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# Wear in Partial Slip Contact

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*An analytical method that evaluates the evolution of stress and surface profile in fretting under the partial slip conditions is presented. The repeated slip occurring near the edges of contact generates wear that changes the contact geometry and contact stresses. The method is based on two scales of time: time for one cycle of the oscillating tangential force and time corresponding to the number of cycles. Archard's wear law is used to evaluate wear and gap variation within the slip zones during one cycle. The governing integral equations are reduced to calculate the contact pressure after each cycle. Evolution of the contact characteristics (contact pressure and shear stress, contact width, gap and slip functions) in fretting is calculated using a stepwise procedure. It is shown that the size of stick zone does not change in wear process of bodies with similar elastic properties under the constant amplitude load conditions, and that an asymptotic solution corresponding to the number of cycles approaching to infinity exists. Analytical expressions for the asymptotic contact pressure, shear and tensile stress, and the gap function are presented. It is proved that the asymptotic contact pressure and shear stress are singular at the ends of stick zone. Detailed results are given for two initial shapes of elastic indenter contacting with an elastic half-space: for the parabolic cylinder and for the indenter having a flat base with rounded edges.*

## 1 Introduction

Partial slip contacts occur in machine components under conditions of small amplitude vibrations. Partial slip is characterized by existence of stick and slip zones within the contact area. The necessary condition to realize partial slip contact is  $Q < \mu P$ , where  $Q$  and  $P$  are the tangential and normal forces applied to the components, and  $\mu$  is the friction coefficient.

If  $Q$  oscillates then repeating slip, which is opposite to the tangential force direction, occurs within the slip zones. This small scale repeated slip displacement is called fretting and it leads to mechanical wear of the contacting bodies. Figure 1 represents the traces through the contact width of the initial surface and the worn surface after 86500 cycles in a fretting fatigue experiment conducted during partial slip conditions. The traces of the pad profiles show that a gap due to wear is formed near the ends of the contact region. Note that wear at the trailing slip zone is considerably more than wear at the leading edge because of the asymmetric stress distribution induced by the bulk tensile load applied to the specimen (see Szolwinski and Farris [1], Szolwinski et al. [2], and McVeigh et al. [3]).

The shape variation of the interacting surfaces results in redistribution of the contact pressure and a change of contact width. It was shown by Rabinowicz [4] and Stowers and Rabinowicz [5] that the wear rate is proportional to the contact pressure and the relative sliding velocity. Thus, the redistribution of the contact pressure in turn influences the wear rate in the slip zones.

The evolution of contact pressure due to wear in fretting was considered by Johansson [6] using the finite element method to perform the contact stress analysis. The time scale was discretized to allow for the application of the shear stress,  $Q$ , as a function of time. Archard's wear equation was coded into the algorithm, and it was assumed that all the wear particles leave the contact zone. The results obtained by Johansson for the cylindrical indenter indicate that the pressure distribution after a period of fretting differs considerably from the initial Hertz type pressure distribution.

Analytical investigations of wear contact problems which take into account the irreversible change of surface geometry due to wear were developed by Korovchinsky [7], Galin [8], Galin and Goryacheva [9], Goryacheva [10,11], etc. All functions (pressure distribution, wear and elastic displacements of the surface, etc.) in the wear contact problem are time-dependent and inter-connected.

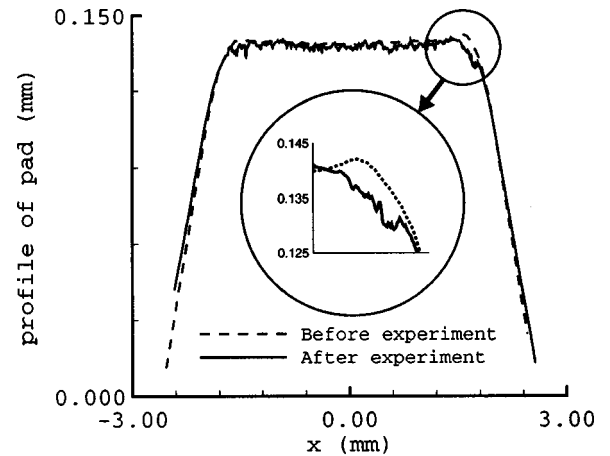
In the present paper, an analytical model of wear contact problems is used to investigate the pressure and geometry evolution in fretting under conditions of partial slip contact. Following Johansson [6], it is assumed that the wear particles do not accumulate between the contacting surfaces. The pressure evolution in wear process is considered for two different shapes of the indenter, i.e., for the parabolic cylinder and for an indenter having a flat base with rounded edges.

## 2 Problem Formulation

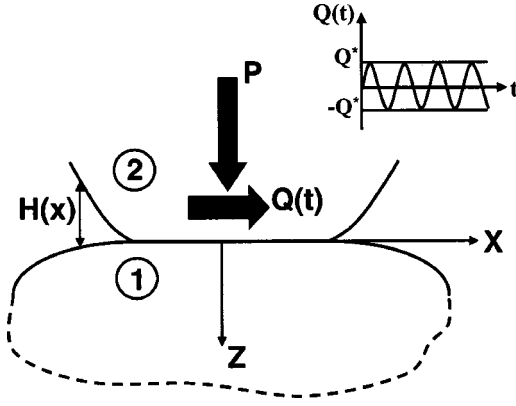
The two-dimensional contact problem illustrated in Fig. 2 for two elastic bodies having undeformed shapes specified by the functions  $z = f_1(x)$  and  $z = -f_2(x)$  related to the system of coordinates  $Oxz$  ( $f_i(x) = f_i(-x)$ ) is considered. The  $Oz$  axis coincides with the axis of symmetry of the contacting bodies with normals common to the two surfaces at  $O$ . The bodies are subjected to the constant normal force  $P$  and the oscillating tangential force  $Q(t)$  so that  $-Q^* \leq Q(t) \leq Q^*$  and there is no moment.

The following condition is satisfied:

$$|Q^*| < \mu P, \quad (1)$$



**Fig. 1** Profilometer traces of a fretting pad after 86500 cycles in fretting fatigue experiment involving partial slip contact. The applied loads were  $P = 1.2 \times 10^6$  N/m,  $Q^* = 3.7 \times 10^5$  N/m.



**Fig. 2 Schematic of two-dimensional partial slip contact between two elastic bodies subject to a normal load and an oscillating tangential force**

where  $\mu$  is the friction coefficient. It is observed in fretting fatigue experiments that  $\mu$  increases with the number of cycles before reaching a steady-state value for most of the fretting fatigue life. In the model presented here we assume that  $\mu$  is constant (steady-state value) and that the static and kinetic coefficients of friction are the same.

It follows from Eq. (1) that this is a partial slip contact with a central stick zone surrounded by two slip zones. Due to relative displacements, wear of the contacting bodies occurs within the slip zones. For different mechanisms of wear the wear rate,  $\partial w / \partial t$ , is approximated by the following equation:

$$\frac{\partial w}{\partial t} = K_w p^\alpha(x,t) V^\beta(x,t), \quad (2)$$

where  $p(x,t)$  is pressure,  $V(x,t)$  is sliding velocity, and  $K_w$  is known as the wear coefficient.

