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# A power consensus controller with overvoltage protection for meshed DC microgrids

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**Abstract:** A novel distributed power consensus control approach with overvoltage protection is proposed and analysed for meshed direct current (DC) microgrids (MGs) with constant power loads (CPLs). The DC MG considered herein consists of source and load nodes connected over an undirected weighted graph induced by the electrical circuit network, namely the conductance matrix. When deploying, the proposed controller features a second graph, that models the communication network over which the source nodes exchange information such as the instantaneous powers, and which is used to adjust the power injection accordingly to achieve power sharing. Additionally, one aims to maintain the voltage at each source below operator-set limits. This feature is critical given the power and voltage dependency. By addressing the occurrence of abnormal voltage values at different nodes in the network, one would guarantee a relatively safer power consensus policy and microgrid operation. To accommodate both objectives, we developed a nonlinear power consensus-based control system, with a voltage-limiting component, by means of Lyapunov analysis and ultimate boundedness theory. Asymptotic closed-loop stability is also established around a set of equilibria. Finally, numerical simulations align with and validate our theoretical findings.

*Keywords:* power consensus-based control, overvoltage protection, DC microgrids, CPLs.

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## 1. INTRODUCTION

The integration of renewable energy sources and storage systems, that are intrinsically of DC nature, drives the interest forward and maintains the focus on the design, control and operation of DC MG networks (Braitor, 2022). This is owed mainly to their unparalleled advantages such as higher efficiency and reliability, reduced energy conversion steps, no synchronisation issues, simpler control structure and expandability (Zolfaghari et al., 2022), and the prospect that they could be a future solution in replacing the existing grid to satisfy the local energy demand in remote areas through arrays of distributed generation units (DGUs).

From a standalone small-scale DC microgrid having a simple photovoltaic (PV) array installation, DC MGs can evolve into complex mesh topologies with DGUs that require autonomous control to increase reliability and improve power quality, whilst avoiding centralisation (Braitor et al., 2022). The current applications are ranging from road, air and maritime transportation, smart industry concepts and data centres, to household and community deployments (see Ahmed et al. (2020)). Moreover, the potential for upscaling has no ceiling.

Given the DC nature of renewable energy generation and with the number of DC end-consumers on the rise, we need to develop a deeper understanding of this class of dynamical networked systems. In this paper, we introduce and investigate a power consensus-based approach with

an overvoltage protection for standalone small-scale DC MGs with CPLs, that imposes power sharing among source nodes and ensures output voltage limitation.

### 1.1 Literature review

Consensus, or the state in which all agents in a network agree on a common state-dependent value, of multi-agent systems has been studied at large, in both static and dynamic form; see for instance (Tuna, 2009; Zhang et al., 2011; Bingqiang et al., 2021; Axelson-Fisk and Knorn, 2022). Implementing a similar policy in DC MG networks seemed a natural choice in attaining power sharing.

In retrospect, numerous research works have explored consensus-based strategies to achieve power sharing in DC networks to avoid the overusage of their sources, with decent results. A distributed control strategy to enforce power sharing in a cluster of DC MGs has been introduced by Moayedi and Davoudi (2016). A power consensus algorithm has been developed by De Persis et al. (2018) for DC microgrids with ZIP loads. Secondary consensus-based approaches to guarantee accurate power sharing have been reported in Tucci et al. (2017); Liu et al. (2018); Braitor et al. (2022). A plethora of research works have aimed to tackle the challenges that consensus-based methods in DC microgrids impose, owed to the interaction between the cyber and physical networks. Moreover, the instabilities introduced into the DC MG systems by constant power loads that could eventually lead

to voltage collapse have been also extensively characterised in Simpson-Porco et al. (2016) and Liu et al. (2022). The conditioning at the load side is typically hard to overcome without proper control in place.

