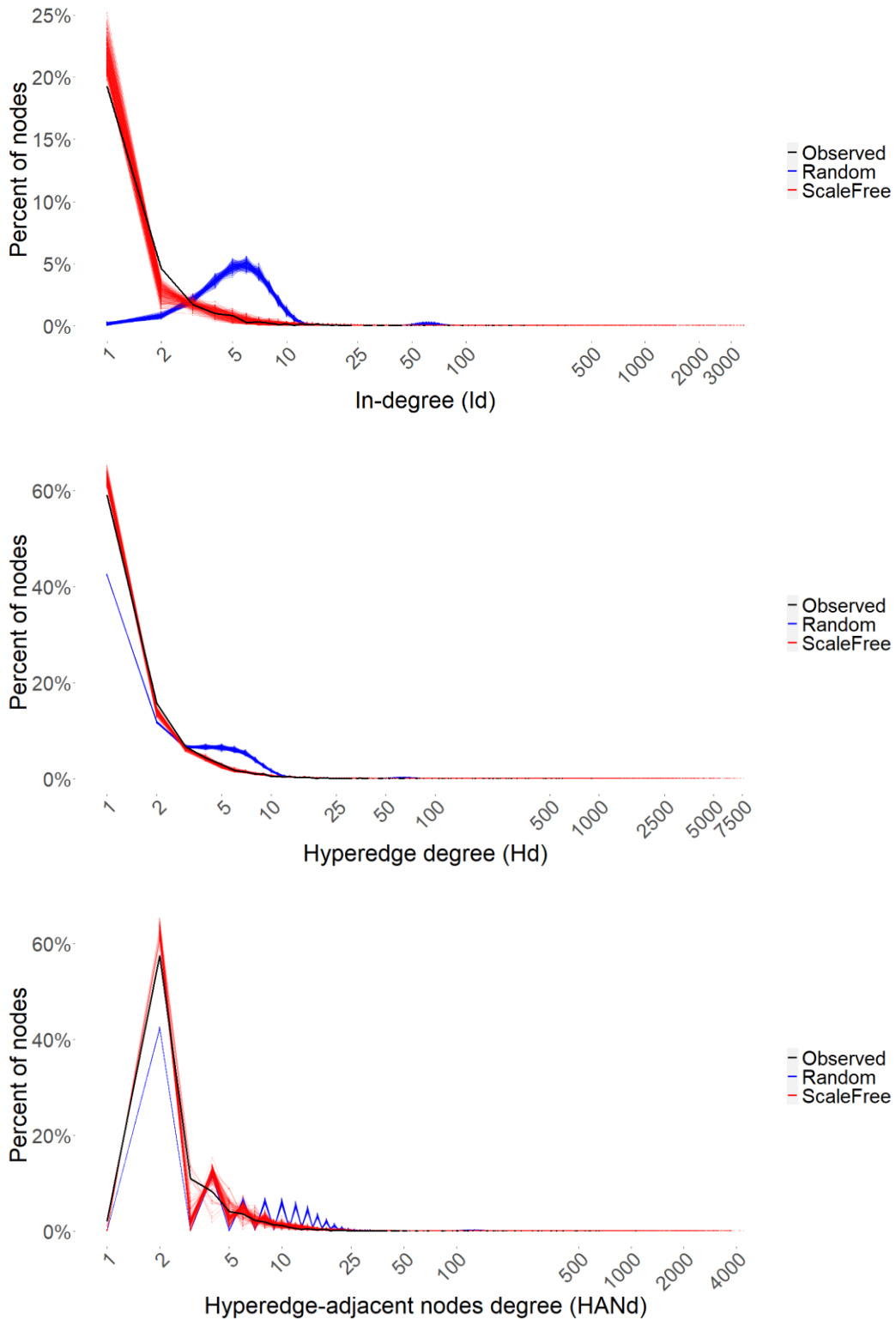
Notes: This figure presents a schematic representation of the relationships between actors involved in the origination and distribution of a bill of exchange. See Accominotti et al. (2021) for a detailed description of the functioning of bills of exchange.

Figure 2. Representation of chains



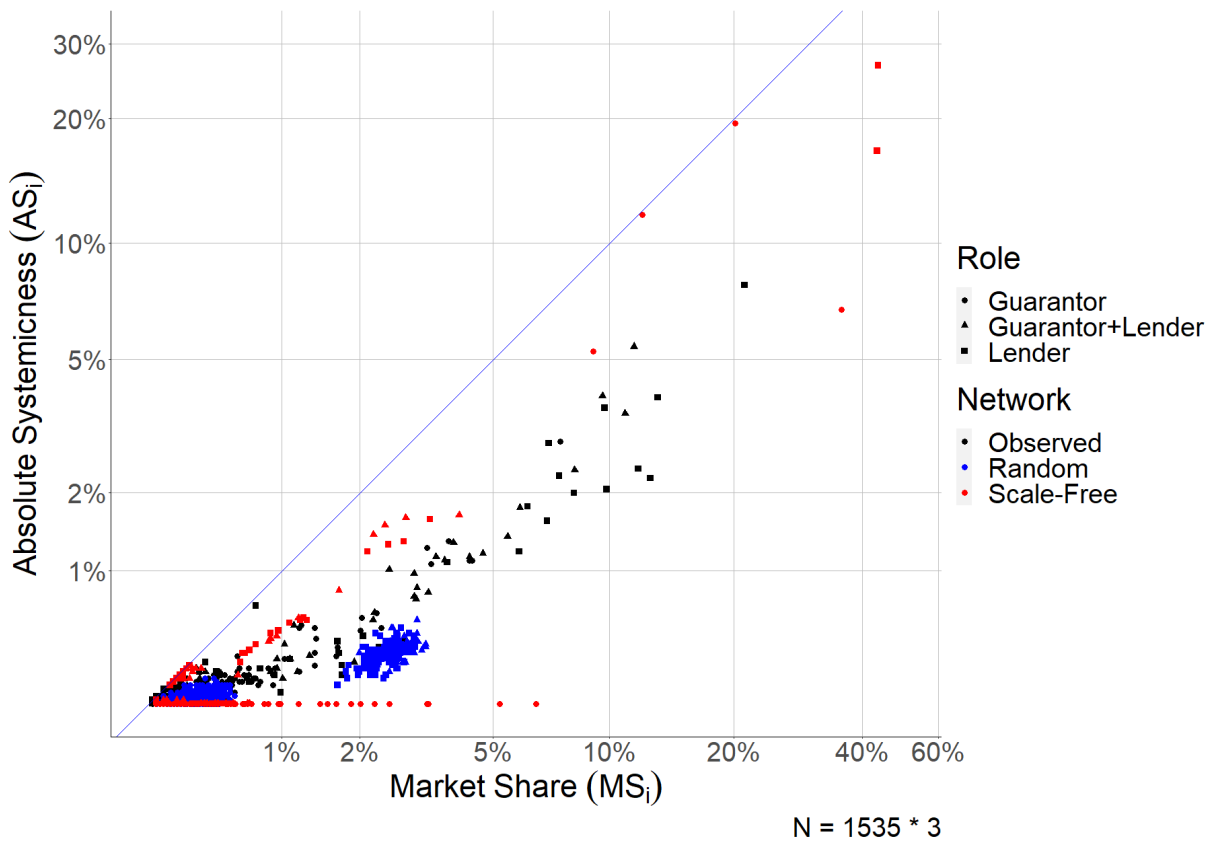
Notes: This figure presents a hypothetical example of a higher-order network involving four chains and six nodes. The four chains are (A,B,C), (D,B,C), (D,B,F) and (D,E,F). Each combination of two same-coloured arrows constitutes a hyperedge that associates the three nodes and their links in a given chain.

