Pollution in strategic multilateral exchange: taxing emissions or trading on permit markets?

Ludovic A. Julien, Anicet Kabre, Louis de Mesnard

EconomiX

Pollution in strategic multilateral exchange: taxing emissions or trading on permit markets?

Ludovic A. Julien
Anicet Kabre
Louis de Mesnard

2023-14 Document de Travail/ Working Paper

EconomiX - UMR 7235 Bâtiment Maurice Allais
Université Paris Nanterre 200, Avenue de la République
92001 Nanterre Cedex

Site Web : economix.fr
Contact : secreteriat@economix.fr
Twitter : @EconomixU

cnrs Université Paris Nanterre

Pollution in strategic multilateral exchange: taxing emissions or trading on permit markets?

Ludovic A. Julien[1], Anicet Kabre[2], Louis de Mesnard[3]

Version: June, 29th 2022

We introduce polluting emissions in a sequential noncooperative oligopoly model of bilateral exchange. In one sector a leader and a follower use polluting technologies which create negative externalities on the payoffs of strategic traders who belong to the other sector. By modeling emissions as a negative externality, we show that the leader pollutes more (less) than the follower when strategies are substitutes (complements). Then, we consider the implementation of public policies to control the levels of emissions, namely two taxation mechanisms and a permit market. We study the effects of these public policies. Moreover, we determine the conditions under which these public policies can implement a Pareto-improving allocation.

Key Words: Stackelberg competition, pollution, fiscal policy, permit market
Subject Classification: C72, D43, Q50

1. INTRODUCTION

One can consider that there are two main types of pollution: (i) *global pollution*, mainly the CO2 emissions, but also the GWP gases (high global warming potential gases) we can cite HFCs (hydrofluorocarbons), PFCs (perfluorocarbons), SF6 (sulfur hexafluoride), NF3 (nitrogentrifluoride), CF4 (tetrafluoromethane or carbon tetrafluoride), and so on-; and (ii) *local pollution*, where some firms pollute some consumers.[4] In this paper, we consider the second type of pollution, which can be composed of fine particles, toxic products, all can be considered as fatal products of the production process-, etc., and even acid rains in Germany some years ago. Local pollution concerns one or more polluters (often firms) and one or more polluted (often consumers), all being well identified, unlike global pollution. Pigou's carbon tax (Pigou 1932; Barnett 1980; Weisbach and Metcalf 2009; Metcalf and Weisbach 2013; Metcalf 2019) is considered as the main tool to regulate local pollution, even if the carbon taxes are not as all taxes very well accepted by the firms, but also by the consumers who understand that the price charged by the firms will necessarily increase. Indeed, the principle of carbon tax is to raise by a tax the marginal cost of the polluter to obtain the equalization of the marginal cost of polluters and of polluted at equilibrium. It should be underlined that the new equilibrium does not implies that pollution is eradicated: to the contrary, at equilibrium, both sides polluters and consumers commonly agree to accept a non-zero level of pollution.

[1]EconomiX, UPL, Univ Paris Nanterre, CNRS, 200 avenue de la République, F92000 Nanterre, France. E-mail: ludovic.julien@parisnanterre.fr
[2]EconomiX, UPL, Univ Paris Nanterre, CNRS. E-mail: kabre.anicet@yahoo.fr
[3]CREGO (EA 7317), univ. Bourgogne - Franche-Comté, 2 Bd Gabriel, 21000 Dijon, France. E-mail: louis.de-mesnard@u-bourgogne.fr
[4]According to the Intergovernmental Panel on Climate Change (IPCC) report 2022, since IPCC AR5, human influence on the Earth's climate has become unequivocal.

Most studies have examined the relation polluter-polluted, by using a partial equilibrium approach (see Montero 2009; Lehmann 2010; de Vries and Hanley 2016; and Hintermann 2017, for surveys). Because of the complementarities and substitutabilities between commodities that exist through agents' preferences, it is necessary to take into account the role played by the preferences of agents on the effectiveness of emission regulation policies. Thus, it would be interesting, instead, to consider this relation in a framework which analyzes the strategic behavior of agents in interrelated markets. Moreover, it turns out that positive and normative approaches to environmental pollution problems with strategic interactions have been studied mainly in markets where the demand side is assumed to be competitive and the supply side embodies a finite number of firms which have a Cournotian behavior in a simultaneous-moves game (see, for instance, Hintermann 2017 for usual partial equilibrium approaches, and Crettez et al. 2021 for a general equilibrium approach). In this paper, we consider a two-stage game with complete information of a market exchange economy in which *all* traders behave strategically to exploit the potential gains from trade on both the demand and the supply sides. Overall, our main objective is to investigate whether public policy implementation can affect local polluting behavior in a sequential strategic multilateral exchange model with production.

1.1. Motivations

The motivations are twofold. First, we consider polluting emission behaviors in a sequential game in which all agents behave strategically. To this end, we consider polluting emissions, such as those modeled by Crettez et al. (2021), in the sequential bilateral oligopoly model of Julien and Tricou (2012), which extends the bilateral oligopoly model with a finite number of traders introduced by Gabszewicz and Michel (1997), and explored by Bloch and Ghosal (1997), Dickson and Hartley (2013), among others.[5] In a first sector, one leader and one follower have inherited a technology with which they produce one good. In a second sector, a good, which is also used as a production factor, is initially held by a finite number of followers. The production activity generates polluting emissions, i.e., negative externalities on traders' utility who belong to the other sector. The strategic traders compete on quantities, and try to manipulate the relative price through their supplies. As all agents behave strategically, there is no assumption of price-taking behavior and the market demand is not given. Endogenous demands seem to be important for pollution analysis insofar as the space over which traders' preferences are defined includes the polluting commodity. Correlatively, polluting emissions are linked to agents' strategies which are themselves partly determined by their preferences.

In addition, how we implement our strategic game will be critical. Therefore, our model will allow us to define a two-stage quantity setting game where the players are the traders, the strategies are their supplies, and the payoffs are the utility levels they reach in the market outcome. We will get a subgame perfect Nash equilibrium of the two-stage game, namely the Stackelberg-Nash equilibrium with emissions,

[5] The class of strategic bilateral exchange models represents a two-commodity version (a two-sector model) with resource specialization (agents have corner endowments) of the strategic market games models with a *finite* number of traders introduced by Shubik (1973), and Shapley and Shubik (1977). In bilateral oligopoly, each trader has corner endowment but wants to consume both goods. There is a market price which aggregates the strategic supplies of all traders and allocates the amounts traded to each trader. The noncooperative equilibrium concept is the Cournot-Nash equilibrium (CNE henceforth). For a survey on bilateral oligopoly, see Dickson and Tonin (2021).

which we will compute.[6] In our bilateral oligopoly model with Stackelberg-Nash competition, strategies could be either substitutes or complements, while if competition is of the Cournot-Nash type, strategies can only be complements. We will determine whether the possibility for strategies to be complements or substitutes can affect the emission's behavior. Thus, this will lead us to compare the Stackelberg-Nash equilibrium with emissions with the Cournot-Nash equilibrium with emissions which would have resulted if the game were instead a simultaneous move game.

Second, we analyze and compare three kinds of regulation to control the levels of emissions in our sequential model: a competitive market of permits, and two different kinds of taxation mechanisms, namely, a tax on emissions, and a per unit tax on the strategic supplies when exchange takes place. We will determine whether polluting emissions can be limited either by the permit market or by any taxation mechanism. We focus on the effects and the performance of these three kinds of regulatory policies by comparing the effects of these public policies, and we will determine the conditions under which these public policies will be Pareto-improving. To this end, we study whether the sequential strategic behavior setting affects the effectiveness and the efficiency of public policy. More specifically, we will wonder whether the possibility for strategies to be complements or substitutes matters for the effectiveness and efficiency of public policy. This will lead us to compare the effects of these public policies with the effects which would have been obtained in the simultaneous move game in which all traders had instead a Cournotian behavior.

1.2. Contribution

Our contribution to the literature is twofold. The first contribution concerns the introduction of pollution in a sequential strategic market game with production. The second contribution concerns the comparative statics exercises which relate to public policies in our setting.

First, strategic interactions belong to Stackelberg competition. Thus, we would like to determine whether the sequential strategic decisions will affect the polluting behavior in bilateral oligopoly. The strategic two-stage structure will capture some specific features such as heterogeneity in market power. Indeed, the emissions will differ across traders who produce the polluting commodity insofar as the game can display either strategic complementarity or strategic substitutability within a given sector. We will notably wonder whether the leader pollutes more (less) than the follower when strategies are substitutes (complements). In addition, we will compare the levels of polluting emissions at the Stackelberg-Nash equilibrium with those obtained at the Cournot-Nash equilibrium.

Some strategic market games with environmental issues have already been studied under Cournot competition. For instance, Godal (2011) analyze various models of non-cooperative pure exchange (without production), and consider how they should be adapted to emission markets. By using a strategic bilateral pure exchange model with interior endowments, he shows that a strategic seller (resp. buyer) had a marginal payoff that is below (resp. above) the equilibrium market price. Here, we will rather consider a strategic bilateral exchange model in which

[6] It is common knowledge that, in strategic market games, and thus in bilateral oligopoly games, the trivial equilibrium is always a Nash equilibrium of the game (Cordella and Gabszewicz 1998; Busetto and Codognato 2006). Therefore, a critical issue concerns the existence of an equilibrium with trade. To simplify the analysis of the strategic equilibrium, and to perform some comparative statics exercises, we will consider a simple model with non-trivial strategic equilibria.

production generates pollution. Therefore, an important point is that pollution is endogenous and the permit market is only one possible instrument among others to reduce polluting emissions.

Second, our model is, to the best of our knowledge, the first strategic sequential model in interrelated markets that compares three kinds of regulations: two taxation mechanisms along with a permit market. We will examine the effects and the performance of these three kinds of regulatory policies. We will consider first two taxation mechanisms: an ad valorem tax on emissions, and a per unit tax on the supplies when strategic exchange takes place. Taxation mechanisms have been studied in pure exchange bilateral oligopolies under the assumption of Cournot competition by Gabszewicz and Grazzini (1999, 2001), Grazzini (2006), and Elegbede et al. (2021).[7] Here, for each taxation mechanism, we will assume that the total product of the tax is used to finance some given public expenditure. We will compare the efficiency of the two taxation mechanisms. Then, we will introduce a competitive permit market, and we will determine the conditions under which the polluting emissions of the leader and the follower decrease with the price of permits. Additionally, we will study the effect of an increase in the price of permits on the payoffs of traders who bear negative externalities due to pollution. Besides, we will study the extent to which the effectiveness and the efficiency of these public policies implemented to remedy market failures - indeed negative externalities and inefficiencies caused by imperfectly competitive behavior - are specific to the sequential decision setting.

1.3. Literature review

There is a huge literature devoted to polluting emissions under strategic interactions in a partial equilibrium framework (see Montero 2009 and Hintermann 2017 for surveys, and Hintermann 2017 for empirical evidence). The models consider either that the supply side of the single commodity market embodies a dominant firm which interacts with a competitive fringe (Hahn 1984) or focus on market power stemming from oligopolistic firms that manipulate the allocation of emission rights for predatory purposes (Von der Fehr 1993; Eshel 2005), which opens the possibility of making market power endogenous (Lange 2012). Moreover, the welfare implications of permits markets are studied notably in Sartzadakis (1997, 2004) and Gersbach and Requate (2004).[8] Christin et al. (2021) show that the

[7] Gabszewicz and Grazzini (2001) study three kinds of fiscal policies with transfers: taxing trade and endowments by subsidzing an outside agent who is deprived from any commodity; and taxing endowments and subsidizing both the outside agent and the traders. By assuming that agents' preferences are represented by the same Cobb-Douglas utility function, they show that the first two kinds of lump-sum taxes with transfers can only reach a second-best, whilst the third one leads to a first-best. Gabszewicz and Grazzini (1999) focus on endowment taxation with incentive transfers in the linear, Cobb-Douglas, and CES bilateral oligopoly models. They show that such a taxation mechanism with transfer can lead to a Pareto-optimal allocation. By using a Cournot-Walras equilibrium model, Grazzini (2006) shows that per unit taxation welfare dominates ad valorem taxation when the number of competitive traders is sufficiently high compared to the number of strategic traders. Elegbede et al. (2021) consider a bilateral oligopoly model in which the preferences of traders are represented by CES utility functions with non unitary shares on consumption. They show that both fiscal policies with transfers implement a first-best allocation only when commodities are perfect complements or perfect substitutes.

[8] Gersbach and Requate (2004) study how refunding schemes should be designed to create incentives for oligopolistic firms to produce an efficient level of output, to optimally abate emissions and to invest in cleaner technologies.

type of pollution abatement technology that is used in an industry has an impact on the way the cap-and-trade system affects the product market oligoplistic equilibrium. The literature explores mainly the link between abatement technologies and emissions (Haita 2014). Our model rather focuses on (sequential) emission's behavior and the effects of regulatory policies such as taxation mechanisms and trading emission rights in interrelated markets.

Taxation mechanisms in a strategic general equilibrium framework with environmental issues have already been investigated. Crettez et al. (2021) consider a two-sector model with pollution permits in which some traders behave *a la* Cournot while a representative consumer behaves as a price-taker. Strategic traders have inherited some polluting technology which specifies how to produce some amount of one commodity with some amount of the other commodity which is initially held by a competitive trader. The authors show that a supply subsidy can be Pareto improving when the agents sufficiently value the produced good insofar as it enlarges the size of trades. Our model differs from the previous one in several ways. First, here all agents behave strategically; the game includes one leader and several followers, which introduces heterogeneity in strategic behavior. Second, producers have different technologies, which introduces a second source of heterogeneity. Third, pollution creates negative externalities on the utility of some traders. Fourth, fiscal policies differ insofar as they consist either of taxing supplies when exchange takes place or taxing emissions, and, no supply subsidy mechanism is proposed.

Our model also complements sequential games with a permit market (Godal and Holtsmark 2010; Dickson and MacKenzie 2018). Godal and Holtsmark (2010) consider a finite set of jurisdictions, with a government and a firm per jurisdiction. In the first stage, each government determines an acceptable amount of emission which is redistributed within its jurisdiction to firms, and it levies tax on domestic emissions. In the second stage, firms determine emissions levels and pay taxes. They show that the only major effects of permits exchange are that taxes on domestic emissions are reduced and some income is redistributed, so non-cooperative behavior yields inefficient outcomes. Here, in addition to taxation on emissions, we will consider another mechanism of taxation, which consists of taxing supplies when exchange takes place, which we want to compare with the tax on emissions. Dickson and MacKenzie (2018) consider the implications of strategic trade in pollution permit markets in a two-stage game. In a first stage, the permit market is modeled as a bilateral oligopoly with strategic firms, and their roles as buyers or sellers of permits are determined. In the second stage, firms transact either competitively or strategically in a commodity market. One point is that comparative statics exercises are difficult to handle when the firms have a Cournotian behavior on the commodity market as the market outcome depends on each firm's marginal cost in relation to those of its rivals. As a consequence, the overall effect of strategic behavior in the product market cannot be ascertained. However, in our model, strategic behavior only holds on the commodity markets, and we will rather focus on the effects of pollution as well as on the (comparative) efficiency of the two taxation mechanisms and the permit market.

The remainder of the paper unfolds as follows. In Section 2, we describe the model. In Section 3, we compute the non-cooperative sequential equilibrium with emissions. Section 4 is devoted to the implementation and the effects of two taxation mechanisms. In Section 5, we study the effects of the implementation of a permit market. In Section 6 we conclude. An Appendix collects some useful computations.

2. THE MODEL

Consider an economy with two divisible homogeneous commodities labeled X and Y. Let p_X and p_Y be the corresponding unit prices. We assume that commodity Y is the numeraire, so $p_Y = 1$. The economy embodies $n + 2$ agents of two types: two agents of type I, who are consumers and producers, and n agents of type II, with $n \geqslant 2$, who are pure consumers.[9] Type I agents are indexed by i, $i \in \{1, 2\}$, and type II agents are indexed by j, $j \in \{1, ..., n\}$.

2.1. Endowments, preferences and technologies

The endowments of the two agents of type I and the n agents of type II are given by:

$$\omega_i = (0, 0),\ i = 1, 2, \tag{1}$$

$$\omega_j = (0, \frac{1}{n}),\ j = 1, ..., n. \tag{2}$$

Therefore, commodity X does not exist initially and must be produced. Thus, we assume, like in Gabszewicz and Michel (1997), that type I traders have inherited some technology which specifies how to produce some amount z_i of good X with some amount k_i of good Y. The production function $F_i(k_i)$ of agent i is given by the linear technology:

$$z_i = F_i(k_i) = \frac{1}{\beta_i} k_i,\ \beta_i \geqslant 1,\ i = 1, 2, \tag{3}$$

where $1/\beta_i$ measures the marginal productivity of the factor of production, so β_i measures the marginal cost (recall $p_Y = 1$).