Above all, achieving power sharing remains a sought-after result as it eliminates the overusage instances of identical sources, and enforces the sources to inject power into the network based on their power ratings. Andreasson et al. (2014) proposes a distributed control approach to achieve decent voltage regulation and power sharing; a tight voltage regulation and accurate power sharing has been attained in the secondary distributed controller proposed by (Braiton et al., 2022). A passivity-based strategy has been employed in Zonetti et al. (2015) for HVDC systems to reach global asymptotic stability, while in Zonetti et al. (2018) the authors study the feasibility and power sharing under decentralised droop control. Stability and power sharing under decentralised droop control has also been pursued in (Su et al., 2018). An array of other schemes can achieve power sharing, but at the moment, most are lacking formal theoretical analysis.

Droop controllers to ensure power sharing, with an overvoltage protection for both parallel and mesh configurations with CPLs, have been proposed in Braiton et al. (2020, 2021). However, when employing droop controllers, power sharing becomes inaccurate particularly in high loading conditions.

### 1.2 Main contributions

The objective of this paper is to develop a novel consensus approach that achieves critical objectives. They can be briefly summarised as follows:

- enforce power injection at each source node based on operator-set ratios;
- simultaneously ensure an overvoltage protection regardless of the system parameters, network configuration, or loading conditions;
- additionally, the proposed method must stabilise the CPLs negative impedance, well-known to introduce instabilities into the system.

Compared to the power consensus developed in De Persis et al. (2018) that can also guarantee the voltages staying within a compact set around an operating point, the proposed method can rigorously prove an inherent overvoltage protection at all times, even during transients. Also, unlike Braiton et al. (2021), the proposed method can achieve accurate power sharing regardless of the load power demand. Although both use a resistive DC MG network, the present approach considers a different network model.

### 1.3 Paper organisation

The rest of the paper is organised as follows. The remainder of this section briefly revisits necessary notations and relevant preliminaries. Section 2 presents the meshed DC microgrid configuration and dynamics to be subjected to further investigation. In Section 3, the proposed power consensus method with overvoltage protection is developed and analysed. Herein, one provides the methodology to attain both power consensus control and voltage limitation. In Section 4, one investigates the closed-loop stability

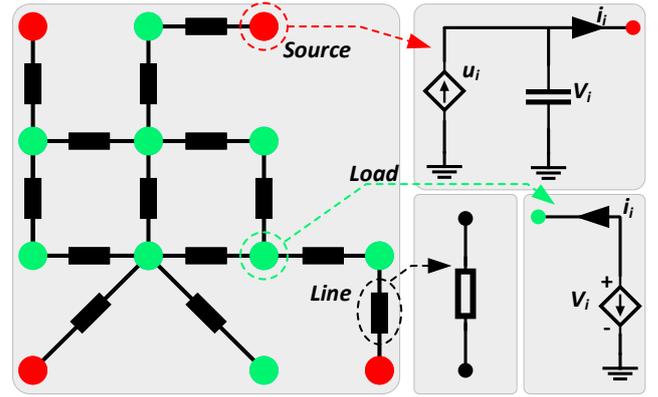


Fig. 1. Meshed DC microgrid network featuring load (●) and source (●) nodes

of the overall system. For testing purposes, a standalone small-scale microgrid example is then numerically simulated in Section 5 to prove the concept. Closing remarks and conclusions are drawn in Section 6.

### 1.4 Notation and preliminaries

*Notations.* Let  $\mathbf{0}_n$  be the zero vector of size  $n$ . For an  $n$ -tuple sequence  $(x_1, \dots, x_n)$ , let  $\mathbf{x} \in \mathbb{R}^n$  be the associated vector, and  $[\mathbf{x}] \in \mathbb{R}^{n \times n}$  the diagonal matrix whose diagonal entries are the elements of vector  $\mathbf{x}$ . For  $A$  and  $B$  matrices with the same number of columns, then  $col(A \ B)$  denotes the matrix  $[A \ B]'$ .