Figure 3. Node degree distribution: observed versus *null* model networks



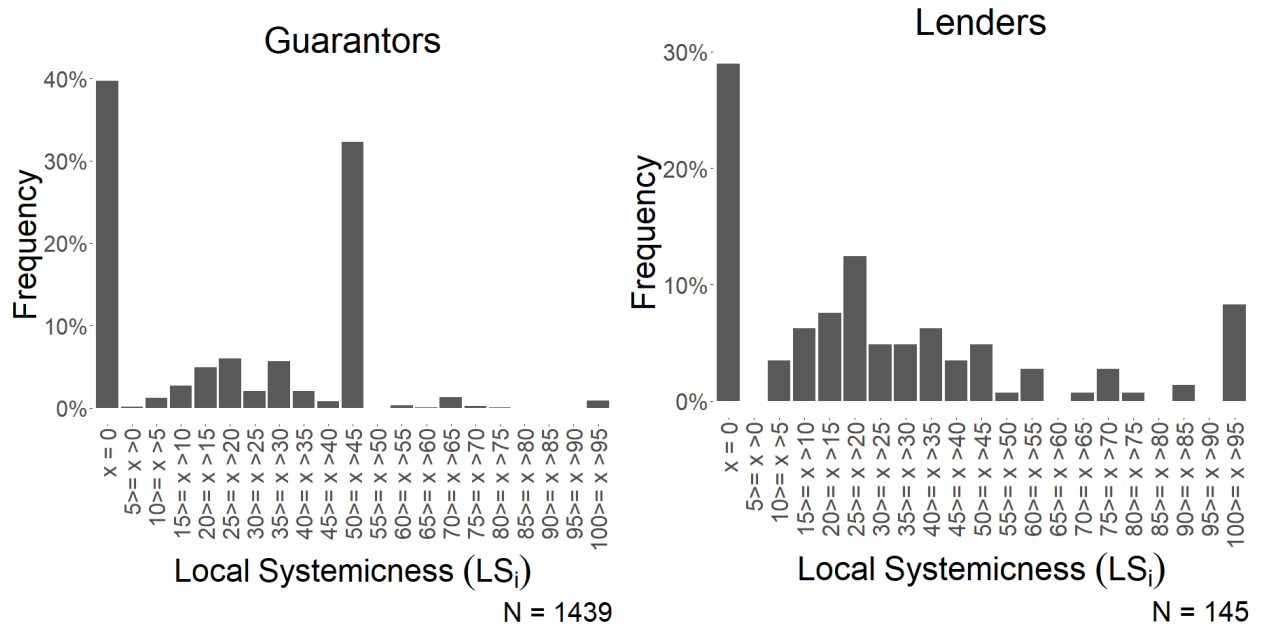
Notes: For each of the three network degree centrality metrics, the figure shows the frequency distribution of nodes according to their degree in the observed network (black line), in 250 simulated random networks (blue lines), and in 250 simulated scale-free networks (red lines). Panel A reports the frequency distribution of nodes according to their *in-degree* (Id_i). Panel B reports the frequency distribution of nodes according to their *hyperedge degree* (Hd_i). Panel C reports the frequency distribution of nodes according to their *hyperedge-adjacent nodes degree* ($HAND_i$). The x-axis is in logarithmic scale. Nodes that only played the role of borrower are excluded from panel A as, by construction, $Id_i = 0$ for those nodes. See text.

Figure 4. Absolute systemicness



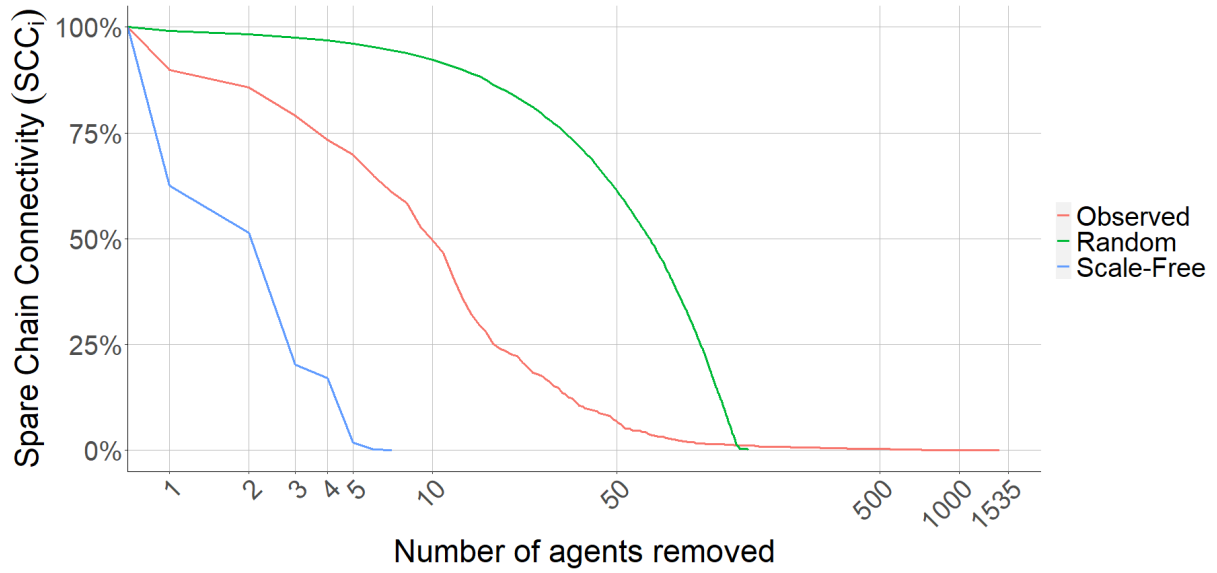
Notes: The figure reports the *absolute systemicness* (AS_i) and *market share* (MS_i) of each intermediary (guarantor or lender) in the observed network as well as in one simulated random network and one simulated scale-free network. On the y-axis, a node i 's *absolute systemicness* corresponds to the percentage of all nodes in the network that remain isolated when i is removed. On the x-axis, a node i 's *market share* corresponds to the percentage of nodes in the network which are hyperedge-adjacent to i . See text for a more detailed definition of these variables. Actors of the observed, random, and scale-free networks are represented in black, blue, and red, respectively. In each network, nodes that only played the role of guarantor are represented by a dot, nodes that only played the role of lender are represented by a square, and nodes that played the role of both guarantor and lender are represented by a triangle. Both axes are in logarithmic scale. See text.