Production is a polluting activity. Following Stockey (1998), Sanin and Zanaj (2012), and Crettez et al. (2021), we assume that a quantity k_i of input generates a quantity e_i of emissions, with $e_i \in \mathbb{R}_+$, given by:

$$e_i = \frac{1}{\gamma} k_i,\ \gamma > 1,\ i = 1, 2, \tag{4}$$

where γ measures the magnitude of pollution. Therefore, commodity X is a polluting consumption good. From (3) and (4), we can express the production z_i of good X by each agent i, $i = 1, 2$, in terms of the volume of emissions e_i, namely:

$$z_i = \frac{\gamma}{\beta_i} e_i,\ i = 1, 2. \tag{5}$$

The preferences of agents are represented by the following utility functions:

$$u_i(x_i, y_i) = x_i^{\alpha}.y_i^{1-\alpha},\ \alpha \in (0, 1),\ i = 1, 2, \tag{6}$$

$$u_j(x_j, y_j, e_1, e_2) = x_j^{\alpha}.y_j^{1-\alpha} - \mu(e_1 + e_2),\ \alpha, \mu \in (0, 1),\ j = 1, ..., n, \tag{7}$$

[9] We could consider a game with several leaders and followers, but, to simplify, we consider a duopoly in the productive sector.

where x (resp. y) are the amount consumed of commodity X (resp. Y), and μ is the disutility of pollution as emissions display negative externalities on the individual welfare of agent $j = 1, ..., n$.

As benchmark cases, let us compute the competitive equilibria of this economy. The competitive equilibrium supply is always equal to the amount produced z_i^*. When $\beta_1 \neq \beta_2$, the relative price is $p_X^* = \min(\beta_1, \beta_2)$. For instance, if $\beta_1 < \beta_2$, then $p_X^* = \beta_1$. Then, the only active producer is $i = 1$, and her vector of emission and production levels is $(e_1^*, z_1^*) = (\frac{\alpha}{\gamma}, \frac{\alpha}{\beta_1})$, and her allocation is $(x_1^*, y_1^*) = (0, 0)$, with utility level $u_1^* = 0$. For each $j = 1, ..., n$, the allocation is $(x_j^*, y_j^*) = (\frac{\alpha}{\beta_1 n}, \frac{1-\alpha}{n})$, with utility level $u_j^* = (\frac{\alpha}{\beta_1})^\alpha (1-\alpha)^{1-\alpha} \frac{1}{n} - \mu \frac{\alpha}{\gamma}$. Likewise, when $\beta_1 > \beta_2$, then $p_X^* = \beta_2$. Then, the only active producer is $i = 2$, and her vector of emission and production levels is $(e_1^*, z_1^*) = (\frac{\alpha}{\gamma}, \frac{\alpha}{\beta_2})$, her allocation is $(x_2^*, y_2^*) = (0, 0)$, with utility level $u_2^* = 0$. For each $j = 1, ..., n$, the allocation is $(x_j^*, y_j^*) = (\frac{\alpha}{\beta_2 n}, \frac{1-\alpha}{n})$, with utility level $u_j^* = (\frac{\alpha}{\beta_2})^\alpha (1-\alpha)^{1-\alpha} \frac{1}{n} - \mu \frac{\alpha}{\gamma}$. Finally, when $\beta_1 = \beta_2 = \beta$, the relative price is $p_X^* = \beta$. There is a continuum of competitive equilibria parameterized by the share θ, $0 \leqslant \theta \leqslant 1$. Indeed, the vectors of emission and production levels are $(e_1^*, z_1^*) = (\theta \frac{\alpha}{\gamma}, \theta \frac{\alpha}{\beta})$ and $(e_2^*, z_2^*) = ((1-\theta)\frac{\alpha}{\gamma}, (1-\theta)\frac{\alpha}{\beta})$, with $(e_i^*, z_i^*) = (\frac{1}{2}\frac{\alpha}{\gamma}, \frac{1}{2}\frac{\alpha}{\beta})$, for each $i = 1, 2$, when $\theta = \frac{1}{2}$. In any case, the allocation is $(x_i^*, y_i^*) = (0, 0)$, with utility level $u_i^* = 0$. For each $j = 1, ..., n$, the allocation is $(x_j^*, y_j^*) = (\frac{\alpha}{\beta n}, \frac{1-\alpha}{n})$, and the utility level is $u_j^* = (\frac{\alpha}{\beta})^\alpha (1-\alpha)^{1-\alpha} \frac{1}{n} - \mu \frac{\alpha}{\gamma}$. At each competitive equilibrium, the total amount of emissions is given by $e_1^* + e_2^* = \frac{\alpha}{\gamma}$.

2.2. The associated game

To this economy, we associate a two-stage non-cooperative game $\mathbf{\Gamma}$ in which the players are the traders, the strategies are their supplies, and the payoffs are the utility levels. This finite game is an extension of the bilateral oligopoly model with a finite number of traders introduced by Gabszewicz and Michel (1997), and which introduces pollution in the sequential model of Julien and Tricou (2012).

The game $\mathbf{\Gamma}$ displays two stages of decisions and the timing of positions is given. There is one leader, indeed trader 1 of type I, and the $(n+1)$ other traders behave as followers. No trader makes a choice in two subgames. In addition, traders meet once and cannot make binding agreements. By precluding binding agreements, we consider that each trader acts independently and without communication with any of the others. We also assume there is no discounting. Finally, information is assumed to be complete, but information is imperfect between the $(n+1)$ followers.[10] In each decision node, any follower will make an optimal choice, so sequential rationality prevails. As sequential rationality is common knowledge, the game is solved by backward induction.

The strategy set of the leader is given by:

$$Q_1 := \{q_1 \in \mathbb{R}_+ : q_1 \leqslant z_1\}, \tag{8}$$

where q_i represents the pure strategy of the leader. The strategy q_i represents the amount of commodity X the leader sells in exchange for commodity Y. The strategy sets of followers are given by:

[10] Any leader perfectly knows the behavior of all followers, and followers perfectly know the optimal strategy of the leader, but information is imperfect in the subgame between followers.

$$Q_2 := \{q_2(q_1) : Q_1 \to [0, z_1]\}, \tag{9}$$

$$B_j := \{b_j(q_1) : Q_1 \to [0, \frac{1}{n}]\},\ j = 1, ..., n, \tag{10}$$

where $q_2(q_1)$ is the pure strategy of follower 2 of type I, and $b_j(q_1)$ is the pure strategy of follower j of type II, $j = 1, ..., n$. For all $q_1 \in Q_1$, the strategy $q_2(q_1)$ represents the amount of commodity X type I follower offers in exchange for commodity Y, and the strategy $b_j(q_1)$ represents the amount of commodity Y type II follower j offers in exchange for commodity X.

A strategy profile is a vector $(q_1, q_2(q_1); \mathbf{b}(q_1)) = (q_1, q_2(q_1); b_1(q_1), ..., b_n(q_1))$, with $(q_1, q_2(q_1); \mathbf{b}(q_1)) \in \prod_i Q_i \times \prod_j B_j$, where $\prod_i Q_i \times \prod_j B_j = Q_1 \times Q_2 \times B_1 \times ... \times B_n$. Let $\mathbf{b}_{-j}(q_1)$ denote the strategy profile of all followers of type II but j. It is worth noting that, for each type I trader, the strategic decision relates to the supply of goods and not to the quantity of emissions. Indeed, the level of emissions e_i, with $e_i \in \mathbb{R}_+$, for each $i = 1, 2$, is not a strategy like the supply q_i, but merely a decision variable which depends upon the technology of emissions (4), and which is linked to the production technology (3).[11] Therefore, the leader and her direct follower behave, when they make their production choices, as if they would not take into consideration the emissions made by her direct rival.

Given a price vector $(p_X, 1)$ and a strategy profile $(q_1, q_2(q_1); \mathbf{b}(q_1)) \in \prod_i Q_i \times \prod_j B_j$, the market clearing price $p_X(q_1, q_2(q_1); \mathbf{b}(q_1))$ is determined according to the following price mechanism which aggregates the strategic supplies of all traders:

$$p_X(q_1, q_2(q_1); \mathbf{b}(q_1)) = \frac{\sum_{j=1}^{n} b_j(q_1)}{q_1 + q_2(q_1)}. \tag{11}$$

To lighten notations, in what follows, let $(.)$ for the vector $(q_1, q_2(q_1); \mathbf{b}(q_1))$. Agents behave strategically, and are aware of their influence on the relative price $p_X(.)$.

Any agent $i \in \{1, 2\}$ has two decisions to make: which quantity q_i of good X to sell on the market; and, which quantity of good X to produce, which through (3) and (4) determines the level of emissions e_i. Thus, the income of the leader is equal to her profit Π_1, where

$$\Pi_1(e_1, (.)) := p_X(.)q_1 - \gamma_1 e_1. \tag{12}$$

With this income, the leader finances her purchase of commodity Y which is equal to Π_1. She ends up with the bundle of commodities $(x_1(e_1, (.)), y_1(.)) = (\frac{\gamma}{\beta_1} e_1 - q_1\ , \Pi_1(e_1, (.))$. Her corresponding utility level is $u_1(\frac{\gamma}{\beta_1} e_1 - q_1, \Pi_1(e_1, (.))$. Likewise, the income of type I follower is equal to his profit Π_2, where

$$\Pi_2(e_2, (.)) := p_X(.)q_2(q_1) - \gamma_2 e_2. \tag{13}$$

With this income, the follower finances his purchase of commodity Y which is equal to Π_2. He ends up with the bundle of commodities $(x_2(e_2, (.)), y_2(.)) = (\frac{\gamma}{\beta_2} e_2 - q_2\ , \Pi_2(e_2, (.))$. Her corresponding utility level is $u_2(\frac{\gamma}{\beta_2} e_2 - q_2(q_1), \Pi_2(e_2, (.))$.

[11] For each trader $i \in \{1, 2\}$, emissions can be considered as consequences of their own supply decisions, so each trader's emissions can be written as function her equilibrium strategy (see Remark 1 in Section 3).

Any agent $j \in \{1, ..., n\}$ has one decision to make: which quantity b_j of good Y to sell on the market. Follower $j \in \{1, ..., n\}$ obtains in exchange for $b_j(q_1)$ a share $\frac{b_j(q_1)}{B}$ of the aggregate supply Q, i.e. a quantity of commodity X equal to $\frac{1}{p_X(.)} b_j(q_1)$ (recall $p_Y = 1$), and ends up with the bundle of commodities $(x_j(.), y_j(.)) = (\frac{1}{p_X(.)} b_j(q_1), \frac{1}{n} - b_j(q_1))$, with corresponding utility level $u_j(\frac{1}{p(.)} b_j(q_1), \frac{1}{n} - b_j(q_1))$.

Finally, let us define the payoffs of traders. Define the function $\pi_1 : \to \mathbb{R}_+ \times \prod_i Q_i \times \prod_j B_j$, $(e_1, (.)) \mapsto \pi_i(e_1, (.))$. Likewise, define $\pi_2 : \mathbb{R}_+ \times \prod_i Q_i \times \prod_j B_j \to \mathbb{R}$, $(e_2, (.)) \mapsto \pi_2(e_1, (.)))$. Finally, for each $j = 1, ..., n$, define $\pi_j : \to \mathbb{R}_+^2 \times \prod_i Q_i \times \prod_j B_j$, $(e_1, e_2, q_1, q_2(q_1); b_j(q_1), \mathbf{b}_{-j}(q_1)) \mapsto \pi_j(e_1, e_2, q_1, q_2(q_1); b_j(q_1), \mathbf{b}_{-j}(q_1))$. To lighten notations, let $\pi_j(.)$ for $\pi_j(e_1, e_2, q_1, q_2(q_1); b_j(q_1), \mathbf{b}_{-j}(q_1))$. Therefore, the utility levels of agents may be written as payoffs. For type I agents, we have:

$$\pi_1(e_1, (.)) = \left(\frac{\gamma}{\beta_1} e_1 - q_1\right)^{\alpha} \left(\frac{\sum_{j=1}^n b_j(q_1)}{q_1 + q_2(q_1)} q_1 - \gamma e_1\right)^{1-\alpha}, \tag{14}$$

$$\pi_2(e_2, (.)) = \left(\frac{\gamma}{\beta_2} e_2 - q_2(q_1)\right)^{\alpha} \left(\frac{\sum_{j=1}^n b_j(q_1)}{q_1 + q_2(q_1)} q_2(q_1) - \gamma e_2\right)^{1-\alpha}, \tag{15}$$

and, for each $j \in [1, ..., n\}$, we have:

$$\pi_j(.) = \left(\frac{q_1 + q_2(q_1)}{b_j(q_1) + \sum_{-j} b_{-j}(q_1)} b_j(q_1)\right)^{\alpha} \left(\frac{1}{n} - b_j(q_1)\right)^{1-\alpha} - \mu(e_1 + e_2). \tag{16}$$

A Stackelberg-Nash equilibrium (SNE hereafter) of $\mathbf{\Gamma}$ is given by a $(n+2)$-tuple of strategies $(\tilde{q}_1, \tilde{q}_2(\tilde{q}_1); \tilde{\mathbf{b}}(\tilde{q}_1))$ and an emission profile $(\tilde{e}_1, \tilde{e}_2)$ such that:

$\pi_1(\tilde{e}_1, \tilde{q}_1, \tilde{q}_2(\tilde{q}_1); \tilde{\mathbf{b}}(\tilde{q}_1)) \geqslant \pi_1(e_1, q_1, q_2(q_1); \mathbf{b}(q_1)) \ \forall \mathbf{b}(q_1) \in \prod_j B_j \ \forall q_2(q_1) \in Q_2$ $\forall q_1 \in Q_1, \ \forall e_1 \in \mathbb{R}_+$;

$\pi_2(\tilde{e}_2, \tilde{q}_1, \tilde{q}_2(\tilde{q}_1); \tilde{\mathbf{b}}(\tilde{q}_1)) \geqslant \pi_2(e_2, \tilde{q}_1, q_2(\tilde{q}_1); \tilde{\mathbf{b}}(\tilde{q}_1)) \ \forall q_2 \in Q_2, \ \forall e_2 \in \mathbb{R}_+$;

$\pi_j(\tilde{e}_1, \tilde{e}_2, \tilde{q}_1, \tilde{q}_2(\tilde{q}_1); \tilde{b}_j(\tilde{q}_1), \tilde{\mathbf{b}}_{-j}(\tilde{q}_1)) \geqslant \pi_j(\tilde{e}_1, \tilde{e}_2, \tilde{q}_1, \tilde{q}_2(\tilde{q}_1); b_j(\tilde{q}_1), \tilde{\mathbf{b}}_{-j}(\tilde{q}_1)) \ \forall b_j \in B_j$, for each $j = 1, ..., n$.

3. NON-COOPERATIVE EQUILIBRIA WITH EMISSIONS

Let us now turn to the computation the SNE. We also determine the properties of the equilibrium supplies and emissions, and we show that the SNE is not Pareto-optimal. Then, we compute the Cournot-Nash equilibrium (CNE henceforth)) of Γ, which we compare to the SNE.

3.1. SNE: computation

PROPOSITION 1. *The Stackelberg-Nash equilibrium supplies and emissions of* Γ *are given by*:

$(\tilde{q}_1, \tilde{q}_2) = \left(\frac{\alpha}{4} \frac{\beta_2}{(\beta_1)^2} \frac{n-1}{n-\alpha}, \frac{\alpha}{2\beta_1}(1 - \frac{1}{2}\frac{\beta_2}{\beta_1}) \frac{n-1}{n-\alpha}\right)$;

$(\tilde{e}_1, \tilde{e}_2) = \left(\frac{\alpha(1+\alpha)}{4\gamma} \frac{\beta_2}{\beta_1} \frac{n-1}{n-\alpha}, \frac{\alpha}{2\gamma} \frac{2\alpha\beta_1 + (1-\alpha)\beta_2}{\beta_1}(1 - \frac{1}{2}\frac{\beta_2}{\beta_1}) \frac{n-1}{n-\alpha}\right)$;

$\tilde{b}_j = \frac{\alpha}{n} \frac{n-1}{n-\alpha}$, $j = 1, ..., n$.

PROOF. See Appendix A.■

From (11) and Proposition 1, the relative price is given by $\tilde{p}_X = 2\beta_1$.

The next proposition provides a result about the nature of strategic interactions between the leader and the follower of type I, which will be useful later in studying the implications of market power. The strategies of the leader and the follower are said to be substitutes (resp. complements) when the best response of the follower decreases (resp. increases) with the strategy of the leader.[12] It transposes to our sequential quantity setting game the analysis made by Bulow et al. (1985) for the Cournot market.