*Definition 1.* (Synchronisation). The network system  $\dot{x} = Ax + Bu$ , with  $u = \mathcal{L}x$  is said to achieve asymptotic synchronisation<sup>1</sup> if and only if

$$\lim_{t \rightarrow \infty} (x_i(t) - x_j(t)) = 0, \quad i, j: (v_i, v_j) \in \mathcal{E}.$$

## 2. DC MICROGRID MODEL AND CONFIGURATION

The DC microgrid portrayed in Fig.1 induces an undirected connected graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , with  $\mathcal{V}$  representing the set of vertices (bus nodes), and  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  the set of edges, representing the resistive interconnection lines in the network. The set of nodes  $\mathcal{V}$  is divided in two subsets; the sources subset  $\mathcal{V}_S$ , and the loads subset  $\mathcal{V}_L$ , such that  $\mathcal{V} = \mathcal{V}_S + \mathcal{V}_L$ . Then, let the current and voltage vectors be denoted as  $i = col(i_S \ i_L) \in \mathbb{R}^{|\mathcal{V}_S + \mathcal{V}_L|}$  and  $V = col(V_S \ V_L) \in \mathbb{R}^{|\mathcal{V}_S + \mathcal{V}_L|}$ , with  $i_S = \{i_i: i \in \mathcal{V}_S\}$ ,  $i_L = \{i_i: i \in \mathcal{V}_L\}$  and  $V_S = \{V_i: i \in \mathcal{V}_S\}$ ,  $V_L = \{V_i: i \in \mathcal{V}_L\}$ . The current-voltage relation will be given by the expression  $i = YV$ , where  $Y$  represents the conductance matrix, as follows

$$\begin{bmatrix} i_S \\ i_L \end{bmatrix} = \begin{bmatrix} Y_{SS} & Y_{SL} \\ Y_{LS} & Y_{LL} \end{bmatrix} \begin{bmatrix} V_S \\ V_L \end{bmatrix}. \quad (1)$$

Note that the diagonal entries  $Y_{SS}$  and  $Y_{LL}$  are positive definite matrices since they are principal submatrices of the Laplacian of a connected undirected graph  $\mathcal{G}$ .

*Remark 1.* As shown previously in Braiton et al. (2021), matrix  $Y_{LL}$ , when having CPLs, can be expressed as a sum

<sup>1</sup> Asymptotic synchronisation, as defined above, is the most intuitive case of synchronisation, and it corresponds to consensus among agents  $x_i = x_j$ ,  $j \in \mathcal{N}_i$ .

between a positive semi-definite and a positive-definite matrix. However, the positive-definiteness property of both matrices  $Y_{SS}$  and  $Y_{LL}$  is more or less obvious. Nonetheless, one can trivially prove this property using Cauchy's interlacing theorem.

Based on this fact, one is allowed to eliminate the load voltages as  $V_L = Y_{LL}^{-1}(i_L - Y_{LS}V_S)$  and reduce the network to the source nodes  $\mathcal{V}_S$  with balance equations

$$i_S - Y_{SL}Y_{LL}^{-1}i_L = Y_{red}V_S \quad (2)$$

where  $Y_{red} = Y_{SS} - Y_{SL}Y_{LL}^{-1}Y_{LS}$  is known as the Kron-reduced conductance matrix (Dorfler and Bullo, 2013), and  $-Y_{SL}Y_{LL}^{-1}i_L$  is the mapping of the load current injections to the sources.

By applying Kirchhoff's laws for the source nodes in Fig. 1, one can write the voltage dynamics, at each source, as described below

$$C_i \frac{dV_i}{dt} = -i_i + u_i, \quad i \in \mathcal{V}_S \quad (3)$$

where  $V_i$  and  $i_i$  represent the capacitor voltage and line current, respectively, while  $u_i$  is the converter current, also utilised as the control input.

### 3. POWER CONSENSUS CONTROL DESIGN

One assumes that all sources  $\mathcal{V}_S$  are controlled as a function of the measured local current  $i_i$  and network power injections  $P_i$ , as well as the injected power  $P_j$  at neighbouring sources that need to be communicated among their respective neighbours.