In the problem under consideration

$$V(x,t) = \left| \frac{\partial s(x,t)}{\partial t} \right|, \quad (3)$$

where  $s(x,t)$  is the slip at the point  $x$  for the instant  $t$ . It is determined by

$$s(x,t) = \begin{cases} 0, & \text{if } |x| < c(t), \\ (u_{x1} - u_{x2}) - \delta_x, & \text{if } c(t) \leq |x| \leq a(t), \end{cases} \quad (4)$$

where  $u_{x1}$  and  $u_{x2}$  are the tangential displacements of the contact surfaces at the point  $x$ ,  $\delta_x$  is the relative tangential displacement of the contacting bodies,  $c(t)$  is the size of the stick zone at the instant  $t$ . It follows from Eqs. (2), and (3) that for  $\alpha = \beta = 1$

$$\frac{\partial w}{\partial t} = K_w p(x,t) \left| \frac{\partial s(x,t)}{\partial t} \right|. \quad (5)$$

Due to wear the shape of contacting bodies varies, which gives rise to the redistribution of the contact stresses. Note that in the problem under consideration there are two scales of time. The first scale is connected with the time of one cycle. During one cycle the redistribution of shear contact stress and variation of the stick zone size take place in response to the variation of tangential force  $Q(t)$ . The redistribution of shear stress does not influence the contact pressure for similar contacting bodies. For dissimilar materials the redistribution of the normal contact stress due to shear stress is negligibly small. Taking also into account that the shape variation is negligibly small in one cycle, it follows that the changes of the contact pressure and the contact width are also negligibly small in one cycle. The changes of these values are connected with the second time scale which is the number of cycles. In what follows the wear process is calculated first in one

cycle, and then from cycle to cycle using a stepwise procedure. For the sake of simplicity of analysis we have assumed that the wear constant,  $K_w$ , is independent of the amount of wear between the contacting bodies (constant with time). It is shown in Section 5 that the asymptotic distributions of pressure, shear, and wear do not depend on the wear constant  $K_w$ . Hence the rate of wear is affected by  $K_w$  but not the magnitude of wear.

### 3 Wear During One Cycle

The value  $\Delta w(x,N)$  denotes the increment of wear of both bodies at the  $N^{\text{th}}$  cycle. Thus the wear after  $N$  cycles is calculated by

$$w(x,N) = \sum_{n=1}^N \Delta w(x,n). \quad (6)$$

The pressure at the  $N^{\text{th}}$  cycle is denoted by  $p(x,N)$ , and the contact half-width by  $a(N)$ .

In partial contact wear occurs only in the slip zones. The sizes of stick and slip zones change during a cycle. If  $Q = Q^*$  the shear stresses at the  $N^{\text{th}}$  cycle are distributed according to the following formula:

$$q_N(x,c^*) = \begin{cases} \mu p(x,N) - q^*(x,N), & \text{if } |x| < c^*, \\ \mu p(x,N), & \text{if } c^* \leq |x| \leq a(N), \end{cases} \quad (7)$$

where  $c^*$  is the size of the stick zone. For contacting bodies of similar materials ( $E_1 = E_2 = E$ ,  $\nu_1 = \nu_2 = \nu$ ) the function  $q^*(x,N)$  is determined from the equation:

$$\int_{-c^*}^{c^*} \frac{q^*(t,N) dt}{x-t} = \frac{\pi \mu E^*}{2} H'(x,N), \quad |x| < c^*, \quad (8)$$

where  $E^* = E/2(1-\nu^2)$ . The function  $H(x,N)$  is the unloaded gap between the bodies at the  $N^{\text{th}}$  cycle. The function  $H'(x,N)$  is the derivative of the function  $H(x,N)$  with respect to  $x$ .

If  $Q(t) < Q^*$  the half-width of the stick zone  $c(t)$  changes from  $c^*$  to  $a(N)$ . During one cycle the time variation of the shear contact stress  $q_N$  depends on the function  $c(t)$ , i.e.,  $q_N = q_N(x,c(t))$ . The size of a stick zone  $c(t)$  is determined from the equilibrium condition

$$Q(t) = \int_{-a(N)}^{a(N)} q_N(x,c(t)) dx. \quad (9)$$

The value of slip at any point  $x$  ( $c(t) \leq |x| \leq a(N)$ ) is determined by

$$s_N(x,c(t)) = -\frac{2}{\pi E^*} \int_{-a(N)}^{a(N)} q_N(x',c(t)) \ln \left| \frac{x-x'}{c(t)-x'} \right| dx'. \quad (10)$$

Since  $s_N(x,c(t))$  is a function of  $x$  and  $c$ , it follows from Eq. (2) that

$$\frac{\partial w(x,t)}{\partial t} = K_w p(x,t) \left| \frac{\partial s(x,c)}{\partial c} \cdot \frac{dc}{dt} \right|. \quad (11)$$

Integrating Eq. (11) during the time  $\Delta t$  of one cycle and using the mean value theorem, we obtain

$$\begin{aligned} \Delta w(x,N) &= K_w \int_{(N-1)\Delta t}^{N\Delta t} p(x,t) \left| \frac{\partial s(x,c)}{\partial c} \cdot \frac{dc}{dt} \right| dt \\ &\approx 2K_w \bar{p}(x,N) \left| \int_x^{c^*} \frac{\partial \bar{s}_N(x,c)}{\partial c} dc \right| \\ &= 2K_w \bar{p}(x,N) |\bar{s}_N(x,c^*)| \\ &\approx K_w [p(x,N-1) |s_{N-1}(x,c^*)| \\ &\quad + p(x,N) |s_N(x,c^*)|] \quad |x| > c^*, \end{aligned} \quad (12)$$

where  $\bar{p}(x, N)$  and  $\bar{s}_N(x, c^*)$  are values of pressure and slip at some intermediate point of the  $N^{\text{th}}$  cycle. The last relation is obtained using the condition  $\bar{s}_N(x, x) = 0$ , and also the assumption that the function  $|\bar{s}_N(x, c)|$  is the same during the loading and unloading of the tangential force  $Q(t)$ . The function  $\Delta w(x, N)$  determined by Eq. (12) satisfies the condition

$$\Delta w(c^*, N) = \Delta w(a(N), N) = 0. \quad (13)$$

We use this condition only at the right hand side of the contact zone because the functions  $w(x, N), p(x, N), q(x, c)$ , etc. are symmetric.

Thus to determine the increment of wear at the  $N^{\text{th}}$  cycle we must calculate the relative slip  $s_N(x, c^*)$  corresponding to the maximum tangential force  $Q^*$ .