Figure 5. Local systemicness: guarantors and lenders



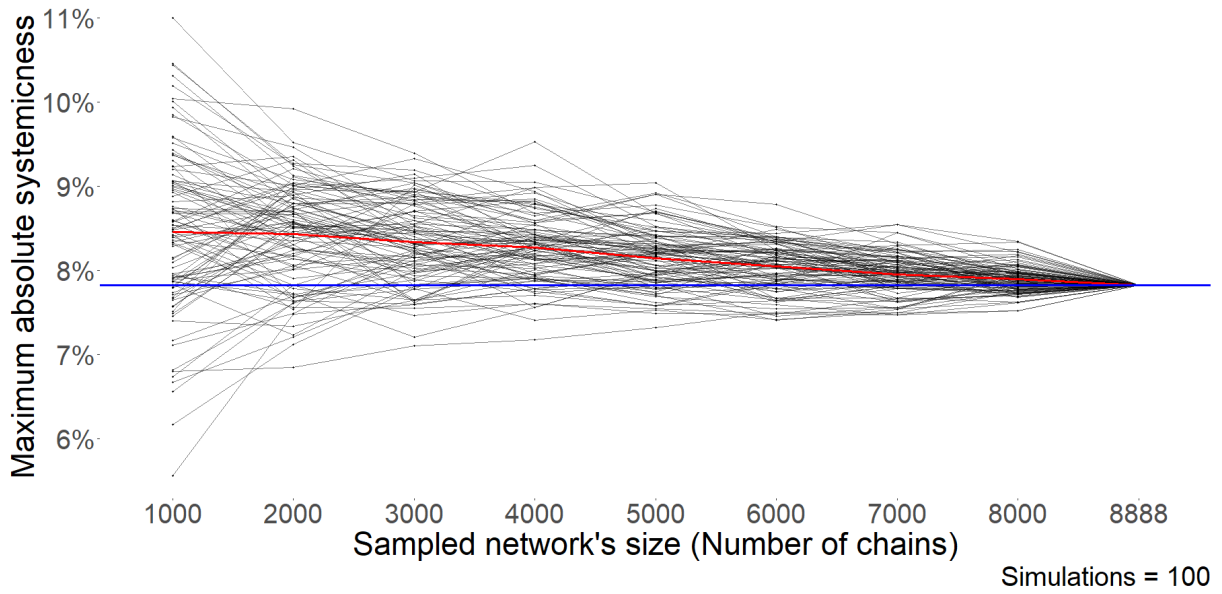
Notes: The figure reports the frequency distribution of guarantors (left panel) and lenders (right panel) according to their *local systemicness* (LS_i). Values on the y-axis correspond to the percentage of guarantors/lenders whose LS_i falls within any given value range reported on the x-axis. See text.

Figure 6. Spare Chain Connectivity



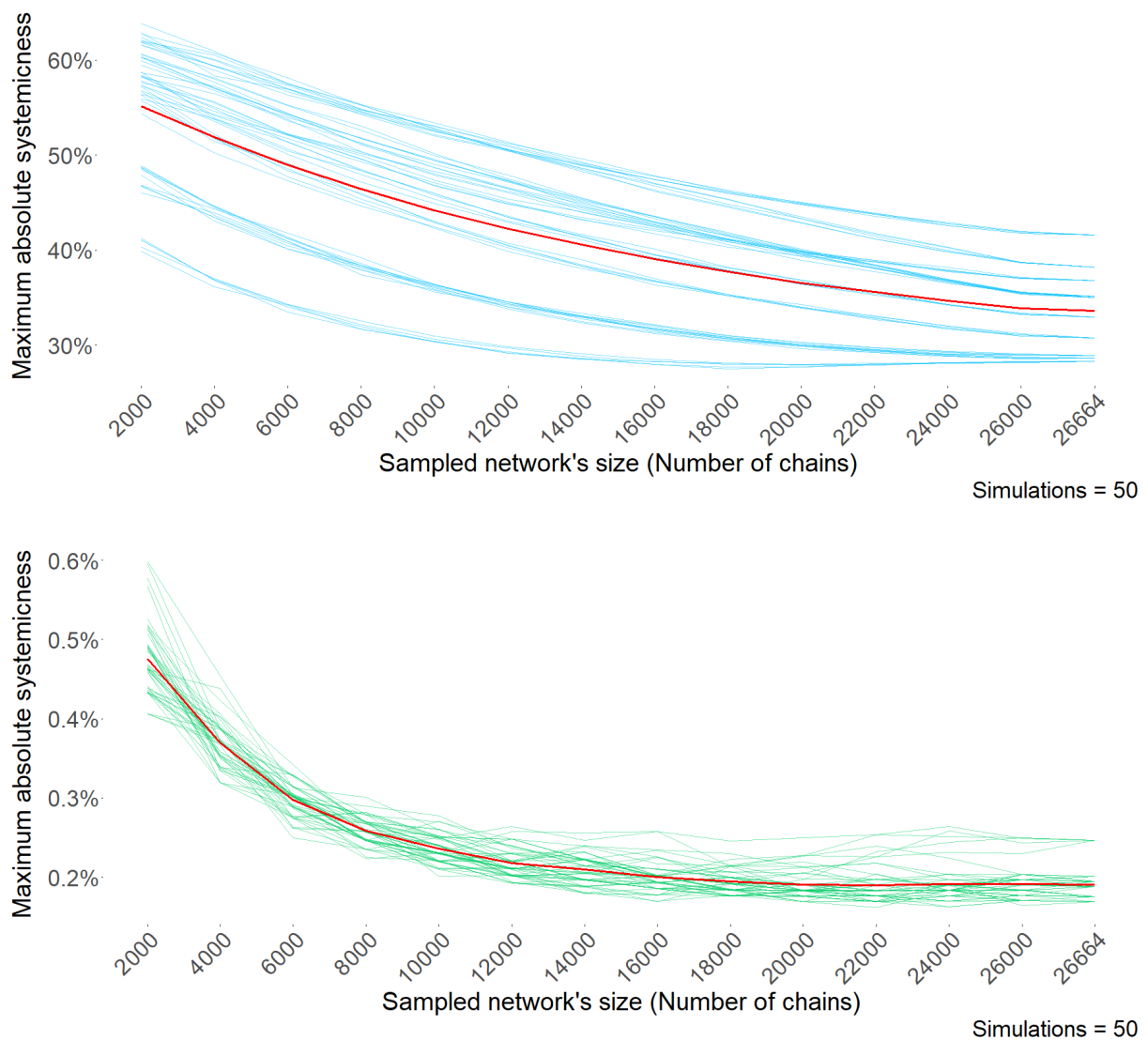
Notes: For each of the three networks (observed network, simulated random network, and simulated scale-free network), the y-axis reports the percentage of original chains that are preserved after sequentially removing a certain number of nodes. Nodes are sequentially removed in the order of their absolute systemicness (AS_i). The x-axis is in logarithmic scale. See text.

Figure 7. Sampling effects and systemicness in the observed network



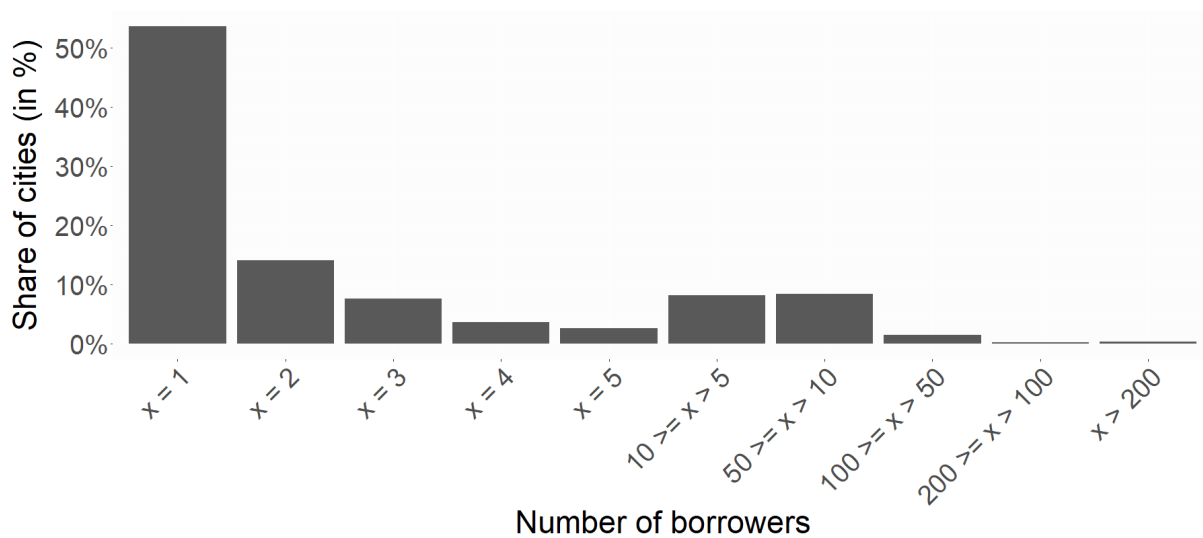
Notes: The figure reports the outcome of simulations performed to assess the effect of sampling biases on maximum absolute systemicness in the observed network. Each of the 100 black lines reports the *maximum absolute systemicness* ($\max(AS_i)$) recorded in eight randomly-sampled portions of the observed network. Sampled networks are increasing in size from 1,000 to 8,000 chains. The observed network contains 8,888 chains. The number of chains included in each sampled portion of the observed network is reported on the x-axis and $\max(AS_i)$ is reported on the y-axis. For any given sample size, the red line corresponds to the mean of $\max(AS_i)$ across all 100 sampled networks. The horizontal blue line crosses the y-axis at 7.83% (i.e., the maximum value of AS_i in the observed network). See text.