*Objective 1.* The proposed power consensus control design must achieve weighted proportional power sharing proportional to operator-set gains  $m_i > 0$ . That is, given  $P_i = i_i V_i, \forall i \in \mathcal{V}_S$ , the first objective is to guarantee that at steady state the following identities hold

$$m_j P_j = m_i P_i, \quad i, j \in \mathcal{V}_S, j \in \mathcal{N}_i. \quad (4)$$

*Objective 2.* At the same time, one seeks to enforce an upper limit for the output voltage at each source, i.e.

$$V_i \leq V_{max,i}, \quad i \in \mathcal{V}_S. \quad (5)$$

A unified approach that incorporates the weighted power consensus-based control with the voltage limitation is being proposed in this section. In achieving the set dual-objective, that is to guarantee identity (4) and satisfy inequality (5), the control input  $u_i$  takes the following expression

$$u_i = -g_i (V_i - \sigma_i), \quad i \in \mathcal{V}_S, \quad (6)$$

where  $g_i$  is a positive constant, and  $\sigma_i$  follows the nonlinear dynamics

$$\dot{\sigma}_i = \left( 1 - \frac{(2\sigma_i - V_{max,i})^2}{V_{max,i}^2} \right) \sum_{j \in \mathcal{N}_i} (m_j P_j - m_i P_i) \quad (7)$$

with  $i, j \in \mathcal{V}_S$ , that incorporates a control state-dependent bounded nonlinear gain and the power consensus.

*Remark 2.* (Physical implementation). The controller deployment is straightforward as it can be implemented at each source node by measuring the local voltage and current injection. The computed power is then shared on the cyber network to the neighbouring source nodes.

To better understand the dynamics in equation (7), consider the following proposition.

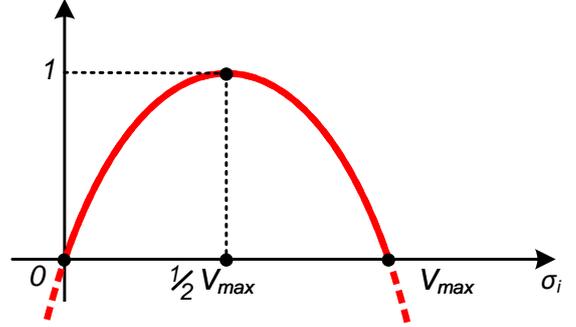


Fig. 2. Graphical representation of the set  $W_{i0}$

*Proposition 1.* The set  $W_{i0}$  defined as

$$W_{i0} = \left\{ \sigma_i \in \mathbb{R}: 1 - \frac{(2\sigma_i - V_{max,i})^2}{V_{max,i}^2} > 0 \right\}$$

is a positive invariant set for any selection of the initial condition of the controller state  $\sigma_{i0}$  in the set  $W_{i0}$ .

**Proof.** Consider the following continuously differentiable Lyapunov function candidate

$$W = \frac{\sigma_i^2}{2V_{max,i}^2}, \quad i \in \mathcal{V}_S. \quad (8)$$

Then by calculating the time derivative, and using the controller expression in (7), one gets

$$\begin{aligned} \dot{W} &= \frac{\sigma_i}{V_{max,i}^2} \dot{\sigma}_i \\ &= \frac{\sigma_i}{V_{max,i}^2} \left( 1 - \frac{(2\sigma_i - V_{max,i})^2}{V_{max,i}^2} \right) \sum_{j \in \mathcal{N}_i} (m_j P_j - m_i P_i) \end{aligned} \quad (9)$$

with  $i, j \in \mathcal{V}_S$ . From equation (9), one notices that excluding synchronisation (as described in Definition 1) when consensus is reached (i.e.  $m_j P_j = m_i P_i$ ), or at the origin where  $\sigma_i = 0$ ,  $\dot{W}_i$  becomes zero at the roots of the parabola  $1 - \frac{(\sigma_i - V_{max,i})^2}{V_{max,i}^2}$ . This indicates that if the initial condition of the controller state,  $\sigma_{i0}$ , is selected in the set  $W_{i0}$  then, intuitively from equation (9), it becomes clear that  $\dot{W}_i$  will eventually become zero as it travels on the described parabola, i.e.  $\dot{W}_i \rightarrow 0, \forall t \geq 0$ .