PROPOSITION 2. *The strategies of the leader and the follower are substitutes (resp. complements) when $\beta_1 < \beta_2$ (resp. $\beta_1 > \beta_2$).*

PROOF. Consider the equation of the best-response given by (A7), i.e., $q_2(q_1, \mathbf{b}) = -q_1 + \sqrt{\frac{1}{\beta_2}\sum_{j=1}^{n} b_j q_1}$. For all $q_1 > 0$, we have that:

$$\frac{\partial q_2(.)}{\partial q_1} = -1 + \frac{1}{2}\left(\frac{1}{\beta_2}\sum_{j=1}^{n} b_j\right)^{\frac{1}{2}} q_1^{-\frac{1}{2}}.$$

By using Proposition 1, it is easy to check that, at a SNE, where $\tilde{q}_1 = \frac{\alpha}{4}\frac{\beta_2}{(\beta_1)^2}\frac{n-1}{n-\alpha}$ and $\tilde{b}_j = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1, ..., n$, we have:

$$\frac{\partial q_2(.)}{\partial q_1} \mid_{q_1=\tilde{q}_1} = -1 + \frac{\beta_2}{\beta_1}.$$

Then, we deduce $\frac{\partial q_2(.)}{\partial q_1} \mid_{q_1=\tilde{q}_1} \lesseqgtr 0$ when $\beta_1 \lesseqgtr \beta_2$. So, the game displays strategic substitutability (resp. complementarity) between the leader and her direct follower when $\beta_1 < \beta_2$ (resp. $\beta_1 > \beta_2$).■

It is worth noting that the possibility for strategies to be substitutes or complements is specific to our sequential setting insofar as if the two agents behave *a la* Cournot, in this model where utility functions are such that goods are imperfectly substitutable, the strategies of type I agents are complements.[13] Indeed, by using the strategies at the CNE (see Proposition 4 thereafter), the best response are increasing functions, i.e., we always have $\frac{\partial q_2(.)}{\partial q_1} \mid_{q_1=\hat{q}_1} > 0$ and $\frac{\partial q_1(.)}{\partial q_2} \mid_{q_2=\hat{q}_2} > 0$ as $\beta_1 + \beta_2 > \sqrt{\beta_1\beta_2}$.

Proposition 2 has a direct implication in terms of market power. Let the market power of traders be measured by their market shares at the SNE. For the leader and the follower, the market share at the SNE is given by the quantity $\frac{\tilde{q}_i}{\tilde{q}_1+\tilde{q}_2}$, $i = 1, 2$. Indeed, as $\tilde{q}_1 - \tilde{q}_2 = \frac{\alpha}{2}\frac{1}{\beta_1}\frac{n-1}{n-\alpha}(\frac{\beta_2}{\beta_1} - 1)$, then, $\tilde{q}_1 \gtreqless \tilde{q}_2$ when $\beta_1 \lesseqgtr \beta_2$. Otherwise, when $\beta_1 = \beta_2$, they have the same market power.

We now explore the issue of emissions in this two-stage quantity setting game.

REMARK 1. The emissions of the leader and the follower increase with their supplies as $\tilde{e}_1 = \frac{1+\alpha}{\gamma}\beta_1\tilde{q}_1$ and $\tilde{e}_2 = \frac{2\alpha\beta_1+(1-\alpha)\beta_2}{\gamma}\tilde{q}_2$.

[12] Indeed, when the best response is decreasing (resp. increasing) there are strategic substitutabilities (resp. complementarities).

[13] In a two-commodity exchange economy in which preferences of traders are represented by a CES utility function, Bloch and Ferrer (2001) showed that when commodities are substitutes (resp. complements) then strategies between traders who belong to the same sector are complements (resp. substitutes).

REMARK 2. The emissions are lower at the SNE than at the competitive equilibrium: we have $\tilde{e}_1 + \tilde{e}_2 = \frac{\alpha}{\gamma}\left[\frac{1+\alpha}{4}\frac{\beta_2}{\beta_1} + (1 - \frac{1}{2}\frac{\beta_2}{\beta_1})\frac{2\alpha\beta_1+(1-\alpha)\beta_2}{2\beta_1}\right]\frac{n-1}{n-\alpha} < \frac{\alpha}{\gamma}$ as $\frac{1+\alpha}{4}\frac{\beta_2}{\beta_1} + (1 - \frac{1}{2}\frac{\beta_2}{\beta_1})\frac{2\alpha\beta_1+(1-\alpha)\beta_2}{2\beta_1} < 1$ and $\frac{n-1}{n-\alpha} < 1$.

We are now able to state the following result which compares the leader and follower emissions in the presence of strategic complementarities or substitutabilities.

PROPOSITION 3. *At the SNE, the leader's emissions are higher (resp. lower) than the follower's emissions when their strategies are substitutes (resp. complements).*

PROOF. From Remark 1, we have $\tilde{e}_1 = \frac{1+\alpha}{\gamma}\beta_1\tilde{q}_1$ and $\tilde{e}_2 = \frac{2\alpha\beta_1+(1-\alpha)\beta_2}{\gamma}\tilde{q}_2$. Let $\beta_2 = \delta\beta_1$, with $0 < \delta < 2$ as $\frac{\beta_2}{\beta_1} \in (0, 2)$. Then, we have that:

$$\tilde{e}_1 - \tilde{e}_2 = \frac{\beta_1}{\gamma}\{(1+\alpha)\tilde{q}_1 - [2\alpha + (1-\alpha)\delta]\tilde{q}_2\}.$$

From Proposition 2, we have that $\delta \gtrless 1$ if and only if $\tilde{q}_1 \lessgtr \tilde{q}_2$. In addition, as $1 < \delta < 2$ and $\alpha \in (0, 1)$, we have $1 + \alpha \lessgtr 2\alpha + (1-\alpha)\delta$ if and only if $\delta \gtrless 1$. Then, we have $\tilde{e}_1 \lessgtr \tilde{e}_2$ if and only if $\delta \gtrless 1$.■

Proposition 3 may be interpreted as follows. The source of pollution comes from the production of commodity X. Less marginal cost means higher production and more emissions. When strategies are substitutes, the leader has higher market power (her market share is higher because her marginal cost is lower). Therefore, the leader produces more and pollutes more than her direct follower.

We determine now the allocations and the payoffs at the SNE. The individual allocations are given by:

$$(\tilde{x}_1, \tilde{y}_1) = \frac{\alpha}{4}\frac{\beta_2}{\beta_1^2}\frac{n-1}{n-\alpha}\left(\alpha, (1-\alpha)\beta_1\right); \tag{17}$$

$$(\tilde{x}_2, \tilde{y}_2) = \frac{\alpha}{\beta_2}\left(1 - \frac{1}{2}\frac{\beta_2}{\beta_1}\right)^2\frac{n-1}{n-\alpha}\left(\alpha, (1-\alpha)\beta_2\right); \tag{18}$$

$$(\tilde{x}_j, \tilde{y}_j) = \left(\frac{1}{2\beta_1}\frac{\alpha}{n}\frac{n-1}{n-\alpha}, \frac{1-\alpha}{n-\alpha}\right),\ j = 1, ..., n. \tag{19}$$

Then, we deduce the corresponding payoffs for each type I trader:

$$\tilde{\pi}_1 = \frac{\alpha^{\alpha+1}}{4}\frac{\beta_2}{\beta_1}\left(\frac{1}{\beta_1}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{n-1}{n-\alpha}; \tag{20}$$

$$\tilde{\pi}_2 = \alpha^{\alpha+1}\left(1 - \frac{1}{2}\frac{\beta_2}{\beta_1}\right)^2\left(\frac{1}{\beta_2}\right)^{\alpha}(1-\alpha)^{1-\alpha}\frac{n-1}{n-\alpha}; \tag{21}$$

and, for each type II trader:

$$\tilde{\pi}_j = \frac{(\frac{\alpha(n-1)}{2\beta_1 n})^{\alpha}(1-\alpha)^{1-\alpha}}{n-\alpha} - \mu\frac{\alpha}{\gamma}\frac{\frac{\beta_2}{2} + (1 - \frac{1}{2}\frac{\beta_2}{\beta_1})(2\alpha\beta_1 + (1-\alpha)\beta_2)}{2\beta_1}\frac{n-1}{n-\alpha}. \tag{22}$$

Let us consider the welfare properties of the SNE. First, the SNE is not Pareto-optimal. To see this, consider the marginal rate of substitution of trader k given by $MRS^k(\tilde{x}_k, \tilde{y}_k) = \frac{\partial u_k/\partial x_k}{\partial u_k/\partial y_k}$, $k = i, j$. By using (17)-(19), we see that the marginal rates of substitution differ accross traders, i.e., $MRS^i(\tilde{x}_i, \tilde{y}_i) = \beta_i$, for $i = 1, 2$, and $MRS^j(\tilde{x}_i, \tilde{y}_i) = \frac{2\beta_1}{n-1}$, for $j = 1, ..., n$. The reason stems from the strategic behavior of traders who restrict their supplies to increase the relative price. Second, it is easy to see that the leader's payoff is higher than her direct follower's payoff as $\tilde{\pi}_1 - \tilde{\pi}_2 = \frac{\alpha^{\alpha+1}}{4}(\frac{1}{\delta\beta_1})^{\alpha}(1-\alpha)^{1-\alpha}\frac{n-1}{n-\alpha}[\delta^{\alpha+1} - (2-\delta)^2] \geqslant 0$, as $\alpha \in (0, 1)$ and $0 < \delta < 2$. Third, there is no Pareto domination between the SNE and competitive equilibria insofar as, at a SNE (resp. competitive equilibirum), traders of type I have higher (resp. lower) payoffs but traders of type II have lower (resp. higher) payoffs.

3.2. Comparison with the CNE

To underline the importance of the two-stage structure of the game Γ, let us consider the simultaneous move version of the game. To this end, we determine now the Cournot-Nash equilibrium (CNE) with emissions.

PROPOSITION 4. *The Cournot-Nash equilibrium supplies and emissions are given by*:

$(\hat{q}_1, \hat{q}_2) = \left(\alpha\frac{\beta_2}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}, \alpha\frac{\beta_1}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}\right)$;

$(\hat{e}_1, \hat{e}_2) = \left(\frac{\alpha}{\gamma}\frac{\beta_2(\beta_1+\alpha\beta_2)}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}, \frac{\alpha}{\gamma}\frac{\beta_1(\beta_2+\alpha\beta_1)}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}\right)$;

$\hat{b}_j = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1, ..., n$.

PROOF. See Appendix B.■

The next proposition compares the emissions at both strategic equilibria.

PROPOSITION 5. *The level of emissions at the SNE is lower (resp. higher) than at the CNE when the strategies of type I traders are substitutes (resp. complements). Otherwise, when strategies are neither substitutes nor complements, the SNE emission level coincides with the CNE emission level.*

PROOF. At a SNE, we have:

$$\tilde{e}_1 + \tilde{e}_2 = \frac{\alpha}{4\gamma}\frac{4\alpha(\beta_1)^2 + 3(1-\alpha)\beta_1\beta_2 - (1-\alpha)(\beta_2)^2}{(\beta_1)^2}\frac{n-1}{n-\alpha}.$$

At a CNE, we have:

$$\hat{e}_1 + \hat{e}_2 = \frac{\alpha}{\gamma}\frac{\alpha(\beta_1)^2 + 2\beta_1\beta_2 + \alpha(\beta_2)^2}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}.$$

Let $\beta_2 = \delta\beta_1$, with $0 < \delta < 2$. We deduce:

$$(\tilde{e}_1 + \tilde{e}_2) - (\hat{e}_1 + \hat{e}_2) = \frac{\alpha(1-\alpha)\delta(\delta-1)(5-\delta^2)}{4\gamma(1+\delta)^2}\frac{n-1}{n-\alpha} > 0.$$

Recall from Proposition 2 that strategies are substitutes (resp. complements) when $\beta_1 < \beta_2$ (resp. $\beta_1 > \beta_2$). Therefore, if $\delta \gtrless 1$, i.e., $\beta_1 \lesseqgtr \beta_2$, then $(\tilde{e}_1 + \tilde{e}_2) \gtrless (\hat{e}_1 + \hat{e}_2)$. Finally, if $\delta = 1$, then $(\tilde{e}_1 + \tilde{e}_2) = (\hat{e}_1 + \hat{e}_2)$.

■

Let us compare the individual polluting behavior at the SNE and at the CNE. When strategies are substitutes (resp. complements), trader 1's emissions are higher (resp. lower) at the SNE than at the CNE, i.e., $\tilde{e}_1 > \hat{e}_1$ when $\beta_1 < \beta_2$ (resp. $\tilde{e}_1 < \hat{e}_1$ when $\beta_1 > \beta_2$). Indeed, as $\tilde{q}_1 \gtrless \hat{q}_1$ when $\delta = \frac{\beta_2}{\beta_1} \gtrless 1$, and by using (17) and (B12) in Appendix B, we have $\tilde{x}_1 - \hat{x}_1 = \frac{\alpha^2}{4\beta_1} \frac{\delta(1-\delta)(1+3\delta)}{(1+\delta)^2} \frac{n-1}{n-\alpha} \gtrless 0$ when $\delta \gtrless 1$ But then, we have $\tilde{e}_1 \gtrless \hat{e}_1$, whenever $\beta_1 \lesseqgtr \beta_2$. Therefore, trader 1 pollutes more when she behaves as a leader at a SNE than when she has a Cournotian behavior at the CNE. In addition, as we have, on the one hand $\tilde{q}_2 - \hat{q}_2 = -\frac{\alpha}{4\beta_1} \frac{(\delta-1)^2(2+\delta)}{(1+\delta)^2} \frac{n-1}{n-\alpha} \leqslant 0$ when $0 < \delta < 2$, and, other other hand, $\tilde{x}_2 - \hat{x}_2 = \frac{\alpha^2}{4\beta_1} \frac{(\delta-1)(\delta^2-\delta-4)}{(1+\delta)^2} \frac{n-1}{n-\alpha} \leqslant 0$ when $1 \leqslant \delta < 2$. But $\tilde{x}_2 - \hat{x}_2 = \frac{\alpha^2}{4\beta_1} \frac{(\delta-1)(\delta^2-\delta-4)}{(1+\delta)^2} \frac{n-1}{n-\alpha} > 0$ when $0 < \delta < 1$. Therefore, trader 2 pollutes less when she behaves as a follower at the SNE than when she has a Cournotian behavior at the CNE provided $1 \leqslant \delta < 2$. Otherwise, the sign of $(\tilde{e}_2 - \hat{e}_2)$ is undetermined.

REMARK 3. Some computations yield $\tilde{\pi}_1 \geqslant \hat{\pi}_1$ and $\tilde{\pi}_2 \leqslant \hat{\pi}_2$, whenever $\beta_1 \leqslant \beta_2$.

The problem is now to determine whether the pollution could be decreased, either with a taxation mechanism or with a permit market.

4. NONCOOPERATIVE EQUILIBRIA WITH TAXATION

In this section, we introduce two fiscal policies, namely, ad valorem taxation on emissions, and per unit taxation.[14] Taxation mechanisms have been introduced in bilateral oligopoly under Cournot competition by Gabszewicz and Grazzini (1999), Grazzini (2006), and Elegbede et al. (2021).[15] To simplify, we assume that the total tax product T is used to finance some exogenous public expenditure, namely G, subject to a balanced budget rule, i.e., such that $T = G$. The introduction of taxation mechanisms modifies the bilateral oligopoly game Γ.

4.1. Two taxation mechanisms

First, consider a tax $t \in (0,1)$ is levied on the emissions of the leader and the follower, with $T(e_1, e_2) \equiv t(e_1 + e_2) = G$, and $G > 0$. Given an $n+2$-tuple of strategies $(q_1, q_2(q_1); \mathbf{b}(q_1)) \in \prod_i Q_i \times \prod_j B_j$, and a tax $t \in (0,1)$, the resulting post tax allocation is given by $(x_1, y_1) = (\frac{\gamma}{\beta_1}(1-t)e_1 - q_1, \frac{\sum_{j=1}^n b_j(q_1)}{q_1+q_2(q_1)} q_1 - \gamma(1-t)e_1)$ for the leader, and by $(x_2, y_2) = (\frac{\gamma}{\beta_2}(1-t)e_2 - q_2(q_1), \frac{\sum_{j=1}^n b_j(q_1)}{q_1+q_2(q_1)} q_2(q_1) - \gamma(1-t)e_2)$ for the follower. With a slight abuse of notations, let $\pi_i(.)$ for $\pi_i(q_i, e_i)$. Then, their payoffs in $\mathbf{\Gamma}$ with taxation on emissions may be written:

$$\pi_1(.) = \left(\frac{\gamma}{\beta_1}(1-t)e_1 - q_1\right)^{\alpha} \left(\frac{\sum_{j=1}^n b_j(q_1)}{q_1 + q_2(q_1)} q_1 - \gamma(1-t)e_1\right)^{1-\alpha}, \tag{23}$$

$$\pi_2(.) = \left(\frac{\gamma}{\beta_2}(1-t)e_2 - q_2(q_1)\right)^{\alpha} \left(\frac{\sum_{j=1}^n b_j(q_1)}{q_1 + q_2(q_1)} q_2(q_1) - \gamma(1-t)e_2\right)^{1-\alpha}. \tag{24}$$

[14] Through numerical simulations in a Stackelberg duopoly model with linear demand, Zhou et al. (2019) show that flat carbon tax and block carbon tax can control carbon emissions.