**Calculation of the Function  $s_N(x, c^*)$  at the  $N^{\text{th}}$  Cycle.** The function  $s_N(x, c^*)$  is determined using Eq. (10) as

$$s_N(x, c^*) = -\frac{2}{\pi E^*} \int_{-a(N)}^{a(N)} q_N(x', c^*) \ln \left| \frac{x - x'}{c^* - x'} \right| dx', \quad (14)$$

where  $q_N(x, c^*)$  is given by Eq. (7). The function  $q^*(x, N)$  and the size of stick zone  $c^*$  can be found from the integral Eq. (8) and the conditions

$$\int_{-c^*}^{c^*} q^*(x, N) dx = \mu P - Q^* \quad (15)$$

and

$$q^*(c^*, N) = 0. \quad (16)$$

Since the function  $H(x, N)$  does not change inside the stick zone  $|x| \leq c^*$  during the wear process, i.e.,  $H(x, N) = H(x, 0) = f(x) = f_1(x) + f_2(x)$ , and the system of Eqs. (8), (15), and (16) has a unique solution, the function  $q^*(x, N)$  and the width  $c^*$  of the stick zone do not change with time, i.e.,  $q^*(x, N) = q^*(x, 0) = q^*(x)$ .

The solution of this system of equations can be written in the following form:

$$q^*(x) = \frac{\mu E^* \sqrt{(c^{*2} - x^2)}}{2\pi} \int_{-c^*}^{c^*} \frac{f'(t) dt}{\sqrt{(c^{*2} - t^2)(t - x)}}, \quad |x| \leq c^*, \quad (17)$$

where the size of the stick zone  $c^*$  is determined from the condition

$$-\frac{\mu E^*}{2} \int_{-c^*}^{c^*} \sqrt{\frac{c^* - t}{t + c^*}} f'(t) dt = \mu P - Q^*. \quad (18)$$

#### 4 Contact Problem Solution for the $N^{\text{th}}$ Cycle

It is assumed that all of the wear particles detached from the surface leave the contact zone so that the shape of contacting bodies varies. The unloaded gap  $H(x, N)$  between the bodies after  $N$  cycles is defined by the relationship

$$H(x, N) = f(x) + w(x, N), \quad (19)$$

where  $f(x) = f_1(x) + f_2(x)$  and the function  $w(x, N)$  is determined by Eq. (6). The contact condition for the  $N^{\text{th}}$  cycle takes the form

$$u_{z1}(x, N) + u_{z2}(x, N) = D(N) - H(x, N), \quad (20)$$

where  $u_{z1}, u_{z2}$  are the normal displacements of the contacting bodies due to their deformations and  $D(N)$  is the approach of the bodies.

We assume that the irreversible surface displacement  $w(x, N)$  due to wear is small, and comparable to the elastic displacement  $u_{zi}$  ( $i = 1, 2$ ). Hence, the boundary conditions are posed on undeformed surfaces, neglecting both the elastic displacement  $u_{zi}$  and the surface wear  $w(x, N)$ . Under this assumption the contact pres-

sure  $p(x, N)$  and the elastic displacement  $u_{zi}$  for a given instant in time are related by an operator which is analogous to the operator relating the pressure and elastic displacement in the corresponding contact problem when wear does not occur. For a two-dimensional contact problem for an elastic half-space, this relation takes the form

$$u_{zi}(x, N) = -\frac{2(1 - \nu_i^2)}{\pi E_i} \int_{-a(N)}^{a(N)} \ln|x - x'| p(x', N) dx' + \text{const}. \quad (21)$$

If  $f_1(x)$  and  $f_2(x)$  are smooth functions the unknown contact width  $a(N)$  is obtained at each instant of time from the condition

$$p(a(N), N) = 0. \quad (22)$$

The contact pressure  $p(x, N)$  also satisfies the equilibrium condition

$$\int_{-a(N)}^{a(N)} p(x, N) dx = P. \quad (23)$$

For any cycle, the gap between the bodies can be calculated using the following relationship:

$$H(x, N) = H(x, N - 1) + \Delta w(x, N). \quad (24)$$

Substituting Eqs. (21), and (24), into Eq. (20) gives

$$\begin{aligned} & -\frac{2}{\pi E^*} \int_{-a(N)}^{a(N)} \ln|x - x'| p(x', N) dx' + \Delta w(x, N) \\ & = D(N) - H(x, N - 1), \quad |x| \leq a(N). \end{aligned} \quad (25)$$

At  $x = a(N)$ , Eq. (25) takes the form

$$\begin{aligned} & -\frac{2}{\pi E^*} \int_{-a(N)}^{a(N)} \ln|a(N) - x'| p(x', N) dx' \\ & = D(N) - H(a(N), N - 1). \end{aligned} \quad (26)$$

Subtracting Eq. (26) from Eq. (25) and taking into account Eq. (12) gives

$$\begin{aligned} & \frac{2}{\pi E^*} \int_{-a(N)}^{a(N)} \ln \left| \frac{a(N) - x'}{x - x'} \right| p(x', N) dx' + K_w |s(x, N)| p(x, N) \\ & = H(a(N), N - 1) - H(x, N - 1) \\ & - K_w |s(x, N - 1)| p(x, N - 1), \quad |x| \leq a(N), \end{aligned} \quad (27)$$

where  $s(x, N) \equiv s_N(x, c^*)$ , and  $p(x, N)$  is the symmetric function satisfying Eqs. (22) and (23).

Equation (27), using Eqs. (19), (24), and (25), can also be written as

$$\begin{aligned} & \frac{2}{\pi E^*} \int_{-a(N)}^{a(N)} \ln \left| \frac{a(N) - x'}{x - x'} \right| p(x', N) dx' + w(x, N) \\ & = f(a(N)) - f(x). \end{aligned} \quad (28)$$

Eqs. (22), (23), and (27) (or (28)) are used to determine the contact pressure  $p(x, N)$  and the contact half-width  $a(N)$  at the  $N^{\text{th}}$  cycle. The functions  $H(x, N - 1)$ ,  $w(x, N - 1)$  and  $s(x, N - 1)$  are found based on the known function  $p(x, N - 1)$  at the  $(N - 1)^{\text{th}}$  cycle.

The wear  $w(x, N)$ , the shear stress  $q(x, N)$ , the slip  $s(x, N)$ , and the gap  $H(x, N)$  between contacting bodies at the  $N^{\text{th}}$  cycle are calculated from Eqs. (6) and (12), (7), (14), and (19) (or (24)), respectively. Then the stress component  $\sigma_x(x, N)$  at  $z = 0$  can be calculated using the formula (see Muskhelishvili [12]):

$$\sigma_x(x, N) = -p(x, N) + \frac{2}{\pi} \int_{-a(N)}^{a(N)} \frac{q(x', N) dx'}{x' - x}, \quad -\infty < x < +\infty. \quad (29)$$

## 5 Asymptotic Analysis

As the gap between contacting surfaces increases, the contact pressure  $p(x, N)$  within the slip zones tends to zero as  $N \rightarrow \infty$ . It also follows from Eq. (14) that  $s'_N(x, c^*)$  is a discontinuous function at  $x = c^*$ ; therefore, the functions  $\Delta w'(x, N)$  and  $H'(x, N)$  (see Eq. (24)) are also discontinuous at  $x = c^*$ .