Figure 8. Sampling effects and systemicness in random and scale-free networks



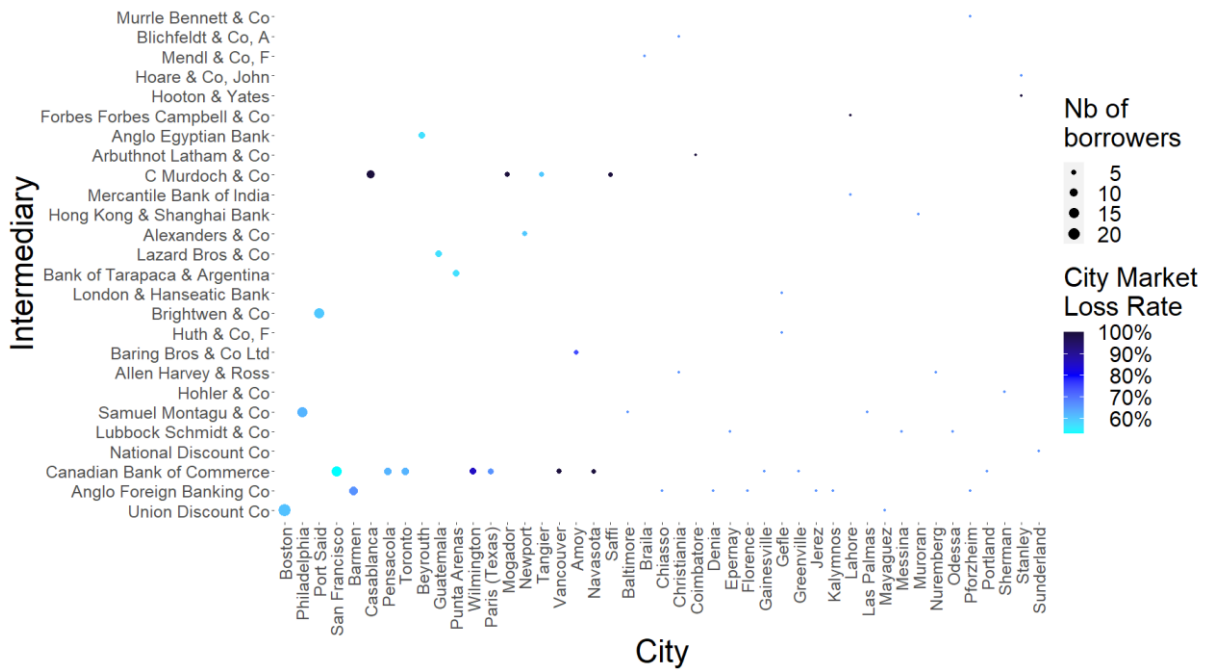
Notes: The figure reports the outcome of simulations performed to assess the effect of sampling biases on maximum absolute systemicness in random and scale-free networks. Each of the 50 blue (green) lines reports the *maximum absolute systemicness* ($\max(AS_i)$) recorded in fourteen randomly-sampled portions of ten simulated random (scale-free) networks. Sampled networks are increasing in size from 2,000 to 26,664 chains. Each of the ten simulated networks contains 26,664 chains. The x-axis reports the number of chains included in each sample of the simulated random (scale-free) network. The y-axis reports $\max(AS_i)$ for each sample. For any given sample size, the red line corresponds to the mean of $\max(AS_i)$ across all 50 sampled networks. See text.

Figure 9. Frequency distribution of cities according to their number of borrowers



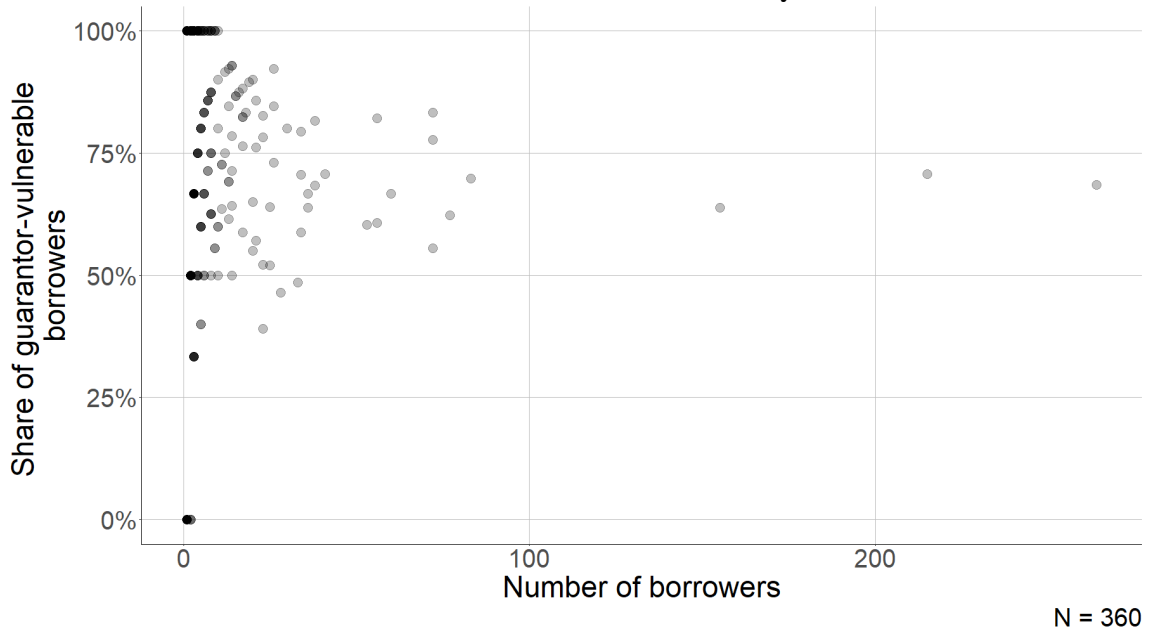
Notes: The figure reports the frequency distribution of cities according to the number of borrowers they feature. Values on the y-axis correspond to the percentage of cities that comprise a number of borrowers falling within any given value range reported on the x-axis. See text.