[15] Collie (2019) showed that fiscal policies that correct market distortions caused by imperfectly competitive behavior and are used for redistributive purposes can lead to a second-best allocation.

Second, consider a tax $\tau \in (0,1)$ is levied on the supply q_i of commodity X, with $T(q_1,q_2) \equiv \tau(q_1+q_2) = G$, and $0 < G < \frac{\alpha}{2}\frac{1}{1+\beta_1}\frac{n-1}{n-\alpha}$. Given an $n+2$-tuple of strategies $(q_1, q_2(q_1); \mathbf{b}(q_1)) \in \prod_i Q_i \times \prod_j B_j$, and a tax $\tau \in (0,1)$, the resulting post tax allocation is given by $(x_1, y_1) = (\frac{\gamma}{\beta_1}e_1 - q_1, (\frac{\sum_{j=1}^n b_j(q_1)}{q_1+q_2(q_1)} - \tau)q_1 - \gamma e_1)$ for the leader, and by $(x_2, y_2) = (\frac{\gamma}{\beta_2}e_2 - q_2(q_1), (\frac{\sum_{j=1}^n b_j(q_1)}{q_1+q_2(q_1)} - \tau)q_2(q_1) - \gamma e_2)$ for the follower. Then, their payoffs in $\tilde{\mathbf{\Gamma}}$ with per unit taxation may be written:

$$\pi_1(.) = \left(\frac{\gamma}{\beta_1}e_1 - q_1\right)^{\alpha}\left[\left(\frac{\sum_{j=1}^n b_j(q_1)}{q_1+q_2(q_1)} - \tau\right)q_1 - \gamma e_1\right]^{1-\alpha}, \tag{25}$$

$$\pi_2(.) = \left(\frac{\gamma}{\beta_2}e_2 - q_2(q_1)\right)^{\alpha}\left[\left(\frac{\sum_{j=1}^n b_j(q_1)}{q_1+q_2(q_1)} - \tau\right)q_2(q_1) - \gamma e_2\right]^{1-\alpha}. \tag{26}$$

We now turn to the study of the effects of these two taxation mechanisms.

4.2. The effects of taxation

First, we compute the SNE with pollution for both taxation schemes. Second, we study the effect of taxation on the emissions. Third, we study the welfare properties of the SNE with taxation. To introduce the SNE values for both taxation schemes, with an abuse of notation, the terms $\tilde{q}_i(t,\tau)$ and $\tilde{e}_i(t,\tau)$ will respectively designate the equilibrium strategy and emission of trader i, $i = 1,2$.

PROPOSITION 6. *Consider the two taxation mechanisms in* Γ. *The interior SNE strategy profiles and emissions profiles of* Γ *are given by*:
$(\tilde{q}_1(t,\tau), \tilde{q}_2(t,\tau)) = \left(\frac{\alpha}{4}\frac{\beta_2+\tau}{(\beta_1+\tau)^2}\frac{n-1}{n-\alpha}, \frac{\alpha}{2}\frac{1}{\beta_1+\tau}\left(1 - \frac{1}{2}\frac{\beta_2+\tau}{\beta_1+\tau}\right)\frac{n-1}{n-\alpha}\right)$;
$(\tilde{e}_1(t,\tau), \tilde{e}_2(t,\tau)) = \left(\frac{(1+\alpha)\beta_1+\alpha\tau}{\gamma(1-t)}\tilde{q}_1(t,\tau), \frac{\alpha(2\beta_1+\tau)+(1-\alpha)\beta_2}{\gamma(1-t)}\tilde{q}_2(t,\tau)\right)$;
$\tilde{b}_j(t,\tau) = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1,...,n$.

PROOF. See Appendix C.■

The next proposition provides two comparative exercises which measure the effect of a variation of the tax on emissions.

PROPOSITION 7. *At the SNE with taxation the traders' emissions increase with the tax on emissions.*

PROOF. Immediate by letting $\tau = 0$ in $(\tilde{e}_1(t,\tau), \tilde{e}_2(t,\tau))$.■

We can explain Proposition 7 as follows. Consider type I traders at the SNE, and let $\tau = 0$. To the extent that traders' offers (and therefore the relative price) are not affected by the tax, the revenue from the sale of commodity X, i.e., $\tilde{p}_X(t)\tilde{q}_i(t)$, $i = 1,2$, is not modified. Then, the traders consume less of good X (resp. more of good Y as trader i's income Π_i increases). In order to restore their optimal consumption (more good X and less good Y), the traders of type I therefore increase their production, and consequently, they increase the quantity of the input they use. The overall effect is to increase emissions.

Therefore, taxing emissions may not be the appropriate tools to regulate the pollution caused by the leader and her direct follower. This result leads us to

consider some policy regulation via the implementation of a per unit tax. We are able to state the following proposition.

PROPOSITION 8. *At the SNE with taxation, the emissions of the leader (resp. follower) decrease (resp. increase) with the per unit tax if the strategies are substitutes (resp. complements), i.e.,* $\beta_1 < \beta_2$ *(resp.* $\beta_1 > \beta_2$*). Additionally, the emissions of both traders decrease with the per unit tax if their strategies are neither substitutes nor complements, i.e.,* $\beta_1 = \beta_2$*.*

PROOF. See Appendix E.

The conditions under which the emissions decrease depend on whether strategies are substitutes or complements. Indeed, by virtue of Proposition 2, when the marginal cost of the leader is lower than the marginal cost of the follower, i.e., when $\beta_1 < \beta_2$, the best response of the follower is decreasing, reflecting strategic substitutability. In such a case, consider the revenue from sales of the leader, i.e., $R_1(\tau) \equiv \tilde{p}_X(\tau)\tilde{q}_1(\tau)$. When $\beta_1 < \beta_2$, her revenue decreases with the tax, i.e., $\frac{\partial R_1(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} = \frac{\alpha}{4}\frac{n-1}{n-\alpha}\frac{\beta_1-\beta_2}{(\beta_1+\tilde{\tau})^2} < 0$. Therefore, the leader consumes more of good X and less of good Y. In order to restore her optimal consumption of good Y, she decreases her demand for the input, i.e., she decreases her production of good X, and consequently decreases her emissions. Likewise, consider the revenue from sales of the follower $R_2(\tau) \equiv \tilde{p}_X(\tau)\tilde{q}_2(\tau)$. When $\beta_2 > \beta_1$, her revenue decreases with the tax, i.e., $\frac{\partial R_2(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} = \frac{\alpha}{4}\frac{n-1}{n-\alpha}\frac{\beta_2-\beta_1}{(\beta_1+\tilde{\tau})^2} < 0$. The reason stems from the fact that the strategies are substitutes, so when the leader decreases her supply, the follower increases his supply, i.e., $\frac{\partial \tilde{q}_2(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} > 0$ (reason: by differentiating (C5) at the SNE, we have $\frac{\partial \tilde{q}_2(.)}{\partial \tilde{q}_1(\tau)} = 1 - \frac{\beta_1+\tau}{\beta_2+\tau} > 0$ when $\beta_1 < \beta_2$). Therefore, the follower consumes more of good Y and less of good X. In order to restore his optimal consumption of good X, the follower increases his demand for the input, i.e., he increases his production of good X, and consequently increases his emissions.

Otherwise, when $\beta_1 = \beta_2$, from Proposition 7, we have that $(\tilde{q}_1(\tau), \tilde{q}_2(\tau)) = (\frac{\alpha}{4}\frac{1}{\beta_1+\tau}\frac{n-1}{n-\alpha}, \frac{\alpha}{4}\frac{1}{\beta_1+\tau}\frac{n-1}{n-\alpha})$. Then, we have $R_i(\tau) = \frac{\alpha}{4}\frac{n-1}{n-\alpha}$, $i = 1, 2$. In this case, the only effect of an increase in the per unit tax is to increase the consumption of good X, which leads both traders to decrease their production of good X, and thus to reduce their emissions.[16]

We compute now the post-tax equilibrium allocations. To this end, let $\tilde{\tau} \in (0, 1)$ be the unit tax which solves $\tau(\tilde{e}_1(\tau) + \tilde{e}_2(\tau)) = G$, i.e., $\tilde{\tau} = \frac{2\beta_1 G}{\alpha\frac{n-1}{n-\alpha} - 2G}$ (see (E6) in Appendix E). Therefore, the relative price is given by $\tilde{p}_X(\tilde{\tau}) = 2(\beta_1 + \tilde{\tau}) = 2\beta_1\frac{\alpha\frac{n-1}{n-\alpha}}{\alpha\frac{n-1}{n-\alpha} - 2G}$ (see (C12) in Appendix C). The allocations to type I traders are given by $(\tilde{x}_1(\tilde{\tau}), \tilde{y}_1(\tilde{\tau})) = \frac{\alpha}{4\beta_1}\frac{\beta_2+\tilde{\tau}}{\beta_1+\tilde{\tau}}\frac{n-1}{n-\alpha}(\alpha, (1-\alpha)\beta_1)$, and $(\tilde{x}_2(\tilde{\tau}), \tilde{y}_2(\tilde{\tau})) = \frac{\alpha}{2\beta_2}\frac{2\beta_1-\beta_2+\tilde{\tau}}{\beta_1+\tilde{\tau}}\left(1 - \frac{1}{2}\frac{\beta_2+\tilde{\tau}}{\beta_1+\tilde{\tau}}\right)\frac{n-1}{n-\alpha}(\alpha, (1-\alpha)\beta_2)$, so we deduce:

$$(\tilde{x}_1(\tilde{\tau}), \tilde{y}_1(\tilde{\tau})) = \frac{\frac{\beta_2}{\beta_1}(\alpha\frac{n-1}{n-\alpha} - 2G) + 2G}{4\beta_1}(\alpha, (1-\alpha)\beta_1)\,; \tag{27}$$

$$(\tilde{x}_2(\tilde{\tau}), \tilde{y}_2(\tilde{\tau})) = \frac{\alpha}{2\beta_2}\frac{2\beta_1 - \beta_2 + \tilde{\tau}}{\beta_1 + \tilde{\tau}}\left(1 - \frac{1}{2}\frac{\beta_2 + \tilde{\tau}}{\beta_1 + \tilde{\tau}}\right)\frac{n-1}{n-\alpha}(\alpha, (1-\alpha)\beta_2)\,, \tag{28}$$

[16] The effects of the per unit tax on emissions are computed in the supplement to Appendix E.

and the allocation to trader $j \in \{1, ..., n\}$ is $(\tilde{x}_j(\tilde{\tau}), \tilde{y}_j(\tilde{\tau})) = \left(\frac{1}{2(\beta_1+\tilde{\tau})} \frac{\alpha}{n} \frac{n-1}{n-\alpha}, \frac{1-\alpha}{n-\alpha}\right)$, so we deduce:

$$(\tilde{x}_j(\tilde{\tau}), \tilde{y}_j(\tilde{\tau})) = \left(\frac{\alpha \frac{n-1}{n-\alpha} - 2G}{2\beta_1 n}, \frac{1-\alpha}{n-\alpha}\right), \; j = 1, ..., n. \tag{29}$$

Therefore, the marginal rates of substitution are such that:

$$MRS^i(\tilde{x}_1(\tilde{\tau}), \tilde{y}_1(\tilde{\tau})) = \; \beta_i, \; i = 1, 2, \tag{30}$$

and

$$MRS^j(\tilde{x}_j(\tilde{\tau}), \tilde{y}_j(\tilde{\tau})) = 2\alpha\beta_1 \frac{n}{n-\alpha} \frac{1}{\alpha \frac{n-1}{n-\alpha} - 2G}, \; j = 1, ..., n. \tag{31}$$

Therefore, the taxation policy does not lead to a Pareto-optimal allocation.[17] The reason stems from the fact that the per unit tax is not sufficiently strong enough to resorb the inefficiency caused by the strategic behavior of traders. Therefore, at the SNE, as commodities are imperfectly substitutable, no fiscal policy is sufficiently powerful to eliminate the market inefficiencies caused by the strategic interactions. Proposition 8 extends the results of Gabszewicz and Grazzini (2001), and Elegbede et al. (2021) in a sequential strategic market game with pollution.

Finally, the next result concerns the effect of an increase of the per unit tax on the payoffs of traders who suffer from the negative externality caused by pollution.

PROPOSITION 9. *At the SNE, if the strategies of type I traders are neither substitutes nor complements, then the payoffs of type II traders increase under per unit taxation whenever the negative effect of the per unit tax on emissions dominates the marginal decrease of indirect utility caused by the increase of the relative price.*

PROOF. The payoff of trader $j \in \{1, ..., n\}$ is given by:

$$\tilde{\pi}_j(\tau) = \frac{\left(\frac{\alpha}{2(\beta_1+\tau)} \frac{n-1}{n}\right)^{\alpha} (1-\alpha)^{1-\alpha}}{n-\alpha} - \mu\chi \frac{n-1}{n-\alpha},$$

where the term $\chi \equiv \alpha \frac{[(1+\alpha)\beta_1+\alpha\tau](\beta_2+\tau)+[\alpha(2\beta_1+\tau)+(1-\alpha)\beta_2](2\beta_1-\beta_2+\tau)}{4\gamma(\beta_1+\tau)^2} > 0$ is derived from the expressions (E1)-(E2) in Appendix E. Assume $\beta_1 = \beta_2 = \beta$. Then, the preceding payoff may be written:

$$\tilde{\pi}_j(\tau) = \frac{\left(\frac{\alpha}{2(\beta+\tau)} \frac{n-1}{n}\right)^{\alpha} (1-\alpha)^{1-\alpha}}{n-\alpha} - \mu\alpha \frac{(1+\alpha)\beta + \alpha\tau}{2\gamma(\beta+\tau)} \frac{n-1}{n-\alpha}.$$

Therefore, as $\tilde{e}_1(\tau) + \tilde{e}_2(\tau) = \frac{(1+\alpha)\beta+\alpha\tau}{2\gamma(\beta+\tau)}$, then, from (7), we deduce:

$$\frac{\partial \tilde{\pi}_j(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} = -\frac{(\frac{\alpha}{\beta+\tilde{\tau}})^{\alpha+1} (\frac{n-1}{2n})^{\alpha} (1-\alpha)^{1-\alpha}}{n-\alpha} - \mu \left(\frac{\partial \tilde{e}_1(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} + \frac{\partial \tilde{e}_2(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} \right).$$

[17] For comparison, at the CNE, we have $MRS^i(\hat{x}_1(\hat{\tau}), \hat{y}_1(\hat{\tau})) = \; \beta_i, \; i = 1, 2$, and $MRS^j(\hat{x}_j(\hat{\tau}), \hat{y}_j(\hat{\tau})) = \frac{n(\beta_1+\beta_2+2\hat{\tau})}{n-1}, \; j = 1, ..., n.$

If $\beta_1 = \beta_2$, then, by Proposition 8, we have $\frac{\partial \tilde{e}_1(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} < 0$ and $\frac{\partial \tilde{e}_2(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} < 0$. Then, we deduce $\frac{\partial \tilde{\pi}_j(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} \gtrless 0$ when

$$-\mu \left(\frac{\partial \tilde{e}_1(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} + \frac{\partial \tilde{e}_2(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} \right) \gtrless \frac{(\frac{\alpha}{\beta+\tilde{\tau}})^{\alpha+1} (\frac{n-1}{2n})^{\alpha} (1-\alpha)^{1-\alpha}}{n-\alpha}.$$

■

Proposition 9 suggests that a per unit tax is Pareto improving for the traders who suffer from the polluting emissions when the effect of the tax on emissions is stronger than the decrease in their utility caused by the strategic behavior of the polluter traders who manipulate the relative price. Such a result holds provided the leader has the same market power than her direct follower. Otherwise when $\beta_1 \neq \beta_2$ nothing can be asserted about the effect of the tax on emissions. This result differs from Crettez et al. (2021) who consider a two-sector equilibrium model with pollution permits in which some traders behave *a la* Cournot while a representative consumer behaves as a price-taker. The competitive trader is taxed and the product of the tax is transferred as a subsidy to the oligopolists. This supply subsidy can be Pareto improving when the agents sufficiently value the polluting commodity insofar as it enlarges the size of trades. Here, the tax policy is Pareto improving whichever is the preference for the polluting commodity.

We now turn to the regulation of emissions with a permit market.

