On singularities in vortex sheets in 2D Euler flows using a high resolution characteristic mapping method

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This numerical study aims at getting further insight into singular solutions of the 2D Euler equations for non-smooth initial data, in particular for vortex sheets, similar to Caflisch et al[1]. For this, high resolution simulations of vortex layers in 2D incompressible Euler flows are performed using the characteristic mapping method[2]. An example of such a flow can be found in figue 1a. The method is a semi-Lagrangian method that evolves the flow map using the gradient-augmented level set method. Due to the semi-group structure of the flow map, it can be decomposed into sub-maps (each over a finite time interval), and thus the error can be controlled by choosing appropriate remapping times. Composing the flow map then yields exponential resolution in linear time and thus fine scale flow structures can be resolved in unprecedented detail. Here the roll-up process of vortex layers is studied varying the thickness of the layer showing its impact on the growth of palinstrophy and curvature of the center-line. The self-similar structure of the vortex core is investigated in the vanishing thickness limit, as can be seen in figure 1b. Conclusions on the non-uniqueness of weak solutions of 2D Euler for non-smooth initial data are drawn and the presence of flow singularities is revealed.

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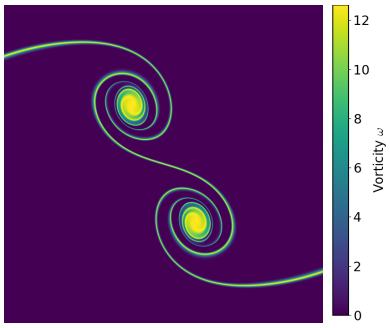
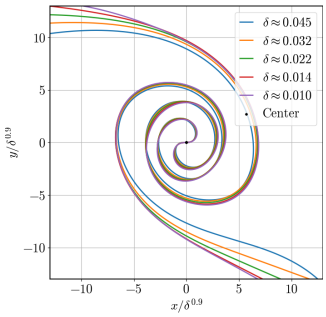


(a) Vorticity of vortex roll-up at time $t = 5.4$ for $\delta = 0.032$.



(b) Center-line of the vortex sheet for different sheet thicknesses illustrating the self-similarity.

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[1]Caflisch et al., *J. Fluid Mech.* **932**, (2022).
[2]Yin et al., *J. Comput. Phys.*, **424**, 109781 (2021)