Figure 10. Geographically systemic intermediaries

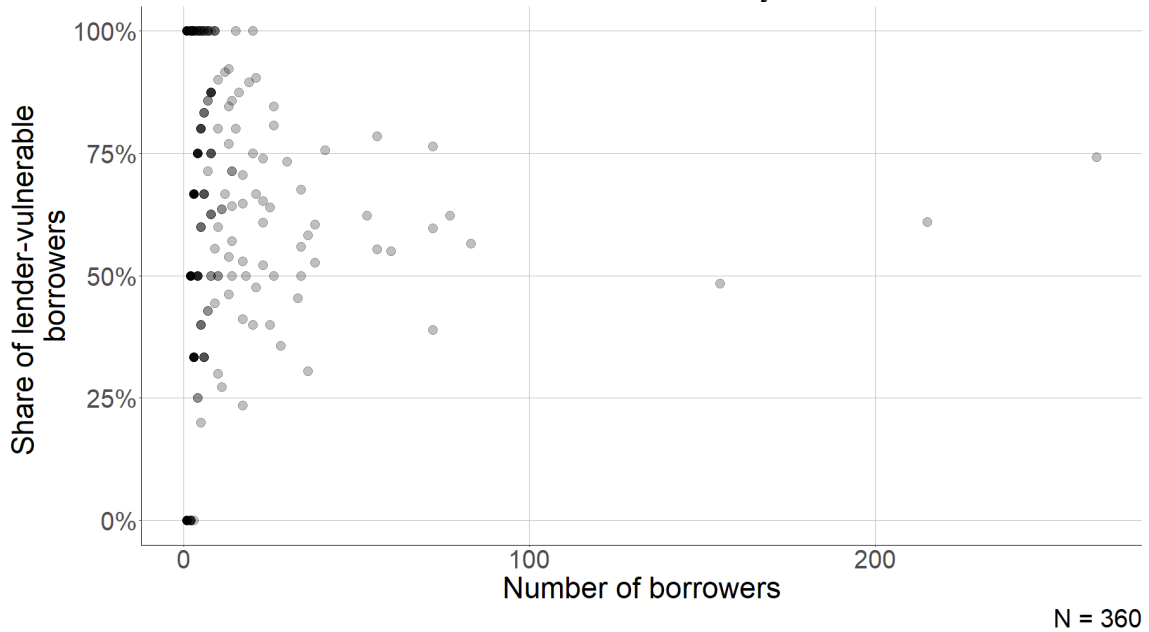


Notes: The figure represents all intermediaries (guarantors and lenders) that were geographically systemic with respect to cities that appear on the x-axis. All cities featuring at least two borrowers are included. An intermediary is considered geographically systemic for a given city if more than 50% of borrowers of that city lose market access as a consequence of its removal. For every city, geographically systemic intermediaries are represented by a blue dot. The dot's size varies according to the number of borrowers located in the city and its darkness varies according to the intermediary's city market loss rate (CMLR_c) with regards to that city. On the x-axis, cities are ranked (from left to right) according to their overall number of borrowers. On the y-axis, intermediaries are ranked (from bottom to top) according to their absolute systemicness (AS_i). See text.

Figure 11. City vulnerability
Panel A. Guarantor-vulnerability



Panel B. Lender-vulnerability



Notes: For each city with at least two bills in the network, the figure reports its overall number of borrowers (x-axis) and the percentage of these borrowers that are *guarantor-vulnerable* (Panel A) and *lender-vulnerable* (Panel B) (y-axis). A borrower is considered guarantor- (lender-)vulnerable if it is dependent on one single guarantor (lender) for her market access. Cities that appeared in one single chain in the network are removed from the analysis as, by construction, 100% of their borrowers were vulnerable. Each dot on the figure corresponds to one city. Several cities exhibit the exact same number of overall and vulnerable borrowers, in which case their dots are superimposed. Superimposed dots appear darker on the figure. See text.

Algorithm 1: Absolute systeminess, market share, and local systeminess

Input:

Data as an edgelist E where each row is a chain (a bill in our case) and each column is a role (three columns in our case). The set of chains (rows) in E is C , each row is unique in E (no two chains are alike), and V is the set of agents in E .

Procedure:

1. Identify all agents i in guarantor and/or lender role (subset $V_{GL} \in V$): $\{i\} \in V_{GL} \forall \{i\} \in V : Id_i > 0$
 2. FOR i in V_{GL} , subset from E all rows where the agent i has the guarantor and/or lender role. The result is the edgelist E_i
 3. Obtain the edgelist R_i via the subtraction of the edgelist E_i from E , so $R_i = E \setminus E_i$
 4. Obtain the subset V_{pi} of agents included in E_i excluding i : $V_{pi} = \{j \in V : j \in E_i\} \setminus \{i\}$
 5. Obtain the subset V_{si} of agents who are included in V_{pi} but not in R_i (i.e., the agents who depend on i for market access): $V_{si} = V_{pi} \setminus \{j \in V : j \in R_i\}$
 6. Compute the *absolute systeminess* of an agent i (AS_i) as the proportion of agents of V_{si} in V less one (the agent i): $AS_i = (|V_{si}| / (|V| - 1))$
 7. Compute the *market share* of an agent i (MS_i) as the proportion of agents of V_{pi} in V less one (the agent i): $MS_i = (|V_{pi}| / (|V| - 1))$
 8. Compute the *local systeminess* of an agent i (LS_i) as the proportion of agents of V_{si} in V_{pi} : $LS_i = (AS_i / MS_i) = (|V_{si}| / |V_{pi}|)$
-

Algorithm 2: Spare chain connectivity

Input:

Data as an edgelist E where each row is a chain (a bill in our case) and each column is a role (three columns in our case). The set of chains (rows) in E is C , each row is unique in E (no two chains are alike), and V is the set of agents in E .

The absolute systemicness AS for all guarantors and lenders V_{GL}

Procedure:

1. Order (permute) V_{GL} elements by decreasing absolute systemicness. The result is the sequence of agents VAS .
 2. FOR $i = 1$ to $|VAS|$
 3. IF $i = 1$: $E_R == E$; $C_R == C$; $V_R == V$
 4. Obtain from E_R all rows where i has the guarantor and/or lender roles. The result is the edgelist E_i .
 5. Redefine E_R as E_R without E_i : $E_R = E_R / E_i$
 6. Obtain the set of chains C_{Ri} from the edgelist E_R .
 7. Compute the *spare chain connectivity* SCC as the proportion of chains remaining in the network after removing the agent i and all its predecessors in the VAS sequence. So, $SCC_i = |C_{Ri}| / |C|$
 8. IF $E_R = \{\}$ ENDFOR
-

Algorithm 3: City market loss rate

Input:

Data as an edgelist E where each row is a chain (a bill in our case) and each column is a role (three columns in our case). The set of chains (rows) in E is C , each row is unique in E (no two chains are alike), and V is the set of agents in E .

Data frame of borrowers by city. The set of borrowers is DR and the set of cities is CT .

Procedure:

1. FOR c in CT , subset from E all rows (chains) whose agents playing the role of borrowers are located in city c . The result is the subset of agents and chains E_c , where C_c is the subset of chains in E_c and DR_c is the subset of agents in borrower role in E_c
 2. IF $|C_c| > 1$:
 3. Identify all actors i with guarantor and/or lender role in E_c . The result is the subset V_{GL_c}
 4. FOR i in V_{GL_c} , subset from E_c all rows (chains) where i has the guarantor and/or lender role. The result is the subset of chains and agents E_{ci}
 5. Obtain the subset of chains and agents R_{ci} which are not included in E_{ci} : $R_{ci} = E_c \setminus E_{ci}$
 6. Obtain the subset DR_{ci} of agents (borrowers) of E_{ci} who are not included in R_{ci} (i.e., the agents who depend on i for market access): $DR_{ci} = E_{ci} \setminus R_{ci}$
 7. Compute the *city market loss rate* of a city c for an agent i ($CMLr_{ci}$) as the proportion of the borrowers in city c (i.e., of DR_c) who depend on agent i for market access: $CMLr_{ci} = (|DR_{ci}|/|DR_c|)$
-

Algorithm 4: City vulnerability

Input:

Same as Algorithm 3.

