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Experiments on the control of differential game of target defense

Mukhtar Sani¹, Ahmad Hably¹, Bogdan Robu^{1*}, Jonathan Dumon¹, Nacim Meslem¹

Abstract—This paper evaluates experimentally a novel strategy for solving a variant of the differential game of target defense in the presence of obstacles. The state-of-the-art approaches mostly employ an offline optimization strategy that is only applicable to holonomic systems. This paper presents an online optimization technique, by designing a trade-off parameter that integrates game theory with the model predictive control which allows a nonholonomic defender to intercept an attacker while simultaneously defending a specific target. Several indoor laboratory experiments validate the performance of the proposed approach and compared with a standard model predictive control approach.

I. INTRODUCTION

Differential game, first studied in [1], is a powerful tool for studying dynamic behavior of multiple systems (or players) that interact with each other either in a conflict or in a cooperative manners and analyzing their decision-making processes. The continuous-time nature of these games is aligned with a lot of robotics problems since robots kinematic equations are naturally and originally in continuous time. Translating players' decision strategies into algorithms is a non-trivial task.

Hamilton–Jacobi–Isaacs (HJI) equation has been used in [2] to address a special class of differential games, called "reach and avoid", including time-varying dynamics, targets, and constraints. In this reach and avoid game, the player attempting to reach the target is treated as an attacker whose objective is to arrive at the goal, and the other player (or the defender) tries to intercept the offensive player (attacker) and to prevent him to from reaching the target. The value of the game is defined as the time required for the attacking player to reach the goal, and the objectives of the attacking and defending players are respectively minimizing and maximizing this value. This game has several applications such as collision avoidance [3], path planning [4], and confrontational situations [5]. In surveillance and security, for tracking malicious evader that is trying to escape, the security robot must ensure that the evader does not escape by maintaining visibility. This game can also be useful in wildlife monitoring, where unmanned surface vehicle is required to navigate in a cluttered environment while tracking different species. In [6], a priority-based evasion system has been used for multiple players or agents in order that they avoid each

other and reach their respective targets. In [7], by exploiting sensitivity analysis within an iterated best response algorithm, an online motion planning algorithm for two-player drone racing has been proposed and validated experimentally.

Model predictive control (MPC) [8] is a feedback implementation of optimal control using finite prediction horizon and online optimization. The technique allows predicting the system's future states using its mathematical model and find the future optimal control sequence minimizing a certain objective function. Only the first part of this control sequence is applied to the system while the remaining is discarded. For its intrinsic capability to deal with stability, disturbances, constraints, and obstacle scenarios, MPC is an appealing scheme to address the navigation problem of mobile robots [9]. In our previous work [10], MPC has been used for non-cooperative competitive games between two unmanned ground robots while incorporating obstacle avoidance techniques. Nevertheless, the information available to each player plays an important role in generating optimal strategies. For this, a new game-theoretic way of using MPC approach to handle pursuit-evasion games in presence of obstacles for nonholonomic mobile robots where each player can only measure the current position of its opponent, is given in [11].

In this present paper, the differential game of target defense consists of two pursuit-evasion games that are coupled to produce a single game with three players: the defender, the attacker, and the target. The objective of the defender is to intercept an attacker before reaching the target, while the objective of the attacker is to reach the target while avoiding the defender. We consider a situation where the speed limits of the players are the same and the distance between each player and the target is equal. We propose an improved formulation of nonlinear model predictive control (NMPC) which we termed as game-theoretic model predictive control (GT-MPC) to handle the differential game of static target defense (DGSTD) in both simple and complex environments. From the perspective of the defender, GT-MPC contains a parameter that allows for a trade-off between the interception of the attacker and protection of the target. Game-theoretic model predictive control (GT-MPC) approach proposed in this paper combines the notions of game-theory and model predictive control. Game theory is a mathematical tool that models systems as intelligent rational decision-makers where an agent considers the opponent's strategy when selecting its optimal strategy. Each player in a game seeks to find its Nash equilibrium that is, considered the best response a particular player can have given that the opponent is also doing his best. For more details on the relationship between game theory

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and control, the reader can refer to the nice paper [12].