Thus, as  $N \rightarrow \infty$  the contact pressure approaches zero within the slip zones, and it tends to the following function inside the stick zone  $|x| \leq c^*$  (see Muskhelishvili [12]):

$$p_\infty(x) = \frac{E^*}{2\pi\sqrt{(c^{*2}-x^2)}} \int_{-c^*}^{c^*} \frac{\sqrt{(c^{*2}-t^2)}f'(t)dt}{t-x} + \frac{P}{\pi\sqrt{(c^{*2}-x^2)}}. \quad (30)$$

This formula is valid because the gap function has the discontinuous derivative at the point  $x = c^*$ . The pressure, given by the function (30) within the stick zone, and equal to zero outside the stick zone, exactly satisfies Eqs. (22), (23), and (28). The pressure,  $p_\infty(x)$ , is the asymptotic solution of these system of equations.

The asymptotic shear stress  $q_\infty(x)$  is determined from the condition  $s'(x) = 0$  if  $|x| \leq c^*$ . For similar materials it can be written in the form

$$\int_{-c^*}^{c^*} \frac{q_\infty(t)dt}{x-t} = 0, \quad |x| < c^*. \quad (31)$$

The solution of Eq. (31) satisfying the equilibrium condition

$$\int_{-c^*}^{c^*} q_\infty(t)dt = Q^*$$

is

$$q_\infty(x) = \frac{Q^*}{\pi\sqrt{(c^{*2}-x^2)}}, \quad |x| < c^*. \quad (32)$$

The half-width  $c^*$  of the stick zone in Eqs. (30) and (32) is determined from Eq. (18).

It should be noted that the asymptotic complementary function  $q_\infty^*(x)$  which is

$$\begin{aligned} q_\infty^*(x) &= \mu p_\infty(x) - q_\infty(x) \\ &= \frac{\mu E^*}{2\pi\sqrt{(c^{*2}-x^2)}} \int_{-c^*}^{c^*} \frac{\sqrt{(c^{*2}-t^2)}f'(t)dt}{t-x} \\ &\quad + \frac{\mu P - Q^*}{\pi\sqrt{(c^{*2}-x^2)}}, \quad |x| < c^*, \end{aligned} \quad (33)$$

coincides with the function  $q^*(x)$  (see Eq. (17)), and satisfies the following conditions:

$$\begin{aligned} q_\infty^*(x) &= q^*(x) > 0, \quad x \in (-c^*, c^*) \\ q_\infty^*(-c^*) &= q_\infty^*(c^*) = 0. \end{aligned}$$

Using Eq. (32) we can reduce the asymptotic slip function to

$$s_\infty(x) = -\frac{2Q^*}{\pi^2 E^*} \ln \frac{\sqrt{x^2 - c^{*2}} + x}{x}, \quad c^* \leq x \leq a_\infty. \quad (34)$$

Substitution of the asymptotic pressure  $p_\infty(x)$  into Eq. (28) gives the following expression to calculate the asymptotic wear distribution within the following slip zones:

$$\begin{aligned} w_\infty(x) &= f(a_\infty) - f(x) - \frac{2}{\pi E^*} \int_{-c^*}^{c^*} \ln \left| \frac{a_\infty - x'}{x - x'} \right| p_\infty(x') dx', \\ c^* &\leq x \leq a_\infty. \end{aligned} \quad (35)$$

The asymptotic stress component  $\sigma_x^\infty(x)$  at  $z=0$  is determined by the following expression reduced from Eqs. (29), (31), and (32)

$$\sigma_x^\infty(x) = \begin{cases} \frac{2Q^*}{\pi\sqrt{x^2 - c^{*2}}}, & \text{if } -\infty < x < -c^* \\ -p_\infty(x), & \text{if } -c^* < x < c^* \\ -\frac{2Q^*}{\pi\sqrt{x^2 - c^{*2}}}, & \text{if } c^* < x < \infty. \end{cases} \quad (36)$$

Note that the last expression has been obtained under the assumption that contacting components are elastically similar. It shows that the asymptotic function  $\sigma_x^\infty(x)$  is discontinuous at  $x = -c^*$ ,  $z = 0$ .

## 6 Method of Calculation

The following dimensionless parameters and functions are introduced:

$$\begin{aligned} \bar{p}(\bar{x}, N) &= \frac{2}{\pi E^*} p(\bar{x}R, N), \quad \bar{q}^*(\bar{x}) = \frac{2}{\pi \mu E^*} q^*(\bar{x}R), \\ \bar{K}_w &= \frac{\pi \mu E^*}{2} K_w, \quad \bar{P} = \frac{2P}{\pi R E^*}, \quad \bar{Q} = \frac{2}{\pi \mu R E^*} (\mu P - Q^*), \\ \bar{x} &= \frac{x}{R}, \quad \bar{a} = \frac{a}{R}, \quad \bar{c} = \frac{c}{R}, \end{aligned} \quad (37)$$

$$\bar{H}(\bar{x}, N) = \frac{H(\bar{x}R, N)}{R}, \quad \bar{s}(\bar{x}, N) = \frac{s(\bar{x}R, N)}{R},$$

$$\bar{f}(\bar{x}) = \frac{f(\bar{x}R)}{R}, \quad \bar{w}(\bar{x}, N) = \frac{w(\bar{x}R, N)}{R},$$

where  $R$  is some scale distance specified for each particular problem.

Since the function  $p(x, N)$  is symmetric the following relation is valid:

$$\begin{aligned} \int_{-a(N)}^{a(N)} \ln \left| \frac{x - x'}{a(N) - x'} \right| p(x', N) dx' \\ = \int_0^{a(N)} \ln \left| \frac{x^2 - x'^2}{a^2(N) - x'^2} \right| p(x', N) dx'. \end{aligned}$$

Using this relation, we reduce Eqs. (22), (23), and (28) to the following dimensionless system of equations:

$$\begin{aligned} \int_0^{a(N)} \ln \left| \frac{a^2(N) - x'^2}{\bar{x}^2 - x'^2} \right| \bar{p}(x', N) dx' + \bar{w}(\bar{x}, N) \\ = \bar{f}(\bar{a}(N)) - \bar{f}(\bar{x}), \quad |\bar{x}| \leq \bar{a}(N) \\ 2 \int_0^{\bar{a}(N)} \bar{p}(\bar{x}, N) d\bar{x} = \bar{P}, \quad \bar{p}(\bar{a}(N), N) = 0, \end{aligned} \quad (38)$$

where

$$\begin{aligned} \bar{w}(\bar{x}, N) &= \bar{K}_w [|\bar{s}(\bar{x}, N)| \bar{p}(\bar{x}, N) + |\bar{s}(\bar{x}, N-1)| \bar{p}(\bar{x}, N-1)] \\ &\quad + \bar{w}(\bar{x}, N-1), \quad \bar{w}(\bar{x}, 0) = 0, \quad N = 1, 2, \dots \end{aligned} \quad (39)$$

$$\bar{s}(\bar{x}, N) = \begin{cases} 0, & \text{if } 0 \leq \bar{x} < \bar{c}^* \\ \int_0^{\bar{a}(N)} \ln \left| \frac{\bar{c}^{*2} - x'^2}{\bar{x}^2 - x'^2} \right| \bar{q}(x', N) dx', & \text{if } \bar{c}^* \leq \bar{x} \leq \bar{a}(N) \end{cases} \quad (40)$$



$$\bar{q}(\bar{x}, N) = \begin{cases} \bar{p}(\bar{x}, N) - \bar{q}^*(\bar{x}), & \text{if } 0 \leq \bar{x} < \bar{c}^* \\ \bar{p}(\bar{x}, N), & \text{if } \bar{c}^* \leq \bar{x} \leq \bar{a}(N). \end{cases} \quad (41)$$