That essentially means that  $\sigma_i$  will start and remain in the set  $W_{i0}$  for all  $t \geq 0$ . In other words, the set  $W_{i0}$  is a positive invariant set. Hence, the proof is complete.  $\square$

Therefore, by virtue of the above proof, a sensible choice for the initial condition  $\sigma_i$  could be anywhere within  $0 < \sigma_i < V_{max,i}$ . For instance, by considering a typical initial condition of the integral state  $\sigma_i$ , i.e.  $\sigma_i(0) = \frac{1}{2}V_{max,i}$ , then the controller state remains following the equilibria of equation (7), within the interval  $\sigma_i(t) \in [0, V_{max,i}]$ , for all  $t \geq 0$ .

*Remark 3.* The motivation to keep the control state on well-defined curves stemmed from existing bounded control methods. More details and further insights are found in Konstantopoulos et al. (2016); Konstantopoulos (2017); Konstantopoulos and Baldivieso-Monasterios (2019).

It must be highlighted that the control variable  $\dot{\sigma}_i$  becomes zero

i) at asymptotic synchronisation where one has

$$\sum_{j \in \mathcal{N}_i} (m_j P_j - m_i P_i) = 0$$

which ensures accurate power consensus among all source nodes, or

ii) when  $\sigma_i = 0$  and  $\sigma_i = V_{max,i}$ , the latter corresponding to the overvoltage protection as it shall be explained in Proposition 2.

*Remark 4.* (A similar power consensus). With a virtually similar closed-loop system, but a potentially simpler method inspired by Zhao and Dörfler (2015) and based on a distributed averaging integral control, one has

$$\begin{aligned} C_i \dot{V}_i &= -i_i + p_i \\ D_i \dot{p}_i &= i_i - p_i + \sum_{j \in \mathcal{N}_i} (K_j^{-1} V_j p_j - K_i^{-1} V_i p_i). \end{aligned}$$

with  $k_{i,j}$  being positive gains. However, numerical results show that Zhao and Dörfler (2015) performs similarly to (De Persis et al., 2018), based on tests carried out in the latter. Additionally, (De Persis et al., 2018) guarantees that all voltages stay within a compact set around an operating point. However, the strategy has not incorporated an analytic proof for the overvoltage capability.

The present strategy aims to fill that gap in the following subsection, by formalising a proof.

### 3.1 Overvoltage protection

By replacing the control input  $u_i$  from (6) into the open-loop current dynamics (3), one obtains the following closed-loop current dynamics

$$C_i \frac{dV_i}{dt} = -i_i - g_i V_i + g_i \sigma_i, \quad i \in \mathcal{V}_S. \quad (10)$$

Now, let us define the following proposition that ensures voltage limitation at each source node.

*Proposition 2.* The solution  $V_i(t)$  of (10) with the initial condition  $V_i(0) \leq V_{max,i}$  is uniformly ultimately bounded, i.e.  $|V_i(t)| < V_{max,i}$ , at all future times  $t \geq 0$ .

**Proof.** Consider the following continuously differentiable Lyapunov function candidate

$$E_i = \frac{1}{2} C_i V_i^2, \quad i \in \mathcal{V}_S. \quad (11)$$

The time derivative of  $E$  yields

$$\dot{E}_i = C_i V_i \dot{V}_i = -i_i V_i - g_i V_i^2 + g_i \sigma_i V_i, \quad i \in \mathcal{V}_S \quad (12)$$

and provided the bounded state  $\sigma_i \in [0, V_{max,i}]$  from Proposition 1, one has

$$\dot{E}_i \leq -i_i |V_i| - g_i V_i^2 + g_i V_{max,i} |V_i|, \quad i \in \mathcal{V}_S. \quad (13)$$

Given the direction of the power-flow, i.e. from source nodes to load nodes, the current  $i_i > 0$ , which then implies

$$\dot{E}_i \leq -i_i |V_i|, \quad \forall |V_i| \geq V_{max}, \quad i \in \mathcal{V}_S. \quad (14)$$

By virtue of (14), the solution  $V_i(t)$  is uniformly ultimately bounded. Therefore, if initially  $|V_i(0)| \leq V_{max,i}$ , then it holds that

$$|V_i(t)| \leq V_{max,i}, \quad \forall t > 0, \quad i \in \mathcal{V}_S \quad (15)$$

due to invariant set property. The proof is complete.  $\square$

Notice that any selection of the positive constants  $V_{max,i}$  results in the desired overvoltage protection (15). That is, an upper limit for the source nodes output voltage is ensured regardless of the network configuration or system parameters.