Procedure:

1. FOR c in CT , subset from E all rows (chains) whose agents playing the role of borrowers are located in city c . The result is the edgelist E_c , where C_c is the subset of chains in E_c and DR_c is the subset of agents in borrower role in E_c
 2. IF $|C_c| > 1$:
 3. FOR i in DR_c identify both subsets of agents in guarantor role V_{G_c} and in lender role V_{L_c} . If $|V_{G_c}| = 1$, the borrower i is *guarantor-vulnerable*. If $|V_{L_c}| = 1$, the borrower i is *lender-vulnerable*.
 4. Define $DR_{G_c} \in DR_c$ as the set of *guarantor-vulnerable* borrowers in city c . Define $DR_{L_c} \in DR_c$ as the set of *lender-vulnerable* borrowers in city c .
 5. Compute city c 's *guarantor-vulnerability* VUL_{G_c} as the proportion of the borrowers in city c who are guarantor-vulnerable: $VUL_{G_c} = (|DR_{G_c}|/|DR_c|)$. Compute city c 's *lender-vulnerability* VUL_{L_c} as the part of the borrowers in city c who are lender-vulnerable: $VUL_{L_c} = (|DR_{L_c}|/|DR_c|)$
-

Table 1: Profiles of agents on the sterling money market

Profile	Number	% of all agents
Pure Borrower (outside London)	3,290	66.20
Pure Borrower (in London)	145	2.92
Pure Guarantor	1,326	26.68
Hybrid (Borrower+Guarantor)	64	1.29
Pure Lender	61	1.23
Hybrid (Borrower+Lender)	35	0.70
Hybrid (Guarantor+Lender)	29	0.58
Hybrid (Borrower+Guarantor+Lender)	20	0.40
Total	4,970	100.00

Notes: This table presents the number of agents of different profiles on the sterling money market as well as their share in the total population of agents. “Pure” refers to nodes playing only one role (borrower or guarantor or lender) in the various bills in which they are involved. “Hybrid” refers to nodes that play different roles in the various bills on which they appear. Note that any agent playing the role of guarantor or lender had to be located in the UK. See text.

Table 2: Degree centrality measures in a hyperstructure: an example

Node (i)	In-degree (Id_i)	Hyperedge degree (Hd_i)	Hyperedge-adjacent nodes degree ($HAND_i$)
A	0	1	2
B	2	3	4
C	1	2	3
D	0	3	4
E	1	1	2
F	2	2	3

Notes: This table illustrates three network degree centrality metrics for nodes in a hyperstructure. For each node A, B, C, D, E, F, in the hypothetical network represented in Figure 2, the table reports their *in-degree*, *hyperedge degree* and *hyperedge-adjacent degree*. See text for the three degree definitions.

Table 3. Maximum node degree value in observed and simulated networks

Measure	Network type	Nb of networks	Min	Max	Mean	Median
Maximum In-degree ($\max(Id_i)$)	Observed	1	357	357	357	357
	Random	250	77	97	85	84
	Scale-Free	250	895	3555	2192	2020
Maximum Hyperedge degree ($\max(Hd_i)$)	Observed	1	900	900	900	900
	Random	250	89	127	108	108
	Scale-Free	250	896	7666	3434	3100
Maximum Hyperedge-Adjacent Nodes degree ($\max(HAND_i)$)	Observed	1	1055	1055	1055	1055
	Random	250	171	239	204	203
	Scale-Free	250	1218	4417	2684	2566

Notes: This table presents descriptive statistics (minimum, maximum, mean and median) on the maximum degree values observed in a. the observed network, b. the 250 simulated random networks, and c. the 250 simulated scale-free networks. See text for the definition of the three degree values and for details on the simulations.

Table 4. Maximum absolute systemicness and maximum market share in observed and simulated networks

Measure	Network type	Nb of networks	Min	Max	Mean	Median
Maximum Absolute Systemicness ($\max(AS_i)$)	Observed	1	7.828%	7.828%	7.828%	7.828%
	Random	250	0.442%	0.684%	0.530%	0.523%
	Scale-Free	250	9.297%	68.554%	31.967%	29.070%
Maximum Market Share ($\max(MS_i)$)	Observed	1	21.231%	21.231%	21.231%	21.231%
	Random	250	3.099%	4.004%	3.399%	3.380%
	Scale-Free	250	16.312%	76.533%	53.815%	53.547%

Notes: This table presents descriptive statistics (minimum, maximum, mean and median) on the maximum values of absolute systemicness and market share observed in a. the observed network, b. the 250 simulated random networks, and c. the 250 simulated scale-free networks. See text for the definition of the two variables and for details on the simulations.

Table 5. Group systemicness

Group of intermediaries	Number of impacted nodes (V_{sG_x})	Absolute systemicness (AS_{G_x})	Market share (MS_{G_x})
Discount houses (N=20)	2094	42.1%	65.7%
Anglo-Foreign Banks (N=45)	1053	21.2%	40.6%
Top-10 Merchant Banks (N=10)	569	11.5%	22.3%
Clearing Banks (N=11)	281	5.7%	11.6%

Notes: For each of the four historical groups of intermediaries (discount houses, Anglo-foreign banks, top-10 merchant banks, and clearing banks), the table reports the number of nodes that remain isolated when the entire group is removed from the network ($|V_{sG_x}|$), the group's *absolute systemicness* (AS_{G_x}), as well as the group's *market share* (MS_{G_x}). See text and Algorithm 1 for the formal definition of each indicator.

Table 6. Demography of cities

Number of borrowers per city	Number of cities in that category	City names
200+	2	London; New York.
101-200	1	Calcutta.
51-100	9	Alexandria; Bombay; Buenos Aires; Colombo; Hamburg; Manchester; Memphis; New Orleans; Yokohama.
11-50	52	Amsterdam; Antwerp; Bahia; Barmen; Batavia; Belfast; Berlin; Bordeaux; Boston; Bradford; Bremen; Cairo; Chicago; Constantinople; Copenhagen; Dallas; Galveston; Glasgow; Havana; Hong Kong; Houston; Iquique; Karachi; Kobe; Lima; Liverpool; Madras; Malaga; Manila; Melbourne; Montevideo; Montreal; Norfolk; Para; Paris; Pernambuco; Philadelphia; Port Said; Rangoon; Riga; Rio de Janeiro; San Francisco; Santos; Sao Paulo; Savannah; Shanghai; Singapore; Smyrna; St Petersburg; Stockholm; Sydney; Valparaiso.
6-10	50	Antofagasta; Arequipa; Augusta; Baghdad; Barbados; Beyrouth; Bilbao; Brussels; Bucharest; Casablanca; Dundee; Foochow; Fort Worth; Frankfurt; Genoa; Goteborg; Guatemala; Guayaquil; Hankow; La Paz; Leeds; Leipzig; Lisbon; Manaus; Marseille; Montgomery; Moscow; Oporto; Oruro; Paris (Texas); Patras; Penang; Pensacola; Port Elizabeth; Port of Spain; Punta Arenas; Rosario; Rotterdam; Salonica; San Jose; Santiago; Shimonoseki; St Louis; Surabaya; Tientsin; Toronto; Valencia; Waco; Wilmington; Zurich.
2-5	172	Adelaide; Aden; Algiers; Alicante; Almeria; Amoy; Amritsar; Athens; Atlanta; Auckland; Baltimore; Bangkok; Barcelona; Bari; Basle; Bassora; Birmingham (Alabama); Bogota; Braila; Brisbane; Brooklyn; Budapest; Campinas; Canton; Carrara; Cartagena; Castries; Cavalla; Ceara; Cedar Rapids; Ceylon; Charleroi; Charlotte; Chiasso; Christchurch; Christiania; Cienfuegos; Cochabamba; Coimbatore; Cologne; Como; Concepcion; Corfu; Corsicana; Crefeld; Curacao; Daitotei; Danzig; Demerara; Denia; Dunedin; Dusseldorf; Epernay; Fazilka; Florence; Fremantle; Gainesville; Galatz; Gefle; Greenville; Grenada; Halifax; Hamilton; Helena (Arkansas); Herisau; Hiogo; Invercargill; Iquitos; Jerez; Johannesburg; Jumet; Kalymnos; Kansas City; Keighley; Kristiansand; Labuan; Ladysmith; Laguna; Lahore; Langerfeld; Las Palmas; Le Havre; Leghorn; Leicester; Little Rock; Lodz; Lyon; Macassar; Macon; Malmoe; Managua; Mangalore; Maracaibo; Maranhao; Mauritius; Mayaguez; Mazagan; Medellin; Messina; Mexico; Milan; Minneapolis; Mobile; Mogador; Mosgiel; Muroran; Nagasaki; Naples; Navasota; Neckarau; Newcastle; Newport; Nuremberg; Odessa; Oklahoma City; Orizaba; Palermo; Panama; Parahyba; Paris (Arkansas); Passaic; Perth; Pforzheim; Ponce; Portland; Porto Alegre; Potosi; Prague; Puerto Gallegos; Quebec; Rio Grande; Rustchuk; Saffi; San Antonio; San Juan; San Salvador; Sandakan; Seattle; Sevilla; Sherman; Sorata; St Etienne; St Gall; St Vincent; Stanley; Sunderland; Syra; Tacna; Tangier; Teheran; Temple; Tenerife; The Hague; Townsville; Trieste; Tripoli; Troy; Tupiza; Turin; Vancouver; Venice; Veracruz; Verviers; Vicksburg; Victoria; Vienna; Vostizza; Warsaw; Wellington; Yazoo City; Zante; Zanzibar.
1	331	Aarau; Abilene; Abo (Turku); Ada (Oklahoma); Aguadilla; Akyab; Albany (Australia); Albany (Georgia); Americus; Amotfors; Andros; Ansbach; Antigua; Ardmore (Oklahoma); Arecibo; Arica; Ashgabat; Asuncion; Athens (Georgia); Aymeries; Bahama; Bamberg; Barnaul; Barrow in Furness; Barry; Bassein; Bayonne; Bergen; Bermondsey; Bielefeld; Binche; Birmingham; Blackburn; Bocholt;