5. NONCOOPERATIVE EQUILIBRIA WITH A PERMIT MARKET

To control the pollution caused by production activities, we introduce a permit market (see Montero 2009; Godal 2011; Lange 2012; De Feo et al. 2013; Schwartz and Stahn 2013; Hintermann 2017; Dickson and MacKenzie 2018, Christin et al. 2021, among others). Let r be the permit price in terms of good Y. The price vector is now given by $(p_X, 1, r)$. To simplify, we assume perfect competition on the permit market, so the price r is given (for a justification, see notably Montero 2009; Crettez et al. 2021). In this context, each trader $i \in \{1, 2\}$ is initially endowed with an amount $\bar{e}_i$ of pollution permits.[18]

Given an $n+2$-tuple of strategies $(q_1, q_2(q_1); \mathbf{b}(q_1)) \in \prod_i Q_i \times \prod_j B_j$, and a vector of endowment of permits $(\bar{e}_1, \bar{e}_2)$, the incomes of traders (12) and (13) are now given by $\Pi_1(e_1, (.)) := p_X(.)q_1 - \gamma e_1 + r(\bar{e}_1 - e_1))$ and $\Pi_2(e_2, (.)) := p_X(.)q_2(q_1) - \gamma_2 e_2 + r(\bar{e}_2 - e_2))$, where $r(\bar{e}_i - e_i)$ represents trader i's net purchase of emission rights, $i = 1, 2$. The resulting allocation is given by $(x_1, y_1) = (\frac{\gamma}{\beta_1} e_1 - q_1, \frac{\sum_{j=1}^{n} b_j(q_1)}{q_1 + q_2(q_1)} q_1 - \gamma e_1 + r(\bar{e}_1 - e_1))$ for the leader, and by $(x_2, y_2) = (\frac{\gamma}{\beta_2} e_2 - q_2(q_1), \frac{\sum_{j=1}^{n} b_j(q_1)}{q_1 + q_2(q_1)} q_2(q_1) - \gamma e_2 + r(\bar{e}_2 - e_2))$ for the follower. With a slight abuse of notations, let $\pi_i(.)$ for $\pi_i(q_i, e_i)$. Therefore, the payoffs of trader $i \in \{1, 2\}$ in $\mathbf{\Gamma}$ with a pemit market may be written:

$$\pi_1(.) = \left(\frac{\gamma}{\beta_1} e_1 - q_1 \right)^{\alpha} \left(\frac{\sum_{j=1}^{n} b_j(q_1)}{q_1 + q_2(q_1)} q_1 - \gamma e_1 + r(\bar{e}_1 - e_1) \right)^{1-\alpha}, \qquad (32)$$

[18] Indeed, we could write $\bar{e}_1 = \lambda e$ and $\bar{e}_2 = (1-\lambda)e$, $\lambda \in [0, 1]$, with e as the legal maximum aggregate level of pollution.

$$\pi_2(.) = \left(\frac{\gamma}{\beta_2}e_2 - q_2(q_1)\right)^{\alpha}\left(\frac{\sum_{j=1}^{n} b_j(q_1)}{q_1 + q_2(q_1)}q_2(q_1) - \gamma e_2 + r(\bar{e}_2 - e_2)\right)^{1-\alpha}. \quad (33)$$

With an abuse of notation, the terms $\tilde{e}_1(r)$ and $\tilde{e}_2(r)$ will respectively designate the emission of the leader and the follower at the CNE with a permit market.[19] Therefore, the market clearing condition on the permit market may be written as:

$$\tilde{e}_1(r) + \tilde{e}_2(r) = \bar{e}_1 + \bar{e}_2. \quad (34)$$

We will state three kinds of results. First, we compute the SNE with a pollution permits market. Second, we study the effect of an increase in the price of permits on the emissions of traders. Third, we study the effect of an increase in the price of permits on the payoffs of type II traders.

PROPOSITION 10. *The interior SNE strategy profiles and emissions profiles of* Γ *with a permit market are given by:*
$(\tilde{q}_1(r), \tilde{q}_2(r)) = \left(\frac{\gamma}{\gamma+r}\tilde{q}_1, \frac{\gamma}{\gamma+r}\tilde{q}_2\right)$;
$(\tilde{e}_1(r), \tilde{e}_2(r)) = \left(\frac{\alpha}{\gamma+r}r\bar{e}_1 + \frac{\gamma}{\gamma+r}\tilde{e}_1, \frac{\alpha}{\gamma+r}r\bar{e}_2 + \frac{\gamma}{\gamma+r}\tilde{e}_2\right)$;
$\tilde{b}_j(r) = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1, ..., n$.

PROOF. See Appendix F.■

REMARK 5. The net purchase of emissions, i.e., the quantity $(\tilde{e}_i(r) - \bar{e}_i)$, is such that $\tilde{e}_i(r) - \bar{e}_i \gtreqless 0$ whenever $\bar{e}_i \gtreqless \frac{\gamma}{\gamma+(1-\alpha)r}\tilde{e}_i$, with $\frac{\gamma}{\gamma+(1-\alpha)r} < 1$, where $(\tilde{e}_i(r) - \bar{e}_i) = \frac{\gamma}{\gamma+r}\tilde{e}_i - \frac{\gamma+(1-\alpha)r}{\gamma+r}\bar{e}_i$.[20]

We turn now to the effect of an increase in the price of permits on trader i's emissions.

PROPOSITION 11. *The emissions of trader i increases (resp. decreases) with the price of permits whenever* $\bar{e}_i > \frac{\tilde{e}_i}{\alpha}$ *(resp.* $\bar{e}_i < \frac{\tilde{e}_i}{\alpha}$*). Moreover, the emissions do not change with the price of permit when* $\bar{e}_i = \frac{\tilde{e}_i}{\alpha}$.

PROOF. Consider the leader's emissions $\tilde{e}_1(r) = \frac{\alpha}{\gamma+r}r\bar{e}_1 + \frac{\gamma}{\gamma+r}\tilde{e}_1$ (a similar reasoning may be handled for the follower). Some computations yield:

$$\frac{\partial \tilde{e}_1(r)}{\partial r} = \frac{\alpha\gamma}{(\gamma+r)^2}\left(\bar{e}_1 - \frac{1+\alpha}{4\gamma}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha}\right).$$

As by Proposition 1 the leader's emissions are given by $\tilde{e}_1 = \frac{\alpha(1+\alpha)}{4\gamma}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha}$, then, we have:

$$\frac{\partial \tilde{e}_1(r)}{\partial r} \gtreqless 0 \text{ whenever } \bar{e}_1 \gtreqless \frac{1+\alpha}{4\gamma}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha} = \frac{\tilde{e}_1}{\alpha}.$$

■

Emission behavior depends on the amount of the initial allocation of pollution rights. The emissions of a single trader increase (resp. decrease) with the price of

[19] Recall that $\tilde{e}_1$ and $\tilde{e}_2$ represent the emissions without a permit market: no confusion will arise.

[20] Obviously we have $\sum_{i=1}^{2}(\tilde{e}_i(r) - \bar{e}_i) = 0$.

permits when the endowment of permits is strictly higher (resp. lower) than the level of emissions *without* the permits market. Indeed, by using Remark 5, trader i's net purchase of emissions increases (resp. decreases) with the price of permits whenever $\bar{e}_i > \frac{\tilde{e}_i}{\alpha}$ (resp. $\bar{e}_i < \frac{\tilde{e}_i}{\alpha}$). Moreover, when the preference for commodity X is low, i.e., when α is low, a rise in the price of permits cannot lead to an increase of the emissions unless the endowment of permits is large enough. Finally, the emission behavior is not modified when the endowment of permits depends linearly on the emissions *without* the permits market. Here the effect of an increase in the price of permits on polluting behavior does not pass through marginal costs, unlike the two taxation mechanisms. Indeed, insofar as permit allocations are fixed, if one trader, either the leader or the follower, increases her emissions, then the other must decrease them. The same kind of results holds in the CNE with pollution permits (see (G11) in Appendix G).

We determine now the SNE relative price, allocations, and the corresponding payoffs with a pollution permit market. The market price is given by:

$$\tilde{p}_X(r) = 2\beta_1 \frac{\gamma + r}{\gamma}. \tag{35}$$

Therefore, the allocations to type I traders are given by:

$$(\tilde{x}_1(r), \tilde{y}_1(r)) = \left(r\bar{e}_1 + \frac{\alpha}{4}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha} \right) \left(\frac{\alpha}{\beta_1}\frac{\gamma}{\gamma + r}, 1-\alpha \right); \tag{36}$$

$$(\tilde{x}_2(r), \tilde{y}_2(r)) = \left(r\bar{e}_2 + \alpha \left(1 - \frac{1}{2}\frac{\beta_2}{\beta_1} \right)^2 \frac{n-1}{n-\alpha} \right) \left(\frac{\alpha}{\beta_2}\frac{\gamma}{\gamma + r}, 1-\alpha \right), \tag{37}$$

and the allocation to type II traders are given by:

$$(\tilde{x}_j(r), \tilde{y}_j(r)) = \left(\frac{1}{2\beta_1}\frac{\gamma}{\gamma + r}\frac{\alpha}{n}\frac{n-1}{n-\alpha}, \frac{1-\alpha}{n-\alpha} \right), \; j = 1, ..., n. \tag{38}$$

It is easy to see that the permits market does not lead to a Pareto-optimal allocation. Indeed, the marginal rates of substitution are given by:

$$MRS^i = \beta_i \frac{\gamma + r}{\gamma}, \; i = 1, 2, \tag{39}$$

$$MRS^j = 2\beta_1 \frac{\gamma + r}{\gamma}\frac{n}{n-1}, j = 1, ..., n, \tag{40}$$

and, they differ across traders. The reason stems once again from the strategic behavior of traders.

Finally, the next proposition considers the effect of an increase in the price of permits on the payoffs of type II traders.

PROPOSITION 12. *The payoffs of type II traders decrease (resp. does not modify) with the price of permits (resp. either when the traders strongly prefer commodity Y or when there is a large number of type II traders).*

PROOF. Immediate. For each $j = 1, ...n$, we have that:

$$\tilde{\pi}_j(r) = \frac{\left(\alpha \frac{1}{2\beta_1}\frac{n-1}{n}\frac{\gamma}{\gamma+r} \right)^\alpha (1-\alpha)^{1-\alpha}}{n-\alpha} - \mu(\tilde{e}_1(r) + \tilde{e}_2(r)).$$

By using the market clearing condition $\tilde{e}_1(r) + \tilde{e}_2(r) = \bar{e}_1 + \bar{e}_2$, we deduce:

$$\forall j \in \{1, ..., n\} \ \frac{\partial \tilde{\pi}_j(r)}{\partial r} = -\frac{\alpha}{\gamma + r} \frac{\left(\alpha \frac{1}{2\beta_1} \frac{n-1}{n} \frac{\gamma}{\gamma + r}\right)^{\alpha} (1-\alpha)^{1-\alpha}}{n - \alpha} < 0.$$

Otherwise, we have that

$$\forall j \in \{1, ..., n\} \ \lim_{\alpha \to 0} \frac{\partial \tilde{\pi}_j(r)}{\partial r} = \lim_{n \to +\infty} \frac{\partial \tilde{\pi}_j(r)}{\partial r} = 0.$$

■

Therefore, the welfare of type II traders cannot be improved with a competitive permit market when the polluting commodity is desirable. Besides, there are two conditions under which trader j's payoff does not decrease with the price of permit. First, when the preference for commodity X is low, a rise in the price of permits cannot lead to an increase of emissions (from a single trader) unless the endowment of permits is large enough (by virtue of Proposition 11). Moreover, since consumers of type II have a low preference for the polluting good, the increase in the price of permits has little impact on their payoffs. In addition, if there were a large number of traders who compete for selling commodity Y, i.e., if this side of the market were perfectly competitive, the increase in the price of permits were dominated by the decrease in emissions. Finally, this result is not specific to Stackelberg competition as it can also hold under Cournot competition (see Appendix G). Nevertheless, Proposition 12 holds as soon as the permits market is competitive, but its conclusion fails when strategic behavior affects the price of permits as in Godal and Holtsmark (2010), and Dickson and MacKenzie (2018).

6. CONCLUDING REMARKS

Our model introduces pollution in a two-stage strategic market game. It can find, for instance, an echo in international trade with resource specialization. Indeed, heterogeneous strategic agents whose market power differ, and who live in two distinct countries would compete on quantity on the world market. The production activity of one country pollutes the other country through a negative externality on some traders. The problem is to determine whether emissions levels can be reduced either via two taxation mechanisms or via a permit market when all traders behave strategically.

The sequential strategic setting is critical. Indeed, with Stackelberg-Nash competition, the strategies of the polluting traders can be either substitutes or complements, while with Cournot competition they are only complements. We show it has some implications on emissions' behavior, through the difference in costs, and also on the effects of regulatory policies. Two kinds of regulation are considered to limit the emissions: two taxation mechanisms (ad valorem taxation on emissions and per unit taxation on the relative price) and one permit market. The main conclusions are as follows.

First, a per unit tax on strategies led to reduce the emissions of the leader (resp. follower) whenever the strategies of the leader and of follower were substitutes (through the difference in marginal costs). In addition, a per unit tax reduced the emissions provided the producers had the same marginal costs. Moreover, the per unit tax was Pareto improving for the nonpolluting traders when the effect of

this tax on emissions was stronger than the decrease in their utility caused by the strategic behavior of the polluting traders.

Second, agents' preferences played a critical role when we considered the effect of the price of permits on emissions' behavior. Indeed, the payoffs type II traders (those who are pure consumers, by respect to type I traders who are consumers and producers at the same time) decreased with the price of permits unless either when there was a large number of traders of type II, who are pure consumers, or when the consumers *strongly* preferred the non-produced commodity. Thus, we showed that the welfare could not be improved with a competitive permit market when the polluting commodity was desirable.

To put in a nutshell: the performance of fiscal policies merely depended on the polluting traders' technologies through the parameters of productivity, while in the case of a permit market, this performance merely depended on agents' preference for the polluting commodity, and on the number of market participants, i.e., on the degree of competition. Moreover, in the case of fiscal policies, the sequentiality of decisions matter, while it did not matter with a competitive permit market.

The model we study is based on certain assumptions whose relaxation could be the subject of future research. Our model was linear in the production and the polluting technologies, and without abatements costs. Nonlinearities in the technology and dynamic analysis such as in the introduction of time in the production activity (with abatements costs) could also be considered.

7. APPENDIX

7.1. Appendix A: proof of Proposition 1

Let us solve the game Γ by backward induction. To this end, consider, in the second stage of the game, the behavior of both types of followers. The problems of follower of type I and of type II may be written:

$$\max_{(e_2,q_2)} \left(\frac{\gamma}{\beta_2}e_2 - q_2\right)^{\alpha} \left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2\right)^{1-\alpha}, \tag{A1}$$

$$\max_{b_j} \left(\frac{q_1+q_2}{b_j + \sum_{-j\neq j} b_{-j}}b_j\right)^{\alpha} \left(\frac{1}{n} - b_j\right)^{1-\alpha} - \mu(e_1+e_2),\ j=1,...,n. \tag{A2}$$

The sufficient first-order conditions for an interior solution for the follower of type I, i.e., $\frac{\partial \pi_2(q_1,q_2;\mathbf{b})}{\partial q_2} = 0$ and $\frac{\partial \pi_2(q_1,q_2;\mathbf{b})}{\partial e_2} = 0$, may be written as:

$$\left[-\alpha\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2\right) + (1-\alpha)\frac{\sum_{j=1}^{n} b_j}{(q_1+q_2)^2}q_1\left(\frac{\gamma}{\beta_2}e_2 - q_2\right)\right] A = 0, \tag{A3}$$

$$\left[\alpha\frac{\gamma}{\beta_2}\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2\right) - (1-\alpha)\gamma\left(\frac{\gamma}{\beta_2}e_2 - q_2\right)\right] A = 0, \tag{A4}$$

where $A \equiv \left(\frac{\gamma}{\beta_2}e_2 - q_2\right)^{\alpha-1}\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2\right)^{-\alpha}$. For the followers of type II, we have $\frac{\partial \pi_j(e_1,e_2;\mathbf{q};b_j,\mathbf{b}_{-j})}{\partial b_j} = 0$, $j \in \{1,...,n\}$, which may be written as:

$$\left[\alpha\frac{\sum_{-j,-j\neq j}b_{-j}}{(b_j+\sum_{-j,-j\neq j}b_{-j})^2}\left(\frac{1}{n}-b_j\right)-(1-\alpha)\frac{b_j}{b_j+\sum_{-j,-j\neq j}b_{-j}}\right]A'=0, \quad \text{(A5)}$$

where $A'\equiv\left(\frac{q_1+q_2}{b_j+\sum_{-j,-j\neq j}b_{-j}}b_j\right)^{\alpha-1}\left(\frac{1}{n}-b_j\right)^{-\alpha}$.