The paper is organized as follows. Section II contains the formulation of the static target defense problem as a differential game. The following Section III details the mathematical formulation of the game-theoretic model predictive control (GT-MPC). The experimental platform is introduced in Section IV while the experimental results are detailed in Section V. The paper ends with conclusions and perspectives in Section VI.

II. DIFFERENTIAL GAME OF STATIC TARGET DEFENSE FORMULATION

In this section, we will present the formulation of the differential game of static target defense and its players. The first player is the attacker (A) and the second is the defender (D). Both these players are nonholonomic since they have turn radius constraints. The third player is the target (T) and it is represented only by its Cartesian position. While the position of the static target is regarded as the center of a particular object approximated by a sphere, the kinematic model of nonholonomic wheeled mobile robots (A) and (D) can be represented in equation (1) as used in [13], [14].

$$\begin{cases} \dot{x}(t) &= v(t) \cos \theta(t) \\ \dot{y}(t) &= v(t) \sin \theta(t) \\ \dot{\theta}(t) &= \omega(t) \end{cases} \quad (1)$$

where the state variable $\mathbf{x} = [x, y, \theta]^T$ represents the position of the robot in the chassis frame of reference and the heading angle, while the control variables $\mathbf{u} = [v, \omega]^T$ represents the linear speed and the angular speed which are also bounded as $\mathbf{v}_{min} \leq \mathbf{v} \leq \mathbf{v}_{max}$ and $\omega_{min} \leq \omega \leq \omega_{max}$ respectively. Since the robots used in this work are not a point mass, the sizes of the defender and the attacker are defined as R_D and R_A respectively. However, this imposes a difficulty that is connected with the point-stabilization task, that is, the task cannot be accomplished with a pure feedback algorithm due to Brockett's condition which implies that the linearized nonholonomic models are not stabilizable [15].

Having defined the players of the game, models, and the game environment, players' objectives and the game termination conditions will be presented in the next paragraph. The objective of the defender is to pursue the attacker and protect the target while the objective of the attacker is to reach the target and avoid the defender. Since the defender and the attacker have the same capabilities but also are at the distance between each other and they are equidistant from the target. Then the defender will never win the game when it blindly tracks the attacker. Without any cooperation from the target, the question is how can the defender intelligently pursue the two objectives explained herein and win the game while simultaneously avoiding all the obstacles whose position and size are defined as x_{obs}, y_{obs} while having a size R_{obs} . The game is played where the attacker tries to minimize the terminal distance between the attacker and the target while the defender tries to maximize it. The first condition is when the attacker is intercepted by the defender. This happens when

the distance between the defender and the attacker is equal to some threshold value, as given in (2):

$$C_1 : \sqrt{(x_A - x_D)^2 + (y_A - y_D)^2} = l_1 \quad (2)$$

where $l_1 = R_A + R_D$. The second termination condition is when the attacker captures the target. This occurs when the attacker eventually reaches the target without being intercepted by the defender as given in (3):

$$C_2 : \sqrt{(x_T - x_A)^2 + (y_T - y_A)^2} = l_2 \quad (3)$$

where $l_2 = R_T + R_A$. Therefore, the termination condition of the game can be defined as (4) which is the combination of the two conditions (2) and (3).