The function  $\bar{q}^*(\bar{x})$  and the parameter  $\bar{c}^*$  are determined from the system of equations:

$$\int_0^{\bar{c}^*} \ln \left| \frac{\bar{c}^{*2} - x'^2}{\bar{x}^2 - x'^2} \right| \bar{q}^*(x') dx' = \bar{f}(\bar{c}^*) - \bar{f}(\bar{x}), \quad 0 \leq \bar{x} \leq \bar{c}^*,$$

$$2 \int_0^{\bar{c}^*} \bar{q}^*(\bar{x}) d\bar{x} = \bar{Q}, \quad \bar{q}^*(\bar{c}^*) = 0 \quad (42)$$

The parameters  $\bar{K}_w$ ,  $\bar{P}$ , and  $\bar{Q}$  are given.

A stepwise procedure is used to calculate the evolution of contact characteristics due to wear. At each step the contact half-width is divided by  $n$  nodes (which are not necessary equidistant) and the function  $\bar{p}(\bar{x}, N)$  is approximated by the linear function  $p(t)$  between each two nodes, i.e.,

$$p(t) = p_j + \frac{p_{j+1} - p_j}{t_{j+1} - t_j} (t - t_j), \quad t_j \leq t \leq t_{j+1}, \quad j = 1, 2, \dots, n,$$

$$0 = t_1 < t_2 < \dots < t_n < t_{n+1} = a, \quad p_j = p(t_j), \quad p_{n+1} = p(a) = 0. \quad (43)$$

Substitution of Eq. (43) into Eq. (38) and integration of the linear functions between the nodes analytically reduces the integral equation to a system of linear equations. The size of the contact zone at the  $N^{\text{th}}$  cycle is determined from Eq. (38) by iteration. The functions  $\bar{s}(\bar{x}, N-1)$ ,  $\bar{w}(\bar{x}, N-1)$  and  $\bar{p}(\bar{x}, N-1)$  are known from the previous  $(N-1)$ th step. The solution of the system gives the dimensionless contact pressure  $\bar{p}(\bar{x}, N)$  and shear stress  $\bar{q}(\bar{x}, N)$ , the dimensionless wear  $\bar{w}(\bar{x}, N)$ , the dimensionless slip function  $\bar{s}(\bar{x}, N)$ , and the contact half-width  $a(N)$  at each  $N^{\text{th}}$  cycle.

## 7 Results and Discussion

The algorithm described above was used to calculate the shape variation and the contact stress evolution in the fretting process. In numerical calculations the following values of the dimensionless parameters were taken:  $\bar{K}_w = 0.0495$ ,  $\bar{P} = 0.631 \cdot 10^{-5}$ ,  $\bar{Q} = 0.788 \cdot 10^{-6}$  or  $\bar{Q} = 0.310 \cdot 10^{-5}$ . It follows from Eq. (37) that  $Q^*/(\mu P) = 1 - \bar{Q}/\bar{P}$ . Thus, for the cases under consideration  $Q^*/\mu P$  is 0.875 and 0.509. These correspond to recent experiments [2].

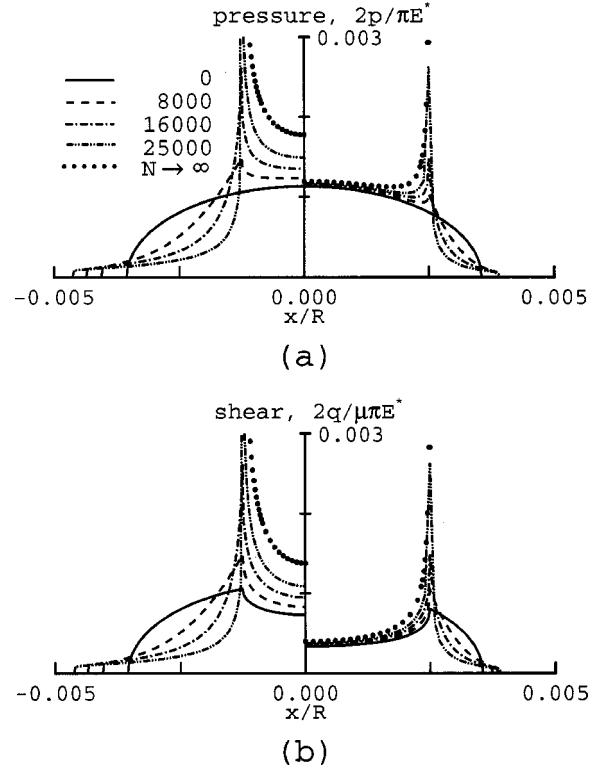
Two different initial shapes of the indenter were considered.

**Wear in the Hertz Contact With Partial Slip.** The shape of the parabolic cylinder is described by the function  $f(x) = x^2/(2R)$ . In dimensionless form this function is  $\bar{f}(\bar{x}) = \bar{x}^2/2$ .

The results of calculations of the contact pressure and shear stress for  $\bar{Q} = 0.788 \cdot 10^{-6}$  and  $\bar{Q} = 0.310 \cdot 10^{-5}$  are presented in Fig. 3. Here and in what follows the left side of the figures corresponds to  $\bar{Q} = 0.788 \cdot 10^{-6}$  and the right side corresponds to  $\bar{Q} = 0.310 \cdot 10^{-5}$ . The results indicate that the pressure (see Fig. 3(a)) changes essentially during the wear process. It increases within the stick zone and decreases inside the slip zones where the wear occurs. The large peak of the pressure occurs at the ends  $\pm c^*$  of the stick zone after 10,000 cycles. The shear stress within the contact exhibits similar behavior as shown in Fig. 3(b).

The evolution of the gap and slip functions is presented in Fig. 4. Figure 4(a) illustrates the gap variation due to wear, i.e., the wear displacements of both contacting bodies. The maximum wear displacements take place in the middle points of the slip zones. The absolute values of the derivatives of the wear function at the end  $c^*$  of the slip zone increase during the wear process. The relative slip (see Fig. 4(b)) increases during the wear process.

Comparison of the results for different values of tangential loads makes it possible to conclude that increase of the tangential



**Fig. 3 Evolution of contact pressure and shear stress due to wear in a partial slip contact between a cylindrical indenter and a half-space. Note that the right halves of the graphs correspond to  $\bar{Q} = 3.104 \times 10^{-6}$  and the left halves to  $\bar{Q} = 7.88 \times 10^{-7}$ . The stresses are symmetric about the  $x=0$  line.**

force  $Q^*$  ( $Q^* < \mu P$ ), i.e., decrease of the value of  $\bar{Q}$ , increases the size of the slip zones and the gap between surfaces formed in the wear process. The contact stress within the stick zone after a number of cycles is larger for a higher value of the tangential force.