From (A3) and (A4), we deduce:

$$\frac{\sum_{j=1}^n b_j}{(q_1+q_2)^2}q_1=\beta_2. \quad \text{(A6)}$$

The solution to (A5) and to (A6) yield the best responses, which we denote with a slight abuse of notations as $q_2=q_2(q_1;\mathbf{b})$ and $b_j=b_j(q_1,q_2;\mathbf{b}_{-j})$, with:

$$q_2(q_1;\mathbf{b})=-q_1+\sqrt{\frac{1}{\beta_2}\sum_{j=1}^n b_j q_1}, \quad \text{(A7)}$$

$$b_j(q_1,q_2;\mathbf{b}_{-j})=\frac{-\sum_{-j,-j\neq j}b_{-j}+\sqrt{(\sum_{-j,-j\neq j}b_{-j})^2+\frac{4\alpha(1-\alpha)}{n}\sum_{-j,-j\neq j}b_{-j}}}{2(1-\alpha)}. \quad \text{(A8)}$$

As all traders of the same type (here of type II) have the same endowments and utility functions, their payoffs are symmetric in b_j and b_{-j}, for all $j\neq -j$. Then, they must adopt the same strategy at equilibrium, i.e. $b_j=b_{-j}$, for all $j\neq -j$, we have that:

$$b_j=\frac{\alpha}{n}\frac{n-1}{n-\alpha},\ j=1,...,n, \quad \text{(A9)}$$

Therefore, in the first stage of the game, the market price may be written as a function of q_1, with $p_X(q_1)=\frac{\sum_{j=1}^n b_j(q_1)}{q_1+q_2(q_1)}=\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1}$, so the problem of the leader may be written:

$$(\tilde{e}_1,\tilde{q}_1)\in\arg\max\ \left(\frac{\gamma}{\beta_1}e_1-q_1\right)^{\alpha}\left(\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1}-\gamma e_1\right)^{1-\alpha}. \quad \text{(A10)}$$

As the price function $\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1}$ is strictly concave in q_1, the first-order conditions , namely $\frac{\partial\pi_1(q_1,q_2(q_1);\mathbf{b})}{\partial q_1}=0$ and $\frac{\partial\pi_1(q_1,q_2(q_1);\mathbf{b})}{\partial e_1}=0$, are sufficient, and they may be written as:

$$\left[-\alpha\left(\sqrt{\beta_2\alpha\frac{n-1}{n-\alpha}q_1}-\gamma e_1\right)+\frac{1-\alpha}{2}\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}}q_1^{-\frac{1}{2}}\left(\frac{\gamma}{\beta_1}e_1-q_1\right)\right]A''=0, \quad \text{(A11)}$$

$$\left[\alpha\frac{\gamma}{\beta_1}\left(\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1}-\gamma e_1\right)-(1-\alpha)\gamma\left(\frac{\gamma}{\beta_1}e_1-q_1\right)\right]A''=0, \quad \text{(A12)}$$

where $A'' \equiv \left(\frac{\gamma_1}{\beta_1}e_1 - q_1\right)^{\alpha-1}\left(\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}q_1} - \gamma_1 e_1\right)^{-\alpha}$.

By considering the terms in brackets in (A3) an (A4), and by equalizing and cancelling, we have:

$$\frac{1}{2}\sqrt{\alpha\beta_2\frac{n-1}{n-\alpha}}q_1^{-\frac{1}{2}} = \gamma. \tag{A13}$$

The solution to (A13) yields the equilibrium strategy of the leader:

$$\tilde{q}_1 = \frac{\alpha}{4}\frac{\beta_2}{(\beta_1)^2}\frac{n-1}{n-\alpha}. \tag{A14}$$

From (A7) and (A9), we deduce the equilibrium strategies of the followers:

$$\tilde{q}_2 = \frac{\alpha}{2\beta_1}\frac{n-1}{n-\alpha}(1 - \frac{1}{2}\frac{\beta_2}{\beta_1}), \tag{A15}$$

$$\tilde{b}_j = \frac{\alpha}{n}\frac{n-1}{n-\alpha},\ j = 1, ..., n. \tag{A16}$$

Then, the equilibrium relative price is given by:

$$\tilde{p}_X = 2\beta_1. \tag{A17}$$

By using (A12), we can deduce the leader's emissions at the SNE:

$$\tilde{e}_1 = \frac{\alpha(1+\alpha)}{4\gamma}\frac{\beta_2}{\beta_1}\frac{n-1}{n-\alpha}. \tag{A18}$$

And, by using (A4), (A15), and (A17), we deduce the follower's emissions:

$$\tilde{e}_2 = \frac{\alpha}{\gamma}(1 - \frac{1}{2}\frac{\beta_2}{\beta_1})(\frac{2\alpha\beta_1 + (1-\alpha)\beta_2}{2\beta_1})\frac{n-1}{n-\alpha}. \tag{A19}$$

The magnitudes given by (A14)-(16) and (A18)-(A19) correspond to the magnitudes given in Proposition 1. Finally, by using (A14)-(A19) we can deduce (17)-(22).■

7.2. Appendix B: proof of Proposition 4

Consider now the computation of the CNE in which all traders behave in a simultaneous move game. The problems of all traders may be written:

$$\max_{(e_i,q_i)} \left(\frac{\gamma}{\beta_i}e_i - q_i\right)^{\alpha}\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_i - \gamma e_i\right)^{1-\alpha},\ i = 1, 2, \tag{B1}$$

$$\max_{b_j} \left(\frac{q_1+q_2}{b_j + \sum_{-j\neq j} b_{-j}}b_j\right)^{\alpha}\left(\frac{1}{n} - b_j\right)^{1-\alpha} - \mu(e_1+e_2),\ j = 1, ..., n. \tag{B2}$$

The sufficient first-order conditions for an interior solution are given by (A5) for $j \in \{1, ..., n\}$, and by (B3)-(B4) for $i \in \{1, 2\}$, with:

$$\left[-\alpha\left(\frac{\sum_{j=1}^{n} b_j}{q_i+q_{-i}}q_i-\gamma e_i\right)+(1-\alpha)\frac{\sum_{j=1}^{n} b_j}{(q_i+q_{-i})^2}q_{-i}\left(\frac{\gamma}{\beta_i}e_i-q_i\right)\right]B=0, \quad \text{(B3)}$$

$$\left[\alpha\frac{\gamma}{\beta_1}\left(\frac{\sum_{j=1}^{n} b_j}{q_i+q_{-i}}q_i-\gamma e_i\right)-(1-\alpha)\gamma\left(\frac{\gamma}{\beta_i}e_i-q_i\right)\right]B=0,\ i=1,2, \quad \text{(B4)}$$

where $B \equiv \left(\frac{\gamma}{\beta_i}e_i-q_i\right)^{\alpha-1}\left(\frac{\sum_{j=1}^{n} b_j}{q_i+q_{-i}}q_i-\gamma e_i\right)^{-\alpha}$.

The solutions to these equations are the best responses, which may be written:

$$q_1(q_2;\mathbf{b})=-q_2+\sqrt{\frac{1}{\beta_1}\sum_{j=1}^{n} b_j q_2}, \quad \text{(B5)}$$

$$q_2(q_1;\mathbf{b})=-q_1+\sqrt{\frac{1}{\beta_2}\sum_{j=1}^{n} b_j q_1}, \quad \text{(B6)}$$

$$b_j(q_1,q_2;\mathbf{b}_{-j})=\frac{\alpha}{n}\frac{n-1}{n-\alpha},\ j=1,...,n, \quad \text{(B7)}$$

where all traders of the same type (here of type II) must adopt the same strategy at equilibrium, i.e. $b_j = b_{-j}$, for all $j \neq -j$.

The solutions to (B5)-(B7) are given by:

$$(\hat{q}_1,\hat{q}_2)=\left(\frac{\alpha\beta_2}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha},\frac{\alpha\beta_1}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}\right), \quad \text{(B8)}$$

$$\hat{b}_j=\frac{\alpha}{n}\frac{n-1}{n-\alpha},j=1,...,n. \quad \text{(B9)}$$

Therefore, the equilibrium relative market price is given by:

$$\hat{p}_X=\beta_1+\beta_2. \quad \text{(B10)}$$

Then, we deduce the emissions:

$$(\hat{e}_1,\hat{e}_2)=\left(\frac{\alpha}{\gamma}\frac{\beta_2(\beta_1+\alpha\beta_2)}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha},\frac{\alpha}{\gamma}\frac{\beta_1(\beta_2+\alpha\beta_1)}{(\beta_1+\beta_2)^2}\frac{n-1}{n-\alpha}\right). \quad \text{(B11)}$$

The allocations are then:

$$(\hat{x}_1,\hat{y}_1)=\left(\frac{1}{\beta_1}\left(\frac{\alpha\beta_2}{\beta_1+\beta_2}\right)^2\frac{n-1}{n-\alpha},\alpha(1-\alpha)\left(\frac{\beta_2}{\beta_1+\beta_2}\right)^2\frac{n-1}{n-\alpha}\right), \quad \text{(B12)}$$

$$(\hat{x}_2,\hat{y}_2)=\left(\frac{1}{\beta_2}\left(\frac{\alpha\beta_1}{\beta_1+\beta_2}\right)^2\frac{n-1}{n-\alpha},\alpha(1-\alpha)\left(\frac{\beta_1}{\beta_1+\beta_2}\right)^2\frac{n-1}{n-\alpha}\right), \quad \text{(B13)}$$

$$(\hat{x}_j,\hat{y}_j)=\left(\frac{\alpha}{\beta_1+\beta_2}\frac{1}{n}\frac{n-1}{n-\alpha},\frac{1-\alpha}{n-\alpha}\right),\ j=1,...,n. \quad \text{(B14)}$$

Therefore, the CNE payoffs of traders $i = 1, 2$ are given by:

$$\hat{\pi}_1 = \alpha^{\alpha+1} \left(\frac{\beta_2}{\beta_1 + \beta_2} \right)^2 \left(\frac{1}{\beta_1} \right)^{\alpha} (1-\alpha)^{1-\alpha} \frac{n-1}{n-\alpha}, \tag{B15}$$

$$\hat{\pi}_2 = \alpha^{\alpha+1} \left(\frac{\beta_1}{\beta_1 + \beta_2} \right)^2 \left(\frac{1}{\beta_2} \right)^{\alpha} (1-\alpha)^{1-\alpha} \frac{n-1}{n-\alpha}, \tag{B16}$$

and, the CNE payoffs of traders $j \in \{1, ..., n\}$ are given by:

$$\hat{\pi}_j = \frac{\left(\frac{\alpha}{\beta_1+\beta_2} \frac{n-1}{n} \right)^{\alpha} (1-\alpha)^{1-\alpha}}{n-\alpha} - \mu\alpha \frac{\beta_2(\beta_1 + \alpha\beta_2) + \beta_1(\beta_2 + \alpha\beta_1)}{\gamma(\beta_1 + \beta_2)^2} \frac{n-1}{n-\alpha}. \tag{B17}$$

■

7.3. Appendix C: proof of Proposition 6

In this Appendix we determine the SNE emissions by encompassing the two taxation mechanisms. To this end, let $(t, \tau) \in [0, 1]^2$, and let us rewrite the payoffs (23)-(26), with $\pi_i(.) \equiv \pi_i(e_i, q_i, \mathbf{q}_{-i}; \mathbf{b}; t, \tau)$, $i = 1, 2$, as follows:

$$\pi_1(.) = \left(\frac{\gamma}{\beta_1}(1-t)e_1 - q_1 \right)^{\alpha} \left(\left(\frac{\sum_{j=1}^n b_j(q_1)}{q_1 + q_2(q_1)} - \tau \right) q_1 - \gamma(1-t)e_1 \right)^{1-\alpha}, \tag{C1}$$

$$\pi_2(.) = \left(\frac{\gamma}{\beta_2}(1-t)e_2 - q_2(q_1) \right)^{\alpha} \left(\left(\frac{\sum_{j=1}^n b_j(q_1)}{q_1 + q_2(q_1)} - \tau \right) q_2(q_1) - \gamma(1-t)e_2 \right)^{1-\alpha}. \tag{C2}$$

Consider the followers. The best response of trader j of type II is given by (A9). Consider the follower of type I. Differentiating the above expression (C2) with respect to q_2 and e_2 leads to the sufficient first-order conditions:

$$\begin{array}{c} \frac{\partial \pi_2(.)}{\partial q_2} = \left\{ -\alpha \left[\left(\frac{\sum_{j=1}^n b_j}{q_1+q_2} - \tau \right) q_2 - \gamma(1-t)e_2 \right] + \right. \\ \left. (1-\alpha) \left[\frac{\sum_{j=1}^n b_j q_1}{(q_1+q_2)^2} - \tau \right] \left(\frac{\gamma}{\beta_2}(1-t)e_2 - q_2 \right) \right\} C = 0 \end{array} \tag{C3}$$

and

$$\begin{array}{c} \frac{\partial \pi_2(.)}{\partial e_2} = \left\{ \alpha \frac{\gamma}{\beta_2}(1-t) \left[\left(\frac{\sum_{j=1}^n b_j}{q_1+q_2} - \tau \right) q_2 - \gamma(1-t)e_2 \right] - \right. \\ \left. (1-\alpha)\gamma(1-t) \left(\frac{\gamma}{\beta_2}(1-t)e_2 - q_2 \right) \right\} C = 0, \end{array} \tag{C4}$$

where $C \equiv \left(\frac{\gamma}{\beta_2}(1-t)e_2 - q_2 \right)^{\alpha-1} \left(\left(\frac{\sum_{j=1}^n b_j}{q_1+q_2} - \tau \right) q_2 - \gamma(1-t)e_2 \right)^{-\alpha}$.

Therefore, from (C2)-(C4) and (A9), the best responses of followers may be written:

$$q_2(q_1; \mathbf{b}; \tau) = -q_1 + \sqrt{\frac{\sum_{j=1}^n b_j q_1}{\beta_2 + \tau}}, \tag{C5}$$

$$b_j(q_1, q_2; \mathbf{b}_{-j}; \tau) = \frac{\alpha}{n}\frac{n-1}{n-\alpha},\ j \in \{1, ..., n\}. \tag{C6}$$

Therefore, in the first stage of the game, the problem of the leader may be written:

$$\max_{q_1}\left(\frac{\gamma}{\beta_1}(1-t)e_1 - q_1\right)^{\alpha}\left(\sqrt{(\beta_2+\tau)\sum_{j=1}^{n} b_j q_1} - \tau q_1 - \gamma(1-t)e_1\right)^{1-\alpha}. \tag{C7}$$

The first-order conditions, i.e., $\frac{\partial \pi_1(q_1, q_2(q_1); \mathbf{b}; t, \tau)}{\partial q_1} = 0$ and $\frac{\partial \pi_1(q_1, q_2(q_1); \mathbf{b}; t, \tau)}{\partial e_1} = 0$, may be written:

$$\begin{array}{c}\left\{-\alpha\left[\sqrt{(\beta_2+\tau)\sum_{j=1}^{n} b_j q_1} - \tau q_1 - \gamma(1-t)e_1\right] + \right. \\ \left.(1-\alpha)\left[\frac{1}{2}\sqrt{\alpha\frac{n-1}{n-\alpha}(\beta_2+\tau)}q_1^{-\frac{1}{2}} - \tau\right]\left(\frac{\gamma}{\beta_1}(1-t)e_1 - q_1\right)\right\} C' = 0,\end{array} \tag{C8}$$

and

$$\begin{array}{c}\left\{\alpha\frac{\gamma}{\beta_1}(1-t)\left[\sqrt{(\beta_2+\tau)\sum_{j=1}^{n} b_j q_1} - \tau q_1 - \gamma(1-t)e_1\right] - \right. \\ \left.(1-\alpha)\gamma(1-t)\left(\frac{\gamma}{\beta_1}(1-t)e_1 - q_1\right)\right\} C' = 0,\end{array} \tag{C9}$$

where $C' \equiv \left(\frac{\gamma}{\beta_1}(1-t)e_1 - q_1\right)^{\alpha-1}\left(\sqrt{(\beta_2+\tau)\sum_{j=1}^{n} b_j q_1} - \tau q_1 - \gamma(1-t)e_1\right)^{-\alpha}$.

The solution to (C7)-(C8) is given by:

$$\tilde{q}_1(t, \tau) = \frac{\alpha}{4}\frac{\beta_2+\tau}{(\beta_1+\tau)^2}\frac{n-1}{n-\alpha}; \tag{C10}$$

Then, from (C4), we deduce:

$$\tilde{q}_2(t, \tau) = \frac{\alpha}{2}\frac{1}{\beta_1+\tau}\left(1 - \frac{1}{2}\frac{\beta_2+\tau}{\beta_1+\tau}\right)\frac{n-1}{n-\alpha}, \tag{C11}$$

which by letting $(\tilde{q}_1', \tilde{q}_2') \equiv (\tilde{q}_1(t, \tau), \tilde{q}_2(t, \tau))$ yields the values of Proposition 5.