$$C = C_1 \cup C_2 \quad (4)$$

III. GAME-THEORETIC MODEL PREDICTIVE CONTROL FORMULATION

In this section, the control formulation will be presented. First a discrete nonlinear model predictive control is formulated by solving the following optimal control problem (OCP):

$$\min_{\mathbf{u} \in \mathbb{R}^{n_{\mathbf{u}} \times N}} J_N(\mathbf{x}_0, \mathbf{u}) \quad (5)$$

subject to

$$\begin{cases} \mathbf{x}(0) = \mathbf{x}_0, \\ \mathbf{x}(k+1) = \mathbf{f}(\mathbf{x}(k), \mathbf{u}(k)); \quad k \in \{0, 1, \dots, N-1\}, \\ \mathbf{x}_{min} \leq \mathbf{x}(k) \leq \mathbf{x}_{max} \quad k \in \{1, 2, \dots, N\}, \\ \mathbf{u}_{min} \leq \mathbf{u}(k) \leq \mathbf{u}_{max} \quad k \in \{0, 1, \dots, N-1\} \end{cases} \quad (6)$$

where:

$$J_N(\mathbf{x}_0, \mathbf{u}) = \sum_{k=0}^{N-1} V(\mathbf{x}(k), \mathbf{u}(k)) + W(\mathbf{x}(N)) \quad (7)$$

The term $V(\mathbf{x}(k), \mathbf{u}(k))$ is called the running cost which can be computed by penalizing the deviation of the system's state $\mathbf{x}(k)$ and control input $\mathbf{u}(k)$ from the reference state $\mathbf{x}^r(k)$ and reference control input $\mathbf{u}^r(k)$ respectively. Generally, the running cost is defined as:

$$V(\mathbf{x}(k), \mathbf{u}(k)) = \|\mathbf{x}(k) - \mathbf{x}^r(k)\|_Q^2 + \|\mathbf{u}(k) - \mathbf{u}^r(k)\|_R^2 \quad (8)$$

where $Q \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{m \times m}$ are positive definite symmetric weighting matrices. N is the prediction horizon, assuming that the length of the prediction and control horizon is the same. The choice of N is not straightforward, as a large values would imply a non-realistic behavior where all the future movements of the agent are known, whereas a small value would imply a myopic reaction only based on the current measurements. Moreover, since the sampling time is smaller than of the robot's dynamics, one can suppose that the estimation of few further positions would not be very far from the reality.

In point stabilization problems, the state reference $\mathbf{x}^r(k)$ is a fixed value, thus the control, reference $\mathbf{u}^r(k) = 0$. In our case where we treat trajectory tracking problem, the state reference $\mathbf{x}^r(k)$ is time-varying, therefore the deviation

of control input from the reference can be penalized due to computational advantages such as rendering the optimal control problem easier, avoiding control values with expensive energy as mentioned in [16].

The second term of equation (7), $W(\mathbf{x}(N))$, normally referred as terminal cost is used for stability purpose. It can be computed by penalizing the last entry from the state prediction $\mathbf{x}(N)$ from its reference $\mathbf{x}^r(N)$. Terminal cost can be defined as:

$$W(\mathbf{x}(N)) = \|\mathbf{x}(N) - \mathbf{x}^r(N)\|_P^2 \quad (9)$$

where $P \in \mathbb{R}^{n \times n}$ is a positive definite weighting matrix. The solution of the optimal control problem (5) is the optimal control sequence of the form:

$$\mathbf{u}^* = (\mathbf{u}^*(0), \mathbf{u}^*(1), \dots, \mathbf{u}^*(N-1)) \quad (10)$$

Now, combining both of the concepts of game theory and MPC, the defender will have information on the attacker's intentions. The first objective of the defender is then to intercept the attacker before it reaches the target, while the second objective is to track the attacker. To satisfy both objectives concurrently, the defender is made to track a reference that depends on the parameter α which makes a trade-off between the attacker's and the target's positions. This parameter will be designed and tuned discretely.