Note that for the case under consideration Eqs. (17) and (18) can be reduced to the following expressions:

$$q^*(x) = \frac{\mu E^*}{2R} \sqrt{c^{*2} - x^2}, \quad |x| < c^*, \quad (44)$$

$$c^{*2} = \frac{4R(\mu P - Q^*)}{\mu \pi E^*}. \quad (45)$$

The increase of the gap between the contacting bodies within the slip zones in the wear process explains the decrease of the contact stress there.

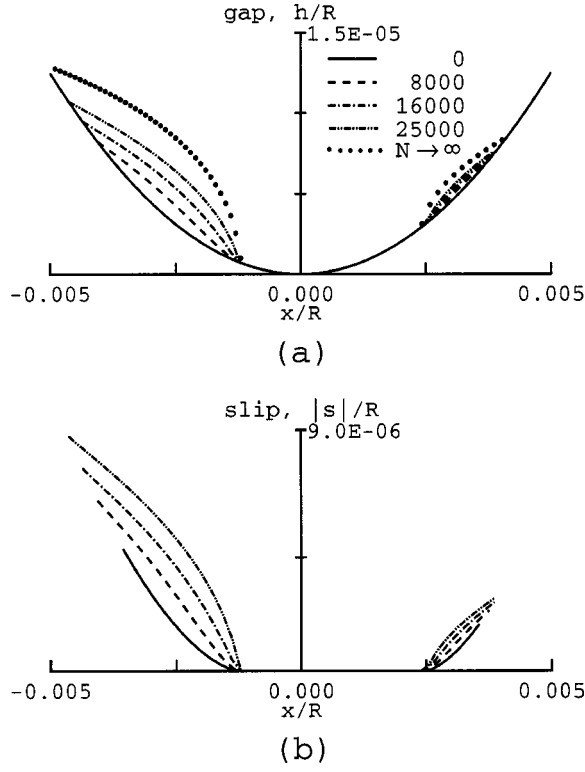
Equations (30) and (35) lead to the following analytical expressions for the asymptotic pressure in contact of the elastic cylinder and the elastic half-space

$$p_\infty(x) = \frac{1}{\sqrt{(c^{*2} - x^2)}} \left[ \frac{E^*}{4R} (c^{*2} - 2x^2) + \frac{P}{\pi} \right], \quad |x| < c^*, \quad (46)$$

where  $p_\infty(x) \geq 0$  since the following condition holds:

$$c^{*2} < a(0) = \frac{4PR}{\pi E^*},$$

and the asymptotic wear distribution is



**Fig. 4 Evolution of gap and slip functions due to wear in partial slip contact between a cylindrical indenter and a half-space. The right halves of the graphs correspond to  $\bar{Q}=3.104 \times 10^{-6}$  and the left halves to  $\bar{Q}=7.88 \times 10^{-7}$ .**

$$w_{\infty}(x) = \frac{2P}{\pi E^*} \ln \frac{x + \sqrt{x^2 - c^{*2}}}{c^*} - \frac{x}{2R} \sqrt{x^2 - c^{*2}}, \quad c^* \leq x \leq a_{\infty} \quad (47)$$

The asymptotic value  $a_{\infty}$  is determined from the condition

$$w_{\infty}(a_{\infty}) = 0 \quad (48)$$

Integrating Eq. (47) gives the total wear if  $N \rightarrow \infty$

$$W_{\infty} = 2 \int_{c^*}^{a_{\infty}} w_{\infty}(x) dx = \left( \frac{a_{\infty}^2}{3R} + \frac{c^{*2}}{6R} - \frac{2P}{\pi E^*} \right) \sqrt{a_{\infty}^2 - c^{*2}}. \quad (49)$$

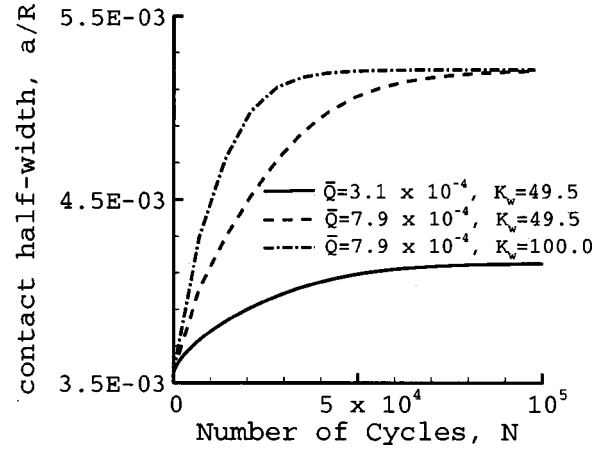
The asymptotic solutions for  $N \rightarrow \infty$  are presented in the figures in dimensionless form by dotted lines.

Note that the pressure distributions for different numbers of cycles presented in Fig. 3(a) are in good agreement with the pressure evolution function obtained by Johansson [6] for the following data:

$$R = 2.8 \text{ m}, \quad P = 3.1 \cdot 10^6 \text{ N/m}, \quad Q^* = 0.875 \mu P, \\ \mu = 0.4, \quad K_w = 7 \cdot 10^{-13} \text{ m}^2/\text{N}, \quad E = 2.05 \cdot 10^{11} \text{ N/m}^2, \quad \nu = 0.3$$

The above value of the wear constant,  $K_w$ , was adopted by Johansson from measurements made by Toth [13]. The values of dimensionless parameters  $\bar{K}_w$  and  $\bar{P}$  can be calculated from Eqs. (38) for the above data. A careful comparison of the contact pressure results obtained by the method presented here to the graph produced by Johansson for  $N=16,000$  revealed that there is a 15 percent difference between the two approaches.

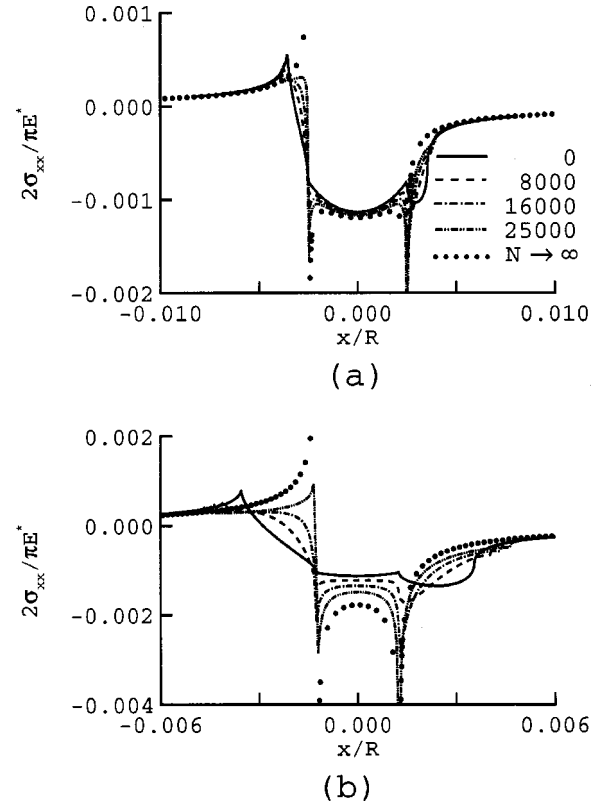
Figure 5 illustrates the evolution of the contact half-width  $a(N)$  in wear process at various values of tangential loads. The asymptotic values of  $a_{\infty}$  are calculated from Eqs. (47) and (48). For the parameters under consideration they are  $\bar{a}_{\infty} = 0.0052$  at  $\bar{Q} = 0.788 \cdot 10^{-6}$  and  $\bar{a}_{\infty} = 0.0042$  at  $\bar{Q} = 3.104 \cdot 10^{-6}$ . Figure 5 de-



**Fig. 5 Evolution of the half-contact width due to wear in partial slip contact between a cylindrical indenter and a half-space**

picts that increasing of the wear coefficient  $K_w$  decreases the run-in stage of wear process, i.e., the time corresponding to the large value of the function  $a'(N)$ .