Bochum; Bolivar; Bonham; Botosani; Bradford (New Zealand); Brenham; Brighthouse; Broe; Brooketon; Broome; Brownwood;
 Brunswick; Burslem; Cadiz; Cairns; Calicut; Canaveral; Candia; Cape Town; Caracas; Cassel; Castlebar; Catacaos; Catania; Cathcart;
 Cawnpore; Cephalonia; Cernobbio; Charleston; Charlottetown; Chemainus; Chemnitz; Chickasha (Oklahoma); Church (Lancashire);
 Cincinnati; Cochin; Colonne; Colquechaca; Columbus (Georgia); Comber; Coquimbo; Cordele; Corocoro; Corumba; Costa Rica;
 Cuiaba; Cuthbert; Danville; Darwin; Dawson; Deerlijk; Denderleeuw; Denton; Derby; Dewsbury; Dison; Dixon; Dordrecht; Dothan;
 Drogheda; Dublin (Georgia); Dumbarton; Durban; Dyersburg; Emelghem; Enkhuizen; Espinho; Falkirk; Farmersville; Faro; Fortin;
 Frangsund; Fray Bentos; Frederiksberg; Fredneks; Fredrikstadt; Funchal; Gais; Galle; Gamleby; Gandia; Gibraltar; Gisborne; Goor;
 Gourdon; Govan; Granada (Nicaragua); Greenock; Greenwood; Grenada (Mississippi); Grimsby; Guerville; Guiria; Halberstadt;
 Hanmore; Harlingen; Heilbronn; Helsingborg; Henderson; Herberton; Hodeidah; Holstebro; Holzheim; Hoogezand; Hoorn; Howrah;
 Huddersfield; Humacao; Indianapolis; Jacksonville; Jaffna; Jaragua; Jersey; Jesselton; Johannegeorgenstadt; Jonkoping; Kalmar; Kazan;
 Keelung; Kerassunde; Kidderminster; Killik Aike; Kimberley; Kingston; Kirkcaldy; Klingenthal; Koenigsberg; Kuching; La Coruna; La
 Plata; La Salada; Langesund; Larnaca; Launceston; Lawrencetown; Leigh on Sea; Lerwick; Limon; Llagostera; Lodelinsart; Lulea;
 Lurgan; Maarssen; Maceio; Madeira; Madrid; Maffersdorf; Malta; Mantua; Marin; Marshalltown; Massena; Masterton; Matanzas;
 Medan; Merida; Middlesbrough; Mirzapur; Mistle; Moji; Moltann; Monro; Mossel Bay; Mulheim; Munich; Muscat; Napier; Narva;
 Natal; Nepal; Nessonvaux; Neustadt; Newbury; Newton; Norrkoping; Nuwara Eliya; Ollioules; Ootacamund; Opobo; Orlando; Otaru;
 Ottignies; Ottumwa; Oude Pekela; Padang; Pahepe; Palafrugell; Partick; Paysandu; Peking; Pepinster; Petit-Goave; Piacenza; Pictou;
 Pine Bluff; Pireus; Pisagua; Plano; Port Antonio; Puerto Cabello; Pyrgos; Qulon; Ravilloles; Rebstein; Remscheid; Reutlingen; Reval;
 Risano; Rochdale; Rockhampton; Rome; Rouen; Roux; Roxburgh; Sacile; Saigon; Sains du Nord; Saint Martin; Salina; Samsoun; Santa
 Ana; Santa Fe; Santander; Saxon; Schemda; Schiedam; Schomberg; Schonheide; Semarang; Seraing; Sfax; Shreveport; Simla; Sioux
 City; Siveveghem; Skelleftea; Skive; Soderhamn; St Anne's Bay; St Avold; St Feliu de Guixols; St Fiden; St John (New Brunswick);
 Strasbourg; Stromstad; Suakin; Sucre; Sulina; Sundsvall; Svendborg; Swansea; Sybadah; Szechwan; Szeged; Taipei; Tananarive;
 Tandragee; Tarija; Taylor; Tellicherry; Terrell; Therezina; Thomasville; Thorn; Tiflis; Timaru; Tocopilla; Tokyo; Toowoomba; Tossa;
 Traben; Trapani; Travancore; Trelleborg; Tsingtau; Uddevalla; Ulrichstal; Union Beach; Vevey; Vigevano; Villers; Vilvorde; Vyborg;
 Wandersbek; Wanganui; Wantage; Werdohl; West Hartlepool; Whiteinch; Wiesbaden; Wilhelmsburg; Wilkes Barre; Winschoten;
 Winterthur; Xanthi; Zaandam; Zofingen.

Notes: This table lists all cities in the network. Cities are classified into different categories according to the number of borrowers they feature. See text.

Online Appendix

“Intermediaries’ Substitutability and Financial Network Resilience: A Hyperstructure Approach”

A.1. The concepts of *substitutability* and *criticality*

In this appendix, we clarify the difference between our concept of *substitutability* and the concept of *criticality* often used in dyadic network analysis in order to measure the impact of a node’s disappearance on other nodes’ network connectivity.

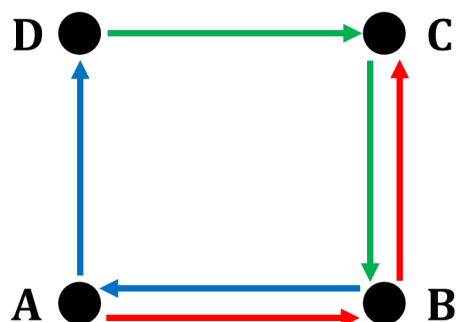
In the paper, we define *substitutability* as follows (p. 13): “*An intermediary is considered substitutable if other nodes are not strictly dependent on it for their access to the money market (or connection to the network). More precisely, one given node A is said to be substitutable with respect to another given node B if A does not appear in all chains to which B belongs. In that case, A’s removal from the network does not leave B isolated, as node B can still connect to the network through other chains that do not involve A. One given actor’s degree of substitutability is therefore inversely proportional to the number of nodes that would remain isolated if the actor was removed from the network*”

The concept of *criticality* has been used with a variety of definitions in the network literature, all of which being related to the notion of *betweenness centrality*. We refer here to the popular definition provided by Goyal (2018, p. 698): “*A node is said to be critical if it lies on all paths between S [a source node] and D [a destination node].*”

This definition of criticality has been developed to study dyadic networks and, as we show below, cannot be directly operationalized in the context of a network featuring higher-order structures. This is because the notion of betweenness centrality cannot be easily transposed to a network featuring supra-dyadic entities, since the presence of such entities imposes constraints on the paths available to connect the various pairs of nodes featured in the network.