A. The defender's control

Following the general formulation presented above, The defender's control can be at every sampling time by solving the following optimal control problem:

$$\min_{\mathbf{u}_D} J = \|\mathbf{x}_D(N) - \mathbf{x}_R\|_{Q_N}^2 + \sum_{k=0}^{N-1} \|\mathbf{x}_D(k) - \mathbf{x}_R\|_Q^2 + \|\mathbf{u}_D(k)\|_R^2 \quad (11)$$

Subject to:

$$\mathbf{x}_D(k+1) = f(\mathbf{x}_D(k), \mathbf{u}_D(k)), \quad k = 0, 1, \dots, N \quad (12)$$

$$\sqrt{(x_D(k) - x_{obs})^2 + (y_D(k) - y_{obs})^2} \geq (R_D + R_{obs}) \quad (13)$$

$$\mathbf{x}_{D_{min}} \leq \mathbf{x}_D(k) \leq \mathbf{x}_{D_{max}} \quad (14)$$

$$\mathbf{u}_{D_{min}} \leq \mathbf{u}_D(k) \leq \mathbf{u}_{D_{max}} \quad (15)$$

with

$$\mathbf{x}_R = (1 - \alpha)\mathbf{x}_A + \alpha\mathbf{x}_T \quad (16a)$$

$$\alpha \in (0, 1) \quad (16b)$$

As we notice from (16b), the cost function depends on the parameter α which varies from 0 to 1. As the parameter increases from 0 towards 1, more importance is given to protect the target than to pursue the attacker. Due to the fact that nonholonomic systems are not linearly stabilizable, a terminal cost was added to asymptotically stabilize the controller. Furthermore, the obstacles are included in the problem through the equation (13).

B. The attacker's control

Conversely, the attacker is interested in reaching the target as fast as possible while avoiding the defender, the problem can be formulated using MPC such that the target's state is regarded as the reference signal while the defender is taken as a dynamic obstacle. Therefore, the attacker will be solving the following optimal control problem at each time instant:

$$\min_{\mathbf{u}_A} J = \|\mathbf{x}_A(N) - \mathbf{x}_T\|_{Q_N}^2 + \sum_{k=0}^{N-1} \|\mathbf{x}_A(k) - \mathbf{x}_T\|_Q^2 + \|\mathbf{u}_A(k)\|_R^2 \quad (17)$$

Subject to:

$$\mathbf{x}_A(k+1) = f(\mathbf{x}_A(k), \mathbf{u}_A(k)), \quad k = 0, 1, \dots, N \quad (18)$$

$$\sqrt{(x_A(k+1) - x_D)^2 + (y_A(k+1) - y_D)^2} \geq (R_A + R_D) \quad (19)$$

$$\sqrt{(x_A(k) - x_{obs})^2 + (y_A(k) - y_{obs})^2} \geq (R_A + R_{obs}) \quad (20)$$

$$\mathbf{x}_{A_{min}} \leq \mathbf{x}_A(k) \leq \mathbf{x}_{A_{max}} \quad (21)$$

$$\mathbf{u}_{A_{min}} \leq \mathbf{u}_A(k) \leq \mathbf{u}_{A_{max}} \quad (22)$$

This cost function (17) comprises the running cost which is the sum of stages predicted over the prediction horizon and the terminal cost for stabilizing the controller. \mathbf{x}_A and \mathbf{u}_A respectively denote the states and control variables of the attacker whereas \mathbf{x}_T represents the position of the target taken as a reference to go. The constraint given by equation (18) represents the nonlinear model of the robot. Two obstacle avoidance constraints are added to the formulation. The constraint (19) is used to allow the attacker to avoid the defender (as a dynamic obstacle), while the second constraint (20) is employed to avoid static obstacles that might be encountered during the experiment. The weight matrices can be tuned and values that stabilize the controller can be chosen, as will be shown in the results section V.

IV. EXPERIMENTAL SET-UP

Before presenting the results of the different experiments, we will present our experimental step-up. We have fabricated, at Gipsa-lab, two nonholonomic mobile robots of uni-cycle type as depicted in Figure 1. They are named Robot 1 and Robot 2 respectively. Each robot consists of two controlled wheels on the right and left sides with a free wheel at the front to support the robot. Two continuous rotation servo motors are coupled to the right and left wheels. Each motor gets the speed command signal (ω_r or ω_l) which commands the robot rotation speed and linear movement direction (forward or backward). The speed commands of each servo-motor are simultaneously received wirelessly from an IRC board (Figure 3) which communicates with the robot's Spectrum DSMX receiver at 2.4GHz, for extra interference protection and faster re-connection times. The communication between the simulation program used in our case (Simulink) and the IRC board is done via UDP-protocol block of Simulink Real-Time toolbox.