The evolution of the distribution of the  $\sigma_x$  stress component in wear process for two different tangential loads is illustrated by Figs. 6(a) and 6(b). As the number of cycles increases the  $\sigma_x$  distribution tends to the asymptotic curve calculated from Eq. (37) which predicts the discontinuity of the  $\sigma_x$  component at  $x = -c^*$ ,  $z=0$ . Recent work correlates this stress component with fatigue life [1–3].



**Fig. 6 Evolution of tensile stress due to wear in partial slip contact between a cylindrical indenter and a half-space for  $\bar{Q} = 3.104 \times 10^{-6}$  (a) and  $\bar{Q} = 7.88 \times 10^{-7}$  (b)**

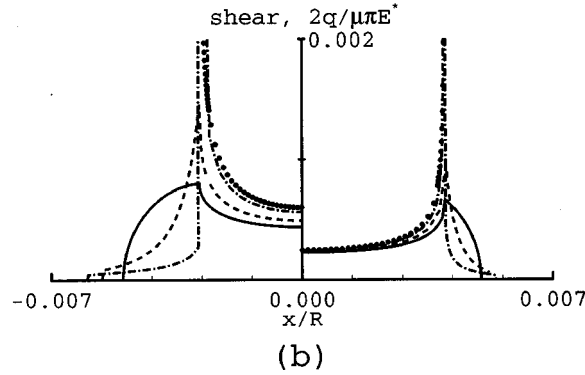
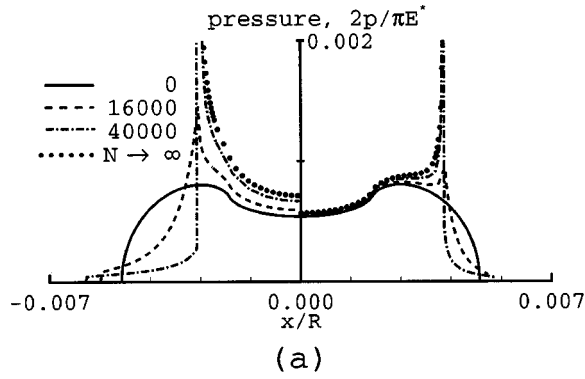


Fig. 7 Evolution of contact pressure and shear stress due to wear in partial slip contact between a flat indenter with rounded edges and a half-space. The right halves of the above graphs correspond to  $\bar{Q}=3.104 \times 10^{-6}$  and the left halves to  $\bar{Q}=7.88 \times 10^{-7}$ .

**Wear Due to the Contact Between an Elastic Indenter With a Flat Base and Rounded Edges and an Elastic Half-Plane.** The shape of the elastic indenter with a flat base within the region  $|x| \leq d$  is described by the function:

$$f(x) = \begin{cases} \frac{(x+d)^2}{2R}, & \text{if } -a(N) \leq x < -d \\ 0, & \text{if } -d \leq x < d \\ \frac{(x-d)^2}{2R}, & \text{if } d \leq x < a(N) \end{cases} \quad (50)$$

The results of calculations for this indenter and the same values of the dimensionless parameters  $\bar{K}_w$ ,  $\bar{P}$ ,  $\bar{Q}$ , and  $\bar{d} = d/R = 0.002$  are presented in Figs. 7, 8, 9, and 10. Note that the initial contact stress distributions differ from the Hertzian case. However, the main features of stress evolution due to the wear process are similar. The large peaks of the contact stresses occur at the ends of the stick zones as the cycles progress. Note, that the point  $x = c^*$  always is outside the flat zone  $|x| \leq d$ , i.e.,  $c^* > d$ . The contact pressure distribution tends to the asymptotic solution which follows from Eq. (30):

$$p(y) = \frac{E^* c^* (1+y^2)}{2\pi R (1-y^2)} \left[ \delta \sqrt{1-\delta^2} + \frac{2PR}{E^* c^{*2}} + \arccos \delta \left( 1 - \frac{8y^2}{(1+y^2)^2} \right) + 2y \frac{1-y^2}{(1+y^2)^2} \ln \left| \frac{(y_0+y)(1-yy_0)}{(y_0-y)(1+yy_0)} \right| \right]$$

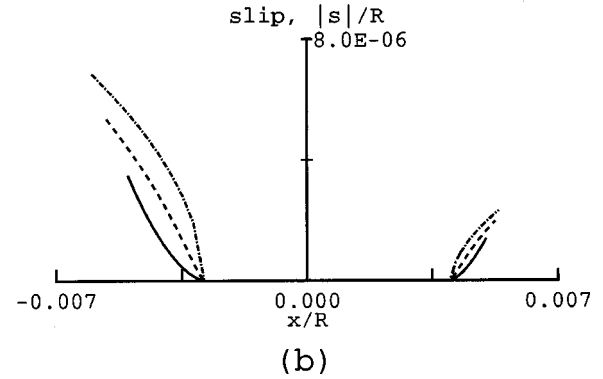
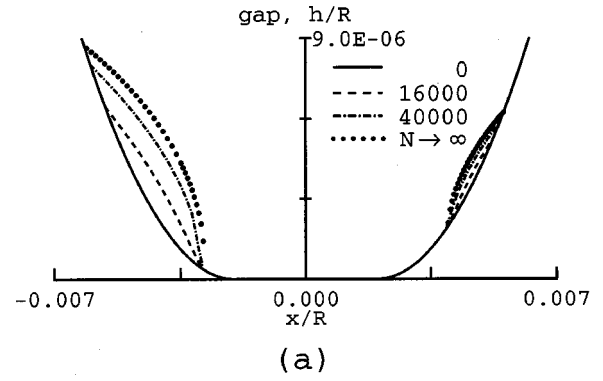


Fig. 8 Evolution of gap and slip functions due to wear in partial slip contact between a flat indenter with rounded edges and a half-space. The right halves of the above graphs correspond to  $\bar{Q}=3.104 \times 10^{-6}$  and the left halves to  $\bar{Q}=7.88 \times 10^{-7}$ .