A key feature of the proposed controller is that it slows down near the imposed limits, therefore, it does not suffer from integrator wind-up which may introduce instability into the system.

*Remark 5.* (Nonlinear consensus method). When comparing the present control strategy with related nonlinear consensus algorithms proposed previously in the literature, such as the work of Cortés (2008), one would firstly notice that the differentiation between sources and loads is ignored, leading to a modified closed-loop system (10). However, similar to others, the developed approach is different in the sense that it uses another layer of weighted averaging introduced by the cyber network, separate from the averaging induced by the physical network.

## 4. STABILITY ANALYSIS

In this section, local stability for the closed-loop system (7),(10) is being studied around the equilibrium point  $V_{ie}, \sigma_{ie}$ . The equilibria of  $\sigma_i$  are trivial to observe; there are two equilibrium points at  $\mathbf{0}_{|\mathcal{V}_S|}$  and  $V_{max,i}$ , the latter being of interest as it corresponds to the maximum allowed voltage.

*Proposition 3.* The system (7),(10) is locally asymptotically stable around the equilibria set pair  $(V_e, \sigma_e)$ .

**Proof.** One should proceed by first looking at the corresponding Jacobian matrix  $J$  expressed below

$$J = \begin{bmatrix} -C^{-1} (Y_{SS} + g) & g \\ \mathbf{0}_{|\mathcal{V}_S|} & -2(2\sigma_e - V_{max}) V_{max}^{-2} \mathcal{L} m P \end{bmatrix}$$

with  $C = \text{diag}\{C_i\}$ ,  $g = \text{diag}\{g_i\}$ ,  $\sigma_e = \text{diag}\{\sigma_{ie}\}$ ,  $m = \text{diag}\{m_i\}$ ,  $P = \text{diag}\{P_i\}$ , while  $\mathcal{L}$  represents the Laplacian matrix induced by the communication network and  $\mathbf{0}_{|\mathcal{V}_S|}$  is the zero matrix of suitable dimensions.

The standard eigenvalue problem (SEP) for  $J$  is  $Jv = \lambda v$ , with  $\lambda$  being an eigenvalue and  $v$  the corresponding eigenvector. Note that matrix  $J$  can be rewritten as a multiplication of symmetric matrices

$$J = \underbrace{\begin{bmatrix} -C^{-1} & \mathbf{0}_{|\mathcal{V}_S|} \\ \mathbf{0}_{|\mathcal{V}_S|} & -2(2\sigma_e - V_{max}) (V_{max})^{-2} \end{bmatrix}}_{X_1} \times \underbrace{\begin{bmatrix} Y_{SS} + g & \mathbf{0}_{|\mathcal{V}_S|} \\ \mathbf{0}_{|\mathcal{V}_S|} & \mathcal{L} \end{bmatrix}}_{X_2} \underbrace{\begin{bmatrix} I_n & \mathbf{0}_{|\mathcal{V}_S|} \\ \mathbf{0}_{|\mathcal{V}_S|} & m P \end{bmatrix}}_{\bar{Q}}, \quad (16)$$

the SEP then becoming

$$X_1 X_2 \bar{Q} v = \lambda v. \quad (17)$$

Given matrix  $\bar{Q} > 0$ , let  $y = \bar{Q} v$  resulting in

$$X_1 X_2 y = \lambda \bar{Q}^{-1} y. \quad (18)$$

The SEP becomes a generalised eigenvalue problem (GEP). Matrix  $X_2$  is positive semidefinite with kernel spanned by  $[\mathbf{0}'_n \mathbf{1}'_n]'$  corresponding to the asymptotic synchronisation (as explained in Definition 1) or the states