Indeed, the market price is:

$$\tilde{p}_X(t, \tau) = 2(\beta_1 + \tau). \tag{C12}$$

By using the first-order conditions (C3)-(C4) and (C8)-(C9), we deduce:

$$\tilde{e}_1(t, \tau) = \frac{(1+\alpha)\beta_1 + \alpha\tau}{\gamma(1-t)}\tilde{q}_1(t, \tau) \tag{C13}$$

$$\tilde{e}_2(t, \tau) = \frac{\alpha[(2(\beta_1+\tau) - \tau)] + (1-\alpha)\beta_2(1-\tau)}{\gamma(1-t)}\tilde{q}_2(t, \tau), \tag{C14}$$

which by letting $(\tilde{e}_1', \tilde{e}_2') \equiv (\tilde{e}_1(t, \tau), \tilde{e}_2(t, \tau))$ yields the values of Proposition 5.■

7.4. Appendix D: CNE with taxations

We determine the CNE emissions by encompassing the two taxation mechanisms. To this end, consider the payoffs given by:

$$\pi_1(.) = \left(\frac{\gamma}{\beta_1}(1-t)e_1 - q_1\right)^{\alpha}\left(\left(\frac{\sum_{j=1}^n b_j}{q_1+q_2} - \tau\right)q_1 - \gamma(1-t)e_1\right)^{1-\alpha}, \tag{D1}$$

$$\pi_2(.) = \left(\frac{\gamma}{\beta_2}(1-t)e_2 - q_2\right)^{\alpha}\left(\left(\frac{\sum_{j=1}^n b_j}{q_1+q_2} - \tau\right)q_2 - \gamma(1-t)e_2\right)^{1-\alpha}. \tag{D2}$$

$$\pi_j(.) = \left(\frac{q_1+q_2}{b_j + \sum_{-j\neq j} b_{-j}}b_j\right)^{\alpha}\left(\frac{1}{n} - b_j\right)^{1-\alpha} - \mu(e_1+e_2),\ j = 1, ..., n. \tag{D3}$$

The sufficient first-order conditions for an interior solution are given by (A5) for $j \in \{1, ..., n\}$, and by (E4) and (E5) for each $i \in \{1, 2\}$:

$$\begin{array}{c}\left\{-\alpha\left[\left(\frac{\sum_{j=1}^n b_j}{q_i+q_{-i}} - \tau\right)q_i - \gamma(1-t)e_i\right] + \right. \\ \left.(1-\alpha)\left(\frac{\sum_{j=1}^n b_j q_{-i}}{(q_i+q_{-i})^2} - \tau\right)\left(\frac{\gamma}{\beta_i}(1-t)e_i - q_i\right)\right\}D = 0\end{array} \tag{D4}$$

$$\begin{array}{c}\left\{\alpha\frac{\gamma}{\beta_i}(1-t)\left[\left(\frac{\sum_{j=1}^n b_j}{q_i+q_{-i}} - \tau\right)q_i - \gamma(1-t)e_i\right] - \right. \\ \left.(1-\alpha)\gamma(1-t)\left(\frac{\gamma}{\beta_i}(1-t)e_i - q_i\right)\right\}D = 0,\end{array} \tag{D5}$$

where $D \equiv \left(\frac{\gamma}{\beta_i}(1-t)e_i - q_i\right)^{\alpha-1}\left(\left(\frac{\sum_{j=1}^n b_j}{q_1+q_2} - \tau\right)q_i - \gamma(1-t)e_i\right)^{-\alpha}$.

The solutions to these equations and to (A5) are given by:

$$q_1(q_2; \mathbf{b}; \tau) = -q_2 + \sqrt{\frac{1}{\beta_1+\tau}\sum_{j=1}^n b_j q_2}, \tag{D6}$$

$$q_2(q_1; \mathbf{b}; \tau) = -q_1 + \sqrt{\frac{1}{\beta_2+\tau}\sum_{j=1}^n b_j q_1}, \tag{D7}$$

$$b_j(q_1, q_2; \mathbf{b}_{-j}; \tau) = \frac{\alpha}{n}\frac{n-1}{n-\alpha},\ j = 1, ..., n, \tag{D8}$$

where all traders of the same type (here of type II) must adopt the same strategy at equilibrium, i.e. $b_j = b_{-j}$, for all $j \neq -j$.

The solution is given by:

$$\hat{q}_1(t, \tau) = \alpha\frac{\beta_2+\tau}{(\beta_1+\beta_2+2\tau)^2}\frac{n-1}{n-\alpha}, \tag{D9}$$

$$\hat{q}_2(t, \tau) = \alpha\frac{\beta_1+\tau}{(\beta_1+\beta_2+2\tau)^2}\frac{n-1}{n-\alpha}, \tag{D10}$$

$$\hat{b}_j(t, \tau) = \frac{\alpha}{n}\frac{n-1}{n-\alpha}, j = 1, ..., n. \tag{D11}$$

Therefore, the equilibrium relative price is given by:

$$\hat{p}_X(t,\tau) = \beta_1 + \beta_2 + 2\tau. \tag{D12}$$

By using the first-order conditions, we deduce the emissions:

$$\hat{e}_1(t,\tau) = \alpha \frac{(\beta_2+\tau)[\alpha(\beta_2+\tau)+\beta_1]}{\gamma(1-t)(\beta_1+\beta_2+2\tau)^2} \frac{n-1}{n-\alpha}, \tag{D13}$$

$$\hat{e}_2(t,\tau) = \alpha \frac{(\beta_1+\tau)[\alpha(\beta_1+\tau)+\beta_2]}{\gamma(1-t)(\beta_1+\beta_2+2\tau)^2} \frac{n-1}{n-\alpha}. \tag{D14}$$

■

7.5. Appendix E: proof of Proposition 8

Consider the SNE with emissions. By using the expressions of $\tilde{e}_1(t,\tau)$ and $\tilde{e}_2(t,\tau)$ given in Proposition 6, and by assuming $t = 0$, we deduce:

$$\tilde{e}_1(\tau) = \alpha \frac{[(1+\alpha)\beta_1 + \alpha\tau](\beta_2+\tau)}{4\gamma(\beta_1+\tau)^2} \frac{n-1}{n-\alpha}; \tag{E1}$$

$$\tilde{e}_2(\tau) = \alpha \frac{[\alpha(2\beta_1+\tau)+(1-\alpha)\beta_2]\left(1-\frac{1}{2}\frac{\beta_2+\tau}{\beta_1+\tau}\right)}{2\gamma(\beta_1+\tau)} \frac{n-1}{n-\alpha}. \tag{E2}$$

Let $\tilde{\tau}$ be the solution to $\tau\tilde{q}_1(\tau) + \tau\tilde{q}_2(\tau) = G$. Some computations lead to:

$$\tilde{\tau} = \frac{2\beta_1 G}{\alpha\frac{n-1}{n-\alpha} - 2G}. \tag{E3}$$

By using (E1) and (E2), we deduce:

$$\frac{\partial\tilde{e}_1(\tau)}{\partial\tau} \mid_{\tau=\tilde{\tau}} = -\alpha \frac{[(1+\alpha)\beta_1 + \alpha\tilde{\tau}](\beta_2-\beta_1) + \beta_1(\beta_2+\tilde{\tau})}{4\gamma(\beta_1+\tilde{\tau})^3} \frac{n-1}{n-\alpha}. \tag{E4}$$

$$\frac{\partial\tilde{e}_2(\tau)}{\partial\tau} \mid_{\tau=\tilde{\tau}} = -\alpha \frac{2[\alpha(2\beta_1+\tilde{\tau})+(1-\alpha)\beta_2](\beta_1-\beta_2) + \beta_2(\beta_1+\tilde{\tau})}{4\gamma(\beta_1+\tilde{\tau})^3} \frac{n-1}{n-\alpha}. \tag{E5}$$

Then, we have that:

$$\frac{\partial\tilde{e}_1(\tau)}{\partial\tau} \mid_{\tau=\tilde{\tau}} = -H\{\alpha(1+\alpha)\frac{n-1}{n-\alpha} - 4G](\beta_2-\beta_1) + \alpha\beta_2\frac{n-1}{n-\alpha}\}, \tag{E6}$$

and

$$\frac{\partial\tilde{e}_2(\tau)}{\partial\tau} \mid_{\tau=\tilde{\tau}} = -H\{2[2\alpha(\alpha\frac{n-1}{n-\alpha}-G)+(1-\alpha)\frac{\beta_2}{\beta_1}(\alpha\frac{n-1}{n-\alpha}-2G)](\beta_1-\beta_2)+\alpha\beta_2\frac{n-1}{n-\alpha}\}, \tag{E7}$$

where $H \equiv \frac{1}{\gamma}\left(\frac{\alpha\frac{n-1}{n-\alpha}-2G}{2\alpha\beta_1\frac{n-1}{n-\alpha}}\right)^2 > 0$.

Then, as $0 < G < \frac{\alpha}{2}\frac{1}{1+\beta_1}\frac{n-1}{n-\alpha}$, we have:

$$\begin{array}{l} \frac{\partial \tilde{e}_1(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} < 0 \text{ if } \beta_2 > \beta_1; \\ \text{and} \\ \frac{\partial \tilde{e}_2(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} < 0 \text{ if } \beta_1 > \beta_2. \end{array} \tag{E8}$$

Finally, consider the emissions at the SNE when $\beta_1 = \beta_2$. Let $\beta_1 = \beta_2 = \beta$ in (E1) and (E2). Then, we have:

$$\tilde{e}_1(\tau) = \alpha \frac{[(1+\alpha)\beta + \alpha\tau]}{4\gamma(\beta+\tau)} \frac{n-1}{n-\alpha}; \tag{E1}$$

$$\tilde{e}_2(\tau) = \alpha \frac{[(1+\alpha)\beta + \alpha\tau]}{4\gamma(\beta+\tau)} \frac{n-1}{n-\alpha}. \tag{E2}$$

Therefore, we have that:

$$\frac{\partial \tilde{e}_i(\tau)}{\partial \tau} \mid_{\tau=\tilde{\tau}} = -\frac{\alpha\beta}{4\gamma} \left(\frac{1}{\beta+\tilde{\tau}} \right)^2 < 0 \text{ if } \beta_1 = \beta_2 = \beta,\ i = 1, 2, \tag{E9}$$

where the equilibrium tax $\tilde{\tau}$ is now given by $\tilde{\tau} = \frac{(1+\alpha)\beta}{2}(\sqrt{1 + \frac{1}{2}\frac{\gamma}{\alpha}\frac{n-\alpha}{n-1}\frac{1}{((1+\alpha)\beta)^2}G} - 1)$.■

7.5.1. Supplement to Appendix E.

Consider the CNE with emissions. By using the expressions of $\hat{e}_1(\tau, t)$ and $\hat{e}_2(\tau, t)$ given by (D13) and (D14) in Appendix D and by assuming $t = 0$, we deduce:

$$\hat{e}_1(\tau) = \frac{\alpha}{\gamma} \frac{(\beta_2+\tau)[\alpha(\beta_2+\tau)+\beta_1]}{(\beta_1+\beta_2+2\tau)^2} \frac{n-1}{n-\alpha} \tag{E10}$$

$$\hat{e}_2(\tau) = \frac{\alpha}{\gamma} \frac{(\beta_1+\tau)[\alpha(\beta_1+\tau)+\beta_2]}{(\beta_1+\beta_2+2\tau)^2} \frac{n-1}{n-\alpha}. \tag{E11}$$

Then, we have that:

$$\frac{\partial \hat{e}_1(\tau)}{\partial \tau} \mid_{\tau=\hat{\tau}} = -\frac{\alpha}{\gamma} \frac{2\alpha(\beta_2+\hat{\tau})(\beta_2-\beta_1) + 2\beta_1\hat{\tau} + \beta_1\beta_2(3-\frac{\beta_1}{\beta_2})}{(\beta_1+\beta_2+2\hat{\tau})^3} \frac{n-1}{n-\alpha}, \tag{E12}$$

$$\frac{\partial \hat{e}_2(\tau)}{\partial \tau} \mid_{\tau=\hat{\tau}} = -\frac{\alpha}{\gamma} \frac{2\alpha(\beta_1+\hat{\tau})(\beta_1-\beta_2) + 2\beta_2\hat{\tau} + \beta_1\beta_2(3-\frac{\beta_2}{\beta_1})}{(\beta_1+\beta_2+2\hat{\tau})^3} \frac{n-1}{n-\alpha}. \tag{E13}$$

Then, if $\beta_1 = \beta_2$, then $\frac{\partial \hat{e}_1(\tau)}{\partial \tau} \mid_{\tau=\hat{\tau}} < 0$ and $\frac{\partial \hat{e}_2(\tau)}{\partial \tau} \mid_{\tau=\hat{\tau}} < 0$. Otherwise, either $\frac{\partial \hat{e}_1(\tau)}{\partial \tau} \mid_{\tau=\hat{\tau}} < 0$ if $\beta_2 \geqslant \beta_1$, or $\frac{\partial \hat{e}_2(\tau)}{\partial \tau} \mid_{\tau=\hat{\tau}} < 0$ if $\beta_1 \geqslant \beta_2$.■

7.6. Appendix F: proof of Proposition 10

Consider, in the second stage of the game, the behavior of the follower of type I (the problem of each follower of type II is not modified), which may be written:

$$\max_{(e_2,q_2)} \left(\frac{\gamma}{\beta_2}e_2 - q_2\right)^{\alpha} \left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2 + r(\bar{e}_2 - e_2)\right)^{1-\alpha}. \tag{F1}$$

Differentiating (F1) with respect to q_2 and e_2 leads to the sufficient first-order conditions:

$$\begin{array}{c} \frac{\partial \pi_2(.)}{\partial q_2} = \left\{-\alpha\left[\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2 + r(\bar{e}_2 - e_2)\right] + \right. \\ \left. (1-\alpha)\frac{\sum_{j=1}^{n} b_j}{(q_1+q_2)^2}q_1\left(\frac{\gamma}{\beta_2}e_2 - q_2\right)\right\} E = 0, \end{array} \tag{F2}$$

and

$$\begin{array}{c} \frac{\partial \pi_2(.)}{\partial e_2} = \left\{\alpha\frac{\gamma}{\beta_2}\left[\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2 + r(\bar{e}_2 - e_2)\right] - \right. \\ \left. (1-\alpha)(\gamma+r)\left(\frac{\gamma}{\beta_2}e_2 - q_2\right)\right\} E = 0, \end{array} \tag{F3}$$

where $E \equiv \left(\frac{\gamma}{\beta_2}e_2 - q_2\right)^{\alpha-1}\left(\frac{\sum_{j=1}^{n} b_j}{q_1+q_2}q_2 - \gamma e_2 + r(\bar{e}_2 - e_2)\right)^{-\alpha}$.

By following the same procedure as in Appendices A and C, the best response of follower j is given by (A9), i.e., $b_j = \frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j = 1, ..., n$.

$$q_2(q_1; \mathbf{b}; r) = -q_1 + \sqrt{\frac{1}{\beta_2}\frac{\gamma}{\gamma+r}\sum_{j=1}^{n} b_j q_1}. \tag{F4}$$

Therefore, in the first stage of the game, the problem of the leader may be written:

$$\max_{(e_1,q_1)} \left(\frac{\gamma}{\beta_1}e_1 - q_1\right)^{\alpha} \left(\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}q_1} - \gamma e_1 + r(\bar{e}_1 - e_1)\right)^{1-\alpha}. \tag{F5}$$

The sufficient first-order conditions (the function $\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}q_1}$ is strictly concave in q_1), namely $\frac{\partial \pi_1}{\partial q_1} = 0$ and $\frac{\partial \pi_1}{\partial e_1} = 0$, may be written:

$$\{-\alpha[\sqrt{\alpha\beta_2\tfrac{n-1}{n-\alpha}q_1} - \gamma e_1 + r(\bar{e}_1 - e_1)] + \frac{(1-\alpha)\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}}q_1^{-\frac{1}{2}}(\frac{\gamma}{\beta_1}e_1 - q_1)}{2}\}E' = 0 \tag{F6}$$

and

$$[\alpha\gamma\frac{\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}q_1} - \gamma e_1 + r(\bar{e}_1 - e_1)}{\beta_1} - (1-\alpha)(\gamma+r)(\tfrac{\gamma}{\beta_1}e_1 - q_1)]E' = 0 \tag{F7}$$

where $E' \equiv \left(\frac{\gamma}{\beta_1}e_1 - q_1\right)^{\alpha-1}\left(\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}q_1} - \gamma e_1 + r(\bar{e}_1 - e_1)\right)^{-\alpha}$.