$$+ \delta \frac{1-y^2}{1+y^2} \ln \left| \frac{y_0^2 - y^2}{1 - y^2 y_0^2} \right|, \quad |y| \leq 1 \quad (51)$$

where

$$\delta = \frac{d}{c^*}, \quad y = \tan \left( \frac{1}{2} \arcsin \frac{x}{c^*} \right), \quad y_0 = \tan \left( \frac{1}{2} \arcsin \frac{d}{c^*} \right).$$

The plots of the function  $\bar{p}(\bar{x}) = 2p(\bar{x}R)/\pi E^*$  calculated from Eq. (51) are shown in Fig. 7(a) by dotted lines. The asymptotic values of shear contact stress, gap, and slip functions are also

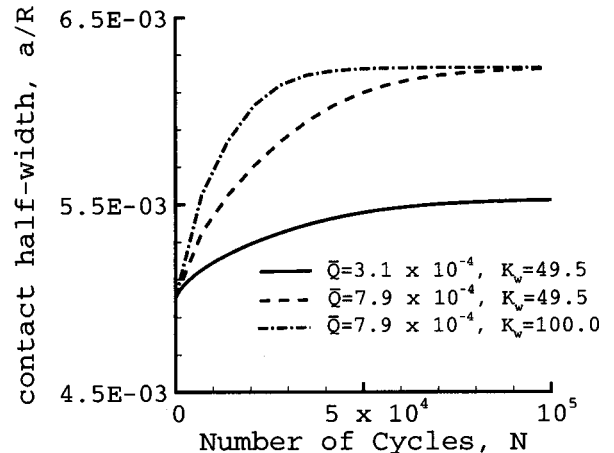
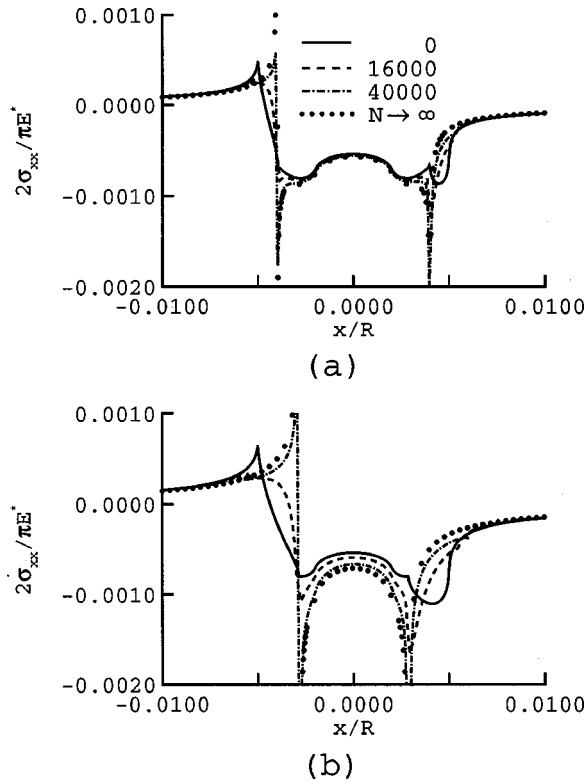


Fig. 9 Evolution of the half-contact width due to wear in partial slip contact between a flat indenter with rounded edges and a half-space





**Fig. 10 Evolution of tensile stress due to wear in partial slip contact between a flat indenter with rounded edges and a half-space for  $\bar{Q}=3.104 \times 10^{-6}$  (a) and  $\bar{Q}=7.88 \times 10^{-7}$  (b)**

presented in Figs. 7(b) and 8 by dotted lines. The asymptotic maximum gap depth predicted from the calculations for the punch with the rounded edges is about 0.013 mm. This result is comparable to the experimental measurements illustrated in Fig. 1. The dependence of the half-width size on the number of cycles is shown in Fig. 9. This dependence is similar to one presented in Fig. 5. For both cases the process of stress evolution can be divided into two stages: run-in and steady-state stage. For the case  $d=0$ , the asymptotic values of the contact half-width are 0.415 when  $\bar{Q}=0.3105 \times 10^{-5}$  and 0.520 when  $\bar{Q}=0.788 \times 10^{-5}$ . The asymptotic contact half-width values for the case  $d=2 \times 10^{-3}$  are 0.552 when  $\bar{Q}=0.3105 \times 10^{-5}$  and 0.622 when  $\bar{Q}=0.788 \times 10^{-5}$ . Figure 10 illustrates the tensile stress redistribution due to the wear process for two different values of the tangential load. The value of the tensile stress at the boundary between stick and slip zones in the trailing part of the contact increases due to gap formation within the slip zones. These peaks cause the high damage accumulation rate at these points for cyclic tangential loading.

## 8 Conclusion

An analytical method for calculation of the contact stress evolution and wear displacements in fretting under the partial slip condition is presented. The method is based on two scales of time: the time for one cycle and time corresponding to the number of cycles. The wear in one cycle is evaluated by Eq. (12). The integral equation is reduced to calculate the contact pressure at the  $N^{\text{th}}$  cycle. A stepwise procedure based on iteration at each step is used to calculate the evolution of the contact characteristics in fretting. This method is computationally less expensive than the time integration process implemented by Johansson [6]. To speed up the calculations Johansson used the larger value of the wear coefficient satisfying the condition of numerical stability. Due to the

analytical estimates used in this work there is no necessity to artificially increase the wear coefficient to decrease the simulated number of cycles.

The analytical study of the governing equations made it possible to conclude that

- 1 the size of the stick zone does not change from cycle to cycle under constant load conditions
- 2 the asymptotic solution exists as the number of cycles tends to infinity

The analytical expressions for the asymptotic contact stresses (normal and shear), the stress component  $\sigma_x$  at  $z=0$  and the wear distribution within the slip zones have been presented. It was shown that asymptotic contact zone coincides with the stick region, and asymptotic contact pressure and shear stress are singular at the ends of the stick zone. The asymptotic pressure, shear, and gap function distributions do not depend on the wear constant but depend on the applied loads and the initial gap function between the contacting bodies.

The contact stress and gap evolution were calculated for two initial shapes of indenter contacting with an elastic half-space; i.e., for the parabolic cylinder and for an indenter having a flat base with rounded edges.

The results of calculation show that the contact stresses change essentially due to wear in both cases. The gap function has a discontinuous derivative at the boundary of stick and slip zones due to wear of contacting surfaces within slip zones. The pressure and shear stress increase at these points and tend to infinity. The contact stresses outside the stick zone tend to zero.

The contact stress concentration at the boundary of the stick and slip zones results in increasing of the tensile stress at these points and in transition of the point of maximum tensile stress from the end of the contact to the boundary between stick and slip zones. Based on the approach formulated by Szolwinski and Farris [1] crack nucleation for a small number of cycles would occur at the edge of contact while the nucleation for a larger number of cycles would occur at the stick/slip boundary. Also note that wear causes the contact size to grow. Thus, post-mortem observation of the crack location does not necessarily correspond to the relative position of the crack from the edge of contact at the time of crack nucleation.

The existence of a third body consisting of detached wear particles must influence on the stress evolution in the wear process. This problem needs to be formulated and studied in future.

## Acknowledgments

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