The position of the robots are recovered using a motion capture system. In this work, this system consists of eight

Miquis M3 Qualisys cameras. These cameras are designed to capture accurate data with very low latency. Each 2MP camera has a resolution of 1824×1088 pixels as well as a recording speed of 340 frames per second. The cameras are light and portable (around 700 grams each) and have been properly positioned to cover all the experimental area, as their maximum capture distance is around 15 meters, which is close to the maximum size of the experimental ground of $16m^2$. The cameras, depicted in Figure 2, are synchronized using an integrated Qualisys software (namely Camera Sync Unit) which communicates with a Raspberry-pi board containing a python code for converting the quaternions to Euler angles. Camera Sync Unit supports a wide range of synchronization and communication protocols including PTP, SMPTE, IRIG. The states of both robots, as well as any moving obstacle, are then sent in real-time to a Matlab/Simulink environment through Simulink Real-Time toolbox via UDP-protocol as mentioned before.

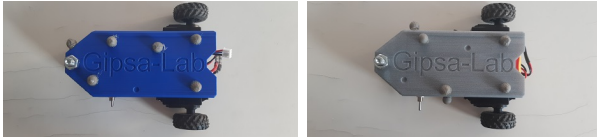


Fig. 1: The two robots (1 left and 2 right)

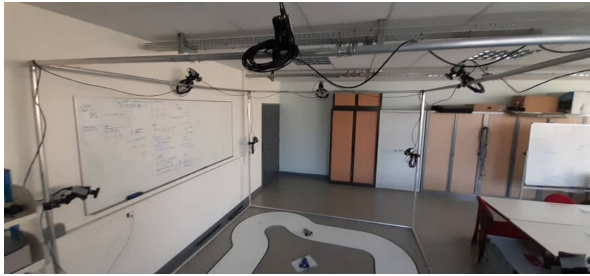


Fig. 2: The motion capture system used to measure the states of the robots and the moving obstacles during the experiments.

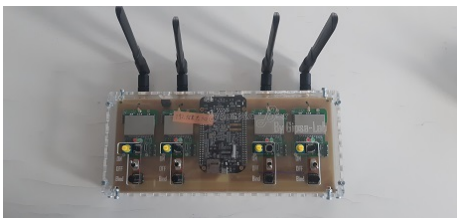


Fig. 3: The IRC Communication system to send the speed commands of each servo-motor.

V. EXPERIMENTAL RESULTS

In this section, the experimental results of different games will be presented. The controllers to be applied to the robots use the states and other information to compute the control inputs v and ω which are converted to the angular speeds of the left (ω_l) and right (ω_r) wheels of the robot by the

following mapping:

$$\begin{cases} \omega_r = (2v + \omega R_{rob})/2r \\ \omega_l = (2v - \omega R_{rob})/2r \end{cases} \quad (23)$$

where R_{rob} is the length of the robot's base from the center and r is the radius of the robot's wheels. Each experiment can terminate in three ways, i.e when the attacker reaches the target, when the defender captured the attacker, or when the game time elapses (120 seconds) which happened when the defender prevented the attacker from reaching the target. Also, the initial positions of each game were selected so that the players are equidistant from the target. The area of the game environment for the experiments is between $-2m$ to $+2m$ on both axis. Moreover, the bounds on the control variables which translates into inequality constraints are defined as $v_{max} = 0.065m/s$, $v_{min} = -0.065m/s$, $\omega_{max} = 0.05rad/s$, and $\omega_{min} = -0.05rad/s$ for both robots. The sample time is 0.1 seconds and the prediction horizon N is equal to 10.