By considering the terms in brackets in (F6) and (F7), and by equalizing and cancelling, leads to:

$$\frac{1}{2}\sqrt{\alpha\beta_2\frac{\gamma+r}{\gamma}\frac{n-1}{n-\alpha}}q_1^{-\frac{1}{2}}=\beta_1\frac{\gamma+r}{\gamma}. \tag{F8}$$

The solution of (F8) yields the equilibrium strategy of the leader:

$$\tilde{q}_1(r)=\alpha\frac{\beta_2}{4(\beta_1)^2}\frac{\gamma}{\gamma+r}\frac{n-1}{n-\alpha}. \tag{F9}$$

From (F4), we deduce the equilibrium strategy of the follower of type I:

$$\tilde{q}_2=\frac{\alpha}{2\beta_1}\frac{n-1}{n-\alpha}\left(1-\frac{1}{2}\frac{\beta_2}{\beta_1}\right) \tag{F10}$$

We also have $\tilde{b}_j=\frac{\alpha}{n}\frac{n-1}{n-\alpha}$, $j=1,...,n$. Therefore, the market price is given by:

$$\tilde{p}_X(r)=\frac{\gamma+r}{\gamma}\tilde{p}_X. \tag{F11}$$

From (F5) and (F9), we deduce the leader's level of emissions:

$$\tilde{e}_1(r)=\frac{\alpha}{\gamma+r}r\bar{e}_1+\frac{\alpha(1+\alpha)}{\gamma+r}\frac{\beta_2}{4\beta_1}\frac{n-1}{n-\alpha}, \tag{F12}$$

and from (F3) and (F10), we deduce the follower's level of emissions:

$$\tilde{e}_2(r)=\frac{\alpha}{\gamma+r}r\bar{e}_2+\frac{\alpha}{\gamma+r}\left(1-\frac{1}{2}\frac{\beta_2}{\beta_1}\right)\left(\frac{2\alpha\beta_1+(1-\alpha)\beta_2}{2\beta_1}\right)\frac{n-1}{n-\alpha}, \tag{F13}$$

which are the magnitudes of Proposition 10. Finally, we deduce (35)-(40).■

7.7. Appendix G: CNE with pollution permit market

The market clearing condition on the permit market may be written here $\hat{e}_1(r)+\hat{e}_2(r)=\bar{e}_1+\bar{e}_2.$, where $\hat{e}_1(r)$ and $\hat{e}_2(r)$ will denote the emissions of tle leader and the follower at the CNE.

The problems of all traders may be written:

$$\max_{(e_1q_1)}\left(\frac{\gamma}{\beta_i}e_i-q_i\right)^{\alpha}\left(\frac{\sum_{j=1}^n b_j}{q_1+q_2}q_i-\gamma e_i+r(\bar{e}_i-e_i)\right)^{1-\alpha},\ i=1,2, \tag{G1}$$

$$\max_{b_j}\left(\frac{q_1+q_2}{b_j+\sum_{-j\neq j}b_{-j}}b_j\right)^{\alpha}\left(\frac{1}{n}-b_j\right)^{1-\alpha}-\mu(e_1+e_2),\ j=1,...,n. \tag{G2}$$

The sufficient first-order conditions for an interior solution are given by (A5) for $j\in\{1,...,n\}$, and by (G3)-(G4) for $i\in\{1,2\}$, with:

$$\{-\alpha[\frac{\sum_{j=1}^n b_j}{q_i+q_{-i}}q_i-\gamma e_i+r(\bar{e}_i-e_i)]+(1-\alpha)\frac{\sum_{j=1}^n b_j}{(q_i+q_{-i})^2}q_{-i}\left(\frac{\gamma}{\beta_i}e_i-q_i\right)\}F=0, \tag{G3}$$

$$\{\alpha\frac{\gamma}{\beta_1}[\frac{\sum_{j=1}^{n}b_j}{q_i+q_{-i}}q_i-\gamma e_i+r(\bar{e}_i-e_i)]-(1-\alpha)\gamma\left(\frac{\gamma}{\beta_i}e_i-q_i\right)\}F=0,\ i=1,2, \quad \text{(G4)}$$

where $F\equiv\left(\frac{\gamma}{\beta_i}e_i-q_i\right)^{\alpha-1}\left(\frac{\sum_{j=1}^{n}b_j}{q_i+q_{-i}}q_i-\gamma e_i+r(\bar{e}_i-e_i)\right)^{-\alpha}$.

The solutions to (G3)-(G4) are the followers' best responses, which are given by:

$$q_1(q_2;\mathbf{b};r)=-q_2+\sqrt{\frac{1}{\beta_1}\frac{\gamma}{\gamma+r}\sum_{j=1}^{n}b_jq_2}, \quad \text{(G5)}$$

$$q_2(q_1;\mathbf{b};r)=-q_1+\sqrt{\frac{1}{\beta_2}\frac{\gamma}{\gamma+r}\sum_{j=1}^{n}b_jq_1}, \quad \text{(G6)}$$

$$b_j(q_1,q_2;\mathbf{b}_{-j};r)=\frac{\alpha}{n}\frac{n-1}{n-\alpha},\ j=1,...,n, \quad \text{(G7)}$$

where all traders of the same type (here of type II) must adopt the same strategy at equilibrium, i.e. $b_j=b_{-j}$, for all $j\neq -j$.

The solutions to (G5)-(G7) are given by:

$$(\hat{q}_1(r),\hat{q}_2(r))=\left(\frac{\alpha\beta_2}{(\beta_1+\beta_2)^2}\frac{\gamma}{\gamma+r}\frac{n-1}{n-\alpha},\frac{\alpha\beta_1}{(\beta_1+\beta_2)^2}\frac{\gamma}{\gamma+r}\frac{n-1}{n-\alpha}\right), \quad \text{(G8)}$$

$$\hat{b}_j(r)=\frac{\alpha}{n}\frac{n-1}{n-\alpha},j=1,...,n. \quad \text{(G9)}$$

Therefore, the equilibrium relative market price is given by:

$$\hat{p}_X(r)=(\beta_1+\beta_2)\frac{\gamma+r}{\gamma}. \quad \text{(G10)}$$

Then, we deduce the emissions:

$$(\hat{e}_1(r),\hat{e}_2(r))=\left(\alpha\frac{r}{\gamma+r}\bar{e}_1+\frac{\gamma}{\gamma+r}\hat{e}_1,\alpha\frac{r}{\gamma+r}\bar{e}_2+\frac{\gamma}{\gamma+r}\hat{e}_2\right), \quad \text{(G11)}$$

where $\hat{e}_1$ and $\hat{e}_2$ are given by (B11) in Appendix B.

The allocations are then:

$$(\hat{x}_1(r),\hat{y}_1(r))=\left(\alpha\frac{\gamma}{\beta_1}\frac{r}{\gamma+r}\bar{e}_1+\frac{\gamma}{\gamma+r}\hat{x}_1,(1-\alpha)r\bar{e}_1+\hat{y}_1\right), \quad \text{(G12)}$$

$$(\hat{x}_2(r),\hat{y}_2(r))=\left(\alpha\frac{\gamma}{\beta_2}\frac{r}{\gamma+r}\bar{e}_2+\frac{\gamma}{\gamma+r}\hat{x}_2,(1-\alpha)r\bar{e}_2+\hat{y}_2\right), \quad \text{(G13)}$$

$$(\hat{x}_j(r),\hat{y}_j(r))=\left(\frac{\gamma}{\gamma+r}\frac{\alpha}{\beta_1+\beta_2}\frac{1}{n}\frac{n-1}{n-\alpha},\frac{1-\alpha}{n-\alpha}\right),\ j=1,...,n. \quad \text{(G14)}$$

Therefore, the CNE payoffs of type I traders are given by:

$$\hat{\pi}_1(r) = \left(\alpha\frac{\gamma}{\beta_1}\frac{r}{\gamma+r}\bar{e}_1 + \frac{\gamma}{\gamma+r}\hat{x}_1\right)^{\alpha}\left((1-\alpha)r\bar{e}_1 + \hat{y}_1\right)^{1-\alpha}, \tag{G15}$$

$$\hat{\pi}_2(r) = \left(\alpha\frac{\gamma}{\beta_2}\frac{r}{\gamma+r}\bar{e}_2 + \frac{\gamma}{\gamma+r}\hat{x}_2\right)^{\alpha}\left((1-\alpha)r\bar{e}_2 + \hat{y}_2\right)^{1-\alpha}, \tag{G16}$$

and, from the market clearing condition on the permits market, i.e., $\hat{e}_1(r)+\hat{e}_2(r) = \bar{e}_1+\bar{e}_2$, the payoffs of type II traders are given by:

$$\hat{\pi}_j(r) = \frac{\left(\frac{\alpha}{\beta_1+\beta_2}\frac{\gamma}{\gamma+r}\frac{n-1}{n}\right)^{\alpha}(1-\alpha)^{1-\alpha}}{n-\alpha} - \mu\frac{\gamma+\alpha r}{\gamma+r}(\bar{e}_1+\bar{e}_2). \tag{G17}$$

Finally, we have that:

$$\frac{\partial\hat{\pi}_j(r)}{\partial r} = -\alpha\frac{(1-\alpha)\gamma}{(\gamma+r)^2}\frac{\left[\frac{\alpha}{(1-\alpha)}\frac{1}{\beta_1+\beta_2}\frac{n-1}{n}\right]^{\alpha}\left(\frac{\gamma+r}{\gamma}\right)^{1-\alpha}}{n-\alpha} < 0. \tag{G18}$$

Therefore, we conclude:

$$\frac{\partial\hat{\pi}_j(r)}{\partial r} = 0 \text{ either when } \alpha\to 0 \text{ or when } n\to+\infty. \tag{G19}$$

■

ACKNOWLEDGMENTS

This paper has been presented (virtually) at the 31rst EURO Conference which held in Athens in July 2021. An earlier version was presented at the FAERE which held in Rennes in September 2019, and at the 9th SI&GE workshop which held in Nanterre in November 2018. A. Dickson, D. Levando, M. Koslovskya, N. Raffin, H. Stahn, M. Tidball, S. Tonin, and F. Tricou are gratefully acknowlegded for their comments on an ealier version. Usual disclaimers apply.

REFERENCES

[1] Barnett AH (1980) The Pigouvian Tax Rule Under Monopoly, *American Economic Review* 70, 1037-1041.

[2] Bloch F, Ferrer H (2001). Strategic complements and substitutes in bilateral oligopolies. *Economics Letters* 70, 83-87.

[3] Bloch F, Ghosal S (1997) Stable trading structures in bilateral oligopolies. *Journal of Economic Theory* 74, 368-384.

[4] Bulow JI, Geanakoplos JD, Klemperer PD (1985) Multimarket oligopoly: strategic substitutes and complements. *Journal of Political Economy* 93, 488-511.

[5] Busetto F, Codognato G (2006) 'Very nice' trivial equilibria in strategic market games. *Journal of Economic Theory* 131, 295-301.

[6] Cordella T, Gabszewicz JJ (1998) 'Nice' trivial equilibria in strategic market games. *Games and Economic Behavior* 22, 162-169.

[7] Collie DR (2019) Taxation under oligopoly in a general equilibrium setting. *Journal of Public Economic Theory* 21, 738-753.

[8] Crettez B, Jouvet PA, Julien LA (2021) Pareto-improving supply subsidy in a simple general oligopoly equilibrium model with pollution permits. *Environmental Modeling and Assessment* 26, 999-1013.

[9] Christin C, Nicolaï JP, Pouyet J (2021). Pollution permits in oligopolies: the role of abatement technologies. Working Paper: https://hal.archives-ouvertes.fr/hal-03328947/document

[10] De Feo, Resende JG, Sanin, ME (2013) Optimal allocation of tradable emissions permits under upstream-downstream strategic interaction. *International Game Theory Review* 14, 1-23.

[11] Dickson A, Tonin S (2021) An introduction to perfect and imperfect competition via bilateral oligopoly. *Journal of Economics*, 2, 103-128.

[12] Dickson A, MacKenzie IA (2018) Strategic trade in pollution permits. *Journal of Environmental Economics and Management* 87, 94-113.

[13] Dickson A, Hartley R (2013) Bilateral oligopoly and quantity competition. *Economic Theory* 52, 979-1004.

[14] De Vries FP, Hanley N (2016) Incentive-based policy design for pollution control and biodiversity conservation: a review. *Environmental and Resource Economics* 63, 687-702.

[15] Elegbede C, Julien L, de Mesnard L (2021) On preferences and taxation mechanisms in strategic bilateral exchange. *Review of Economic Design* 26, 43-73.

[16] Eshel DMD (2005) Optimal allocation of tradable pollution rights and market structures, *Journal of Regulatory Economics* 28, 205-223.

[17] Gabszewicz JJ, Grazzini L (2001) Strategic multilateral exchange and taxes. In Debreu G, Neuefeind W,Trockel W (Eds), *Economic Essays, a Festschrift for Werner Hildenbrand*, Springer-Verlag, 109-126.

[18] Gabszewicz JJ, Grazzini L (1999) Taxing market power. *Journal of Public Economic Theory* 1, 475-497.

[19] Gabszewicz JJ, Michel P (1997) Oligopoly equilibria in exchange economies. In Eaton BC, Harris RG (Eds), *Trade, Technology and Economics. Essays in honour of R.G. Lipsey*, Edward- Elgar, Cheltenham, 217-240.

[20] Gersbach H, Requate T (2004) Emission taxes and optimal refunding schemes. *Journal of Public Economics* 88, 713-725.

[21] Godal O (2011) Noncooperative models of permit markets. Institute for Research in Economics and Business Administration, Bergen, Working Paper 18/11.

[22] Godal O, Holtsmark B (2010) International emissions trading with endogenous taxes. Statistics Norway, Research Department, Discussion Paper 626.

[23] Grazzini L (2006) A note on ad valorem and per unit taxation in an oligopoly model, *Journal of Economics* 89, 59-74.

[24] Hahn RW (1984) Market power and transferable property rights. *Quarterly Journal of Economics* 99, 753-765.

[25] Haita C (2014) Endogenous market power in an emissions trading scheme with auctioning. *Resources and Energy Economics* 37, 253–278.

[26] Hamilton SF, Sand R (1997) Subsidies in oligopoly markets: a welfare comparison between symmetric and asymmetric costs. *Public Finance Review*, 25, 660-668.

[27] Hintermann B (2017) Market power in emission permit markets: theory and evidence. *Environmental and Resource Economics* 66, 89–112.

[28] IPCC (2022). *Climate Change 2022: Impacts, Adaptation, and Vulnerability.* Contribution of Working Group II to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change [Pörtner HO, Roberts DC, Tignor M, Poloczanska ES, Mintenbeck K, Alegría A, Craig M, Langsdorf S, Löschke S, Möller V, Okem A, Rama B (Eds)]. Cambridge University Press. In Press.

[29] Julien LA, Tricou F (2012) Market price mechanisms and Stackelberg oligopoply equilibria: An example. *Bulletin of Economic Research* 64, 239-252.

[30] Lange A (2012) On the endogeneity of market power in emissions markets. *Environmental and Resource Economics* 52, 573–583.

[31] Lehmann P (2010) Justifying a policy mix for pollution control: a review of economic literature. *Journal of Economic Surveys* 26, 71-97.

[32] Metcalf GE (2019) On the Economics of a Carbon Tax for the United States. *Brookings Papers on Economic Activity, BPEA Conference Drafts*, March, 7-8.

[33] Metcalf GE, Weisbach D (2013) Carbon taxes. In: Shogren JF (Eds), *Encyclopedia of Energy, Natural Resource, and Environmental Economics*, Elsevier Science, 9-14.

[34] Montero JP (2009) Market power in pollution permit markets. *Energy Journal* 30, 115–142.

[35] Pigou AC (1932) *The Economics of Welfare*, 4th edn. London: Macmillan.

[36] Sanin ME, Zanaj S (2012) Clean technology adoption under Cournot competition. *Strategic Behavior and the Environment* 2, 159-172.

[37] Sartzadakis ES (2004) On the efficiency of competitive markets for emission permits. *Environmental and Resource Economics* 27, 1-19.

[38] Sartzadakis ES (1997) Tradeable emission permits regulations in the presence of imperfectly product markets: welfare implications. *Environmental and Resource Economic*, 9, 65-81.

[39] Schwartz S, Stahn H (2013) Competitive permit markets and vertical structures: the relevance of imperfectly competitive eco-industries. *Journal of Public Economic Theory* 16, 69-95.

[40] Shapley LS, Shubik M (1977) Trade using one commodity as a means of payment. *Journal of Political Economy* 85: 937-968.

[41] Shubik M (1973) Commodity money, oligopoly, credit and bankruptcy in a general equilibrium model. *Western Economic Journal* 11: 24-38.

[42] Stockey N (1998) Are there limits to growth?. *International Economic Review* 39, 1-31.

[43] Von der Fehr NH (1993) Tradable emission rights and strategic interaction. *Environmental and Resource Economics* 3, 129-151.

[44] Weisbach DA, Metcalf GE (2009) The Design of a Carbon Tax. *Harvard Environmental Law Review*, 33, 499-556.

[45] World Bank (1997). *Clear Water, Blue Skies*, World Bank, Washington, DC.

[46] Zhou D, An Y, Zha D, Wu F, Wang Q (2019) Would an increasing block carbon tax be better? A comparative study within the Stackelberg game framework. *Journal of Environmental Management* 235, 328-341.