In order to see this concretely, consider the example below of a network featuring three borrower-guarantor-lender chains: 1) A-B-C (red chain); 2) B-A-D (blue chain); and 3) D-C-B (green chain). Figure A.1.1. provides a hypergraph representation of this network.

Figure A.1.1 Hypergraph representation of a network featuring chains



Notes: This figure represents a higher-order network featuring three chains of three actors: 1) A-B-C (red chain); 2) B-A-D (blue chain); and 3) D-C-B (green chain). Each combination of two same-coloured arrows constitutes a hyperedge that associates the three nodes and their links in a given chain.

Directly applying the definition of criticality presented above while ignoring the presence of higher-order structures would lead us to conclude that node B is *not critical* as it does not lie on all paths between A and C. However, if we now consider that nodes can only be connected to each other if they belong to a common higher-order structure (a chain), we can see that A can only reach C through B (red chain). In the absence of B, the source node A can therefore not reach the destination node C.

More generally, in our empirical setup, B's disappearance would result in the breakdown of all chains to which it belongs (and of all the dyadic links embedded in these chains). This would consequently leave all other nodes (A, D and C) isolated in the network. Our concept of substitutability as defined above allows accounting for this phenomenon and for B's true systemicness in the network. According to our definition, node B is *not substitutable* as its presence is essential to all other nodes' connection to the network. Our definition of substitutability can therefore be easily operationalized to study the impact of a given actor's disappearance on other nodes' connection to the network in a network in which nodes interact within higher-order structures.

It must be noted that being based on the concept of betweenness centrality, the notion of criticality necessarily implies that only nodes that are situated on a path connecting two other nodes can be "critical". This is in stark contrast with our notion of substitutability: as we show in the paper, nodes located at the very end of intermediation chains (i.e., lenders) could indeed be "non-substitutable" in the provision of market access to other nodes.

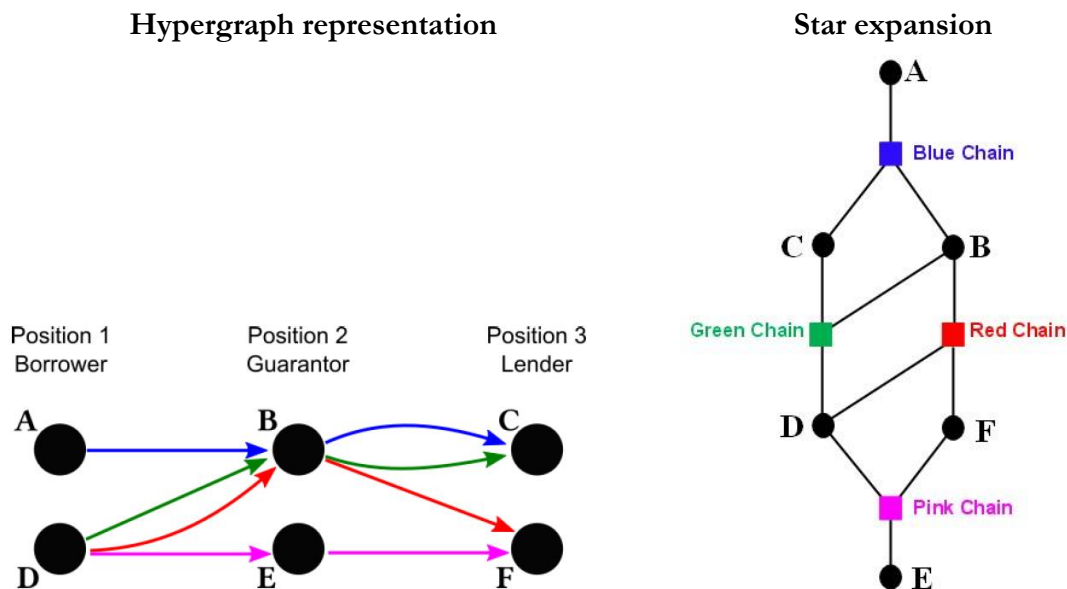
A.2. Hypergraph versus star expansion representation of higher-order networks

In this appendix, we aim to clarify the value-added of a hyperstructure approach compared to a mere star expansion approach for the purpose of the empirical analysis conducted in the paper. We do this through a simple illustrative example.

On the sterling money market, each bill of exchange transaction involved three different actors which each played one of three possible roles in the transaction (borrower, guarantor or lender). Each observed bill of exchange transaction consists of a chain involving three different actors (a borrower, a guarantor and a lender) and two dyadic links (a borrower-guarantor link and a guarantor-lender link). Since any bill of exchange transaction required the presence of three actors, the disappearance of any node in the borrower-guarantor-lender chain results in the breakdown of all dyadic links that compose that chain.

Consider the example of a network composed of four different chains featured in the paper's figure 2. Figure A.2.1 reproduces the hypergraph representation of that network presented in Figure 2 (left panel) and also displays the star expansion of the same hypergraph (right panel).

Figure A.2.1 Hypergraph and star expansion representations of a network featuring chains



Notes: This figure presents two representations of the illustrative higher-order network presented in Figure 2. The network featured involves four chains and six nodes. The four chains are (A,B,C), (D,B,C), (D,B,F) and (D,E,F). The left panel presents the hypergraph representation of the network, while the right panel presents the star expansion of the same hypergraph. In the hypergraph representation, each combination of two same-coloured arrows constitutes a hyperedge that associates the three nodes and their links in a given chain. The star expansion representation does not preserve information on chains but only on the dyadic links between nodes.

The main difference between the network’s hypergraph representation and its star expansion is that the latter does not preserve the internal structure of the network’s supra-dyadic entities. In particular, the star expansion representation does not preserve information about the roles (borrower, guarantor or lender) played by each individual node within each of the chains (bill transactions) in which they are involved. It is not possible to infer from this representation whether nodes A-F are playing the role of borrower, guarantor and lender in each of the chains to which they belong.

This difference has implications for our empirical analysis. While the star expansion approach allows simulating the impact of an individual node’s disappearance on overall network connectivity, it does not allow distinguishing between the substitutability of intermediaries playing various roles on the money market—especially in view of the fact that, as shown in Table 1 of the paper, a non-negligible number of nodes were “hybrid”, i.e. they played *more than one role* on the money market. Such an analysis necessitates having information on the roles played by the different nodes in the various chains featured in the network. Since the star expansion representation does not provide that information, it cannot be used to assess the various degrees of substitutability of different types of intermediaries (borrowers, guarantors, lenders).

Preserving information about the internal structure of supra-dyadic entities can be important in order to assess the systemicness of various types of intermediaries in financial networks featuring higher-order structures. For example, on the underwriting market, lenders and underwriters might not be equally systemic and this is why it can be important to distinguish between the roles played by each intermediary in such a network.

Reference (online appendix)

Goyal, Sanjeev (2018), “Heterogeneity and Networks”, in Hommes, Cars, and LeBaron, Blake (eds), *Handbook of Computational Economics*, North Holland, vol. 4, chapter 12, 687-712.