The Q and R matrices introduced in section III are found to be diagonal matrices with diagonal elements obtained as $(1, 1, 0.001)$ and $(1, 1)$ respectively. The weight on the terminal penalty cost is found to be $1000 * Q$. In a general case, as we are on a problem of distance minimization between the defender and attacker, and not on an angle minimization problem, the weights of the first two elements in Q and R matrices should be the same. In this case we chose them to be both equal to 1 as it is the simplest case. Concerning the third element in the state space vector, namely θ , as it is shown in our previous work [11], [10] that its weight should be much smaller in order to avoid unrealistic behavior where the defender, even though very far, will first turn around itself, in order to match the angle, before deciding to pursue the attacker.

A. Experiments in an obstacle-free environment

Both controllers are implemented on the defender in a bounded-free environment. First the defender employs the classical MPC. The game's trajectory in Figure 4 clearly shows that the defender tracks the attacker but would not intercept or prevent it from reaching the target. This means that the defender blindly tracks the attacker, thus unless with speed advantage, the defender can never intercept this attacker in this case. The control strategy profiles of the defender and the attacker are respectively shown in figures 5 and 6. In the second experiment, the defender's strategy is improved by employing the GT-MPC scheme. The game's trajectories are shown on Figure 7. One can clearly see that the defender prevents the attacker from reaching the target until the game terminated. The attacker considers the defender as a dynamic obstacle, that is why it moves away from the defender to avoid capture. The strategy profiles of the defender and the attacker are shown in figures 8 and 9 respectively. The slight oscillations seen on the real speeds is because the speeds are not measured directly but rather obtained off-line from the real-time state measurements.

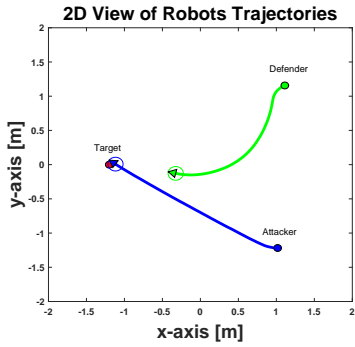


Fig. 4: The game trajectories by applying MPC.

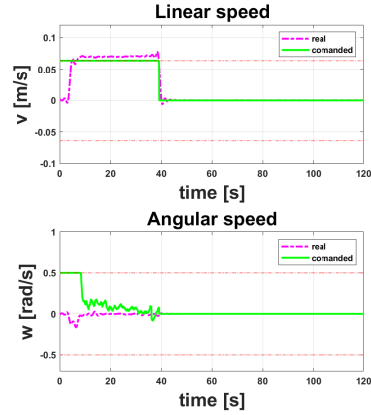


Fig. 6: The attacker's strategy with MPC.

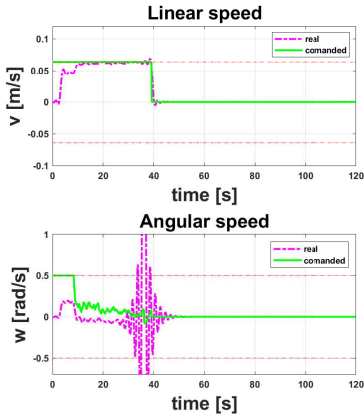


Fig. 5: The defender's strategy with MPC.

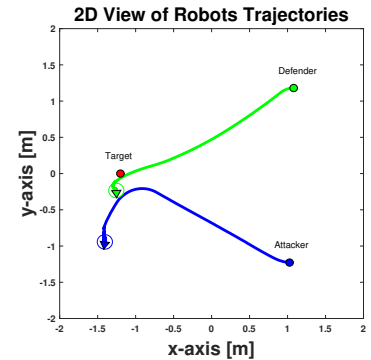


Fig. 7: The game's trajectories using GT-MPC strategy.

B. Experiments in an environment with obstacles

To add more complexity, obstacles' avoidance is incorporated in the formulation of the proposed controller. In this section, three virtual obstacles are placed at strategic positions within the area of the game environment as shown in Figure 10. The trajectories of the robots show that the defender intercepts the attacker before it reaches the target even in presence of different obstacles that reduce the space of admissible trajectories. The strategy profiles of the different players are shown in figures 11 and 12. When the game was repeated with the defender using MPC, it could not intercept the attacker.