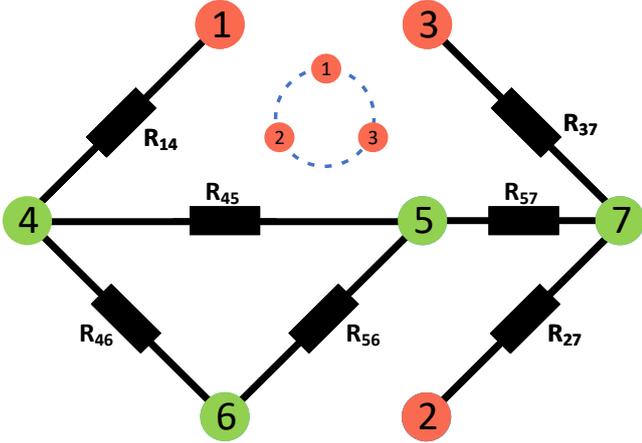


Fig. 3. Power (—) and communication (---) network configurations considered for simulation testing, featuring 3 source (●) and 4 load (●) nodes

reaching consensus, while matrix  $X_1$  is positive definite with kernel spanned by  $\mathbf{0}_{2n}$ . Similarly to the proof of (Braitor et al., 2022, Th.2) and (Simpson-Porco et al., 2013, Th.8), by applying the Courant-Fischer Theorem to the eigenvalue problem, for asymptotic synchronisation, all eigenvalues of matrix  $J$  are real and negative since  $\text{Im}(\mathcal{L}) = \mathbf{1}_n^\perp$ , and  $\text{Im}(X_1) \cap \text{ker}(X_2) = \mathbf{0}_{2n}$ , which means that  $X_2 y$  is never in the kernel of  $X_1$ .

Therefore, it is clear that  $\text{ker}(J) = \text{ker}(X_2)$ . Since the image of matrix  $X_1$  excludes  $\text{span}([\mathbf{0}'_n \mathbf{1}'_n]')$ , it follows that  $Jy$  is the null vector if and only if  $y \in \text{span}([\mathbf{0}'_n \mathbf{1}'_n]')$  that essentially corresponds to the asymptotic synchronisation. Thus, matrix  $J$  is Hurwitz.

The system is locally asymptotically stable. This completes the proof  $\square$

## 5. SIMULATION RESULTS

To test and highlight the capabilities of the proposed control approach on a small-scale MG concept, consider the low-power 7-node meshed DC microgrid concept depicted in Fig. 3, with the parameters specified in Table 1, being simulated for 0.2 s. All network nodes are interconnected with their respective neighbours via a resistive DC network, operating at a nominal voltage of 20 V. Notice that each source node shares information about instantaneous injected power with its respective neighbours via a diffusive-coupling connected communication network, as displayed in Fig. 3.

For the numerical validation of the proof of concept, the main task is to ensure power sharing among the 3 source nodes in a 1 : 2 : 1 ratio, that is at synchronisation one has  $m_1 P_1 = m_2 P_2 = m_3 P_3$ , whilst ensuring that their output voltages remain below their specified limit, i.e.  $V_i \leq V_{max,i}, \forall i \in \{1, 2, 3\}$ . The dynamic response of the system states is displayed in Fig 4.

Starting at  $t = 0$  s, the sources are feeding the load power demand  $P_L$  specified in Table 1, and accurately share their injected power in 1 : 2 : 1 manner, having  $P_1 = 15.35$  W,  $P_2 = 30.7$  W, and  $P_3 = 15.35$  W (Fig 4c). Note that the output voltages remain below their maximum set limit  $V_{max,i}$ .