VI. CONCLUSION AND FUTURE WORK

This paper studied differential games under the context of static target defense and proposed a solution using the combination of nonlinear model predictive control and game theory. The drawbacks of the previous approaches are leveraged to propose a novel game-theoretic model predictive control. A parameter α which provides a trade-off between the pursuing objective and the defending objective was introduced and tuned such that the best value of the parameter that makes it possible for a single defender to intercept the attacker before reaching the target. Even though the algorithms are applied on a nonholonomic mobile robot, it can still be useful for other mobile robots such as unmanned aerial vehicles, unmanned

surface vehicles with known dynamics. Several simulations and real-time experiments in the presence and the absence of obstacles are conducted. It can be concluded that the proposed approach performed better than tracking methods such as NMPC.

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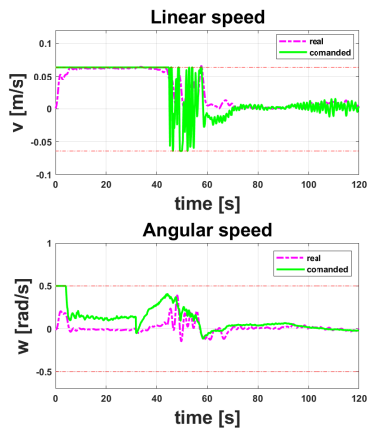


Fig. 8: The defender's strategy by applying the GT-MPC.

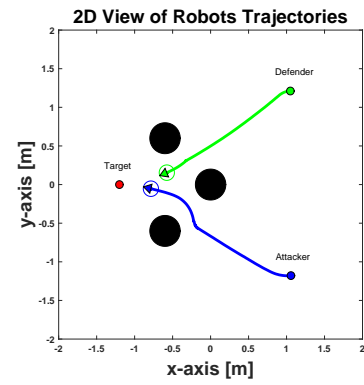


Fig. 10: Trajectories of the game by using GT-MPC in presence of obstacles.

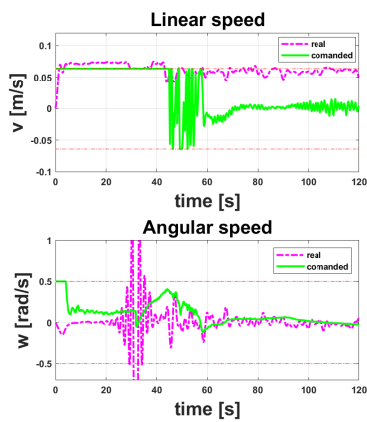


Fig. 9: The attacker's strategy by applying the GT-MPC.

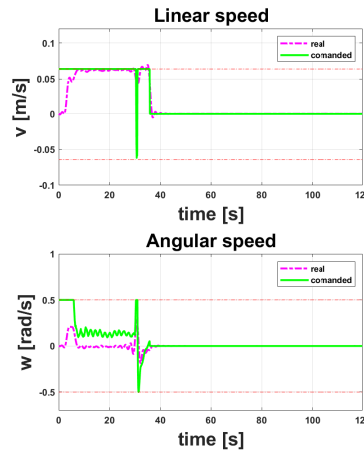


Fig. 11: The defender's strategy by using GT-MPC in presence of obstacles.

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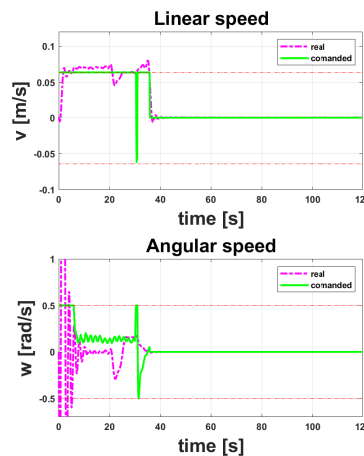


Fig. 12: The attacker's strategy by using GT-MPC in presence of obstacles.