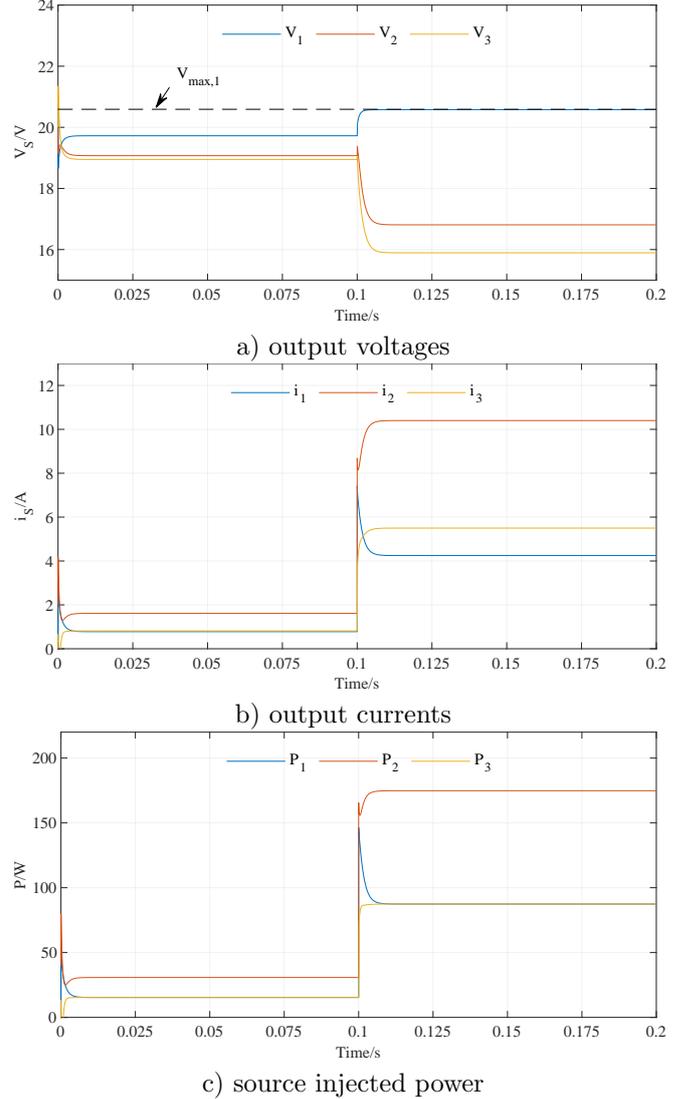


Fig. 4. Dynamic response of the system states

The load power changes at  $t = 0.1$  s, with the new load power demand vector being  $P_L = [60 \ 72 \ 54 \ 30]$  W. At node 1, the voltage forces the bound  $V_{max,1} = 20.5$  V, but it is promptly limited (Fig 4a). The power consensus is kept accurate having  $P_1 = 87.4$  W,  $P_2 = 174.8$  W, and  $P_3 = 87.4$  W as one can observe in (Fig 4c).

The proposed approach manages to successfully enforce accurate power sharing among source nodes, with an inherent overvoltage protection.

## 6. CONCLUSIONS

A power consensus algorithm with overvoltage protection has been presented in this paper aimed for meshed DC microgrids that include CPLs. Apart from the achievable desired power sharing, by using nonlinear analysis tools, an overvoltage protection can be analytically guaranteed at each source node, independently of the system states or parameters. The closed-loop stability of the DC system has been established around an equilibria set. For the proof of concept, a low-power meshed DC microgrid, with diffusive-coupling cyber network for the source nodes, has been tested to highlight the superiority of the strategy and its theoretical findings.

Table 1. System and control parameters

System Parameters	Numerical Values
$R_{14}$	0.8 $\Omega$
$R_{27}$	0.3 $\Omega$
$R_{37}$	0.9 $\Omega$
$R_{45}$	0.7 $\Omega$
$R_{46}$	0.4 $\Omega$
$R_{56}$	0.5 $\Omega$
$R_{57}$	0.5 $\Omega$
$g$	[20 20 20]
$V_{max}$	[20.5 21 24] V
$m$	[0.42 0.21 0.42]
$C$	[250 230 220] $\mu F$
$P_L$	[10 12 9 1] W

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